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Can plasma physics establish a significant bound on long-range dark matter interactions?

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Dark matter has been theorized to be charged under its own "dark electromagnetism" (dark-EM). Under this hypothesis, dark matter can behave like a cold collisionless plasma of self-interacting dark matter particles and exhibit plasmalike instabilities with observational consequences. Using the results published in [Phys. Rev. E **105**, 035204 (2022)], which studied the degree of slowdown between two interpenetrating e^-e^+ plasma clouds due to plasma instabilities, estimates of similar interactions for colliding "dark plasmas" are explored. Comparison with astronomical observations reveals strong new constraints on dark-EM with the dark electromagnetic self-interaction $\alpha_D < 4 \times 10^{-25}$.

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Introduction. The nature of dark matter, which comprises more than 80% of the mass of the Universe, remains a mystery [1,2]. Having been observed only through its gravitational interactions, we know very little about the underlying particle physics of dark matter [3–7]. A huge array of theoretical possibilities remains open, with a range of potential masses spanning 50 orders of magnitude from $\sim 10^{-22}$ eV to $\sim 10^{19}$ GeV. Dark matter could have a wide variety of possible nongravitational interactions both with itself and with other particles, although all we can currently say is that they must be weak enough to have evaded observation [8,9]. Discovering such interactions would be a huge leap forward in understanding dark matter and is one of the biggest goals of modern particle physics.

Interestingly, as we explore in this paper, the most minimal type of dark matter interaction can have rich astrophysical consequences over the entire allowed mass range. An unbroken U(1) gauge force, similar to electromagnetism but acting only between dark matter particles, would mediate long-range dark-matter self-interactions. We refer to such a force as *dark electromagnetism* (dark-EM) [10,11]. This is a very natural possibility that arises in a wide range of underlying dark matter models [6,12–20]. The stability of dark matter constrains these models as the conservation of dark-EM charge could forbid dark matter decay. In general, constraints on this scenario or observable consequences are of broad interest and merit thorough study. Assuming the interactions are weak enough not to bind dark matter particles together, dark matter would then consist of a net-neutral plasma of dark-EM charged particles. In this paper, we investigate the effects this would have on halo dynamics.

The dynamics of dark matter halos is the natural place to observe dark matter self-interactions. A recent observation of the Bullet Cluster (1E 0657-558), where a collision between a subcluster and the main cluster confirmed that the dark matter halo of the subcluster passes through the main cluster with no visible offset between the stars and the DM, thus indicating weak self-interactions. The seemingly clean passage of one halo through another in the Bullet Cluster led to well-known bounds being placed on $2 \to 2$ hard scattering (i.e., short-range interactions) [21,22]. Dark-EM also gives rise to conceptually different effects, which result in far stronger bounds, due to the collective dynamics of the dark-matter plasma. The dark-EM interaction would act as a $N \to N$ rather than a $2 \to 2$ scattering process, where N is extremely large. The bounds from $2 \to 2$ scattering require that a typical dark matter particle has never undergone a hard scattering [12,23]. This indicates that the dark-matter plasma is in the "collisionless" regime [24,25]. Under this hypothesis, we shall see whether the most minimal type of dark matter self-interaction via a new long-range force analogous to the electromagnetic interaction in the standard model could have an impact on DM self-interactions. This possibility

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raises the question of whether plasmalike collisionless instabilities may have a significant impact on galaxy and cluster dynamics. Resolving this will determine whether or not such an interaction is consistent with current observations, and the impact of plasmalike collisionless instabilities on galactic dynamics.

In this work, we consider the simple possibility of an electromagnetic self-interaction between two collisionless dark matter plasma slabs. The equations that govern DM self-interactions are identical to that of a collisionless e^-, e^+ plasma. Following the work done in reference [10], we analyze simulations of interacting equal mass nonrelativistic plasmas from Ref. [26] to draw our conclusions. Collisionless plasma dynamics is both a well-studied field and an area of active research with rich dynamics that are still not fully understood. It is known that two counterpropagating plasmas are subjected to several microinstabilities that generate growth of electromagnetic fields, involving transverse and parallel modes. The full unstable wave number k spectrum has been intensively studied in the cold plasma limit [27–30]. Three main dominating instabilities exist with different wave vectors with respect to the flow. First, the two-stream instability, which has a wave vector aligned with the flow, is driven by the two-peaked nature of the velocity distribution [31]. Second, anisotropy in the velocity spread in different directions (larger along the flow direction) excites the Weibel/current filamentation instability with a wave vector normal to the flow [32–36]. Finally, a hybrid of these two modes, with a wave vector with an angle oblique to the flow, is known as the oblique instability [27,29].

Plasma instabilities are well understood analytically in the linearized regime with small perturbations to an infinite homogeneous plasma. Furthermore, theoretical estimates of the linear growth of electromagnetic field generated via plasma instabilities, complemented by numerical simulations, have been well studied during the interaction of two plasma slabs [37,38]. The electromagnetic fields driven by these plasma instabilities will lead to bulk slowdown of the counterpropagating plasma slabs as long as there is sufficient time for the instabilities to grow. However, the exact time required before a significant slowdown occurs depends on nonlinear effects not included in such an analysis and can only be captured via numerical simulations, as performed in this paper. Our studies, therefore, can place a limit on the interaction strength by considering the full nonlinear dynamics associated with these instabilities.

Parametric estimate of bounds on dark-em. Based on our simulation findings and astrophysical observations such as the Bullet Cluster [21], Abell 520 [39], and other galaxy cluster collisions [40], we place a very strong constraint on the parameter space (α_D, m_D) for DM particles. Here $\alpha_D \equiv e_D^2/\hbar c$ and m_D are the respective dark electromagnetic coupling constant and mass, and e_D is the dark electromagnetic charge of the particle.

Our numerical simulations reported in Ref. [26] showed that the self-interaction of two cold $(v_{\rm fl} \geq v_{\rm th})$ plasma clouds leads to the generation of the Weibel and oblique instabilities, which deflect particle trajectories such that the particles acquire transverse momentum while losing forward momentum. We showed that for a typical plasma slab length larger than $L \approx 10 v_{\rm fl}/\Gamma_{\rm W}$, where $\Gamma_{\rm W}$ is the Weibel growth rate, a significant slowdown occurs. In the case of a hot $(v_{\rm fl} \approx v_{\rm th})$ plasma cloud, we observed a slowdown caused solely by the Weibel instability.

When it comes to a dark matter plasma, this result implies that for sufficiently large $L\omega_{pD}/c$ the DM slab slows down due to the Weibel and oblique instabilities, where $\omega_{pD} = \sqrt{4\pi\rho_D\alpha_D\hbar c/m_D^2}$ is the dark plasma frequency. For simplicity, we have considered symmetric dark matter slabs, but in principle similar results should occur as long as the less dense slab (defining ω_{pD}) passes through a slab with significant $L\omega_{pD}/c$. In any case, the slowdown of the two counterpropagating DM plasma slabs contradicts observations unless the interaction is sufficiently weak. Given some conservative estimates for $L \approx 100$ kpc, $v_{\rm fl} \approx 0.1c$, $m_D \approx 1$ TeV, and the mass density $\rho_D \approx 0.01$ GeV/cm^3 [8,10,21], we can recast our limit of a maximum L to avoid slowdown to a limit on the coupling strength $(\alpha_D \ll 1)$.

First, we estimate the coupling constant based on the Weibel growth rate $\Gamma_{\rm W} \sim \omega_{pD} v_{fl}/c$. We found a significant slowdown occurs if the length $L > 10\Gamma_{\rm W}^{-1} v_{\rm fl}$, or a collision without significant slowdown requires

$$\frac{L\Gamma_{\rm W}}{10 v_{\rm fl}} = \frac{L}{10 v_{\rm fl}} \omega_{pD} \frac{v_{\rm fl}}{c} < 1 \tag{1}$$

Recast in terms of α_D this is equivalent to the constraint

$$\alpha_D < L^{-2} \rho_D^{-1} m_D^2 \left(\frac{100c}{4\pi\hbar}\right) \tag{2}$$

Using our estimates for the parameters, we find a quantitative limit on α_D

$$\alpha_D < 4.2355 \times 10^{-25} \left(\frac{L}{100\ \text{kpc}}\right)^{-2} \left(\frac{\rho_D}{0.01\ \text{GeV/cm}^3}\right)^{-1} \times \left(\frac{m_D}{1\ \text{TeV}}\right)^2 \tag{3}$$

Similarly, we can estimate the coupling constant due to the two-stream/oblique instability, with growth rate $\Gamma_{\rm TS} \sim \omega_{pD}$. The limit on L for the two-stream instability is

$$\frac{L\Gamma_{\rm TS}}{10 v_{\rm fl}} = \frac{L}{10 v_{\rm fl}} \omega_{pD} < 1, \tag{4}$$

which recast in terms of α_D is

$$\alpha_D < L^{-2}\frac{v_{\rm fl}^2}{c^2}\rho_D^{-1}m_D^2\left(\frac{100c}{4\pi\hbar}\right) \tag{5}$$

leading to the constraint

$$\alpha_D < 4.2355\times10^{-27}\left(\frac{L}{100\ \text{kpc}}\right)^{-2}\left(\frac{\rho_D}{0.01\ \text{GeV/cm}^3}\right)^{-1} \times\left(\frac{m_D}{1\ \text{TeV}}\right)^2\left(\frac{v_{\rm fl}}{0.1\ c}\right)^2 \tag{6}$$

The previous bound on α_D that was established assuming a slowdown caused by Coulomb scattering [39] is, in contrast, much weaker. For this limit the collision frequency

$$\nu = \omega_{pD}^4\frac{m_D}{\rho_D}\frac{\ln(\Lambda_C)}{2\pi v_{\rm fl}^3} \tag{7}$$

where $\ln(\Lambda_C)$ is the Coulomb logarithm, is comparable to the crossing time; $\nu L/v_{\rm fl}=1$, and

$$\alpha_D < 1.9454\times10^{4}\left(\frac{L}{100\ \text{kpc}}\right)^{-1/2}\left(\frac{\rho_D}{0.01\ \text{GeV/cm}^3}\right)^{-1/2} \times\left(\frac{m_D}{1\ \text{TeV}}\right)^{3/2}\left(\frac{v_{\rm fl}}{0.1c}\right)^2\left(\frac{\ln(\Lambda_C)}{35}\right)^{-1/2}. \tag{8}$$

Figure 1 shows the two limits from Eqs. (3), (6) in contrast to Eq. (8). In the blue region, a slowdown due to Coulomb scattering is expected. In the green region, a slowdown due to the Weibel and two-stream/oblique instabilities is expected. In the red region, a moderate slowdown due to the two-stream/oblique instability is expected if the plasma is cold ($v_{\rm th}\ll v_{\rm fl}$). Only the white region (and in some cases the red regime) is consistent with observations, where no slowdown is expected.

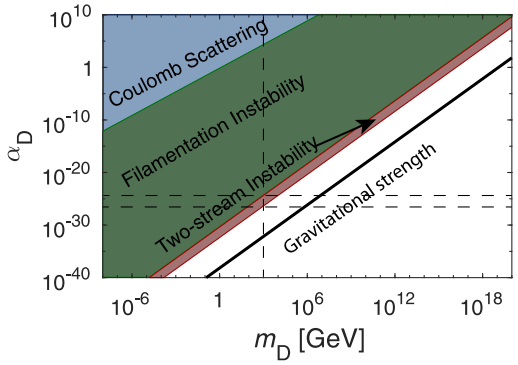


FIG. 1. Constraints on the dark electromagnetic coupling constant α_D as a function of m_D based on the observations from the Bullet Cluster, assuming $\rho_D = 0.01\ \text{GeV/cm}^3$, $L = 100$ kpc, $v_{\rm fl} = 0.1c$, and $\ln(\Lambda_C) = 35$. α_D must lie below the region in blue due to the expected slowdown from Coulomb scattering (Eq. (8), below the region in green from the current filamentation instability [Eq. (3)], and below the region in red from the two-stream/oblique instability [Eq. (6)]. For reference, the dark electromagnetic coupling forces will be equal to the gravitational forces along the bottom black line [Eq. (10)]. The reverence value used in our equations, $m_D = 1$ TeV, is highlighted with a dashed line, along with the limits based on this value from Eqs. (3) and (6).

This establishes a strong upper bound on the strength of a dark electromagnetic self-interaction. To emphasize how strong the bound on α_D this establishes, we also include a line in Fig. 1 corresponding to when the strength of the dark electromagnetic force between two dark matter particles is equal in magnitude to the gravitational force. To ignore the effects of gravity, the coupling constant needs to be sufficiently strong. The coupling constant that satisfies this constraint is

$$\alpha_D > \frac{G}{\hbar c}m_D^2, \tag{9}$$

where G is the gravitational constant. This can be expressed as

$$\alpha_D > 6.7086\times10^{-33}\left(\frac{m_D}{1\ \text{TeV}}\right)^2. \tag{10}$$

We can show that the upper bound is statistically significant by estimating an effective drag force (due to the dark electromagnetic fields) on the dark matter based on two values measured from the main simulation R_1 from Ref. [26]. We can express the estimated drag force in terms of the magnetic isotropization time $\Delta t_{\alpha_B} = 105.4\omega_{Dp}^{-1}$, and the slowdown of the flow during the collision $(v_{\rm fl}-v_{\rm init})/v_{\rm fl} = 0.8556$:

$$D_D = m_D\frac{v_{\rm fl}-v_{\rm init}}{\Delta t_{\alpha_B}}. \tag{11}$$

Equating this with the drag force from Ref. [40] $D_D = \rho_D m_D v_{\rm fl}^2\sigma/m$, we can obtain an effective dark matter cross section $\sigma/m = 5.9\ \text{cm}^2/\text{g}$. Here we have used our estimates of ρ_D and L and assumed a regime with significant slowdown $\omega_{pD} = 10c/L$. As the typical measured value for $\sigma/m = 0\pm1\ \text{cm}^2/\text{g}$ [40], the slowdown in this region is statistically inconsistent with observation.

Larger α_D particles could potentially be explained by more complex theories which include an ionization fraction, where only a select few particles have this larger interaction [39].

Conclusion. In conclusion, by comparing *ab initio* kinetic plasma simulations of the interpenetration of two e^-, e^+ plasma slabs with astronomical observations, we establish a strong upper bound on the strength of the dark electromagnetic self-interaction α_D ($\alpha_D \approx 4\times10^{-25} \ll 1$, for $m_D = 1$ TeV). This bound assumes the most basic unbroken U(1) gauge interaction of the dark plasma. As we have shown that the allowed parameter space of self-interacting DM particles, based on our analysis, exhibits significantly

stronger bounds on α_D than previously established, the electromagnetic interaction between DM is considerably restricted.

It should be noted that the limit put on the self-interaction coupling strength is based on the assumptions that the plasma slabs are step functions in density and that there is no background dark magnetic field.

The two interacting dark matter halos are, out of simplicity, modeled by a step function in density in the spatial direction. The real distributions would vary smoothly, with a representative length scale and density. Therefore, the values ρ_D and L used in Eqs. (3), (6), (8), would correspond to representative values such as the density at half maximum and the full width at half maximum of the distribution. In principle, we are thus overestimating our limit for slowdown; fields responsible for the slowdown begin growing before the density reaches half maximum, and the growth rate of the instability will always be larger when the density is larger than half maximum. This implies that for more realistic distributions, a significant slowdown would occur for a shorter (i.e., weaker interacting) halo. Nevertheless, we acknowledge that the nonlinear shock formation and slowdown of the plasma are modeled differently when using the slab geometry, leading to a degree of uncertainty on the constraint.

While a step function taking the average density and length gives a good estimate of the interaction between two dark matter halos with an arbitrary smooth distribution, one can make an even more conservative estimate on the coupling constant by only considering a small section of the interacting halos where the density gradient can be neglected. A recent complementary work just published [41] concurrent with this publication uses particle-in-cell simulations to find a limit equivalent to $\alpha_D < 7.8 \times 10^{-22}(m_D/1\ \mathrm{TeV})^2$, which is comparable to Eq. (3) if we assume L is approximately 1% smaller, so that the gradient can be neglected. The spatial-temporal effects of the current filamentation instability present when the slabs are not initially superimposed causes a weaker growth before the purely temporal growth sets in [42]. Therefore, this constraint gives a more conservative estimate on the limit of the coupling.

Finally, we note that a uniform background dark magnetic field could lead to a suppression of the instability and slowdown. Although a magnetic field perpendicular to drift does not affect the growth rate [43], a sufficiently strong magnetic field along the drift can suppress the instability [44,45]. The criteria for suppression is equivalent to the regime where the system is stable to the firehose instability [46], an instability derived in the magnetized regime also driven by the temperature anisotropy. We conjecture that a misalignment of the temperature and density gradients of the dark matter halos can lead to a large-scale dark magnetic field competing with the Weibel/current filamentation instability [47,48], with a plasma beta $\beta \sim L^2\omega_{pD}^2/c^2$. However, even in the hot regime ($v_{\mathrm{fl}} \approx v_{\mathrm{th}}$), this would be too weak to suppress the instability. Nevertheless, the limit on the dark matter coupling relies on the assumption that there are no dark currents, which can sustain a strong dark magnetic field parallel to the relative drift between the two DM halos, and thus no background field is considered.

Although we have had several fruitful discussions with experts in dark matter, the authors are primarily experts in the plasma physics aspects of this paper. We hope that our results may be useful for the dark matter community and that the constraints found in this paper can be further verified statistically using observational measurements and uncertainties.

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Data availability. Data availability is subject to the authors of the respective Refs. [21,22], and [40]. are not publicly available because they are owned by a third party and the terms of use prevent public distribution. Data from our previous work [26] are not publicly available. The data are available from the authors upon reasonable request.

Appendix: Particle-In-Cell (PIC) Method. Particle-in-cell (PIC) codes are simulation tools that enable the modeling of complex kinetic and relativistic plasma phenomena on the smallest timescale, i.e., electron time scales. The simulations were performed with the fully relativistic, massively parallel, PIC code OSIRIS [51–53]. In the PIC method, the relativistic Lorentz force is used to calculate the motion of the plasma particles by interpolating the grid fields to the position of the particle. Current density is deposited onto the grid and used to advance the electric and magnetic fields through Maxwell's equations discretized using a finite-difference scheme. Each particle is pushed to a new position and momentum via self-consistently calculated fields. The method is based on a multistep process and is second-order accurate in time.

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