Prediction of new states from $D^{(*)}B^{(*)}\bar{B}^{(*)}$ three-body interactions

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We study three-body systems composed of $D^{(*)}$, $B^{(*)}$, and $\bar{B}^{(*)}$ in order to look for possible bound states or resonances. In order to solve the three-body problem, we use the fixed center approach for the Faddeev equations considering that the $B^{*}\bar{B}^{*}(B\bar{B})$ are clusterized systems, generated dynamically, which interact with a third particle $D(D^{*})$ whose mass is much smaller than the two-body bound states forming the cluster. In the $DB^{*}\bar{B}^{*}$, $D^{*}B^{*}\bar{B}^{*}$, $DB\bar{B}$, and $D^{*}B\bar{B}$ systems with $I = 1/2$, we found clear bound state peaks with binding energies typically a few tens MeV and more uncertain broad resonant states about ten MeV above the threshold with widths of a few tens MeV.

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I. INTRODUCTION

The heavy flavor sector (both open and hidden) has gained renewed attention in the last years by the hadron physics community, in part spurred by the wide increase of experimental results (see Ref. [1] for a recent review). In the meson sector, specially interesting has been the proliferation of states which cannot be easily accommodated as genuine $q\bar{q}$, like many XYZ-type resonances (see, e.g., Refs. [2–6] for some reviews). In the baryon sector remarkably special was the discovery of the pentaquark $P_c(4450)^+$ by the LHCb collaboration [7]. Most of the non $q\bar{q}$ interpretations of many heavy flavor meson resonances lie within the picture of tetraquarks [5,8–10] or meson-meson molecules [6,11–18]. Recently, several extensions to the heavy flavor sector in three-body systems like $\rho B^{*}\bar{B}^{*}$ [19], $\rho D^{*}\bar{D}^{*}$ [20,21], DKK ($DK\bar{K}$) [22], and BDD (BDD) [23] have been carried out with the prediction of several resonant states. The traditional way to deal with

the three-body scattering amplitude has been to solve the Faddeev equations [24]. However, these equations are usually impossible to solve exactly and one has to resort to approximate methods. This is a feature well known by the nuclear and hadron physics community where the Faddeev equations have been widely used to account for three-nucleon systems [25,26] or systems involving mesons and baryons [27–30] or three-meson systems [31–33].

The three-body problem can be drastically simplified when two of the particles form a bound cluster which is not much altered by the interaction with the third particle. In such a case one can resort to the so-called fixed center approximation (FCA) to the Faddeev equations [34–38]. In the last years the FCA has proved convenient in the study of many three-body systems in the light flavor sector [37,39–43]. The first incursion in the charm sector with three-body resonances was done in Ref. [44] with the study of the NDK , $\bar{K}DN$, and $ND\bar{D}$ systems and also in Ref. [45] for DNN .

More recently, and involving only mesons, the FCA has been used to evaluate possible molecular states with open charm in DKK and $DK\bar{K}$ [22], open bottom, open or hidden charm and double charmed three meson systems BDD and BDD [23]. In the DKK and $DK\bar{K}$ systems the evaluation using the FCA benefits from the fact that the DK system is bound generating the $D_{s0}^{*}(2317)$ [11,46,47] and then the third particle rescatters with the components of the

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DK cluster without breaking it. In the BDD and $BDD\bar{D}$ cases, the situation is analogous to the DKK and $DK\bar{K}$ systems since the BD system also binds [48].

In the present work we analyze the $DB^*\bar{B}^*$, $D^*B^*\bar{B}^*$, $DB\bar{B}$, and $D^*B\bar{B}$ systems with $I = 1/2$ to look for possible bound and/or resonant three-body states. In this case, the use of the FCA to evaluate the three-body scattering amplitude is suitable and appropriate since the $B\bar{B}$ and $B^*\bar{B}^*$ systems in isospin $I = 0$ were found to bind [13], forming states of mass about 10450 and 10 550 MeV, respectively. That corresponds to binding energies of about 100 MeV. The work of Ref. [13] was based on the implementation of coupled channel unitary dynamics with kernels obtained from Lagrangians that combine local hidden gauge symmetry and heavy quark spin symmetry. In addition, for our present problem, we can also benefit from the fact that in Ref. [48] an attractive interaction, even producing bound states, was found for BD , B^*D , BD^* , B^*D^* , $B\bar{D}$, $B^*\bar{D}$, $B\bar{D}^*$, and $B^*\bar{D}^*$ in isospin $I = 0$, with less binding energy than in the $B\bar{B}$ or $B^*\bar{B}^*$ cases. On the contrary the analogous two-body interactions in isospin $I = 1$ are repulsive, when allowed. However, since the $I = 1$ amplitude is nonresonant one could expect *a priori* that the $I = 0$ interaction will prevail, helping to bind the three-body state.

II. THEORETICAL FRAMEWORK

In this section, we explain the formalism for the investigation of the $D^{(*)}B^{(*)}\bar{B}^{(*)}$ system. In the following, and in order to illustrate the process, we focus only on the $DB^*\bar{B}^*$ case since we can obtain the expressions for the other channels in a similar way. As explained in the Introduction, in this study the FCA to the Faddeev equations is employed. This approach is effective when two of the three particles form a bound state, which will be called cluster, and there is not enough energy to excite the cluster [49]. In the present calculation we are indeed in this situation since we are going to move in a range of energies close to the three-body (cluster + third-particle) threshold and also the mass of the third particle, the projectile, is

much smaller than the components of the cluster. In our case, the cluster is the $B^*\bar{B}^*$ system, which according to the findings of Ref. [13] forms a bound state, with a binding energy of about 100 MeV. The projectile is a D meson, whose mass is much smaller than the components of the cluster. This D meson undergoes multiple interactions with each component of the cluster. In this way, we need the two-body DB^* and $D\bar{B}^*$ amplitudes [see Eq. (10) below] which enter as an input in the Faddeev equations. We obtain the DB^* and $D\bar{B}^*$ two-body amplitudes from Ref. [48], based on a vector-meson exchange model from hidden gauge symmetry [50–52] and implementing a unitarization procedure by means of the Bethe-Salpeter equation.

In order to write the Faddeev equations with the FCA for the present case, we need to account for all the three-body diagrams contributing to the $DB^*\bar{B}^*$ interaction. Since the scattering amplitude is independent of the third component of isospin, I_3 , let us take, for example, the $I = 1/2$, $I_3 = -1/2$ case, for which we use the following nomenclature for the different channels needed:

$$\begin{aligned} 1) & D^0[B^{*+}B^{*-}], & 2) & D^0[B^{*0}\bar{B}^{*0}], & 3) & D^+[B^{*0}B^{*-}], \\ 4) & [B^{*+}B^{*-}]D^0, & 5) & [B^{*0}\bar{B}^{*0}]D^0, & 6) & [B^{*0}B^{*-}]D^+, \end{aligned} \quad (1)$$

where the two particles in the brackets form the cluster whose mass will be denoted by M_c , and the external D meson is scattered first by the nearby particle, e.g., the D meson at the left-hand side of the bracket is scattered first by B^* , while the one at the right-hand side is scattered first by $\bar{B}^*$. Following this nomenclature, we can define the partition functions T_{ij} which are the amplitudes for the diagrams accounting for the transition from the i to the j channels aforementioned, [see Eq. (1)]. For instance, the amplitude associated with the transition of $D^0[B^{*+}B^{*-}]$ to itself, denoted by T_{11} , is given by the diagrams depicted in Fig. 1. From this figure, we have

FIG. 1. Diagrams associated with the three-body amplitude for the $D^0B^{*+}B^{*-}$ interaction, contributing to the partition T_{11} of the three-body amplitude.

$$T_{11}(s) = t_1(s_{DB^*}) + t_1(s_{DB^*})G_0(s)T_{41}(s) + t_2(s_{DB^*})G_0(s)T_{61}(s), \quad (2)$$

where s is the total three-body center-of-mass energy squared, while t_1 and t_2 are, respectively, the two-body $t_{D^0 B^{*+}, D^0 B^{*+}}$ and $t_{D^0 B^{*+}, D^+ B^{*0}}$ scattering amplitudes, in the charge basis, which can be easily related to the DB^* amplitudes in isospin basis studied in Ref. [48]. These two-body amplitudes depend on the energy squared of the two-body subsystem, s_{DB^*} , (see Eq. (11) below). The G_0 function in the second and third terms of the right-hand side of Eq. (2) is the Green function of the D meson between the particles of the cluster [40], given by

$$G_0(q^0) = \frac{1}{2M_c} \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{F(\vec{q})}{(q^0)^2 - \omega_D^2(\vec{q}) + i\epsilon}, \quad (3)$$

with $\omega_D(\vec{q}) = \sqrt{|\vec{q}|^2 + m_D^2}$. The energy carried by the D meson between the components of the cluster, denoted by q^0 , is a function of the total energy squared s , defined by

$$q^0 = \frac{1}{2M_c} (s - m_D^2 - M_c^2). \quad (4)$$

The information about the $B^* \bar{B}^*$ bound state is encoded in the form factor $F(\vec{q})$ appearing in Eq. (3), which is related to the cluster wave function, $\Psi_c(\vec{r})$, by means of a Fourier transformation, as it was discussed in Refs. [39,53]:

$$F(\vec{q}) = \int d^3 \vec{r} e^{-i\vec{q} \cdot \vec{r}} \Psi_c^2(\vec{r}), \quad (5)$$

which can be obtained by

$$F(\vec{q}) = \frac{1}{N} \int_V d^3 \vec{q}' \frac{1}{M_c - \omega_{B^*}(\vec{q}') - \omega_{\bar{B}^*}(\vec{q}')} \times \frac{1}{M_c - \omega_{B^*}(\vec{q} - \vec{q}') - \omega_{\bar{B}^*}(\vec{q} - \vec{q}')}, \quad (6)$$

where V specifies the conditions $|\vec{q}'| < \Lambda$ and $|\vec{q} - \vec{q}'| < \Lambda$, with Λ the cutoff chosen to coincide with the value used in the evaluation of the $B^* \bar{B}^*$ bound state [13]. The normalization factor N in Eq. (6) is fixed such that $F(\vec{q} = 0) = 1$, and thus it is given by

$$N = \int_{|\vec{p}| < \Lambda} d^3 \vec{p} \left(\frac{1}{M_c - \omega_{B^*}(\vec{p}) - \omega_{\bar{B}^*}(\vec{p})} \right)^2. \quad (7)$$

In Eqs. (6) and (7) we have $\omega_{B^*}(\vec{p}) = \sqrt{|\vec{p}|^2 + m_{B^*}^2}$.

Following a similar procedure to the one used above to obtain the amplitude T_{11} of Eq. (2), we evaluate all the remaining amplitudes related to the transitions involving

every channel listed in Eq. (1), indicated by the indices i, j . Thus, we get a set of thirty-six coupled equations, since i and j run from 1 to 6, which provide the Faddeev equations with the FCA for the interaction we are concerned with. In matrix form, it reads

$$T = V + \tilde{V} G_0 T, \quad (8)$$

where the matrices V and $\tilde{V}$ are written in terms of the two-body $B^* D$ and $\bar{B}^* D$ amplitudes as follows:

$$V = \begin{pmatrix} t_1 & 0 & t_2 & 0 & 0 & 0 \\ 0 & t_3 & 0 & 0 & 0 & 0 \\ t_2 & 0 & t_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_6 & t_7 \\ 0 & 0 & 0 & 0 & t_7 & t_8 \end{pmatrix}, \quad \tilde{V} = \begin{pmatrix} 0 & 0 & 0 & t_1 & 0 & t_2 \\ 0 & 0 & 0 & 0 & t_3 & 0 \\ 0 & 0 & 0 & t_2 & 0 & t_4 \\ t_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & t_6 & t_7 & 0 & 0 & 0 \\ 0 & t_7 & t_8 & 0 & 0 & 0 \end{pmatrix}, \quad (9)$$

with

$$\begin{aligned} t_1 &= t_{B^{*+} D^0, B^{*+} D^0}; & t_5 &= t_{B^{*+} D^0, B^{*+} D^0}; \\ t_2 &= t_{B^{*+} D^0, B^{*0} D^+}; & t_6 &= t_{\bar{B}^{*0} D^0, \bar{B}^{*0} D^0}; \\ t_3 &= t_{B^{*0} D^0, B^{*0} D^0}; & t_7 &= t_{\bar{B}^{*0} D^0, B^{*+} D^+}; \\ t_4 &= t_{B^{*0} D^+, B^{*0} D^+}; & t_8 &= t_{B^{*+} D^+, B^{*+} D^+}. \end{aligned} \quad (10)$$

These scattering matrix elements correspond to the two-body amplitudes for DB^* and $D\bar{B}^*$ interactions given in Ref. [48]. In that reference the kernel of the unitarization procedure is obtained by the evaluation of mechanisms accounting for vector meson exchange from Lagrangians obtained from suitable extensions of hidden gauge symmetry Lagrangians to the heavy flavor sector, and compatible with the heavy quark spin symmetry (HQSS) of QCD [54]. The unitarization procedure only depends on one independent parameter, the three-momentum cutoff of the meson-meson loop function which turned out to be the largest source of uncertainty in Ref. [48]. We will also consider the uncertainty from that source in the results below. It is worth mentioning that the $I = 0$ potential is attractive [48] to the point to produce bound states for BD , $B^* D$, BD^* , $B^* \bar{D}^*$, $B\bar{D}$, $B^* \bar{D}$, $B\bar{D}^*$, and $B^* \bar{D}^*$. This is not the case for $I = 1$ where the lower order interaction is repulsive for $\bar{B}^{(*)} D^{(*)}$ and zero for $B^{(*)} D^{(*)}$ [48].

The amplitudes $t_1 - t_4$ and $t_5 - t_8$ of Eq. (10) must be multiplied by the normalization factors $c_1 = M_c/m_{B^*}$ and $c_2 = M_c/m_{\bar{B}^*}$ respectively, to match the Mandl-Shaw normalization [55] that we use. In this calculation, the polarization sums of the intermediate vector mesons B^* and $\bar{B}^*$ (or D^* and $\bar{D}^*$ below) can be ignored out in the two-body amplitudes since their consideration only gives a subleading contribution of the order of the squared momentum of the hadron over its mass [56–58].

The two-body amplitudes of Eq. (10) depend on the energy of the corresponding two-body subsystem, s_{ij} , with i the projectile and j the corresponding particle of the cluster involved in the amplitude. In terms of the total three-body invariant mass squared, s , it is given by [19,40]

$$s_{DB^*} = m_D^2 + m_{B^*}^2 + \frac{1}{2M_c^2}(s - m_D^2 - M_c^2)(M_c^2 + m_{B^*}^2 - m_{\bar{B}^*}^2). \quad (11)$$

The two-body energy of the $D\bar{B}^*$ subsystem, $s_{D\bar{B}^*}$, is obtained replacing the B^* mass by the $\bar{B}^*$ one in Eq. (11). [Despite we have in this case $m_{B^*} = m_{\bar{B}^*}$ (obviously), we keep them in Eq. (11) just to know the general expression for other cases which could have different masses].

With all these ingredients, Eq. (8) can be algebraically solved as

$$T = (1 - \tilde{V}G_0)^{-1}V. \quad (12)$$

Finally, the three-body amplitude $T_{DB^*\bar{B}^*}$ with $I = 1/2$, associated with a D meson interacting with the $B^*\bar{B}^*$ ($I = 0$) cluster, in terms of the matrix elements of T^{FCA} in Eq. (12) is

$$T_{DB^*\bar{B}^*} = \frac{1}{2}(T_{11} + T_{12} + T_{14} + T_{15} + T_{21} + T_{22} + T_{24} + T_{25} + T_{41} + T_{42} + T_{44} + T_{45} + T_{51} + T_{52} + T_{54} + T_{55}). \quad (13)$$

This expression can be explicitly worked out in terms of the two-body amplitudes in isospin basis and gives

$$T_{DB^*\bar{B}^*}(s) = \frac{t_{\bar{B}^*D}^0 + 3t_{\bar{B}^*D}^1 + (1 - G_0 t_{\bar{B}^*D}^0)(1 + 3G_0 t_{\bar{B}^*D}^1)t_{B^*D}^0}{4 - G_0^2(t_{\bar{B}^*D}^0 + 3t_{\bar{B}^*D}^1)t_{B^*D}^0} \quad (14)$$

where $t_{\bar{B}^*D}^0 \equiv t_{\bar{B}^*D, \bar{B}^*D}^{I=0}$, $t_{\bar{B}^*D}^1 \equiv t_{\bar{B}^*D, \bar{B}^*D}^{I=1}$, and $t_{B^*D}^0 \equiv t_{B^*D, B^*D}^{I=0}$.

For the other channels, $D^*B^*\bar{B}^*$, $DB\bar{B}$, and $D^*B\bar{B}$, the procedure is analogous but changing the masses of the corresponding particles, and using the proper two-body amplitudes for the particles involved.

III. RESULTS

For the numerical evaluation of the three-body amplitudes we use the following values for the meson masses: $m_B = 5279.0$ MeV, $m_D = 1869.0$ MeV, $m_{B^*} = 5325.0$ MeV, and $m_{D^*} = 2007.0$ MeV. As mentioned above, for the evaluation of the form factor of Eq. (6), which takes into account the clustering effect in the meson exchange between the constituents of the cluster, Eq. (3), we need the regularization cutoff Λ which is chosen to coincide with the regularization cutoff used in the unitarization of the $[B\bar{B}]$ and $[B^*\bar{B}^*]$ in Ref. [13]. In general, the cutoff is a free parameter of the two-body model, and one has to resort to some experimental result to constrain it. For instance, in Ref. [11] a cutoff of 415 MeV was required to get a bound state at the experimental value of 3720 MeV for the $D\bar{D}$ system. In Ref. [59] it was justified that heavy quark symmetry implies that the value of the cutoff is independent of the heavy flavor, up to corrections of order $\mathcal{O}(1/m_Q)$, with m_Q the mass of the heavy quark. Therefore, in this line, in Ref. [13] a range of values between 415–830 MeV for the $[B\bar{B}]$ and $[B^*\bar{B}^*]$ cutoff was justified, when compared to the cutoffs needed to obtain the $D\bar{D}$ resonance in Ref. [11] and the $D\bar{D}^*$ producing the $X(3872)$ in Ref. [59]. We will call this cutoff Λ_{BB} in the following. Similar arguments were used in Ref. [48] to justify the use of a cutoff in the range 400–600 MeV for the regularization of the BD -type interactions (BD , B^*D , BD^* , B^*D^* , $B\bar{D}$, $B^*\bar{D}$, $B\bar{D}^*$, and $B^*\bar{D}^*$). We will call this cutoff Λ_{BD} in the following. Therefore, the variation of the cutoffs within the ranges $\Lambda_{BB} \sim 415$ –830 MeV and $\Lambda_{BD} \sim 400$ –600 MeV will be used to estimate the uncertainties in our approach. We also need the masses of the clusters $[B\bar{B}]$ and $[B^*\bar{B}^*]$, which are given by $M_{B\bar{B}} = 10523$ MeV and $M_{B^*\bar{B}^*} = 10613$ MeV for $\Lambda_{BB} = 415$ MeV and $M_{B\bar{B}} = 10380$ MeV and $M_{B^*\bar{B}^*} = 10469$ MeV for $\Lambda_{BB} = 830$ MeV [13].

As an example of the shape of the three-body amplitudes that we obtain, we show in Fig. 2 the squared amplitude of

FIG. 2. The three-body amplitude $|T_{DB^*\bar{B}^*}|^2$ with $\Lambda_{DB} = 400$ MeV and $\Lambda_{BB} = 415$ MeV.

TABLE I. Pole position $\sqrt{s_p}$ and the binding energy E_B of $D^{(*)}B^{(*)}\bar{B}^{(*)}$. The units are MeV.

	$\sqrt{s_p}$	E_B
$DB^*\bar{B}^*$	$12\,384 \pm 65$	27 ± 23
$D^*B^*\bar{B}^*$	$12\,520 \pm 65$	28 ± 25
$DB\bar{B}$	$12\,294 \pm 64$	27 ± 24
$D^*B\bar{B}$	$12\,430 \pm 64$	28 ± 25

the $DB^*\bar{B}^*$ three-body system, $|T_{DB^*\bar{B}^*}|^2$, as a function of $\sqrt{s}$ for some particular values of the regularization cutoffs. As we can see, we get a sharp peak at $\sqrt{s} = 12\,466$ MeV, which is below the $D[B^*\bar{B}^*]$ threshold, at $12\,482$ MeV. This peak can be considered as a three-body $D[B^*\bar{B}^*]$ bound state, with a binding energy of 16 MeV. Qualitatively similar plots, with different positions of the peaks, are obtained for the other three-body channels and different values of the regularization parameters. This is summarized in Table I, where we show the positions of the poles below threshold for the different channels obtained averaging over the results for the different values of the cutoffs within the ranges explained above. We also show in the last column the corresponding binding energies, E_B . The emergence of these three-body bound states is quite robust in our approach since we obtain poles for all the values of the different cutoffs considered. Indeed the value for the upper limit of the Λ_{BB} range (830 MeV) is a very conservative overestimation [13] of this parameter and in spite of that we still get poles for that value of this cutoff.

It is important to note that the binding energies of these systems are almost the same between different channels for the same set of regularization cutoffs. This is a nontrivial result and it is a consequence of the fact that the vector-meson exchange approach for the two-body interactions of B^*D , $\bar{B}^*D$, and $B^*\bar{B}^*$ respects the HQSS. Thus we can understand this coincidence of the binding energy as a manifestation of the HQSS which has already seen in the two-body systems [13,48,60–62].

On the other hand, in Fig. 2, we find a broad bump located around $12\,500$ MeV and a width of the order of 10 MeV in addition to the bound state previously discussed. Although this resonant state is above the $D[B^*\bar{B}^*]$ threshold, it is still below the uncorrelated $DB^*\bar{B}^*$ threshold. In Table II, we summarize the energy of the peak position m_R and the width Γ for the $DB^*\bar{B}^*$, $D^*B^*\bar{B}^*$, $DB\bar{B}$, and $D^*B\bar{B}$ systems. The results on this table are obtained for the two extreme values of the cutoff Λ_{BD} but only for one value of the cutoff $\Lambda_{BB} = 415$ MeV. This is because we do not find a resonant structure above threshold for $\Lambda_{BB} = 830$ MeV. Therefore, the existence of the possible resonant state appearing above threshold in some specific cases are more uncertain than the bound states found below threshold, and further study would be necessary for clarification.

TABLE II. Peak position and width of the resonances. Λ_{BB} is always fixed to 415 MeV.

	$\Lambda_{DB} = 400$ MeV $m_R \Gamma$	$\Lambda_{DB} = 600$ MeV $m_R \Gamma$
$DB^*\bar{B}^*$	$12\,497 10$	$12\,494 15$
$D^*B^*\bar{B}^*$	$12\,634 15$	$12\,632 20$
$DB\bar{B}$	$12\,407 10$	$12\,403 15$
$D^*B\bar{B}$	$12\,544 15$	$12\,542 20$

At this point it is worth clarifying some issues regarding the origin of the poles and resonances obtained above. First of all it is curious to note that for $D[B^*\bar{B}^*]$ with $\Lambda_{BB} = 415$ MeV and $\Lambda_{DB} = 400$ MeV the pole at $\sqrt{s} = 12\,466$ MeV that is shown in Fig. 2 coincides exactly with the value of $\sqrt{s}$ for which $\sqrt{s_{DB^*}} = 7175$ MeV, which is the pole position of $t_{B^*D}^0$. Usually, in other three-body problems, if there is a pole in the two-body amplitude, it does not manifest in the three-body amplitude since it cancels between the numerator and the denominator of the analogous expression to Eq. (12) or Eq. (14). However this is not the case for the present channels due to a subtle accidental coincidence. Indeed, for $\sqrt{s} = 12\,466$ MeV, $t_{B^*D}^0(s_{DB^*})$ has a pole and thus close to this energy Eq. (14) reduces to

$$T_{DB^*\bar{B}^*}(s) \simeq -\frac{(1 - G_0 t_{B^*D}^0)(1 + 3G_0 t_{B^*D}^1)t_{B^*D}^0}{G_0^2(t_{B^*D}^0 + 3t_{B^*D}^1)t_{B^*D}^0} \quad (15)$$

Note that $t_{B^*D}^0$ cancels between numerator and denominator and therefore Eq. (14) should have no pole. However, it turns out that, by coincidence, $t_{B^*D}^0 + 3t_{B^*D}^1$ has a zero at exactly the same value where $t_{B^*D}^0$ has the pole. In order to see why this happens, let us note that the B^*D potentials have the structure

$$V_{BD}^I = \alpha a \quad (16)$$

where $a = -\frac{1}{8f^2}[3s_{DB^*} - 2(m_{B^*}^2 + m_D^2) - \frac{(m_{B^*}^2 - m_D^2)^2}{s_{DB^*}}]$ and α is a coefficient which value is $\alpha = 2$ for $V_{B^*D}^0$, $\alpha = 1$ for $V_{B^*D}^1$ and $\alpha = -1$ for $V_{B^*D}^2$ (see Eqs. (15)–(17) in Ref. [48]). Therefore, the two-body unitarized amplitudes are given by [48]

$$t_{B^*D}^0 = \frac{2a}{1 - 2aG} \quad (17)$$

with G the B^*D loop function. Equation (17) has a pole when

$$1 - 2aG = 0. \quad (18)$$

But, on the other hand we have

$$t_{B^*D}^0 = \frac{a}{1 - aG}, \quad (19)$$

$$t_{B^*D}^1 = \frac{-a}{1+aG} \quad (20)$$

and therefore

$$t_{B^*D}^0 + 3t_{B^*D}^1 = \frac{a}{1-aG} - \frac{3a}{1+aG} = \frac{-2a(1-2aG)}{1-(aG)^2} \quad (21)$$

which is zero when $1-2aG=0$ which is, by accident, exactly the same condition for $t_{B^*D}^0$ to have a pole, (see Eq. (17)). It is worth noting that, if the model for the two-body amplitudes were a bit different, e.g., considering subleading terms in $t_{B^*D}^1$, for instance, then the three-body pole would not coincide exactly with the two-body pole of $t_{B^*D}^0$. We have checked that even changing $t_{B^*D}^1$ by hand about 20%, the three-body pole still appears but at a slightly different position. Therefore this pole has to be considered as an actual three-body state since it corresponds to a pole of Eq. (14), where the two-body pole cancels. Thus the pole in the three-body amplitude has nothing to do with the two-body pole even though it coincides numerically in the position for the channels considered in the present work. On the other hand, we are going to justify that the bump above threshold comes also from the three-body dynamics and is related to a different pole of Eq. (14). Indeed, the possible poles of Eq. (14) would correspond to zeroes of its denominator:

$$4 - G_0^2(t_{B^*D}^0 + 3t_{B^*D}^1)t_{B^*D}^0 = 0. \quad (22)$$

Using Eqs. (17), (19), and (20), one obtains that Eq. (22) has two solutions, one when

$$1 - 2aG = 0, \quad (23)$$

which is the solution that produces the pole below threshold, and the other solution when

$$a^2(G^2 - G_0^2) - 1 = 0, \quad (24)$$

which produces the resonance above threshold. Actually we find that the poles associated with Eq. (24) happen for complex $\sqrt{s}$ since they occur for $\text{Re}[\sqrt{s}]$ above the cluster + third-particle threshold. For the channels we are considering in the present work, we have checked that the $\text{Re}[\sqrt{s}]$ of the solution of Eq. (24) are close to the position of the maximum of the bump found in the three-body amplitudes. Therefore, and in summary, the bumps found above threshold should also be considered as three-body resonances since they correspond to poles of the three-body amplitude.

IV. SUMMARY

We have investigated theoretically the three-body interactions $DB^*\bar{B}^*$, $DB^*\bar{B}^*$, $DB\bar{B}$, and $D^*\bar{B}\bar{B}$ taking into account dynamical models for the $D^{(*)}B^{(*)}$, $D^{(*)}\bar{B}^{(*)}$ and $B^*\bar{B}^*(B\bar{B})$ subsystems studied in previous works. This has allowed us to apply the fixed center approximation to the Faddeev equations where the $B^*\bar{B}^*(B\bar{B})$ two-body subsystems are bound forming clusters, which then interact with a $D^{(*)}$ meson. As a result, we have found three-body bound states for each one of these systems with binding energies around 20–30 MeV. This similarity in the binding between the different channels is a clear manifestation of the heavy quark spin symmetry. Furthermore, we have also found resonant bumps above the $D^{(*)}[B^{(*)}\bar{B}^{(*)}]$ threshold with width about 10 MeV, however these bumps are not stable under the uncertainties that come from the cutoff values used to regularize the two-body meson-meson loops and then their existence are not so clear than the bound states below threshold.

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