

Shedding light on the intrinsic transversal momentum distributions of pions and kaons

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We propose to introduce the intrinsic transversal momentum distribution functions (iTMDs), in conjunction with the light-cone distribution amplitudes, to elucidate the probability amplitude of encountering a meson state wherein the partons swiftly traverse along the longitudinal axis while gently oscillating in the transversal plane. The primary motivation stems from the oversight of soft transverse dynamics within the k_T factorization formalism of an exclusive QCD process, which confines perturbative QCD (pQCD) predictions to scenarios involving large-momentum transfers. We meticulously investigate the π and K electromagnetic form factors using the iTMD-improved pQCD calculation at next-to-leading order. By analyzing data in the timelike physical regions, we obtain the transversal size parameters $\beta_\pi^2 = 0.51 \pm 0.04 \text{ GeV}^{-2}$ and $\beta_K^2 = 0.30 \pm 0.05 \text{ GeV}^{-2}$. We then extract the chiral mass of pions to be $m_0^\pi(1 \text{ GeV}) = 1.84 \pm 0.07 \text{ GeV}$ and explain the precise measurements of kaon form factors in the perturbative timelike region. As a remarkable byproduct, we found that the incorporation of iTMDs improves the pQCD predictions for electromagnetic form factors, extending the applicable range to a few GeV^2 . This improvement allows for direct comparison with existing measurements and lattice QCD evaluations.

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Introduction. The form factor is a momentum-dependent function that encapsulates the characteristics of a specific interaction within an appropriate matrix element. Its measurement serves a crucial purpose in either validating or refining theories, such as perturbative QCD (pQCD) and quark confinement [1,2]. Pion and kaon electromagnetic form factors, defined by the simplest hadronic matrix elements, occupy a central role in the exploration of the hadron structure and the breaking of chiral symmetry [3,4].

The pQCD prediction of electromagnetic form factors is written in a factorization formalism [5],

$$\mathcal{F}(Q^2) = \sum_{t_1, t_2} \int du_i db_i \varphi^{(t_1)}(u_1, \mu_{r_1}) \otimes \varphi^{(t_2)}(u_2, \mu_{r_2}) \otimes \mathcal{H}^{(t_1 t_2)}(u_i, b_i, \mu_f) \times e^{-S(u_i, b_i, \mu_f)}. \quad (1)$$

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In the formalism, contributions from different twists' light-cone distribution amplitudes (LCDAs) of initial and final pions, denoted by φ^{t_1} and φ^{t_2} , respectively, are summed up to give the total result. LCDA at a certain twist φ is a function of the longitudinal momentum fraction u carried by the antiquark inside the meson and the renormalization scale μ_r . $\mathcal{H}^{(t_1 t_2)}$ are the perturbatively calculable hard kernels ordered by twists too, which indicate the probability of momenta redistributions at the quark-gluon level due to the electromagnetic interaction that happened within a small distance. Besides the momentum fraction u_i and the hard factorization scale μ_f , it also depends on the coordinate b conjugated to the transversal momentum k_T , which is introduced in the energetic propagator to regulate the end point singularity. The loop integral of k_T results in several large logarithms, and they are subsequently resummed up to the well-known Sudakov factor e^{-S} . This exponent turns out to suppress the soft contribution, meanwhile highlighting the hard scattering mechanism.

The nonperturbative LCDAs are universal objects defined by matrix elements [6–8],

$$\langle 0 | \bar{u}(z) \Gamma[z, 0] d(0) | \pi^-(p) \rangle \propto \int du e^{iup \cdot z} \varphi(u, \mu_r), \quad (2)$$

where the nonlocal quark current with a suitable Dirac matrix Γ is sandwiched between π and the QCD vacuum, and the path-ordered gauge factor (abbreviated as square brackets) is along the light cone $z^2 = 0$. LCDAs is usually obtained by utilizing the conformal symmetry in massless QCD. Keep in mind that the conformal symmetry of a quantum theory implies a vanished β function, hence it is powerful when used in a QCD process with large-momentum transfers and large energies. So, roughly speaking, the LCDAs embodied in the factorization formalism are the wave functions at zero transverse separations, and this means that the soft transversal dynamics is actually missed in the factorization formalism shown in Eq. (1).

In the previous pQCD study of pion form factors [9], the chiral mass $m_0^\pi(1 \text{ GeV}) = 1.37 \pm 0.30 \text{ GeV}$ is obtained by fitting the next-to-leading-order (NLO) prediction to the data-driven spacelike form factor. The result is much smaller than that obtained from chiral perturbative theory (ChPT), at 1.89 GeV [10], and from the $\overline{\text{MS}}$ current quark mass $m_0^\pi = m_\pi^2/(m_u + m_d) \sim 1.74 \text{ GeV}$ [11]. This deviation necessitates a reevaluation of the pQCD factorization formalism, and the next-to-next-to-leading-order (NNLO) corrections and the high-twist contributions are the first considerations. We convince ourselves that the NNLO would not resolve the m_0^π problem since the NLO calculations [12,13] have showed a good perturbative convergence (about 10% increases). In addition, besides the chiral enhancement from the two-particle twist-three LCDAs, higher-twist contributions are expected to be tiny due to the heavy power suppression [14]. In these senses, the m_0^π problem reveals a possible large effect from the transversal momentum distribution inside pions when they are moving outside the scope of hard scattering.

In Fig. 1, we show the sketch map of electromagnetic interaction of pions. The strength of electromagnetic interactions is divided into two distinct regions by double-dashed curves. Within the circle lies the realm of hard scattering (central region of the electromagnetic potential field), described by the Sudakov-multiplied hard kernel $\mathcal{H}e^{-S}$, which picks up the hard radiations of partons on the

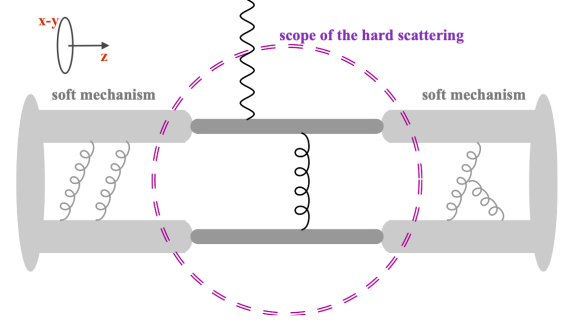


FIG. 1. The sketch map of pion electromagnetic form factors.

transversal plane, as shown by the thin gray stick. Outside the scope of hard scattering, energetic pions are moving fast along the z direction, accompanied by soft bremsstrahlung radiations. The bremsstrahlung radiations along the light cone are absorbed into the effects of high-twist LCDAs. In the factorization formalism in Eq. (1), the hard transversal dynamics of LCDAs has been defined by selecting the renormalization scale of the nonperturbative parameters at the hard factorization scale, which aligns with the marginal region of the central potential field. In the exterior region, situated far from the central potential field, the soft radiations in the transversal plane (depicted by the thick light-gray stick) are notably absent from the definition of LCDAs and have often been overlooked in previous pQCD studies. This is the main motivation of this work.

In this letter, we introduce the intrinsic transversal momentum distribution (iTMD) to describe the soft transversal degree of freedom associated with LCDAs. We ingeniously study the iTMDs shapes of pions and kaons via the electromagnetic form factors. The central goal is to mark up the missed soft transversal dynamics clarified above and, hence, to construct a comprehensive factorization for an exclusive QCD process.

Intrinsic transversal momentum-dependent functions. The soft pion wave function of the valence quark state is generally written by a product of LCDA and iTMDs [15],

$$\langle 0 | \bar{u}(x) \Gamma [x^-, x_\perp; 0, 0_\perp] d(0) | \pi^-(p) \rangle \propto \int du dk_\perp^2 e^{i u p^+ x^- - i k_\perp \cdot x_\perp} \psi(u, k_T), \quad \psi(u, k_T) = \frac{f_\pi}{2\sqrt{6}} \varphi(u, \mu_r) \Sigma(u, k_T). \quad (3)$$

Since iTMDs encapsulate the soft transverse dynamics, deriving their expression from first principles is infeasible. Instead, we adopt a simple Gaussian function to parametrize them while preserving rotational invariance [16,17],

$$\Sigma(u, k_T) = 16\pi^2 \frac{\beta^2}{u(1-u)} e^{-\frac{\beta^2 k_T^2}{u(1-u)}}. \quad (4)$$

It can be figurative, considered by a transversal harmonic oscillator with the transversal size parameter

β^2 . The transversal radius of the valence quark state should be no larger than the mean electric charge radius of the corresponding mesons, so the average value of the conjugate transversal momentum satisfies

$$[\langle k_T^2 \rangle]^\frac{1}{2} \equiv \left[\frac{\int du d^2 k_T k_T^2 |\psi(u, k_T)|^2}{\int du d^2 k_T |\psi(u, k_T)|^2} \right]^\frac{1}{2} \gtrsim \frac{0.2}{\langle r^2 \rangle^\frac{1}{2}}. \quad (5)$$

Performing Fourier transformation, we obtain the soft transversal function in the conjugated coordinate space

$$\hat{\Sigma}(u, b_T) = 4\pi e \frac{b_T^2 u(1-u)}{4\beta^2}. \quad (6)$$

In the pQCD calculation, twist-three LCDAs of the valence quark state give the dominant contributions to pion and kaon electromagnetic form factors due to the chiral enhancement effect. We highlight that the transversal motion in the high-twist LCDAs is definitely independent of the iTMDs introduced in this work, since they transfer different dynamics in the factorization formalism, as stated at the end of the last section. For the two-particle twist-three LCDAs, we propose to introduce the iTMDs in the following way:

$$\begin{aligned} \psi^{p,\sigma}(u, \mu) = & \int \frac{d^2 k_T}{16\pi^3} \phi_{2p}^{p,\sigma}(u, \mu) \Sigma(u, k_T) \\ & + \int \frac{d^2 k_{1T} d^2 k_{2T}}{64\pi^5} \rho_+ \phi_{3p}^{p,\sigma}(u, \mu) \\ & \times \int \mathcal{D}\alpha_i \Sigma'(\alpha_i, k_{iT}). \end{aligned} \quad (7)$$

Here, the superscript p (σ) indicates the twist-three LCDAs induced by the pseudoscalar (tensor) current, and the subscripts $2p$ and $3p$ denote the contributions from the valence quark state and three-particle state, respectively. The three-particle contribution is proportional to the quark mass, so a dimensionless quantity $\rho_+ = (m_{q_1} + m_{q_2})/m_0^P$ is defined with the chiral mass m_0^P . The integral measure over the longitudinal momentum fractions $\alpha_{i=1,2,3}$ reads as $\int \mathcal{D}\alpha_i = \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1}{1-\alpha_1-\alpha_2}$. Three-particle iTMDs are chosen again in a Gaussian function, and in the conjugate coordinate space it reads as

$$\hat{\Sigma}'(\alpha_i, b_1, b_2) = 4\pi e \frac{2\alpha_3(b_1^2 + b_2^2) + (\alpha_1 + \alpha_2)(b_1 - b_2)^2}{16\beta'^2}. \quad (8)$$

Electromagnetic form factors and pQCD prediction. Electromagnetic form factors of pions have been investigated for several decades. They are a touchstone of QCD-based approaches, such as the light-cone sum rules (LCSRs) [18,19], the Dyson-Schwinger equation (DSE) [20,21], the pQCD [9,12], etc., not only in their initial stage of establishment, but also in the well-developed period. An impressive study recently came from the lattice QCD (LQCD), where the authors improved the first-principle evaluation from the small-momentum transfers $-1 \text{ GeV}^2 \leq q^2 \leq 0$ [22] to the large values $-10 \text{ GeV}^2 \leq q^2 \leq 0$ [23]. Meanwhile, the two-loop computation of the leading-twist contribution in the hard-collinear factorization shows an enormous correction to the short-distance coefficient function [24,25]. Among these approaches, the pQCD has a unique advantage to predict the form factors in both spacelike and timelike regions. What is more, it provides a systematical analysis of the contributions from different twist LCDAs. From the experimental side, the precise data

of pion electromagnetic form factors are mainly obtained in the physical regions by electron-positron collider experiments [26–28]. In contrast, the data of spacelike form factors [29–31] are available only in the small-momentum transfers $[-2.50, -0.25] \text{ GeV}^2$.

People usually take a dispersion relation to connect the QCD predictions of spacelike form factors to the precise measurements in timelike regions. Recently, we proposed the modular dispersion relation [9,14]

$$\begin{aligned} \mathcal{F}_\pi(q^2 < 0) = & \exp \left[\frac{q^2 \sqrt{s_0 - q^2}}{2\pi} \int_{s_0}^\infty ds \frac{\ln |\mathcal{F}_\pi(s)|^2}{s \sqrt{s - s_0} (s - q^2)} \right], \\ |\mathcal{F}_\pi(s)|^2 = & \Theta(s_{\max} - s) |\mathcal{F}_{\pi, \text{Inter}}^{\text{data}}(s)|^2 \\ & + \Theta(s - s_{\max}) |\mathcal{F}_\pi^{\text{pQCD}}(s)|^2. \end{aligned} \quad (9)$$

The data term is currently available from the threshold value $s_0 = 4m_\pi^2$ to $s_{\max} \sim 8.7 \text{ GeV}^2$ [26]; meanwhile, the pQCD prediction is powerful in the large-momentum transfers. Hence the modular dispersion relation is model-independent.

The state-of-the-art pQCD calculation has been performed at NLO up to twist-three and leading-order at twist-four levels [14]. As shown in Eq. (1), the result includes contributions from different twist LCDAs, in which the high-twist contributions are conceptually power suppressed by $\mathcal{O}(1/Q^2)$. For the form factors of pseudoscalar mesons, however, the terms proportional to two-particle twist-three LCDAs are dominant in the intermediate- and not-too-large-momentum transfers [$Q^2 \sim \mathcal{O}(10, 10^2)$] due to the chiral enhancement $\mathcal{O}((m_0^P/\tilde{Q})^2)$. Here, the chiral mass $m_0^P \sim \mathcal{O}(1)$ evolves mildly on a hard scale, and the effective longitudinal virtuality $\tilde{Q}^2 \sim u_i Q^2$ in the hard scattering grows much more slowly than the momentum transfer Q^2 itself due to the Sudakov effect.

We now pick up the soft transversal dynamics, which is overlooked in the previous pQCD predictions. For the goldstone pion, the terms proportional to the quark mass can be safely neglected in the soft wave function in Eq. (7), and the soft transversal dynamics is hence described solely by the iTMDs associated with the valence quark state. In order to obtain the transversal size parameter β_π , we consider the constraint from the asymptotic behavior of double photon transition $\pi \rightarrow \gamma\gamma$ [17,32],

$$\beta_\pi^2 = \frac{1}{8\pi^2 f_\pi^2 (1 + a_2^\pi + a_4^\pi + \dots)}. \quad (10)$$

We take the result obtained from the joint analysis of the electromagnetic form factor with the modular dispersion relation and precise LCSRs calculation [14], namely, $a_2^\pi = 0.28 \pm 0.05$, $a_4^\pi = 0.19 \pm 0.06$. With the decay constant $f_\pi = 0.13 \text{ GeV}$ taken from Particle Data Group [11], we obtain $\beta_\pi^2 = 0.51 \pm 0.04 \text{ GeV}^{-2}$. This value corresponds to the mean transversal momentum $358 \pm 15 \text{ MeV}$ from

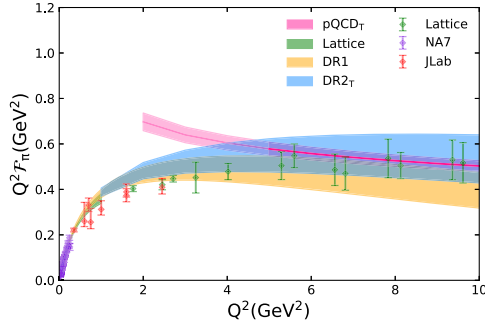


FIG. 2. The iTMD-improved pQCD prediction of the pion electromagnetic form factor $Q^2 F_\pi(Q^2)$.

Eq. (5), revealing the soft transversal dynamics in the soft wave function.

We perform the fit of spacelike form factors obtained from the iTMD-improved pQCD calculation to the modular dispersion relation and obtain the chiral mass $m_0^\pi(1 \text{ GeV}) = 1.84 \pm 0.07 \text{ GeV}$. This value is thirty percent larger than the previous pQCD result [9] but consistent with the ChPT result [10], indicating a significant decrease of the form factor due to the soft transversal dynamics, especially in the small- and intermediate-momentum transfers. In Fig. 2, we plot the iTMD-improved pQCD prediction (magenta) of the pion form factor $Q^2 F_\pi(Q^2)$. We compare it to the LQCD evaluation (green) [22,23] and the measurements (purple and red) [29–31] and find a good consistency between them. More impressively, we find that the power of pQCD prediction is improved down to a few GeV^2 . In addition, we superpose the dispersion relation result in blue and yellow bands. The blue band is obtained by considering the two terms in Eq. (9); the yellow band, however, is obtained by considering only the first term. We find a visible gap between them in the large-momentum transfers. This is traced back to the logarithm expression in the modular dispersion relation in Eq. (9) that weights the high-energy tail contribution.

For the kaon electromagnetic form factor, there is no measurement in the spacelike region so far, but there are abundant data in the timelike region [33–35] spanning from

the physical resonances to large-momentum transfers. We can understand the soft transversal dynamics in the kaon wave function by comparing the precise pQCD calculation directly to the timelike data.

With the well-known ChPT relations [10], the chiral mass of kaon $m_0^K = 1.90_{-0.12}^{+0.08} \text{ GeV}$ is obtained without involving light quark masses. Other parameters in the kaon LCDAs are taken the same as in Refs. [5,8]. The remaining unknown inputs are the transversal size parameters β_K^2 and $\beta_K'^2$ in the iTMDs. In the soft wave functions of kaons, the three-particle contribution is still small, $\mathcal{O}(m_s/m_0^K)$, when compared to the valence quark term. In this case, the current accuracies of measurement and pQCD calculation prevent us from extracting the transversal size parameter $\beta_K'^2$ associated with the three-particle state. So, we settle for the second best and take it equal to the transversal size parameter associated with the valence quark state, namely, $\beta_K'^2 = \beta_K^2$. By fitting the iTMD-improved pQCD prediction to the timelike data, we obtain $\beta_K^2 = 0.30 \pm 0.05 \text{ GeV}^{-2}$. This value corresponds to the mean transversal momentum $0.55 \pm 0.07 \text{ MeV}$ for the valence quark state at leading twist.

We show the result of the kaon electromagnetic form factor in Fig. 3. For the timelike form factor, we compare the iTMD-improved pQCD prediction (magenta) to the pQCD result without considering the soft transversal dynamics (yellow). We see that the iTMD is indispensable to explain the data in the intermediate q^2 . We see from the right subgraph that the iTMD-improved pQCD result (magenta) of the spacelike form factor is smaller than the lattice data [23]. In order to clarify the large $SU(3)$ flavor breaking, from which emerges an additional term proportional to s -quark mass m_s in the twist-three LCDAs, we depict the pQCD result, taking $m_s = 0$ in the yellow band. We find that the m_s term decreases the kaon form factor by about thirty percent. Our result shown in the magenta band agrees with the DSE approaches [36] and the collinear QCD factorization [37].

In Table I, we list the mean transversal momenta and the conjugated distances associated with different LCDAs of π and K . All the mean momenta on the transversal plane are

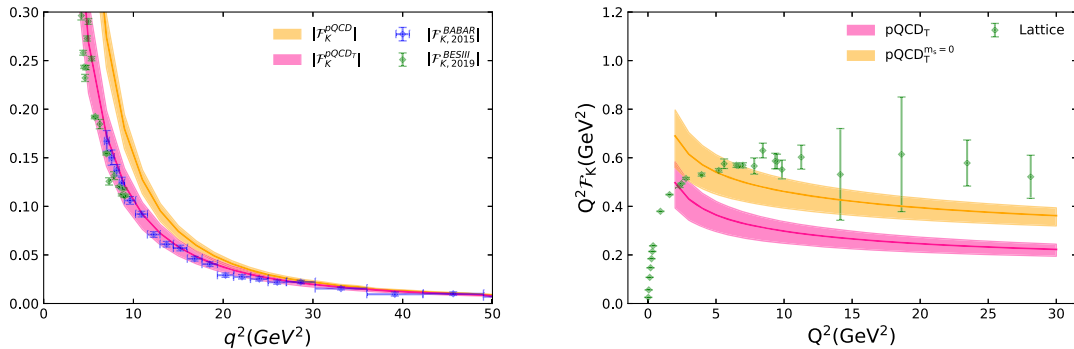


FIG. 3. The pQCD predictions of the kaon electromagnetic form factor in the timelike (left) and spacelike (right) regions.

TABLE I. The mean transversal momenta and the conjugated distances of π and K defined by Eq. (5).

Mean value	ϕ	ϕ^p	ϕ^σ
$\langle k_T^2 \rangle_\pi^{1/2}$ (GeV)	0.36 ± 0.02	0.40 ± 0.02	0.40 ± 0.02
$\langle b_T^2 \rangle_\pi^{1/2}$ (fm)	0.56 ± 0.02	0.50 ± 0.02	0.50 ± 0.02
$\langle k_T^2 \rangle_K^{1/2}$ (GeV)	0.55 ± 0.07	0.53 ± 0.07	0.52 ± 0.07
$\langle b_T^2 \rangle_K^{1/2}$ (fm)	0.37 ± 0.05	0.38 ± 0.05	0.39 ± 0.05

found at soft scales. The conjugated distances are smaller than their electric charge radius ($\langle r_\pi^2 \rangle^{1/2} = 0.67 \pm 0.08$ fm, $\langle r_K^2 \rangle^{1/2} = 0.56 \pm 0.03$ fm [11]). The slight difference in the mean values between leading twist and twist-three LCDAs comes from the three-particle contributions.

k_T factorization revitalization. Considering the soft transversal dynamics via iTMDs, the pQCD factorization formalism in Eq. (1) is improved to

$$\mathcal{F}(Q^2) = \sum_{t_1, t_2} \int du_i db_i \psi^{(t_1)}(u_1, b_1, \mu_{r_1}) \otimes \psi^{(t_2)}(u_2, b_2, \mu_{r_2}) \otimes \mathcal{H}^{(t_1 t_2)}(u_i, b_i, \mu_f) \times e^{-S(u_i, b_i, \mu_f)}. \quad (11)$$

Here, $\psi(u_i, b_i, \mu)$ is the soft distribution amplitude of hadrons formed outside the central scope of the electromagnetic field, including both the longitudinal and transversal components, as shown in Eqs. (3) and (7).

In Fig. 4, we show the state-of-the-art pQCD predictions of pion and kaon electromagnetic form factors. For the convenience of comparison, we depict both the spacelike and timelike results within the same horizontal coordinate $Q^2 = |q^2| \in [4, 30]$ GeV². In order to show the high-twist contributions (mainly from the chiral-enhanced twist-three LCDAs), we also depict the leading twist contributions (denoted “T2”). We find that (a) for the timelike pion form factor, the chiral enhancements decrease quickly with the growing of momentum transfers, and the leading twist

contribution begins to dominate when Q^2 approaches dozens of GeV², (b) for the spacelike kaon form factor, the leading twist contribution begins to be comparable to the high-twist contribution at $Q^2 \sim 20$ GeV², and (c) for the spacelike pion form factors and the timelike kaon form factors, the chiral enhancements from twist-three LCDAs hold to much larger momentum transfers.

These results can be understood from the factorization formalism in Eq. (11). They are written in a convolution of the Sudakov-multiplied hard amplitudes $\mathcal{H}e^{-S}$ with the soft hadron distribution functions ψ . Here, $\mathcal{H}e^{-S}$ favors a small longitudinal fraction $u_i \sim \mathcal{O}(0.1)$ and a small transversal momentum (a few GeV), while ψ highlights the moderate u_i and large k_T . The result is finally a compromise between them. To clarify it quantitatively, we define the mean transversal momentum of the electromagnetic form factor as

$$|k_{iT}| \equiv \left[\sum_{t_1, t_2} \int du_i dk_{iT} \frac{k_{iT}^2 \psi^{(t_1)} \mathcal{H}^{(t_1 t_2)} e^{-S} \psi^{(t_2)}}{\psi^{(t_1)} \mathcal{H}^{(t_1 t_2)} e^{-S} \psi^{(t_2)}} \right]^{\frac{1}{2}} \quad (12)$$

and plot the result in Fig. 5. We see that:

- (1) The mean transversal momenta of spacelike form factors consistently reside at a hard scale, whereas the mean transverse momenta of timelike form factors start to become hard at a large-momentum transfer, specifically $Q^2 \sim 10$ GeV².
- (2) In the electromagnetic interaction of pions, high-twist terms lead to an increase in $|k_T|$, subsequently enhancing the longitudinal component of the propagator’s virtuality within the hard scattering amplitude. Notably, the growth rate in the timelike interaction is much larger than that in the spacelike region, and this explains the better convergence of twist expansion of the timelike form factor, as evident in the left subgraph of Fig. 4.
- (3) In the electromagnetic interaction of kaons, high-twist contributions, conversely, reduce the mean transversal momentum of the timelike form factor by approximately 1 GeV, leading to the apparent chiral enhancement, as illustrated in the right subgraph of Fig. 4. Meanwhile, the high-twist terms cause a slight

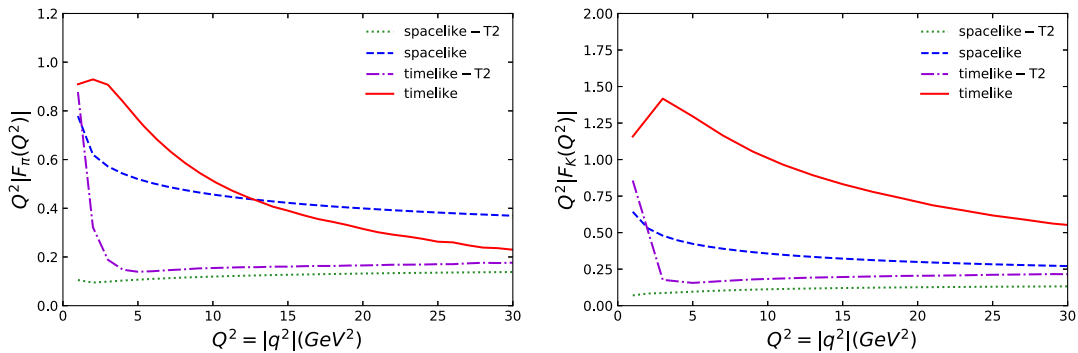


FIG. 4. The state-of-the-art pQCD predictions of pion (left) and kaon (right) electromagnetic form factors.

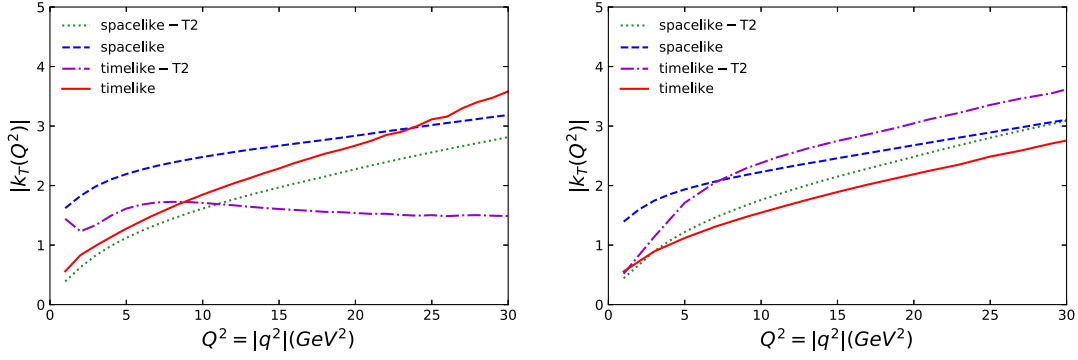


FIG. 5. The mean transversal momenta in the pion (left) and kaon (right) electromagnetic form factors.

increase in the mean transverse momentum of the spacelike form factor, which explains why the leading twist contribution is comparable to that of high twists in the spacelike kaon form factor.

From the perspective of physical imagery, the soft transversal oscillation inside pions (outside the scope of the electromagnetic potential field) is highly excited by the electromagnetic interaction after the partons move into the scope of the potential field, especially in the timelike region. The interaction energy is subsequently transferred to the spectator quark via hard gluons and, hence, the Feynman picture of exclusive reaction converts completely to the hard mechanism. For the electromagnetic interaction of kaons, the energy transfers from the struck parton to the spectator parton; the intensity of the energy transfer, however, is lower than that in the pion due to the larger mass and inertia.

Summary. In the exclusive QCD processes with large- but not infinite-momentum transfers, the hadrons are not moving as a rigid body because the partons inside are “oscillating” simultaneously in the transversal plane. In this letter, we proposed an improved factorization for an exclusive QCD process, incorporating the soft transverse degree of freedom. The hard/hard-collinear dynamics were clearly embodied in the hard scattering amplitudes and, meanwhile, the soft dynamics were absorbed into the iTMD-associated LCDAs. With the combined analysis of electromagnetic form factors from the precise

measurements, the iTMD-improved pQCD predictions, as well as the modular dispersion relation, we extracted the transversal size parameters in the Gaussian expression of iTMDs. They were $\beta_\pi^2 = 0.51 \pm 0.04 \text{ GeV}^{-2}$ and $\beta_K^2 = 0.30 \pm 0.05 \text{ GeV}^{-2}$. We observed a significant decrease in electromagnetic form factors as a result of the iTMDs effect, especially when the momentum transfer was not excessively large. The updated result of the chiral mass of pions, $m_0^\pi = 1.84 \pm 0.07 \text{ GeV}$, was consistent with the ChPT. The iTMD-improved pQCD calculation also explained the precise measurement of timelike kaon form factors far away from the resonances very well. More impressively, it improved the pQCD prediction power of electromagnetic form factors down to a few GeV^2 .

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Data availability. The data supporting this study’s findings are available within the article.

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