Letter

Luminal scalar-tensor theories for a not so dark dark energy

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In general the speed of gravitational waves (GWs) in scalar-tensor modifications of Einstein's gravity is different from the speed of light. Nevertheless, it has been measured that their speeds are nearly the same. For the most general scalar-tensor theories classified to date that do propagate a graviton—DHOST, *including Horndeski and beyond Horndeski (BH) theories*—we show that, remarkably, up to five self-consistent couplings of the scalar of dark energy (DE) to the photon are enough to make their GWs luminal in a wide set of cases. We find at least one luminal BH theory for which the GW decay into DE is suppressed in any cosmological background.

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Introduction. With the new era of multimessenger astronomy, the initial impression was that a large class of scalar-tensor modifications of Einstein's gravity is ruled out by the strict coincidence between the speed of light (c) and gravity (c_g). In particular, the almost simultaneous detection of the gravitational wave (GW) signal from the event GW170817 [1] and the gamma ray burst GRB170817A [2,3] placed the strong constraint,

$$\left|\frac{c_g}{c}-1\right| \leq 5\times 10^{-16}. \tag{1}$$

Nevertheless, it is clear that this constraint essentially indicates a relation between gravity and light. It, nevertheless, does not directly rule out modified gravity theories that could be relevant on cosmological scales, without assuming in first place something about light also on those scales. The standard approach is to take the following assumption:

(b) The photon of Maxwell electrodynamics (EM) remains minimally coupled even at the scales where general relativity (GR) may need modification. Namely, $c=1$ even at the scales where the scalar of dark energy dominates the expansion of the Universe,

thus, we see $c_g = c = 1$. However, *gravity couples universally to all matter*, and in principle, one could also explore an alternative assumption to (b), where the *scalar modification of gravity* at cosmological scales shares this *universal coupling* property, e.g.,

(a) The scalar of dark energy (DE) couples to both, the graviton and the photon in a specific way, *such that we see the luminality of GWs*,

$$\frac{c_g(t)}{c(t)}=1. \tag{2}$$

The assumption (b) conveniently fits EM at *all* scales, mainly to be consistent with laboratory experiments. However, it also quickly forbids additional input to more objectively constrain modified gravity theories.

The assumption (a)—which we take in this letter, and that clearly contains (b) as a particular case—opens a new set of observational possibilities. If DE is not really dark and also couples to the photon, new types of laboratory and astrophysical tests are required [4].

Furthermore, (a) reopens the path to nonminimally coupled theories for DE that were previously thought to be ruled out [5–11]. Interestingly, nonminimal couplings may be relevant in the wake of the recent DESI baryonic acoustic oscillations (BAO) data, favoring dynamical DE [12]. To that end a Horndeski theory with nonminimal couplings of the scalar to gravity would be necessary to safely cross the phantom divide [13,14] (See however the discussions in e.g., [15–19]). Although these possibilities are not conclusive [20], the theories shown in this paper—a broad generalization of the theories used in [13,14]—open new opportunities for the cosmologist.

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We consider the most general degenerate higher-order scalar-tensor modifications of gravity (DHOST) that have been classified to date, which are by construction free of Ostrogradsky ghosts [21–25]. We deduce the DE-photon couplings that are necessary for the observed luminality of GWs in these DHOST (2), with *Horndeski and beyond Horndeski* as particular cases. We find that only two types of DE-photon couplings are necessary. One of them cannot be removed by a conformal/disformal transformation of the metric. It is involved in a new luminal beyond Horndeski (BH) theory that we show below, for which the GW decay to DE is suppressed. Altogether passing the strong constraints on both the luminality [6–11] and nondecay of GW [26].

The model. In the usual parametrization, we consider 19 potentials depending on a scalar field π. They generalize the Einstein-Hilbert action in four dimensions (4D) with minimal and nonminimal couplings of π to gravity. Let us denote the 19 scalar potentials as a_i, b_j, f_k, G_k with $i = 1\ldots5$, $j = 1\ldots10$ and $k = 2, 3$. In principle, we allow all of these potentials to be functions of a scalar field π and $X = \pi_\mu\pi^\mu$, where $\pi_\mu = \nabla_\mu\pi$. However, some of these potentials are not free. There are relations among them in order to not propagate the Ostrogradsky ghost. These relations, known as degeneracy conditions, separate the theory space of DHOST into distinct classes. Thus, in all the theories we consider there are always less than 19 free scalar potentials of π and X, with the specific number of free functions depending on the class. For instance in Horndeski theory there are up to four free functions [27,28]. A complete classification with the number of free functions, and properties is given in [22–24]. Below we only give the degeneracy conditions for the most physically relevant cases.

The Lagrangian is written as

$$\mathcal{L}_{\mathrm{DHOST}_\pi} = f_2 R + f_3 G_{\mu\nu}\pi^{\mu\nu} + \mathcal{L}_{\mathrm{Linear}} + \mathcal{L}_{\mathrm{Quad}} + \mathcal{L}_{\mathrm{Cubic}}, \quad (3)$$

where R is the Ricci scalar, $G_{\mu\nu}$ is the Einstein tensor and ∇ is the covariant derivative computed with the ambient metric of the D-dimensional manifold (of signature $-,+,+,+,\ldots$), and $\mu = 0, 1, \ldots D-1$. The main results in this paper will be in the usual $D = 4$, however, as we explain latter on, they are most easily derived starting from $D = 5$, as we will explicitly state when needed.

The last three terms in (3) contain diffeomorphism invariant combinations of respectively, (up to) linear, quadratic and cubic in $\nabla^2\pi$ terms. Explicitly,

$$\mathcal{L}_{\mathrm{Linear}} = G_2 + G_3\Box\pi, \quad (4)$$

$$\mathcal{L}_{\mathrm{Quad}} = \sum_{i=1}^{5} a_i(\pi, X) L_i^{(2)}, \quad (5)$$

where $L_i^{(2)}$ are of order $(\nabla^2\pi)^2$,

$$L_1^{(2)} = (\pi_{\mu\nu})^2, \quad L_2^{(2)} = (\Box\pi)^2, \quad L_3^{(2)} = \Box\pi(\pi_{\mu\nu}\pi^\mu\pi^\nu)$$
$$L_4^{(2)} = (\pi_{\mu\rho}\pi^\mu)^2, \quad L_5^{(2)} = (\pi_{\mu\nu}\pi^\mu\pi^\nu)^2, \quad (6)$$

and

$$\mathcal{L}_{\mathrm{Cubic}} = \sum_{j=1}^{10} b_j(\pi, X) L_j^{(3)}, \quad (7)$$

where $L_j^{(3)}$ are of order $(\nabla^2\pi)^3$,

$$L_1^{(3)} = (\Box\pi)^3, \quad L_2^{(3)} = \Box\pi(\pi_{\mu\nu})^2, \quad L_3^{(3)} = (\pi_{\mu\nu})^3$$
$$L_4^{(3)} = (\Box\pi)^2(\pi_{\mu\nu}\pi^\mu\pi^\nu), \quad L_5^{(3)} = \Box\pi(\pi_{\mu\nu}\pi^\mu)^2,$$
$$L_6^{(3)} = (\pi_{\rho\sigma})^2(\pi_{\mu\nu}\pi^\mu\pi^\nu), \quad L_7^{(3)} = \pi^{\mu\nu}\pi_{\nu\rho}\pi^{\rho\sigma}\pi_\mu\pi_\sigma,$$
$$L_8^{(3)} = (\pi^{\mu\nu}\pi_\mu)^2(\pi^{\rho\sigma}\pi_\rho\pi_\sigma), \quad L_9^{(3)} = \Box\pi(\pi^{\rho\sigma}\pi_\rho\pi_\sigma)^2,$$
$$L_{10}^{(3)} = (\pi^{\rho\sigma}\pi_\rho\pi_\sigma)^3. \quad (8)$$

It was thought that a large set of DHOST theories—including Horndeski and beyond Horndeski [29,30]—are constrained to some extent in order to satisfy (2). As we noted, this belief assumed (b). In this paper, however, we work on hypothesis (a), and thus, to $\mathcal{L}_{\mathrm{DHOST}_\pi}$ we must add the precise scalar of DE-photon couplings such that we see the luminality of GWs (2). It was initially shown in [5] that a simple way to obtain them in 4D is to start from a 5-dimensional (D) setup; thus, consider the action of DHOST only for a brief moment in 5D,

$$\int \sqrt{-{}^{(5)}g}\, d^5\mathrm{x}\, \mathcal{L}_{\mathrm{DHOST}_\pi}. \quad (9)$$

Writing the 5D metric ${}^{(5)}g$ as

$${}^{(5)}g_{BC} = \begin{pmatrix} g_{\mu\nu} + A_\mu A_\nu & A_\mu \\ A_\nu & 1 \end{pmatrix}, \quad (10)$$

where the latin indices are $B = 0, \ldots4$ and Greek $\mu = 0, \ldots3$. Seen simply as a tool for our purpose in 4D, we compactify the fifth dimension with Kaluza's cylinder condition [31], where we have assumed right away in Eq. (10) a constant dilaton, and such that the 4D fields g and A_μ do not depend on the fifth dimension. We further rescale the 4D fields to reabsorb the $\int dx^4$, and thus we rewrite (9) in terms of 4D fields only.

All in all, after compactification, the theory (9) takes the form of the usual DHOST plus a scalar-photon sector in 4D (11). As usual the $U(1)$ gauge invariance in the vector sector is inherited from diffeomorphisms in 5D.

It is clear that because the 4-vector A_μ and the 4D metric are just but components of the same metric in 5D, their speed in 4D is generally bound to be *the same*. The caveat is that we have broken isotropy in 5D by compactifying one

spatial dimension and ignoring[1] the dynamics of a dilaton. Thus, there are special cases with unequal speeds which we single out below.

DHOST *with dark energy-photon couplings.* From now on in 4D, the complete DHOST action with DE-photon couplings reads,

$$\int \sqrt{-g} d^4x (\mathcal{L}_{\mathrm{DHOST}_\pi} + \mathcal{L}_{\mathrm{DHOST}_A}), \tag{11}$$

with $\mathcal{L}_{\mathrm{DHOST}_\pi}$ given in (3). The DE-photon sector is

$$\mathcal{L}_{\mathrm{DHOST}_A} = \frac{f_3}{8}(4F_{\mu\nu}\nabla_\rho F^{\nu\rho}\pi^\mu + F^2\Box\pi - 4F_\mu{}^\nu F^{\mu\rho}\pi_{\nu\rho}) - \frac{f_2}{4}F^2 + l_{\mathrm{Quad}_A} + l_{\mathrm{Cubic}_A}, \tag{12}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and with obvious notation parallel to (5) and (7),

$$l_{\mathrm{Quad}_A} = a_1(\pi, X) l_1^{(2)}, \tag{13}$$

$$l_{\mathrm{Cubic}_A} = \sum_{j=\{2,3,6\}} b_j(\pi, X) l_j^{(3)}, \tag{14}$$

where $l_1^{(2)} = l_j^{(3)} = \frac{1}{2}(F_{\mu\nu}\pi^\mu)^2$ for $j = 2, 6$, and

$$l_3^{(3)} = \frac{3}{4}F_{\mu\nu}F_{\rho\sigma}\pi^{\mu\rho}\pi^\nu\pi^\sigma. \tag{15}$$

One identifies in principle three types of DE-photon couplings $F^2\nabla^2\pi$, $F^2(\nabla\pi)^2$, and $F^2\nabla^2\pi(\nabla\pi)^2$. However, the latter—proportional to b_3—will be removed below by the luminality condition (2). It is essential to note that the DE-photon couplings $f_3F^2\nabla^2\pi$ *cannot be removed by a conformal/disformal transformation* of the metric that depends on up to first derivatives of π. Namely, one cannot obtain the f_3 DE-photon couplings by such metric redefinition in the Maxwell term $-1/4F^2$.

In short, for each of the six contributions to $\mathcal{L}_{\mathrm{DHOST}_\pi}$ labeled by the scalar potentials $f_2, f_3, a_1, b_2, b_3, b_6$ there is a corresponding DE-photon sector in Eq. (12).

Luminal DHOST *around the corner.* As declared, not all Lagrangians in Eq. (11) propagate gravitational and electromagnetic waves at the same speed. The approach below is to compute the graviton and photon speeds on a cosmological background for the action (11) and find the Lagrangians that can satisfy Eq. (2). Note that previous cases suggest that the results below could also hold on (at least) spherically symmetric backgrounds [33,34].

[1]If we had considered the dilaton, with background $\Phi(t)$, then $\frac{c_g^2}{c^2} = 1 - 3\frac{\dot\pi X(H - \frac{\ddot\Phi}{\dot\Phi})\dot\Phi b_3}{\mathcal{G}_\tau}$. Restoring isotropy $\dot\Phi = a$, we would see that (2) always holds. Although this choice is *unphysical*, this is at the very least a cross-check of our results. See [32] Sec. V C for a discussion.

The scalar mode of DHOST is *not modified* by the new terms $\mathcal{L}_{\mathrm{DHOST}_A}$ on the cosmological background. Thus, we do not discuss any further the scalar sector in this paper. Furthermore, we will assume the DHOST classes that actually propagate a graviton [21–25].

We consider first-order perturbations on a spatially flat Friedmann-Lemaître-Robertson-Walker background. With the perturbed metric $\mathrm{d}s^2 = (\eta_{\mu\nu} + \delta g_{\mu\nu})\mathrm{d}x^\mu\mathrm{d}x^\nu$ where $\eta_{\mu\nu} = -\mathrm{d}t^2 + a(t)^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$, we write only the symmetric, traceless and transverse tensor perturbation h_{ij} and the two transverse vector perturbations S_i, F_i as

$$\delta g = (2S_i\mathrm{d}t\mathrm{d}x^i + (\partial_i F_j + \partial_j F_i + 2h_{ij})\mathrm{d}x^i\mathrm{d}x^j), \tag{16}$$

where we denote spatial indices with lowercase Latin indices, $i = 1, 2, 3$. The perturbed DHOST scalar $\pi(x^\mu)$ is written as $\pi(t) + \chi(t, \vec{x})$ in the linearized expressions, within which $\pi(t)$ is the background scalar field. Finally, on the cosmological medium the photon amounts to the transverse perturbation $A_i(t, \vec{x})$, with vanishing background due to isotropy.

The quadratic action for the graviton reads,

$$\mathcal{S}_{\mathrm{Tensor}} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left(\mathcal{G}_\tau\dot{h}_{ij}^2 - \frac{\mathcal{F}_\tau}{a^2}(\partial_k h_{ij})^2\right), \tag{17}$$

with $f_{3,X} = \frac{\partial f_3}{\partial X}$ and so on,

$$\mathcal{G}_\tau = 2f_2 + 2\ddot\pi X f_{3,X} - X f_{3,\pi} - 2Xa_1 + 2X(3\dot\pi H + \ddot\pi)b_2 + 6\dot\pi XHb_3 + 2\ddot\pi X^2 b_6, \tag{18}$$

$$\mathcal{F}_\tau = 2f_2 - 2\ddot\pi X f_{3,X} + X f_{3,\pi}, \tag{19}$$

while the action for the photon is written as

$$\mathcal{S}_{\mathrm{Vector}} = \frac{1}{4}\int \mathrm{d}t\,\mathrm{d}^3x\,a\left(\mathcal{G}_A\dot{A}_i^2 - \frac{\mathcal{F}_A}{a^2}(\partial_k A_i)^2\right), \tag{20}$$

where

$$\mathcal{G}_A = \mathcal{G}_\tau - 3\dot\pi XHb_3, \tag{21}$$

$$\mathcal{F}_A = \mathcal{F}_\tau. \tag{22}$$

As expected, by construction, the coefficients in the quadratic actions are similar, e.g., as in (22). Now, with their speeds squared, respectively, $c_g^2 = \frac{\mathcal{F}_\tau}{\mathcal{G}_\tau}$ and $c^2 = \frac{\mathcal{F}_A}{\mathcal{G}_A}$, we find their ratio,

$$\frac{c_g^2}{c^2} = 1 - 3\frac{\dot\pi XHb_3}{\mathcal{G}_\tau}. \tag{23}$$

Thus, in principle, the DHOST theories with

$$b_3 = 0, \tag{24}$$

would preserve the unit ratio of speeds (2). However, let us recall that depending on the degenerate class of DHOST

being considered, the scalar potential b_3 *may not be a free function* but it may be fixed by the also crucial degeneracy conditions (see footnote 1).

Degenerate and luminal DHOST. Thus, to apply the luminality condition (24) in DHOST, one is left with the task of establishing whether it is consistent with the degeneracy condition of the class. From the comprehensive classification in [23] Table 1 and [22] it is clear that there are many[2] scalar-tensor theories with a graviton that can be made luminal with Eq. (24). We will focus, however, on the phenomenologically most relevant classes. The simplest successful case is

(i) Every quadratic DHOST with the corresponding DE-photon couplings, $\mathcal{L}_{\mathrm{DHOST}_A} = -\frac{f_2}{4}F^2 + \frac{a_1}{2}(F_{\mu\nu}\pi^\mu)^2$, and with a graviton, satisfies Eq. (2). Namely, the action (11) with $b_i = 0$ and $f_3 = 0$ has luminal GWs. The degeneracy conditions on some of the functions a_i, for multiple classes of theories, are given for instance in [22] Appendix C.

In particular, (i) includes *quadratic Horndeski and beyond Horndeski theory* (BH_4) as special cases. The latter is written with the action (11) and with the following degeneracy relations [21,24],

$$f_2 = G_4, \qquad a_1 = -a_2 = 2G_{4,X} + XF_4,$$
$$a_3 = -a_4 = 2F_4, \qquad a_5 = 0. \tag{25}$$

From (12) and (25) the DE-photon couplings that make luminal the quadratic BH theory are $\mathcal{L}_{BH_{4A}}$ in Eq. (26), from which we recover $\mathcal{L}_{4A}$ given in [5] in the particular case $F_4 = 0$. The theory (i) also includes as a luminal class, for instance, the DHOST ^{2}N-III/IIa, which may still be phenomenologically relevant [23], yet disconnected from the Horndeski class.

Another successfully Luminal case is in the mixed quadratic plus cubic DHOST class:

(ii) The quadratic BH plus cubic Horndeski theory, with $f_3 = G_5(\pi)$ and with the corresponding DE-photon couplings propagates luminal GWs. Namely, the action (11) with the relations (25), and with $b_j = 0$ with $j = 1, \ldots 10$. Explicitly,

$$\int \mathrm{d}^4x(\mathcal{L}_{BH_{4\pi}} + \mathcal{L}_{BH_{4A}} + \mathcal{L}_{H_{5\pi}} + \mathcal{L}_{H_{5A}}), \tag{26}$$

with

$$\begin{aligned}
\mathcal{L}_{BH_{4\pi}} &= G_2 + G_3\Box\pi + G_4R - 2G_{4,X}((\Box\pi)^2 - \pi_{\mu\nu}^2) \\
&\quad - F_4(X(\Box\pi)^2 - X\pi_{\mu\nu}^2 + 2(\pi_{\mu\nu}\pi^\mu)^2 \\
&\quad - 2\Box\pi\pi_{\mu\nu}\pi^\mu\pi^\nu), \\
\mathcal{L}_{\mathrm{BH}_{4A}} &= -\frac{G_4}{4}F^2 + \frac{2G_{4,X} + XF_4}{2}(F_{\mu\nu}\pi^\mu)^2, \\
\mathcal{L}_{H_{5\pi}} &= G_5G^{\mu\nu}\pi_{\mu\nu}, \\
\mathcal{L}_{H_{5A}} &= \frac{G_5}{8}(4F_{\mu\nu}\nabla_\rho F^{\nu\rho}\pi^\mu + F^2\Box\pi \\
&\quad - 4F_\mu{}^\nu F^{\mu\rho}\pi_{\nu\rho}).
\end{aligned} \tag{27}$$

G_5 is a function of π only, and we have taken $F_5 = 0$ (in the standard notation of BH [24]).

The theory (ii) generalizes the luminal Horndeski theory with $G_4(\pi, X)$, $G_5(\pi)$ shown in [5] to also include $F_4(\pi, X)$. This is *essential*; namely, with this new theory it becomes possible to suppress the GWs decay to the scalar of DE, by fixing the newly free potential F_4. Let us see how. First note that $b_3 = \frac{2}{3}(G_{5,X} + 3XF_5) = 0$ is satisfied. Then Eq. (2) follows; that is, in the theory (ii) the GWs are automatically luminal *without fixing any of the scalar potentials*. Secondly, this theory is free of Ostrogradsky ghosts: the degeneracy condition[3] in mixed quadratic plus cubic BH

$$F_4G_{5,X}X = -3F_5\left(G_4 - 2XG_{4,X} - \frac{X}{2}G_{5,\pi}\right), \tag{28}$$

is also automatically satisfied by $G_{5,X} = F_5 = 0$ [Note that the theory with $F_5 = -\frac{G_{5,X}}{3X} \neq 0$, which also sets $b_3 = 0$, has no (see foonote 2) tensor and vector modes once we impose Eq. (28)]. Thirdly, it can be easily checked that $\mathcal{L}_{H_{5A}}$ is a vector-scalar Galileon term. Namely, it is of higher order in the Lagrangian but it has second-order equations of motion. Again, no ghosts.

The essential aspect in the theory (ii) is that it has just the necessary amount of freedom, such that in a subclass within it, the GWs decay to DE may be suppressed. Indeed, in [26] it was shown that—in the case when $c_g(t) \neq 1$—the following expression should be negligible [see Eq. (87) in [26]], because if the GWs had considerably decayed to DE, we would have not observed them in first place:

$$\begin{aligned}
&F_4(4G_4 + X(2G_{4,X} + 3G_{5,\pi})) + XF_{4,X}(2G_4 + XG_{5,\pi}) \\
&\quad + 4G_{4,X}^2 + 4G_4G_{4,XX} + G_{5,\pi}(4G_{4,X} + 2XG_{4,XX} + G_{5,\pi}) = 0,
\end{aligned} \tag{29}$$

[2]A counterexample, where the luminality is not compatible with the degeneracy condition, is the *full* mixed quadratic plus cubic BH [29,30]. We see the degeneracy condition is (28). While $b_3 = 0$ implies $F_5 = -\frac{G_{5,X}}{3X}$. *Assuming* $F_5, G_{5,X} \neq 0$ one finds from (28) a relation $F_4(G_4, G_{4,X}, G_{5,\pi})$ that sets $\mathcal{G}_\tau = \mathcal{G}_A = 0$, which is a singular case with no graviton and no photon. However, note that the branch $F_5 = G_{5,X} = 0$ escapes the problem, because (28) is automatically satisfied with a totally free F_4. See the discussion below in the case (ii). Another counterexample is only cubic, full DHOST ^{3}N-I, which contains cubic Horndeski and BH; as noted in [22] in this class $b_3 = 2b_1$, with b_1 free, up to the condition $b_1 \neq 0$. Thus in this class (24) cannot be met. If we nevertheless take $b_1 = 0$, then we would be forced in another degenerate class, DHOST ^{3}N-II [22], which however has no graviton [23].

[3]We take sign convention for F_4, F_5 from [21,24]. Note however, the opposite sign for F_4 taken in [10,26,35].

where we have already used $G_{5,X} = F_5 = 0$ from the definition of (ii). Note that this constraint is independent of H and $\ddot{\pi}$, thus also independent of the matter content.

As all scalar potentials—in particular $F_4(\pi, X)$—remain free (while also satisfying luminality), there are theories in (ii) for which (29) is satisfied and the GWs decay to DE is suppressed. That is, since Eq. (29) is linear in F_4, it has solution [35],

$$F_4 = \frac{1}{2X^2}\left(2G_4 - X(4G_{4,X} + G_{5,\pi}) + \frac{4J_4(\pi)}{2G_4 + XG_{5,\pi}}\right), \quad (30)$$

where $J_4(\pi)$ is an integration "constant" (with respect to X). Notice two essential points to this conclusion; first, the DE-photon couplings (27). They keep free the $F_4(\pi, X)$ function, while also keeping GWs luminal.[4] Thus, we can solve F_4 as (30). Secondly, the fact that the *precise*[5] $G_5F^2\nabla^2\pi$ couplings in $\mathcal{L}_{\mathrm{H}_{5A}}$ cannot be removed by a conformal/ disformal transformation of the metric. This is significant. The disformal invariance of the decay, which was proven in [26]—and used in the argument in [35] to rule out some BH theories with scalar-photon couplings—does not apply to this case.

Similarly, to the best of our knowledge, the GWs decay constraint [26] has only been computed for the BH theory. In particular, additional checks would be needed to rule out the full theory (i) shown above, which includes quadratic BH *only* as a particular case.

Conclusions. We have shown that five sets of self-consistent DE-photon couplings are enough to render luminal the GWs in all DHOST theories (with a graviton) that first, are up to cubic in $\nabla^2\pi$, and second, whose degeneracy conditions are compatible with the sole condition $b_3(\pi, X) = 0$.

For the cosmologist this means the scalar-tensor theories with $b_3 = 0$—such as the beyond Horndeski theory (ii) Eq. (26) or (i)—may be potentially used with minor consideration of the graviton speed, because DE-photon couplings exist that can take care of the luminality of GWs and the experimental bound (1). Naturally, experimental constraints would be necessary on the DE-photon couplings proposed in this paper $\mathcal{L}_{\mathrm{BH}_{4A}}$, $\mathcal{L}_{H_{5A}}$ Eq. (26). Indeed, laboratory and astrophysical constraints have been already put on at least the disformal set of DE-photon couplings [4,35,36].

We showed at least one theory—a subclass of luminal beyond Horndeski—in which the decay of GWs to DE is suppressed on a cosmological background. This aligns with observational evidence and is relevant, because such background is a good description in the bulk of the trajectory of GWs to Earth. We stressed that the essential type of DE-photon coupling which allows the decay to be suppressed cannot be removed by a conformal/disformal transformation, and thus, the disformal invariance of the decay—which was proven in [26]—does not apply to this case.

We also showed some cases of BH and DHOST (see footnote 2) that remain ruled out by the bound (1), as they have no consistent DE-photon coupling.

Discussion: Recovering GR. aside from the experimental constraints on the DE–photon couplings [4,36], let us note that the Vainshtein screening of the extra scalar mode π—which is essential to align with the precise solar system tests—remains largely unaffected for at least some of the theories in this letter.

More precisely, the Vainshtein effect arises when the second-order derivative self-interactions become large compared to the kinetic linear term (See [37] for a detailed discussion).

Because the high *order of derivatives per field* is essential, let us take the parameter $\alpha = \frac{\partial^2\pi}{\Lambda^3}$, which may be large with respect to the scalar, photon, metric perturbations and their first derivatives [38–40] (where Λ is the theory's energy scale). The effective action to analyze the Vainshtein effect is thus built keeping α at all orders. This is equivalent to define an order-of-perturbation operator $[\cdot]$ that gives a weight both to fields and derivatives, such that $[\alpha] = 0$. In other words, α^n adds zero weight in this perturbative expansion and thus, it will not be truncated. $[\alpha] = 0$ amounts to define $[\pi] = 2$, $[\partial] = -1$ and similarly $[A] = [g] = 2$, where π, A, g here denote perturbations of their respective fields. Thus the terms that modify the usual *quadratic action* $(\partial g)^2, (\partial A)^2, (\partial\pi)^2$ are in principle $g\alpha^n, (\partial g)^2\alpha^n, (\partial A)^2\alpha^n, \pi\alpha^n, (\partial\pi)^2\alpha^n$, in accordance with [38–40]. Notice that all of these quantities are of order 2 under our definition of $[\cdot]$.

Now, applied to the quadratic DHOST case—where the DE-photon couplings are of the type $l_1^{(2)} = (F\partial\pi)^2$ in Eq. (13)—there are no order 2 terms that mix the scalar and the photon perturbations, because the lowest order mixing—in the presence of a vanishingly small background vector field—is $[(\partial A\partial\pi)^2] = 4$, and so the Vainshtein radius remains the same to leading order, independent of the DE-photon couplings. Consequently, existing results on the Vainshtein mechanism remain applicable in this case. This includes known constraints and potential issues in e.g., quadratic DHOST and beyond Horndeski theories [37–40]. Nevertheless, further studies may help to constrain DHOST theories with DE-photon couplings. Of special interest are

[4]Note this critical difference to beyond Horndeski without the DE-photon couplings $\mathcal{L}_{\mathrm{BH}_{4A}}$, $\mathcal{L}_{\mathrm{H}_{5A}}$; in that case F_4 is not free to suppress the decay, because $F_4 = \frac{-2G_{4,X}}{X}$ and $G_5 = 0$ are already fixed to preserve luminal GWs [6–11].

[5]Let us note that in [35] a similar looking Lagrangian to $\mathcal{L}_{\mathrm{H}_{5A}}$ was considered with the aim to suppress the GWs decay while keeping their luminality, $\mathcal{L}^{(3)}_{SVT} \propto \bar{g}_{\alpha\beta}(g, \pi, \nabla\pi)\tilde{F}^{\mu\alpha}\tilde{F}^{\nu\beta}\pi_{\mu\nu}$, with $\tilde{F}$ the dual of F. However, $\mathcal{L}^{(3)}_{SVT}$ and $\mathcal{L}_{\mathrm{H}_{5A}}$ are fundamentally different. Their quadratic Lagrangians and thus, their vector speeds are related in a matter dependent way, through combinations of $H, \ddot{\pi}$. Thus, in accordance with [35] it is not possible to find a matter independent solution to (29) and (2) with $\mathcal{L}^{(3)}_{SVT}$.

regions of strong magnetic fields, such as magnetars, where the background vector field becomes relevant or even dominant for these effects.

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