Erratum: Cross sections, relic abundance, and detection rates for neutralino dark matter

Kim Griest

It was pointed out by Lars Bergstrom that in our paper we (inadvertently) left out the $-p^m p^m / m_Z^2$ part of the $Z^0$ propagator. Including this results in additional terms in the matrix element and the annihilation cross section. For much of the supersymmetric parameter space the resulting corrections are very small, but for pure Higgsinos a maximum correction of around 9% can occur. There are no corrections to the elastic scattering cross sections, the pure photino limit of the annihilation cross section, or to the production ($e^+ e^- \rightarrow \tilde{\chi} \tilde{\chi}$) cross sections. The changes to the annihilation cross section can result in up to a 9% change in the dark-matter detection rates. More importantly, as Bergstrom and Snellman point out, when using this annihilation cross section for present-day annihilation of dark matter in the halo, the additional terms can be important. In particular, at the $Z^0$ pole ($m_\chi = m_Z / 2$), the term proportional to $(m_\chi / m_Z)^2$ cancels. Since the remaining terms are proportional to $v^2 \approx 10^{-6}$ there is a very large dip in the cross section here rather than a large enhancement which the published cross section would predict. This dip does not occur for annihilation in the early Universe where $v^2 \approx \frac{1}{\nu}$. The following corrections should be made.

The squared matrix element [Eq. (3)] should include the additional terms

$$|M|^2 = 16 g^4 \left[ \frac{(Z_{13} - Z_{14})^2}{\cos^2 \theta_w (m_Z^2 - s)} (c_L - c_R)^2 m_q^2 m_Z^2 \left( \frac{s^2}{4} - \frac{sm_Z^2}{2} \right) \right]$$

$$+ \frac{Z_{13}^2 - Z_{14}^2}{2 \cos^2 \theta_w (m_Z^2 - s) (c_L - c_R)} m_q^2 m_Z^2 \left[ \frac{1}{M_s^2 - t} + \frac{1}{M_q^2 - u} \right] \left[ w s^2 + 2(u' + v') s m_X m_q \right]$$

$$+ \left( \frac{1}{M_s^2 - t} + \frac{1}{M_q^2 - u} \right) \frac{4w'}{[(p_1 k_1)^2 - (p_1 k_2)^2]} \right],$$

where all symbols were defined in the paper.

The nonrelativistic expansion of the neutralino annihilation cross section [Eq. (4)] should include the additional terms

$$\sigma_{\text{str}} = \frac{4}{\pi} G_F^2 c_m \frac{m_\chi^4}{4} \left[ (Z_{13} - Z_{14})^2 x^4 (c_L - c_R)^2 z^2 m_q^2 m_Z^2 \left( \frac{16 m^2}{m_Z^2} - 8 + 2 v' \right) \right]$$

$$+ 2 (Z_{13} - Z_{14}) (c_L - c_R) x^2 y^2 z^2 \frac{m_q^2}{m_Z^2}$$

$$\times \left[ 2 + \frac{1}{2} v^2 (x^2 - 2x + \frac{1}{4}(p^2 r^2)) \right] $$

$$\times \left[ 4w' + 2z (u' + v') \right] + 2w' v^2 - \frac{1}{4} w' v^2 \beta^2 r \right].$$

Finally, for completeness Eq. (A1) should include

$$\mathcal{M}' = -\frac{2m_X m_q^2 (c_R - c_L) (Z_{13} - Z_{14})}{2 \cos^2 \theta_w (m_Z^2 - s) m_Z^2} \bar{u}(p_2) \gamma_5 u (p_1) \bar{u}(k_1) \gamma_5 v (k_2),$$

and Eq. (A2) should contain the terms

$$|M_p|^2 = 16 g^4 \left[ \frac{(Z_{13} - Z_{14})^2}{\cos^2 \theta_w (m_Z^2 - s)} (c_R - c_L)^2 m_q^2 m_Z^2 \left( \frac{s^2}{4} \right) \right],$$

$$2 \text{Re} \mathcal{M} \mathcal{M}^* = -\frac{8 g^4 (Z_{13} - Z_{14})^2}{\cos^2 \theta_w (m_Z^2 - s)} (c_R - c_L)^2 m_q^2 m_Z^2,$$
2 \text{Re} M_a M'_a^* = \frac{-8g^4(Z_{13}^2 - Z_{14}^2)(c_R - c_L)m_x m_q}{\cos^2 \theta_w m_Z^2 (m_Z^2 - s)(M_{qL}^2 - t)} \left[ 2eab \left( \frac{s^2}{4} + (p, k_1)^2 - (p, k_2)^2 \right) + sm_q m_x (a^2 + b^2) \right],

(A2')

2 \text{Re} M_b M'_b^* = \frac{-8g^4(Z_{13}^2 - Z_{14}^2)(c_R - c_L)m_x m_q}{\cos^2 \theta_w m_Z^2 (m_Z^2 - s)(M_{qR}^2 - t)} \left[ -2eac \left( \frac{s^2}{4} + (p, k_1)^2 - (p, k_2)^2 \right) + sm_q m_x (a^2 + c^2) \right],

2 \text{Re} M_c M'_c^* = \frac{-8g^4(Z_{13}^2 - Z_{14}^2)(c_R - c_L)m_x m_q}{\cos^2 \theta_w m_Z^2 (m_Z^2 - s)(M_{qL}^2 - u)} \left[ 2eab \left( \frac{s^2}{4} - (p, k_1)^2 + (p, k_2)^2 \right) + sm_q m_x (a^2 + b^2) \right],

2 \text{Re} M_d M'_d^* = \frac{-8g^4(Z_{13}^2 - Z_{14}^2)(c_R - c_L)m_x m_q}{\cos^2 \theta_w m_Z^2 (m_Z^2 - s)(M_{qR}^2 - u)} \left[ -2eac \left( \frac{s^2}{4} - (p, k_1)^2 + (p, k_2)^2 \right) + sm_q m_x (a^2 + c^2) \right].

Note that these should be added with signs appropriate for \( M = M_z + M'_z - M_a - M_b + M_c + M_d \).