Letter

Fermion masses and mixings in string theory with Dirac neutrinos

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Analyzing the supersymmetric Pati-Salam landscape on a $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold in IIA string theory, we have found only two models that accurately account for all standard model fermion masses and mixings. The models are dual to each other under the exchange of two SU(2) sectors and feature 12 adjoint scalars, the maximum number allowed in the landscape, whose linear combination yields the two light Higgs eigenstates. Dirac neutrino masses in normal ordering of $(50.6, 10.6, 6.2) \pm 0.1$ meV satisfy both the experimental as well as swampland constraints.

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Introduction. Standard model (SM) fermions appear in chiral representations of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Intersecting D6-branes in type IIA string theory provide a natural mechanism to realize chiral fermions at D-brane intersections [1]. Family replication results from multiple intersections of D6-branes that fill four-dimensional spacetime and extend into three compact dimensions. The volumes of the cycles wrapped by D-branes determine the four-dimensional gauge couplings, while the total internal volume yields the gravitational coupling. Yukawa couplings arise from open world sheet instantons, specifically the triangular world sheets stretched between intersections where fields involved in the cubic coupling reside. These instanton effects are suppressed by $\exp(-A_{ijk}T)$, where A_{ijk} is the area of the triangle bounded by intersections $\{i, j, k\}$ and T is the string tension [2]. This exponential suppression explains the fermion mass hierarchies and mixings.

Intersecting D-branes model building with three families and realistic Yukawa textures naturally favors direct products of unitary gauge groups over the simple unitary groups, and the K-theory conditions [3,4], being mod 4, are more easily satisfied for $U(2N)$ with $N \in \mathbb{Z}$. Consequently, the left-right symmetric Pati-Salam group, $SU(4)_C \times SU(2)_L \times SU(2)_R$, emerges as the most promising choice for realistic models. The rules to construct supersymmetric Pati-Salam models on a $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold from intersecting D6-branes with the requirements of $\mathcal{N} = 1$ supersymmetry, tadpole cancellation, and the K-theory constraints were outlined in [5–7]. Similar construction was employed in recent works [8–13]. In Ref. [14], the complete landscape of consistent three-family supersymmetric Pati-Salam models from intersecting D6-branes on a $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold was fully mapped, comprised of 202,752 models with 33 distinct gauge-coupling relations. The viable models with realistic Yukawas split into classes of either 6, 9, or 12 adjoint scalars from the $\mathcal{N} = 2$ sector, whose linear combination yields the two light Higgs mass eigenstates.[1] The results for the Yukawa couplings and the analysis of soft terms from supersymmetry breaking for all viable models in the landscape are presented in [18,19].

Neutrino sector. Experimentally, two of the mass eigenstates, m_1 and m_2, are found to be close to each other, while

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[1]There also exists a model having three adjoint scalars from the bulk, however, tiny Yukawa couplings from the bulk Higgs are argued to be related to the infinite distance limit [15] in the moduli space, where a light of tower states, dubbed gonions [1], appears, signaling the decompactification of one or two compact dimensions [16,17]. Instead, the Yukawas originating from the $\mathcal{N} = 2$ sector are insensitive to the bulk moduli, and the issue of decompactification does not arise.

the third eigenvalue m_3 is separated from the former pair, where $m_2 > m_1$ by definition. Normal ordering (NO) refers to $m_3 \gg m_2 > m_1$, while inverted ordering (IO) refers to $m_2 > m_1 \gg m_3$ with constraints from NuFIT 6.0 (2024) [20],

$$\begin{aligned}
\Delta m^2_{21} &= 74.9 \pm 1.9\text{meV}^2, \\
\Delta m^2_{31} &= +2513 \pm 20\text{meV}^2(\text{NO}), \\
\Delta m^2_{32} &= -2484 \pm 20\text{meV}^2(\text{IO}), \\
\sum_{i=1}^{3} m_i &> 58(98)\text{meV NO(IO)}, \\
\sum_{i=1}^{3} m_i &< 72\text{meV} \qquad (1)
\end{aligned}$$

where the constraint on the sum of neutrino masses is the strongest to date from 2024 DESI BAO + CMB data [21] at the 95% C.L.

Recent insights from the swampland program, particularly from the non-SUSY AdS (non-supersymmetric anti-de Sitter) instability conjecture [22] and the light fermion conjecture [23], suggests that without additional chiral fermions with tiny masses, neutrinos must be of Dirac-type together with a bound on the lightest neutrino mass, given by the cosmological constant scale as $m_\nu^{\text{lightest}} \lesssim \Lambda^{1/4}$. The 3D Casimir energy of the SM compactified on a circle receives a positive contribution from the lightest neutrino, which is necessary to avoid unstable, nonsupersymmetric AdS vacua. This constraint is only satisfied for Dirac neutrinos, which carry 4 degrees of freedom, unlike Majorana neutrinos, which only have 2 and cannot compensate for the 4 bosonic degrees of freedom from the photon and the graviton. This also avoids the inevitable lepton number violations in the Majorana case. In Refs. [24–31], the 3D Casimir energies corresponding to the compactification of the standard model on a circle were computed, resulting in the following bounds:

$$\begin{aligned}
m_\nu^{\text{lightest}} &< 7.7(2.6) \pm 0.5 \text{ meV NO (IO) (Dirac)} \\
\sum_{i=1}^{3} m_i &= 65(105) \pm 5 \text{ meV NO (IO) (Dirac)}, \qquad (2)
\end{aligned}$$

where the last constraint on the sum of neutrino masses comes from applying the multiple-point criticality principle.[2] Since the AdS and the dS vacua are separated by infinite distance in the moduli space [33], any transition between them is of first order. The multiple-point criticality principle thus requires the 3D dS vacuum to be close to the flat vacuum [27].

[2]In analogy with the first-order transition between ice and water, the slush exists at 0°C. Conversely, if the temperature happens to be close to zero, it is because of the existence of such a slush [32].

Henceforth, it is crucial in string theory to generate tiny Dirac Yukawa couplings while keeping the other Yukawa couplings and SM gauge couplings unsuppressed. Previous efforts to generate tiny neutrino masses have focused on Euclidean D2-brane instantons within local models without realistic Yukawa textures [34–36]. See Ref. [17] for a recent survey on this issue.

The Pati-Salam model. In this letter, we present the only two models in the supersymmetric Pati-Salam landscape from intersecting D6-branes on a $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold that accurately accommodate all standard model fermion masses and mixings, while also providing a unique prediction for the Dirac neutrino masses. This necessitates the inclusion of at least 12 adjoint scalars from the $\mathcal{N} = 2$ sector, which is the maximum available in the landscape [18]. The two light Higgs eigenstates arise from the linear combination of the vacuum expectation values (VEVs) $v^i_{u,d} = \langle H^i_{u,d} \rangle$ of the 12 adjoint Higgs present in the model.

Majorana neutrino masses can always be added via the type-I seesaw mechanism, taking the Dirac neutrino mass matrix as an input [37], whereby the right-handed neutrino masses can be generated via the stringy instanton effects [38–40]. To evade the AdS vacua in the case of Majorana neutrinos, the model has nine SM singlet chiral supermultiplets from the $\text{SU}(2)_L$ and $\text{SU}(2)_R$ antisymmetric representations, which can play roles similar to the sterile neutrinos [26]. Here, we only focus on the minimal simplest case with tiny Dirac neutrinos.

Table I displays the intersection numbers among the three D6-brane sectors (a, b, c) and an O6-plane sector (3) in the model. The dual model is constructed by exchanging the two SU(2) stacks b and c. Pati-Salam gauge symmetry $\text{SU}(4)_C \times \text{SU}(2)_L \times \text{SU}(2)_R$ is Higgsed down to the SM gauge group $\text{SU}(3)_C \times \text{U}(2)_L \times \text{U}(1)_{I3R} \times \text{U}(1)_{B-L}$ by assigning VEVs to the adjoint scalars, which arise as open-string moduli associated with the stacks a and c. Moreover, the $\text{U}(1)_{I_{3R}} \times \text{U}(1)_{B-L}$ gauge symmetry may be broken to $\text{U}(1)_Y$ by giving VEVs to the vectorlike particles with the quantum numbers $(1, 1, 1/2, -1)$ and $(1, 1, -1/2, 1)$ under the $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_{I_{3R}} \times \text{U}(1)_{B-L}$ gauge symmetry [5,41,42]. This brane splitting results in SM quarks and leptons as

$$\begin{aligned}
F_L(Q_L, L_L) &\to Q_L + L, \\
F_R(Q_R, L_R) &\to U_R + D_R + E_R + N_R. \qquad (3)
\end{aligned}$$

Similar to Refs. [40,43], we can decouple the additional exotic particles.

Yukawa couplings. Yukawa couplings arise from open-string world sheet instantons that connect three D-brane intersections [2]. Three-point couplings for the fermions can be read from the following superpotential:

TABLE I. D6-brane configurations and intersection numbers of Model 22, and its minimal supersymmetric standard model gauge coupling relation is $g_a^2 = \frac{5}{6} g_b^2 = \frac{11}{6} g_c^2 = \frac{11}{8} (\frac{5}{3} g_Y^2) = \frac{8}{7\sqrt{3}} \sqrt[4]{25}^{3/4} \pi e^{\phi_4}$.

Model 22		$SU(4)_C \times SU(2)_L \times SU(2)_R \times USp(2)$							
Stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\square\square}$	$n_{\square\atop\square}$	b	b'	c	c'	3
a	8	$(1,-1) \times (1,0) \times (1,1)$	0	0	3	0	−3	0	0
b	4	$(-2,5) \times (0,1) \times (-1,1)$	3	−3	⋯	⋯	0	−8	2
c	4	$(2,1) \times (1,1) \times (1,-1)$	2	6	⋯	⋯	⋯	⋯	2
3	2	$(0,-1) \times (1,0) \times (0,2)$							

$$\mathcal{W}_3 \sim Y^u_{ijk} Q_i U^c_j H^u_k + Y^\nu_{ijk} L_i N^c_j H^u_k + Y^d_{ijk} Q_i D^c_j H^d_k + Y^e_{ijk} L_i E^c_j H^d_k. \tag{4}$$

Yukawa couplings for D6-branes wrapping a compact $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$ space are

$$Y_{ijk} \sim \prod_{r=1}^{3} \vartheta \begin{bmatrix} \delta^{(r)} \\ \phi^{(r)} \end{bmatrix} (\kappa^{(r)}), \tag{5}$$

with $r = 1, 2, 3$ denoting the three 2-tori, and the arguments of the ϑ function are

$$\delta^{(r)} = \frac{i^{(r)}}{I^{(r)}_{ab}} + \frac{j^{(r)}}{I^{(r)}_{ca}} + \frac{k^{(r)}}{I^{(r)}_{bc}} + \epsilon^{(r)} + \frac{s^{(r)}}{d^{(r)}},$$
$$\phi^{(r)} = 0, \qquad \kappa^{(r)} = \frac{J^{(r)}}{\alpha'} \frac{|I^{(r)}_{ab} I^{(r)}_{bc} I^{(r)}_{ca}|}{(d^{(r)})^2}, \tag{6}$$

where $d^{(r)} = g.c.d.(I^{(r)}_{ab}, I^{(r)}_{bc}, I^{(r)}_{ca})$, $\epsilon^{(r)}$ is a total shift that can be absorbed due to reparametrization, $s^{(r)} \equiv s^{(r)}(i,j,k) \in \mathbb{Z}$ is a linear function on the indices i, j, and k, and J is the complex Kähler structure of the compact space $\mathbb{T}^2$ [2].

The diagonal mass matrices for up quarks, down quarks, charged leptons, and the neutrinos with $m_t/m_b = 41.2551$ and $m_\tau/m_b = 0.0424798$ from particle data group [44] are

$$D_u = m_t \begin{pmatrix} 0.0000125167 & 0 & 0 \\ 0 & 0.00737672 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{7}$$

$$D_d = m_b \begin{pmatrix} 0.0011236 & 0 & 0 \\ 0 & 0.0223524 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

$$D_e = m_\tau \begin{pmatrix} 0.00287574 & 0 & 0 \\ 0 & 0.594612 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{9}$$

$$D_\nu = m_\nu \begin{pmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{pmatrix}, \tag{10}$$

where we have parametrized the neutrino masses as (m_3, m_2, m_1) up to an overall constant m_ν. Employing the quark mixing matrix, V_{CKM}, from UTfit (2023) [45] and the lepton mixing matrix, U_{PMNS}, from NuFIT, we express the up quark matrix and the charged lepton matrix in the mixed form as [11]

$$M_u = V^\dagger_{\mathrm{CKM}} D_u V_{\mathrm{CKM}} = m_t \begin{pmatrix} 0.000458944 & 0.0019479 e^{i0.0716746} & 0.00863679 e^{i0.387353} \\ 0.0019479 e^{-i0.0716746} & 0.00868911 & 0.0414746 e^{-i0.0181756} \\ 0.00863679 e^{-i0.387353} & 0.0414746 e^{i0.0181756} & 0.99824 \end{pmatrix}, \tag{11}$$

$$M_e = U_{\mathrm{PMNS}} D_e U^\dagger_{\mathrm{PMNS}} = m_\tau \begin{pmatrix} 0.287128 & 0.221395 e^{i0.37897} & 0.249054 e^{i2.62527} \\ 0.221395 e^{-i0.37897} & 0.553552 & 0.315556 e^{i0.109961} \\ 0.249054 e^{-i2.62527} & 0.315556 e^{-i0.109961} & 0.756807 \end{pmatrix}. \tag{12}$$

To explain all SM fermion masses and mixings, we only need to fit $\{M_u, D_d, D_\nu, M_e\}$ by solving for the 24 Higgs VEVs and arguments $\{\kappa^{(1)}, \epsilon^{(1)}_{3u}, \epsilon^{(1)}_{3d}, \epsilon^{(1)}_{3\nu}, \epsilon^{(1)}_{3e}\}$ of the theta function that fix all Yukawa couplings.

Three-point mass matrices. From Table I, the relevant intersection numbers are

$$I^{(1)}_{ab} = 3, \qquad I^{(1)}_{bc} = -12, \qquad I^{(1)}_{ca} = -3,$$
$$I^{(1)}_{bb'} = -20, \qquad I^{(1)}_{cc'} = 4, \qquad I^{(1)}_{bc'} = -8. \tag{13}$$

Three-point Yukawa couplings arise from the triplet intersections from the branes a, b, c on the first two-torus ($r = 1$) with 12 pairs of Higgs from the $\mathcal{N} = 2$ sector. Yukawa matrices for the Model 22 are of rank 3, and the three intersections required to form the disk diagrams for the Yukawa couplings all occur on the first torus. The other two-tori only contribute an overall constant that has no effect in computing the fermion mass ratios. Thus, it is sufficient for our purpose to only focus on the first torus. The characteristics and the argument of the modular theta function, as defined in (6), become

$$\delta^{(1)} = \frac{i^{(1)}}{3} - \frac{j^{(1)}}{3} - \frac{k^{(1)}}{12} + \frac{s^{(1)}}{3},$$
$$\phi^{(1)} = 0, \qquad \kappa^{(1)} = \frac{12J^{(1)}}{\alpha'}, \tag{14}$$

where $i = \{0, \ldots, 2\}$, $j = \{0, \ldots, 2\}$, and $k = \{0, \ldots, 11\}$, which, respectively, index the left-handed fermions, the right-handed fermions, and the Higgs fields.

The selection rule for the occurrence of a trilinear Yukawa coupling for a given set of indices is

$$i^{(1)} + j^{(1)} + k^{(1)} = 0 \mod 3. \tag{15}$$

Then, the rank-3 mass matrix for the fermions can be determined by taking shift $s^{(1)} = j$ in (14),

$$Z_3 = \begin{pmatrix} T_0v_1 + T_9v_4 + T_6v_7 + T_3v_{10} & T_{10}v_3 + T_7v_6 + T_4v_9 + T_1v_{12} & T_{11}v_2 + T_8v_5 + T_5v_8 + T_2v_{11} \\ T_2v_3 + T_{11}v_6 + T_8v_9 + T_5v_{12} & T_3v_2 + T_0v_5 + T_9v_8 + T_6v_{11} & T_4v_1 + T_1v_4 + T_{10}v_7 + T_7v_{10} \\ T_7v_2 + T_4v_5 + T_1v_8 + T_{10}v_{11} & T_8v_1 + T_5v_4 + T_2v_7 + T_{11}v_{10} & T_6v_3 + T_3v_6 + T_0v_9 + T_9v_{12} \end{pmatrix}, \tag{16}$$

where $v_i = \langle H_i \rangle$, and the three-point coupling functions are given in terms of the Jacobi theta function,

$$T_k \equiv \vartheta \begin{bmatrix} \epsilon^{(1)}_3 + \frac{k}{12} \\ 0 \end{bmatrix} (\kappa^{(1)}), \qquad k = 0, \ldots, 11. \tag{17}$$

The coupling functions (17) in the four mass matrices $\{Z_{3u}, Z_{3d}, Z_{3\nu}, Z_{3e}\}$ are all fixed by setting

$$\epsilon^{(1)}_{3u} = \epsilon^{(1)}_{3d} = 0, \qquad \epsilon^{(1)}_{3e} = \epsilon^{(1)}_{3\nu} = \frac{1}{2}, \qquad \kappa^{(1)} = 66. \tag{18}$$

Since the modular theta function is quasi-doubly periodic, the choice of 0 or 1/2 for $\epsilon^{(1)}$'s is natural for stabilizing the open-string moduli, while $\kappa^{(1)}$ is fixed by the experimental constraints (1).

The 12 up-type Higgs VEVs $v^u_i = v^\nu_i$ are then determined by fitting the nine entries in M_u and three off-diagonal zeros in D_ν. Similarly, nine out of 12 down-type Higgs VEVs $v^d_i = v^e_i$ are determined by D_d, and three can either be determined by the diagonal entries in D_e (if we ignore the leptons' mixings) or the diagonal entries in M_e (if we take account of the leptons' mixings), and the remaining off-diagonal entries can be accounted for by adding the four-point interactions to get

$$\begin{aligned}
v^u_1 &= 0.000458945e^{i0.000033} & v^d_1 &= 0.0000272353 \\
v^u_2 &= 0.000115601e^{i\pi} & v^d_2 &= 0.0000103508e^{i\pi} \\
v^u_3 &= 1.61517e^{-i0.0274002} & v^d_3 &= 0.000779274 \\
v^u_4 &= 6.11128 \times 10^{-6}e^{-i\pi} & v^d_4 &= 3.93336 \times 10^{-6}e^{i\pi} \\
v^u_5 &= 0.00868911 & v^d_5 &= 0.000541809 \\
v^u_6 &= 0.0133327e^{i3.05317} & v^d_6 &= 0.0000103675e^{i\pi} \\
v^u_7 &= 13.1569e^{i0.0181749} & v^d_7 &= 0.000295652 \\
v^u_8 &= 0.027537e^{i\pi/2} & v^d_8 &= 0.0000103508e^{i\pi} \\
v^u_9 &= 0.99824 & v^d_9 &= 0.0242394 \\
v^u_{10} &= 0.00636231e^{-i\pi/2} & v^d_{10} &= 3.93336 \times 10^{-6}e^{i\pi} \\
v^u_{11} &= 2.74778e^{-i0.386158} & v^d_{11} &= 0.000778019 \\
v^u_{12} &= 0.0132806e^{i\pi} & v^d_{12} &= 0.0000103675e^{i\pi}
\end{aligned} \tag{19}$$

$$\Rightarrow \quad Z_{3u} = M_u, \qquad Z_{3d} = D_d,$$
$$|Z_{3e}| = m_\tau \begin{pmatrix} 0.287128 & 0.071824 & 0.000723162 \\ 0.071824 & 0.755588 & 0.000821766 \\ 0.000723162 & 0.000821766 & 0.756807 \end{pmatrix},$$
$$|Z_{3\nu}| = m_\nu \begin{pmatrix} 13.1569 & 0 & 0 \\ 0.0003 & 2.74778 & 0 \\ 0.0065 & 0.0015 & 1.61517 \end{pmatrix}. \tag{20}$$

Hence, the quark sector is matched exactly, thereby explaining all quark masses and mixings. For the charged leptons, we present an approximate fitting for the mixed form of the leptons' matrix $|Z_{3e}|$, which will be supplemented with the four-point contribution to account for leptons' mixings later.

The diagonal neutrino mass matrix $|Z_{3\nu}| \sim m_\nu(T_6^\nu v_7^u, T_6^\nu v_{11}^u, T_6^\nu v_3^u)$ predicts neutrinos to be in normal ordering with coupling $T_6^\nu = 1$ up to an overall scale m_ν, to be fixed by experimental constraints. $T_6^\nu = 1$ is a nice feature, as it avoids extra fine-tuning, given that the neutrinos are already several orders of magnitude lighter than their quark and lepton counterparts. The experimental constraints (1) for the NO are satisfied by setting $m_\nu = 3.848$ meV,

$$\Rightarrow (m_3, m_2, m_1) = (50.6, 10.6, 6.2) \pm 0.1 \text{ meV},$$
$$\Delta m_{21}^2 = 73.2 \text{ meV}^2, \qquad \Delta m_{31}^2 = +2525 \text{ meV}^2,$$
$$\sum_{i=1}^{3} m_i = 67.4 \text{ meV}, \qquad \text{(Dirac with NO)}. \tag{21}$$

The prediction of Dirac neutrino masses is robust, as the ratios of neutrino masses are essentially determined by the up quark matrix (7), which serves as an input into the up quark mixing matrix (11) given that the Cabibbo–Kobayashi–Maskawa (CKM) matrix is now known with high precision. Although the uncertainties in (7) can be significant since the unification scale is not known precisely, the experimental constraints (1) can mitigate these uncertainties. Consequently, the uncertainties in (7) translate into the uncertainty in the Kähler modulus $\kappa^{(1)} = 66 \pm 2$, while the overall uncertainty in neutrino masses remains within ± 0.1 meV.

Comparing the results from (21), our universe avoids AdS vacua in 3D, as the mass of the lightest neutrino turns out to be less than the threshold value of 7.7 meV and the sum of the masses of three Dirac neutrinos also falls within the range given by the multiple-point criticality principle (2).

Leptons' mixings from four-point functions. The four-point couplings in Model 22 can come from considering interactions of a, b, c with b' or c' on the first two-torus, as can be seen from the intersection numbers (13). There are 20 SM singlet fields S_L^i and 8 Higgs-like states $H'_{u,d}$. We consider four-point interactions with b', with the shifts $l = \frac{k}{4}$ and $\ell = \frac{k}{3}$ taken along the index k and using values from (13) [18],

$$\delta = \frac{i}{I_{ab}^{(1)}} + \frac{j}{I_{ca}^{(1)}} + \frac{k}{I_{bc}^{(1)}} + l, \qquad = \frac{i}{3} - \frac{j}{3}, \tag{22}$$

$$d = \frac{\iota}{I_{bb'}^{(1)}} + \frac{J}{I_{bc'}^{(1)}} + \frac{k}{I_{bc}^{(1)}} + \ell, \qquad = -\frac{\iota}{20} - \frac{J}{8}. \tag{23}$$

The matrix elements $a_{i,j,\iota}$ on the first torus from the four-point functions result in the classical four-point contribution to the mass matrix with VEVs u_ι, w_J [18] and the four-point couplings given by

$$F_i \equiv \vartheta \begin{bmatrix} \epsilon_4^{(1)} + \frac{i}{20} \\ 0 \end{bmatrix} (\kappa^{(1)}), \qquad i = 0, \ldots, 19. \tag{24}$$

Since we have already fitted the up quark matrix precisely, thus we set all up-type VEVs u_i^u and w_j^u to be zero. Thus, we are essentially concerned with fitting the charged lepton mixing matrix such that the corresponding corrections for the down-type quarks remain negligible. The desired solution can be readily obtained by setting $\epsilon_{4d}^{(1)} = 1/2$ and $\epsilon_{4e}^{(1)} = 0$ with only considering the following nonzero VEVs:

$$u_3^d = 0.00054705, \qquad u_4^d = 0.000429559,$$
$$u_5^d = 0.01716, \qquad w_8^d = 1,$$
$$F_1^e = F_{19}^e = 0.595495, \qquad F_{17}^e = 0.00941675$$
$$\Rightarrow Z_{4e} = m_\tau w_8^d \begin{pmatrix} 0 & u_5^d F_{17}^e & u_4^d F_{19}^e \\ u_5^d F_{17}^e & u_4^d F_{19}^e & u_3^d F_1^e \\ u_4^d F_{19}^e & u_3^d F_1^e & 0 \end{pmatrix}, \tag{25}$$

which yields the following four-point contribution to be added to the three-point functions $\{Z_{3d}, Z_{3e}\}$ (20):

$$Z_{4e} = m_\tau \begin{pmatrix} 0 & 0.156933 & 0.248425 \\ 0.156933 & 0.248425 & 0.316373 \\ 0.248425 & 0.316373 & 0 \end{pmatrix}$$
$$\Rightarrow Z_{3e} + Z_{4e} = M_e, \qquad Z_{4d} = 0. \tag{26}$$

Conclusion. We have achieved the precise matching of all fermion masses and mixings from three-point couplings alone, whereas the four-point couplings were only needed to account for the leptons' mixings. This constitutes the first precise prediction of Dirac neutrino masses from a consistent string theory setup. The Dirac masses of neutrinos were derived by three-point functions, whereas the leptons' mixing needed four-point functions, which were suppressed by the string-scale M_S. This is quite satisfactory because the four-point couplings only affected the tiny neutrinos, and all other heavier fermions were unaffected by such interactions. An experimental confirmation of the heaviest neutrino mass at ∼50 meV would thus validate the model.

The higher-dimensional four-point operators $\mathcal{W}_4 \supset \frac{1}{M_S}(Y'^d_{ijkl} Q_i D^c_j H'^d_k S^L_l + Y'^e_{ijkl} L_i E^c_j H'^d_k S^L_l)$ needed to explain neutrino mixings can be related to the dark-dimension scenario [46] motivated by the emergent strings conjecture [15]. Dark dimension relates dark matter (5D gravitons), dark energy (Λ), and the axion decay constant ($f_a \lesssim \hat{M}_5$) with the scale of the lightest neutrino (m_1). Taking $m_1 = 6.2$ meV in the relations $\hat{M}_5 = m_1^{1/3} M_{\rm pl}^{2/3}$ and $m_1 = \lambda^{-1}\Lambda^{1/4}$, the species scale in 5D was set at $\hat{M}_5 = 9.74 \times 10^8$ GeV, resulting in the size and the thickness of the dark dimension to be 31.8 μm and 2.0×10^{-23} cm, respectively. No deviations in the gravitational inverse-square law were detected above 38.6 μm at 2σ [47], however, this is to be probed in the near future.

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