Estimating $r_B^{D\pi}$ as an input to the determination of the CKM angle $\gamma$

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I. INTRODUCTION

The CKM description of charged-current quark transitions has been experimentally scrutinized to an impressive accuracy. The CKM angle $\gamma$ encapsulates the relative phase between $b \to c$ and $b \to u$ quark transitions, $\gamma \equiv \arg \left(-\frac{V_{ub}V_{cb}^*}{V_{ub}V_{ub}}\right)$, and is determined with a precision of $7^\circ$, as compared to a precision below $3^\circ$ deduced from indirect measurements [1,2].

The interference between Cabibbo-favored and Cabibbo-suppressed $B \to D\pi$ decay amplitudes provides sensitivity to the CKM angle $\gamma$. The relative size of the interfering amplitudes is an important ingredient in the determination of $\gamma$. Using branching fractions from various $B \to D\ell$ decays, and the measured value for $r_B^{DK}$, the magnitude of the amplitude ratio of $B^+ \to D^0\pi^+$ and $B^+ \to \bar{D}^0\pi^+$ decays is estimated to be $r_B^{D\pi} = 0.0053 \pm 0.0007$.

II. ESTIMATING $r_B^{D\pi}$ FROM BRANCHING FRACTIONS

The expression for the branching fraction takes the following form:

\begin{equation}
\text{BR}(B \to D\pi) = \frac{A(B^+ \to D^0\pi^+)}{A(B^+ \to \bar{D}^0\pi^+)}.
\end{equation}

Using the measured value of $r_B^{DK}$ [7]. A similar approach was used to estimate the ratio of amplitudes for the decays $B^0 \to D^\pm\pi^\mp$ [9]. An overview of the decays used is given in Table I. The amplitudes of the decays that involve a kaon in the final state are denoted by primed symbols.

At tree level, the $B^+ \to \bar{D}^0\pi^+$ amplitude receives contributions from a color-allowed (T) and color-suppressed topology (C), whereas the $B^+ \to D^0\pi^+$ amplitude proceeds predominantly through the color-suppressed topology (C$^{ub}$) and also via the annihilation topology, as illustrated in Fig. 1, where the superscript $ub$ indicates that the decay proceeds through a $b \to u$ transition.

The method to estimate $r_B^{D\pi}$ with $B^0 \to \bar{D}^0K^0$ decays is given in Sec. II A, whereas the use of $B^0 \to D^0\pi^0$ decays is shown in Sec. II B. The effect of the annihilation diagram is estimated in Sec. II C.

<table>
<thead>
<tr>
<th>Decays</th>
<th>Topology</th>
<th>BR ($\times 10^{-4}$)</th>
<th>CKM factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \to D^0\pi^+$</td>
<td>$T + C$</td>
<td>$48.1 \pm 1.5$</td>
<td>$V_{cb}V_{ud}$</td>
</tr>
<tr>
<td>$B^0 \to \bar{D}^0\pi^0$</td>
<td>$(C - E)/\sqrt{2}$</td>
<td>$2.63 \pm 0.14$</td>
<td>$V_{cb}V_{ud}$</td>
</tr>
<tr>
<td>$B^+ \to \bar{D}^0K^+$</td>
<td>$T' + C'$</td>
<td>$3.70 \pm 0.17$</td>
<td>$V_{cb}V_{us}$</td>
</tr>
<tr>
<td>$B^0 \to \bar{D}^0K^0$</td>
<td>$C'$</td>
<td>$0.52 \pm 0.07$</td>
<td>$V_{cb}V_{us}$</td>
</tr>
<tr>
<td>$B^+ \to D^+_s\phi$</td>
<td>$A'$</td>
<td>$0.017^{+0.012}_{-0.007}$</td>
<td>$V_{ub}V_{cs}$</td>
</tr>
</tbody>
</table>

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The following estimate of the ratio of amplitudes can be made:

\[ r_B^{D\pi} = \frac{A(B^+ \to D^0 \pi^+)}{A(B^+ \to D^0 \pi^+)} = \frac{|C^{ab}|}{|T + C|} = \frac{V_{ub}V_{cd}}{|V_{cb}V_{ud}|} z |C| / |T + C| \]  

where \( z \) quantifies the ratio between the hadronic parts of the two color-suppressed tree diagrams proceeding through a \( b \to c \) or \( b \to u \) transition (shown in Fig. 1), \( C^{ab} = z C \times (V_{ub}V_{cd})/(V_{cb}V_{ud}) \). The contribution from the annihilation topology is also absorbed in the quantity \( z \), and will be further discussed in Sec. II.C.

We can estimate \( |C|/(|T| + |C|) \) in two ways.

(A) \( r_B^{D\pi} \sim A(B^0 \to \bar{D}^0 K^0)/A(B^+ \to \bar{D}^0 K^-) \), applying SU(3) symmetry, and correcting for the different CKM elements involved.

(B) \( r_B^{D\pi} \sim A(B^0 \to \bar{D}^0 \pi^0)/A(B^+ \to \bar{D}^0 \pi^+ \), using external estimates for the contribution from \( W \)-exchange topologies (E) to the decay \( B^0 \to \bar{D}^0 \pi^0 \).

The magnitude of \( z \) will be estimated in Sec. III by comparing the result of the amplitude ratio of the decays \( B^+ \to D^0 K^+ \), \( r_B^{D\pi} \), to the measured value by LHCb [7]. For the numerical values of the CKM elements, we use the values listed in Table II.

A. Estimating \( r_B^{D\pi} \) from \( B^0 \to \bar{D}^0 K^0 \)

The decays \( B \to DK \) can be used to estimate the contributions of various \( B \to D \pi \) decay topologies, assuming SU(3) symmetry.

The validity of this assumption was probed by comparing the \( D^{(*)K} \) and \( D^{(*)\pi} \) decay rates, correcting for differences in phase space, CKM elements, form factors and decay constants [8]. This assures that the decays \( B^0 \to \bar{D}^0 K^0 \) and \( B^+ \to \bar{D}^0 K^+ \) can be used to estimate a value for the amplitude ratio, \( |C|/(|T + C| = |C'|/(|T' + C'|, where

\[ \frac{|C'|}{|T' + C'|} = \sqrt{\alpha \frac{BR(B^0 \to \bar{D}^0 K^0)}{BR(B^+ \to \bar{D}^0 K^+)} } \]  

The factor \( \alpha \) quantifies a correction to the quoted value of \( BR(B^0 \to \bar{D}^0 K^0) \) from the Particle Data Group [10]. The measured branching fraction by the BABAR [13] and Belle [14] collaborations is obtained from the sum over the charge-conjugate final states, and therefore the quoted branching fraction represents the sum of the \( B^0 \to \bar{D}^0 K^0 \) and \( B^0 \to D^0 K^0 \) branching fractions. Recently LHCb also performed an analysis of the decays \( B^0 \to \bar{D}^0 K^0 \) [15].

The quoted branching fraction can thus be expressed as the sum of the squares of the two color-suppressed tree amplitudes,

\[ BR(B^0 \to \bar{D}^0 K^0) = A(B^0 \to \bar{D}^0 K^0)^2 + A(B^0 \to D^0 K^0)^2 = |C'|^2 + |C''|^2 \] 

\[ = (1 + z' \left| \frac{V_{ub}V_{cs}}{V_{cb}V_{us}} \right|^2) \times |C'|^2 \] 

where \( z' \) quantifies the ratio between the hadronic parts of the two color-suppressed tree diagrams proceeding through the \( b \to u \) and \( b \to c \) transitions,
\[ |C^{ab}| = \frac{C'}{C} \times |V_{cb}V_{us}/V_{ub}V_{cd}|. \] Hence, we need to correct the quoted branching fraction of the decay \( B^0 \to D^0 K^0 \) to yield an estimate of the amplitude of \( C' \), relative to \( |T' + C'| \) with \( \alpha = 1/(1 + 0.156z') \), to obtain

\begin{equation}
\frac{r_D}{C} = \frac{V_{ub}V_{cd}}{V_{cb}V_{ud}} \sqrt{\frac{BR(B^0 \to D^0 K^0)}{BR(B^0 \to D^0 \pi^0)}}. \tag{4}
\end{equation}

**B. Estimating \( r_D \) from \( B^0 \to D^0 \pi^0 \)**

A second estimate of \( r_D \) can be obtained using the decay \( B^0 \to D^0 \pi^0 \). The decay \( B^0 \to D^0 \pi^0 \) receives contributions from the color-suppressed tree diagram (C) and from the W-exchange diagram (E). The comparison of the \( B^0 \to D^0 \pi^0 \) and \( B^+ \to D^0 \pi^+ \) decay rates gives [8]

\[ \left| \frac{C - E}{T + C} \right| = \sqrt{\frac{BR(B^0 \to D^0 \pi^0)}{BR(B^+ \to D^0 \pi^+)}}, \tag{5} \]

again assuming that CKM elements, form factors, decay constants and phase space factors cancel in the ratio. The uncertainty originates from the uncertainty on the measured branching fractions. The factor \( \sqrt{2} \) originates from the isospin decomposition of the neutral pion. Although the branching fraction \( BR(B^0 \to D^0 \pi^0) \) is determined as the sum of the \( D^0 \) and \( D^0 \) final states, the \( b \to u \) color-suppressed tree amplitude is negligible compared to the \( b \to c \) amplitude, unlike the situation of Eq. (2).

The color-suppressed tree diagram is expected to dominate the total transition amplitude with respect to the W-exchange topology \( E \), which is supported by the comparison of the branching fractions of \( B^0 \to D^0 \pi^0 \) and \( B^0 \to D^0 K^0 \) [8], leading to

\[ \left| \frac{C - E}{C} \right| = 0.913 \pm 0.074. \tag{6} \]

To obtain an independent estimate of \( r_D \) with respect to Eq. (4), i.e. without reusing information on the branching fraction of \( B^0 \to D^0 K^0 \), the size of the W-exchange amplitude can be estimated from the decay \( B^0 \to D_\pi K^+ \) [16], resulting in the following value [8]:

\[ \left| \frac{E}{T + C} \right| = 0.056 \pm 0.004. \tag{7} \]

Without assuming any value for the relative phase between the W-exchange (E) and color-suppressed (C) amplitudes, we assign the full contribution of the W-exchange amplitude as uncertainty to the estimate of \( |C|/|T + C| \).

\[ \frac{C}{T + C} = 0.331 \pm 0.010(BR) \pm 0.056(E). \tag{8} \]

The resulting expression for \( r_D \) then becomes

\[ r_D = \frac{V_{ub}V_{cd}}{V_{cb}V_{ud}} \sqrt{\frac{BR(B^0 \to D^0 \pi^0)}{BR(B^+ \to D^0 \pi^+)}}, \tag{9} \]

C. Effect of annihilation topology

The relative contribution from the annihilation topology with respect to the color-suppressed tree topology for the \( B^+ \to D^0 \pi^+ \) amplitude, is estimated using the measured branching fraction of the decay \( B^+ \to D^- \phi \) [17], relative to the decay \( B^0 \to D^0 K^0 \). At lowest order the \( B^+ \to D^- \phi \) decay proceeds purely through the annihilation topology.

The estimate of \( |A/C| \) for the \( B^+ \to D^0 \pi^+ \) case can be directly obtained from the branching ratios, when corrected for by the appropriate CKM elements and decay constants \( f_X \),

\[ |A/C| = \sqrt{\frac{BR(B^+ \to D^- \phi)}{BR(B^0 \to D^0 K^0)} \left( \frac{V_{ub}V_{cd}}{V_{ub}V_{cs}} \right) \left( \frac{f_D f_K}{f_D f_{\phi}} \right)} \sim 0.25 \]

with a large uncertainty from the branching fraction measurement of \( B^+ \to D^- \phi \); see Table I. It is also noted that the branching fraction \( BR(B^+ \to D^- D_\pi^+) \) deviates from \( BR(B^0 \to D^- D_\pi^+) \), where the main difference is expected to arise from the annihilation contribution [18]. Possible contributions to these final states from rescattering processes are discussed in Ref. [19]. The relative phase between the annihilation and color-suppressed tree topology is unknown, so the annihilation contribution can enhance or reduce the value of \( r_D \). Assuming SU(3) symmetry, this contribution is expected to be equal in the \( B^+ \to D^0 K^+ \) system, and will thus be accounted for in the determination of \( z \) from \( r_D \) in the next section.

**III. CORRECTION USING \( r_D \)**

To quantify the ratio \( z \) between the hadronic parts of the \( b \to u \) and \( b \to c \) color-suppressed tree diagrams, \( C^{ab} = z' C' \times (V_{ub}V_{cs}/V_{ub}V_{cd}) \), an estimate for \( r_D \) can be obtained in a similar way, and be compared to the fitted value for \( r_D \) from the LHCb fit [7]. The quantity \( z \) also contains the correction due to contributions from the annihilation topology; see Fig. 1. We obtain the following expression for \( r_D \):

\[ r_D = \frac{V_{ub}V_{cd}}{V_{cb}V_{ud}} \left( \frac{z'}{1 + 0.156z'} \right) \sqrt{\frac{BR(B^0 \to D^0 K^0)}{BR(B^+ \to D^0 K^+)}}, \]

which differs from Eq. (4) by the different CKM elements involved. Inserting the value for \( r_D \) obtained from the
LHCb fit, $r_{B}^{DK} = 0.101 \pm 0.006$ [7], the following estimate for the ratio of the hadronic parts of the color-suppressed amplitudes is obtained:

$$\frac{z'}{1 + 0.156z} = 0.68 \pm 0.05 \Rightarrow z' = 0.76 \pm 0.07.$$  \hspace{1cm} (10)

The fact that the value of $z'$ is close to unity, indicates that the hadronic parts of the two color-suppressed tree diagrams are of similar magnitude, in particular if the annihilation topology negatively interferes with the color-suppressed tree topology, i.e. if the relative strong phase is close to 180°, which would lead to a value $z' \sim 0.75$. We assume that the deviation from unity is equal for the $D\pi$ case, with an uncertainty of 10% from SU(3) symmetry breaking effects, $z = 0.76 \pm 0.07$ (BR) $\pm 0.02$ (SU(3)).

Inserting the numerical values in Eq. (4) and Eq. (9) leads to the following estimates of $r_{B}^{D\pi}$:

$$r_{B}^{D\pi}(D^0K^0) = 0.0053 \pm 0.0002(V_{\text{CKM}}) \pm 0.0004(\text{BR})$$

$$\pm 0.0005(\text{SU(3)}),$$

$$r_{B}^{D\pi}(D^0\pi^0) = 0.0053 \pm 0.0002(V_{\text{CKM}}) \pm 0.0002(\text{BR})$$

$$\pm 0.0009(E) \pm 0.0005(z)$$  \hspace{1cm} (11)

which are in good agreement, albeit with a large uncertainty from the $W$-exchange contribution to the $B^0 \to D^0\pi^0$ decay rate. The agreement shows the internal consistency of the approach presented here. An additional 10% uncertainty from SU(3) symmetry is assumed in the estimate of Eq. (11), based on the agreement of the relative contributions of the various decay topologies to the $B \to D\pi$ and $B \to D\pi$ decays [8].

IV. CONCLUSIONS

The estimate for the value of the amplitude ratio $r_{B}^{D\pi}$ that is presented here provides a valuable input to the discussion of the measurement of $r_{B}^{D\pi}$ at LHCb. The actual measurement of $r_{B}^{D\pi}$ can be achieved either by a combination of indirect measurements, as presented in Refs. [6,7], or by direct measurement using semileptonic decays of the form $B^+ \to D^0\pi^+$, where $D^0 \to K^-\mu^+\nu_\mu$, and the charge of the kaon and muon can unambiguously tag the $D^0$ flavor. Future determinations of $r_{B}^{D\pi}$ can be compared to the estimate presented here, to assess the validity of the assumptions on rescattering and SU(3) symmetry as used in this paper. The LHCb Collaboration foresees accumulating a 4 times larger data set by the end of 2018, and a 5 times smaller uncertainty at the end of the LHCb upgrade, which will result in an experimental uncertainty of the measured value of $r_{B}^{D\pi}$ that is smaller than the one presented here.

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