

First-order electroweak phase transition with a gauge-invariant approach

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We study the electroweak phase transition with a three-dimensional standard model effective field theory at the two-loop level under a gauge-invariant approach. We observe that the phase transition parameters obtained with the gauge-invariant approach deviate from that obtained in the Landau gauge at around the percent level. We give the strongly first-order phase transition condition for electroweak baryogenesis and find that the predicted gravitational wave signals would be reduced for the new physics scale $\Lambda \lesssim 590$ GeV, and the parameter spaces of $\Lambda \lesssim 570$ GeV can be probed by the space-based interferometers, such as LISA, TianQin, and Taiji.

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Introduction. First-order phase transition (PT) is an important way to study the evolution of the Universe, which provides a chance to generate baryon asymmetry of the Universe through electroweak baryogenesis [1], primordial magnetic field [2–9], and one important stochastic gravitational wave (GW) source to be probed by the next-generation GW detector experiments, such as LISA [10–12]. Such a PT can be achieved in many models beyond the standard model (SM) since the SM of particle physics with the 125 GeV Higgs admits a smooth crossover between the high- and low-temperature phases [13].

The dynamical symmetry breaking process driven by the first-order PT can be quantitatively calculated by the thermal effective potential [11]. The studies of the PT dynamics are conventionally performed based on the effective potential in the Landau gauge and therefore suffer from the gauge-dependent problem due to the element fields not invariant under gauge transformation [14,15]. However, since the physical observables must be gauge independent, the gauge-invariant results from gauge-dependent theory should be achieved. The Nielsen identity ensures a gauge-independent minimal value of the effective potential obtained in Landau gauge [16]. It was proposed to maintain the gauge invariant through performing $\hbar$ expansion in the PT dynamics calculation, where the Nielsen identities guarantee the gauge invariant order by order; see studies in the Abelian Higgs model [17–22] and the SM extended by the scalar

model [23]. An alternative way to achieve gauge-invariant results is to construct a new potential at the minimum of the effective potential. This approach is to define the external source coupling to a gauge-invariant composite operator $\mathcal{L} \rightarrow \mathcal{L} - J\langle\Phi^\dagger\Phi\rangle$ [14,24–26]. Therefore, such an effective potential can be obtained after performing the Legendre transform of the ground state energy density.

Recently, the 3D thermal field theory obtained after the dimensional reduction (DR) procedure has been vastly developed for the PT study since it is suitable for the nonperturbative lattice simulation [27–31]. However, the procedure conducted by DR implicitly assumed the high-temperature expansion, which makes some obtained PT parameters suffer large uncertainties for relatively low new physics scales [22]. To reduce the uncertainty, utilizing the SM effective field theory (SMEFT), we construct the 3D thermal effective field theory with DR after improving the wave function renormalization, called the improved-DR approach. In order to estimate the uncertainty of gauge dependence, based on the improved-DR approach, we further construct the gauge-invariant thermal effective potential at two-loop order after considering that the composite operator $\langle\Phi^\dagger\Phi\rangle$ coupled to the external source, dubbed the GI approach.

In this Letter, we, for the first time, quantitatively compute the PT dynamics and strongly first-order electroweak PT (SFOEWPT) condition with the GI method [32]. We estimate the difference of PT quantities obtained within the Landau gauge and the GI method based on the DR (and improved-DR) approach and show that the strong first-order electroweak PT occurs in the new physics (NP) scale range of $\Lambda \lesssim 800$ GeV. In addition, we reveal the implications of the improved-DR approach for the PT parameters and the corresponding GW predictions.

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The improved-DR approach. Since the potential barrier for SMEFT mostly comes from the dimension-six operator of $(\Phi^\dagger \Phi)^3/\Lambda^2$ with Λ being the NP scale, we consider the SMEFT with that single dimension-six operator in this Letter. The original four-dimensional (4D) Lagrangian under study has the form of

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu \Phi)^\dagger (D_\mu \Phi) + \mu_h^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2 + c_6(\Phi^\dagger \Phi)^3, \quad (1)$$

where $c_6 = \Lambda^{-2}$ and Φ is the Higgs doublet. This model in three dimensions is described by the Lagrangian

$$\mathcal{L} = \frac{1}{4}G_{ij}^a G_{ij}^a + \frac{1}{4}F_{ij}F_{ij} + (D_i \Phi)^\dagger (D_i \Phi) + V, \quad (2)$$

where $G_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g_3 \epsilon^{abc} A_i^b A_j^c$, $F_{ij} = \partial_i B_j - \partial_j B_i$, and $D_i \Phi = (\partial_i - ig_3 \tau^a A_i^a/2 - ig_2' B_i/2)\Phi$ with τ^a being the Pauli matrices. The fields in Eq. (2) have the mass dimension $\text{GeV}^{1/2}$ and couplings are dimensional. This theory works when the gauge couplings fulfill the condition of $g < 1$. The DR theory splits the mass scale into superheavy (πT), heavy (gT), and light mode ($g^2 T$). The theory is simpler than the 4D theory in that the fermions and bosons with masses being larger than gT have been integrated out. The V is the light mode potential, which was obtained at the two-loop order after integrating out superheavy and heavy modes within the Landau gauge and has the form

$$V = V_{\text{tree}} + \hbar V_{\text{1loop}} + \hbar^2 V_{\text{2loop}}, \quad (3)$$

where

$$V_{\text{tree}} = \frac{1}{2}\mu_h^2 \phi^2 + \frac{1}{4}\lambda \phi^4 + \frac{1}{8}c_6 \phi^6, \quad (4)$$

$$V_{\text{1loop}} = (d-1)(2J_{\text{soft}}(m_W) + J_{\text{soft}}(m_Z)) + J_{\text{soft}}(m_\phi) + 3J_{\text{soft}}(m_\chi), \quad (5)$$

with

$$J_{\text{soft}}(m) = -\frac{1}{12\pi}(m^2)^{\frac{3}{2}}. \quad (6)$$

Therein, the field-dependent masses are

$$m_\phi^2 = \mu_h^2 + 3\lambda \phi^2 + \frac{15}{4}c_6 \phi^4, \quad m_W^2 = \frac{1}{4}g^2 \phi^2, \quad (7)$$

$$m_\chi^2 = \mu_h^2 + \lambda \phi^2 + \frac{3}{4}c_6 \phi^4, \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2)\phi^2. \quad (8)$$

The two-loop effective potential V_{2loop} and related parameters can be found in Ref. [41]. To reduced the uncertainties

from high-temperature expansion implicitly adopted in the DR approach, we utilize the improved-DR approach with the contribution of the field-dependent term in the Z factors of the wave function, $Z(T) \rightarrow Z(\phi, T)$ (see Supplemental Material [42]). This substitution would change the mass parameters of the Higgs (μ_h^2) and alter the matching of 3D couplings and 4D parameters, thereby reducing the uncertainty associated with high-temperature expansion [42].

GI thermal effective potential. The thermal effective potential with the GI method can be constructed by introducing a current J coupling to the composite field $\sigma = \phi^\dagger \phi$ [14,24–26],

$$e^{-W[J]} = \int \mathcal{D}A_i^a \mathcal{D}\phi e^{-S - \int d^3x J\sigma}, \quad (9)$$

then the effective potential is given by the Legendre transformation

$$V(\sigma) = V(v(T), \mu_h^2 + J) - J\sigma, \quad \sigma = \partial V(v(T), \mu_h^2 + J)/\partial J. \quad (10)$$

The composite effective potential $V(\sigma)$ is thus gauge independent since it is constructed to be the minimal value of $W[J]$ with respect to J and ϕ [14]. We calculate the effective potential at $\hbar^2$ order; this requires the current J to be calculated to first order in $\hbar$: $J = J_0 + \hbar J_1$ [28]. With the definition

$$v^2 = v_0^2 + \hbar v_1^2 + \hbar^2 v_2^2, \quad (11)$$

the effective potential at the vacuum expectation value (VEV) expands as

$$V(v) = V_{\text{tree}}(v_0^2 + \hbar v_1^2 + \hbar^2 v_2^2) + \hbar V_{\text{1loop}}(v_0^2 + \hbar v_1^2) + \hbar^2 V_{\text{2loop}}(v_0^2). \quad (12)$$

The two solutions of the condition

$$\left. \frac{dV(\phi^2)}{d(\phi^2)} \right|_{\phi^2=v^2} = 0, \quad (13)$$

$\phi_s^2 = 0$ and $\phi_b^2 = v^2$ correspond to the symmetric and broken phase, respectively. For the leading order in $\hbar$, we have

$$v_0^2 = \frac{2}{3c_6} \left(-\lambda + \sqrt{-3c_6 J_0 - 3c_6 \mu_h^2 + \lambda^2} \right). \quad (14)$$

Inserting this into Eq. (10), we can obtain the relation

$$J_0 = -\mu_h^2 - 2\lambda\sigma - 3c_6\sigma^2. \quad (15)$$

To the order of $\hbar$, we have

$$v_1^2 = -\frac{V'_{1\text{loop}}(\phi^2)}{V'_{\text{tree}}(\phi^2)}. \quad (16)$$

By defining

$$\begin{aligned} m_W &= \sqrt{\frac{1}{2}g^2\sigma}, \quad m_Z = \sqrt{\frac{1}{2}(g^2 + g'^2)\sigma}, \\ m_\phi &= \sqrt{4\lambda\sigma + 12c_6\sigma^2}, \end{aligned} \quad (17)$$

the J_1 can be simplified to

$$J_1 = \frac{1}{8\pi} [4(\lambda + 6c_6\sigma)m_\phi + 2g^2m_W + (g^2 + g'^2)m_Z]. \quad (18)$$

The two-loop GI thermal effective potential in the broken phase is the

$$\begin{aligned} V_b(\sigma) &= \mu_h^2\sigma + \lambda\sigma^2 + c_6\sigma^3 - \frac{\hbar}{12\pi} (4m_W^3 + 2m_Z^3 + m_\phi^3) \\ &\quad - \frac{\hbar^2}{64\pi^2} [(5\lambda + 27c_6\sigma)m_\phi^2 + 2g^2m_Wm_\phi \\ &\quad + (g^2 + g'^2)m_Zm_\phi] + \hbar^2 V_{2\text{loop}}(m_\chi = 0), \end{aligned} \quad (19)$$

where the $V_{2\text{loop}}(m_\chi = 0)$ is a function of σ , m_W , m_Z , and m_ϕ with $m_\chi = 0$. For the symmetric phase ($\phi = 0$), the leading order of Eq. (3) gives

$$\frac{\partial V}{\partial J} = \hbar\sqrt{\mu_h^2 + \hbar J_1} = -2\pi\sigma, \quad (20)$$

which requires $\sigma \leq 0$ at the symmetric phase. Then we can obtain the

$$J_1 = (4\pi^2\sigma^2 - \mu_h^2\hbar^2)/\hbar^3, \quad (21)$$

and the two-loop gauge-invariant potential at the symmetric phase is

$$\begin{aligned} V_s(\sigma) &= \mu_h^2\sigma + \frac{3}{16}(3g^2 + g'^2 + 8\lambda)\sigma^2 - \frac{4\pi^2\sigma^3}{3\hbar^3} \\ &\quad + \frac{1}{4}(3g^2 + g'^2)\sigma^2 \log \left[-\frac{\bar{\mu}_3\hbar}{4\pi\sigma} \right], \end{aligned} \quad (22)$$

where $\bar{\mu}_3 = g^2T$ is the light mode renormalization scale and has little effect on the result. We finally get the GI thermal effective potential for the PT dynamics study as

$$V(\sigma) = \theta(\sigma)V_b(\sigma) + \theta(-\sigma)V_s(\sigma). \quad (23)$$

Nucleation criteria. As the Universe cools to the critical temperature T_c , the broken and symmetric phase has the same free energy. With the GI thermal effective potential,

the temperature T_c can be obtained by solving

$$V_b(\sigma_b) = V_s(\sigma_s), \quad \left. \frac{\partial V_b(\sigma)}{\partial \sigma} \right|_{\sigma=\sigma_b} = 0, \quad \left. \frac{\partial V_s(\sigma)}{\partial \sigma} \right|_{\sigma=\sigma_s} = 0. \quad (24)$$

When the temperature of the Universe cools below T_c , the symmetric phase becomes unstable and will decay to the true phase. At the nucleation temperature T_n , the bubble nucleation rate $\Gamma = Ae^{-S_c}$ is equal to the Hubble rate $\Gamma \sim H$, i.e., $S_c \approx 140$. The Euclidean action is

$$S_c = \int \frac{\sigma'}{2\sigma} + V(\sigma) d^3x. \quad (25)$$

The action can be obtained by solving the bounce function

$$\frac{1}{2\sigma} \left(\frac{d^2\sigma}{d\rho^2} - \frac{1}{2\sigma} \left(\frac{d\sigma}{d\rho} \right)^2 + \frac{2}{\rho} \frac{d\sigma}{d\rho} \right) = \frac{dV(\sigma)}{d\sigma}. \quad (26)$$

After inserting $\sigma = \phi^2/2$ into Eq. (26), the bounce function becomes

$$\frac{d^2\phi}{d\rho^2} + \frac{2}{\rho} \frac{d\phi}{d\rho} = \frac{dV(\phi)}{d\phi}, \quad (27)$$

and

$$\begin{aligned} V(\phi) &= \theta(\phi - \sqrt{2|\sigma_s|})V_b(\phi^2/2 + \sigma_s) \\ &\quad + \theta(-(\phi - \sqrt{2|\sigma_s|}))V_s(\phi^2/2 + \sigma_s). \end{aligned} \quad (28)$$

We use the code BubbleDet to solve Eq. (27) and obtain the nucleation temperature T_n [43]. Then, we evaluate the inverse PT duration $\beta/H = T(dS_c/dT)$ and PT strength $\alpha = T(\Delta\rho/\rho_{\text{rad}})$ at the temperature T_n , where $\Delta\rho$ is the difference in the trace anomaly [22,41,44,45] given by

$$\Delta\rho = -\frac{3}{4}\Delta V(\phi, T) + \frac{1}{4}T \frac{d\Delta V(\phi, T)}{dT}, \quad (29)$$

with $\Delta V = V(\phi) - V_s(\sigma_s)$, and $\rho_{\text{rad}} = (\pi^2/30)g_*T^4$ is the radiation energy density with the effective number of relativistic degrees of freedom $g_* = 106.75$.

SFOEWPT condition. The electroweak (EW) sphaleron process provides the baryon number violation condition to explain the baryon asymmetry of the Universe [1,46–48]. The sphaleron process will reach equilibrium at high temperature, and it should be strongly suppressed at low temperature to avoid washout of the generated baryon number, which corresponds to the requirement of a strongly first-order electroweak PT [15], i.e., $\Gamma_{\text{sph}} \propto \exp(-E_{\text{sph}}^{\text{EW}}/T) \ll H(T)$ with $E_{\text{sph}}^{\text{EW}}$ being the electroweak sphaleron energy; this requires that the sphaleron rate in the broken phase be lower

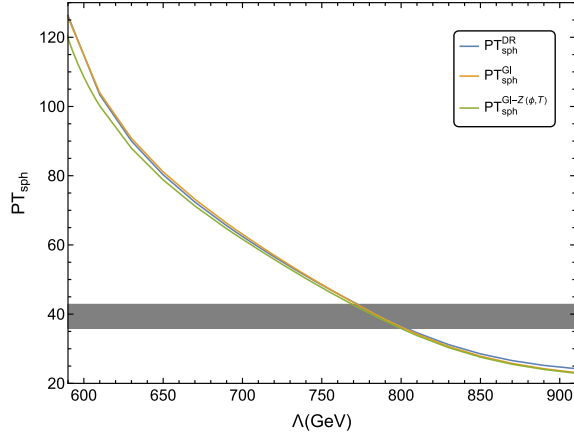


FIG. 1. The parameter PT_{sph} as function of Λ with DR and GI methods based on the DR approach (and improved-DR approach).

than the Hubble rate. After reorganizing the relation among the electroweak sphaleron energy, PT temperature (T_n), and the scalar VEV, the baryon number washout avoidance condition can be obtained to restrict the baryon asymmetry explanation parameter spaces [49,50],

$$PT_{\text{sph}} \equiv \frac{E_{\text{sph}}}{T} - 7 \ln \frac{v}{T} + \ln \frac{T_n}{100 \text{ GeV}} > (35.9 - 42.8). \quad (30)$$

In the Landau gauge 3D perturbative calculations based on DR, the parameter v represents $v/T = \sqrt{2\langle\Phi^\dagger\Phi\rangle}/\sqrt{T}$ with $\langle\Phi^\dagger\Phi\rangle = dV_{\text{eff}}/d\tilde{\mu}_{h,3}^2$ [41], where the $\tilde{\mu}_{h,3}^2$ is the light scale mass parameter ($\tilde{\mu}_{h,3}^2 \equiv \mu_h^2$). In the GI method, the v is the VEV of $V(\phi)$ in Eq. (28).

We show the limitation of the model parameter Λ by the baryon asymmetry with the baryon number violation condition [1,41,46–48] in Fig. 1. The magnitude of the parameter PT_{sph} decreases as the Λ increases, and the difference between the results computed with DR in Landau gauge ($PT_{\text{sph}}^{\text{DR}}$) and GI method ($PT_{\text{sph}}^{\text{GI}}$) is insignificant, i.e., the two approaches give the same region of SFOEWPT, i.e., $\Lambda \lesssim 770\text{--}800$ GeV. Our study shows that the GI method based on the improved-DR approach induces diminution of the electroweak sphaleron energy at low Λ , which further leads to suppression of the PT_{sph} [see the curves of $PT_{\text{sph}}^{\text{GI-Z}(\phi,T)}$]. The suppression effect decreases as the Λ increases, and finally, its effect is negligible for the SFOEWPT condition.

PT thermodynamics and GW predictions. Then, we first show the PT parameters $\alpha, \beta/H, T_n$, and v_w as function of the Λ ($\Lambda \leq 800$ GeV) with Landau gauge and GI approaches based on DR with high-temperature expansion in Fig. 2. Figure 2(a) shows that, as the Λ increases, the magnitude of the α (β/H) decreases (increases). The differences between the results of α and β/H calculated

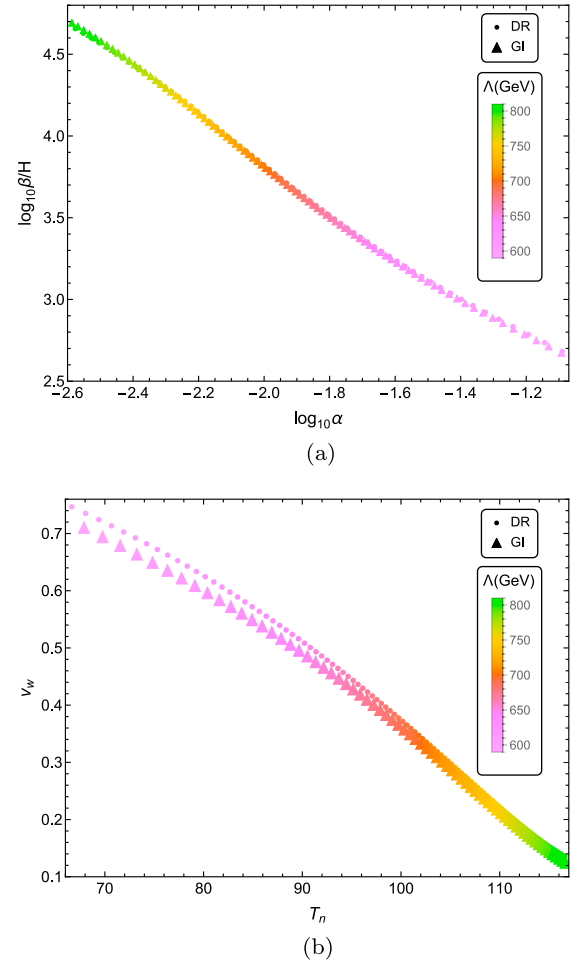


FIG. 2. The PT parameters (a) α and β/H and (b) T_n and v_w as function of Λ . The color bar shows the value of Λ . (a) The relation between the α and β/H . (b) The relation between the v_w and T_n with DR and GI indicating the calculation methods.

with the two methods are negligible. Figure 2(b) shows that the v_w (T_n) decreases (increases) as the NP physics scale Λ increases. The results of v_w computed with the GI method are less than those in the Landau gauge, and the two gradually approach each other at large Λ closing to 800 GeV. This is because $v_w \propto \sqrt{\Delta V}$ (the difference between the broken phase and symmetric phase) and the ΔV in the Landau gauge is greater than that of the GI approach. Here, we notice that, with the $\hbar$ expansion of the 3D effective potential at the minimum, Ref. [22] reveals that the gauge invariant can be reached at the order of $\mathcal{O}(g^4)$, which is confirmed by our results.

Since the high-temperature expansion requires $M_i/T \ll 1$, the temperature T_c is safely calculated by high-temperature expansion, but T_n is not the case because we usually have $M_i/T \sim 1$ when nucleation occurs; see Fig. 1 of Supplemental Material [42]. Figure 3 shows how the effect of field-dependent mass term corrections to the wave function renormalization affects the PT parameters

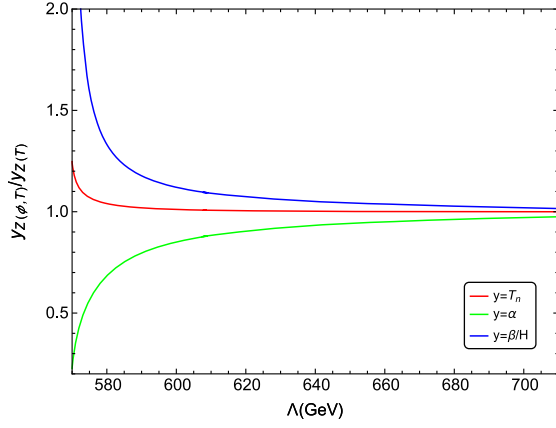


FIG. 3. The difference of PT parameters calculated with GI method. The ratio $y_{Z(\phi,T)}/y_{Z(T)}$ indicates the magnitude of the correction to the high-temperature expansion by the field-dependent mass terms.

calculated with GI method. These mass terms make a large correction to PT parameters in the parameter range of $\Lambda \leq 590$ GeV, which will reduce the magnitude of α and increase the magnitudes of T_n and β/H . The corrections to T_n are small, but large to α and β/H . This effect decreases with increasing Λ and is negligible for $\Lambda \gtrsim 660$ GeV.

Basically, for the GWs from first-order PT, there are three sources, i.e., bubble collision [51,52], sound wave [44,53,54], and magnetohydrodynamics turbulence [55]. To obtain strong GWs to be probed by LISA, one requires strong PT with large α and small $\beta/H \sim \mathcal{O}(10^2)$. Such PT parameters can be reached for low Λ where the divergence of PT parameters calculated with the high-temperature expansion and the case with wave function renormalization

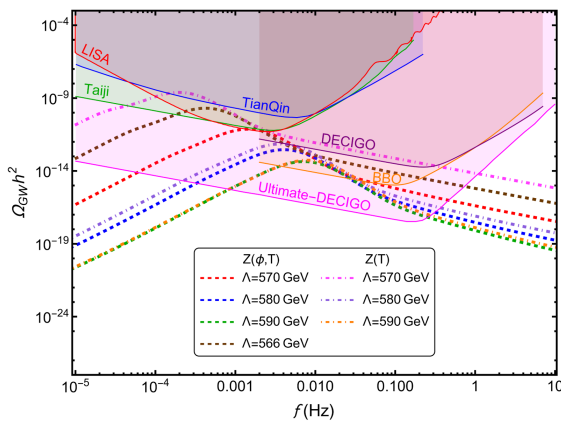


FIG. 4. GW predictions for benchmarks with $\Lambda \leq 590$ GeV. The dashed lines denote the GWs with mass term correction, and the dot-dashed lines denote the GWs with high-temperature expansion.

correction from field-dependent mass terms appear. For comparison, we calculate the PT parameters for GW spectra prediction with two approximations, the GI method based on the traditional DR approach with high-temperature expansion and the improved-DR approach. Figure 4 illustrates the disparity between the computed GW spectra based on the PT parameters and the sensitivity curves of Taiji, TianQin, LISA, BBO, and DECIGO. Our results show that our method based on the improved wave function renormalization would reduce the amplitude of the predicted GWs (with PT parameters obtained through the traditional DR approach) by around 2 orders for $\Lambda = 570$ GeV. As shown in Fig. 4, the effect of the field-dependent part on the improved wave function factor is insignificant at $\Lambda = 590$ GeV, but the effect on the GW prediction becomes significant when $\Lambda < 590$ GeV. Finally, we draw the conclusion that the first-order phase transition parameter spaces with $\Lambda \gtrsim 570$ GeV cannot be probed by Taiji, TianQin, and LISA.

Conclusion. In this Letter, we construct the gauge-invariant thermal effective potential based on the three-dimensional thermal SM effective field theory. We study the PT dynamics and find that the electroweak PT can be strongly first order when the new physics scale $\Lambda \lesssim 770\text{--}800$ GeV. The PT parameters (trace anomaly, PT temperature, duration, and bubble wall velocity) calculated with the GI approach and the Landau gauge based on DR at two-loop order deviate from each other at the percent level. We observe that, in the NP scale range of $\Lambda \leq 590$ GeV, the predicted GW signals in 3D thermal SMEFT obtained without considering the field-dependent correction to the wave function renormalizations can be probed by BBO and Ultimate DECIGO, but this result is doubtful due to the high-temperature expansion. In regions with $\Lambda < 590$ GeV, the field-dependent correction to the wave function factor makes the results of the GWs further reduced, and gravitational waves are only likely to be detected by LISA, TianQin, and Taiji when $\Lambda < 570$ GeV.

We further note that one can quantitatively assess the reliability of the perturbative computations in the GI approach through comparison with the nonperturbative lattice simulation of the bubble nucleation rate in the first-order PT process [56,57].

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