

Understanding gravitational form factors with the Weizsäcker-Williams method

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Understanding the internal structure of nucleons and nuclei has been a topic of enduring interest in high-energy physics. Gravitational form factors (GFFs) provide an important portal for us to probe the energy-momentum/mass distribution of nucleons and nuclei. This paper presents the study of the photon and gluon momentum GFFs, also known as the A-GFFs, of relativistic hadrons using the Weizsäcker-Williams method. To begin, we express the photon A-GFFs in terms of charge form factors and discuss the corresponding photon radius. Furthermore, an integral relation between the gluon A-GFF and the Laplacian of dipole scattering amplitude is derived in the small- x framework, and it allows us to unravel the gluon energy momentum distribution inside hadrons through measurements at the upcoming Electron-Ion Collider. In addition, we generalize the analysis to study the A-GFF of nuclei and propose employing the nuclear gluon mean square radius, together with the charge distribution, to constrain the neutron distribution for large nuclei. This work provides an interesting perspective into the fundamental structure of high-energy hadrons.

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Introduction. Gravitational form factors (GFFs) [1–3], defined as the transition matrix elements of energy-momentum tensor (EMT), encode profound fundamental information of hadrons (see a recent review [4]), such as the distribution of the hadron mass [5–12] and spin [3,13–17] as well as the nature of the internal stress [18–21]. Constraints on hadronic GFFs can be accessed through exclusive processes such as the deeply virtual Compton scattering (DVCS) [3,22–29] and near-threshold heavy-quarkonium productions [30–45] at Jefferson Laboratory [46,47], the electron-ion collider (EIC) [48–51], the large hadron electron collider [52,53], and also ultraperipheral heavy-ion collisions. Lattice simulations [54–59] and perturbative analysis [8,60–64] have provided a lot of

theoretical insights into the understanding of hadronic GFFs [4].

Of particular interest is the study of GFFs in fast-moving hadrons [20,21,65–76]. First, the Lorentz contraction facilitates a probabilistic interpretation of spatial transverse parton densities and associated GFFs, allowing for physical radii to characterize the intrinsic size of hadrons (see the discussions in, e.g., [21,65]). Furthermore, the Wigner distributions [77,78] and generalized transverse momentum distributions (GTMDs) [79–81] provide the full quantum information stored in a quantum system. Currently, the Weizsäcker-Williams (WW) method [82–84], as a powerful and widely used tool in computing these distributions [85–99], remains relatively unexplored in the GFF studies.

The WW method, also known as the equivalent photon approximation [82], has been successful in describing the soft photon radiations from relativistic charged particles in ultraperipheral nuclei collisions [100–103]. Its application to QCD allows us to define and compute the WW small- x gluon distributions radiated from color charges [85–94]. The WW gluon distribution, as an essential ingredient in the color-glass-condensate (CGC) formalism, is vital to describe the gluon saturation phenomena [104–110]. Especially, the small- x WW gluon GTMDs have been

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under intensive study recently [96–99]. Given their close connections to GFFs, it is natural to also use the WW method to compute the gluon GFFs and the associated radii in the small- x formalism.

The objective of this paper is to use the WW method to study the photon and gluon GFFs of high-energy hadrons. As a first attempt, we focus on the A-GFFs, pivotal in determining the light-front momentum radius [21]. First, we compute the photon GFFs as a simple example and proof of principle. Then, through the small- x gluon GTMDs, we relate the gluon A-GFFs to the impact parameter dependent color-dipole scattering amplitude. Our results reveal that the gluon GFFs and radius are sensitive to the saturation momentum of hadron systems and help in understanding the geometry and mass distribution of proton and nuclei. The extension to other GFFs will be addressed in a future study.

The Photon A-GFFs and Radii.

The photon A-GFFs in WW method: Let us first consider the photon GFFs, for which the associated EMT is defined as follows:

$$T_{\gamma}^{\mu\nu} = F^{\mu\alpha} F_{\alpha}^{\nu} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}, \quad (1)$$

where $F^{\mu\alpha}$ represents the photon field strength tensor. As a first step, we assume the target charged hadron is spin-0. The photon GFFs for such a hadron can be parametrized as [19]

$$\begin{aligned} \langle p' | T_{\gamma}^{\mu\nu} | p \rangle = & 2P^{\mu} P^{\nu} A_{\gamma}(t) + C_{\gamma}(t) \frac{\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2}{2} \\ & + 2m^2 \bar{C}_{\gamma}(t) g^{\mu\nu}, \end{aligned} \quad (2)$$

where $p = P - \Delta/2$ and $p' = P + \Delta/2$ are the initial and final state hadron momentum, respectively. P is the average momentum, Δ is the momentum transfer with $t = \Delta^2$, and m denotes the hadron mass. The above photon GFFs, defined as the matrix elements of the EMT operator, provide the vital information on the energy momentum distribution from photons. Specifically, the $A_{\gamma}(t)$ and $C_{\gamma}(t)$ GFFs encode information about mass/momentum and shear pressure distribution, respectively, while $\bar{C}_{\gamma}(t)$ arises from the nonconservation of the photon EMT. In this paper, we mainly focus on the $A(t)$ GFF and comment on the possible extension to other GFFs at the end.

In collider experiments, charged particles often travel relativistically, where the WW method applies [82–84]. In the relativistic regime, perturbed by a small transverse momentum transfer $|t| = \Delta_{\perp}^2$, the initial and final-state hadrons in Eq. (2) travel rapidly along nearly the same direction. According to the WW method, the electromagnetic field of the fast-moving hadron can be treated as a swarm of quasireal photons in the transverse plane. These photons originate from the rapid motion of the hadronic

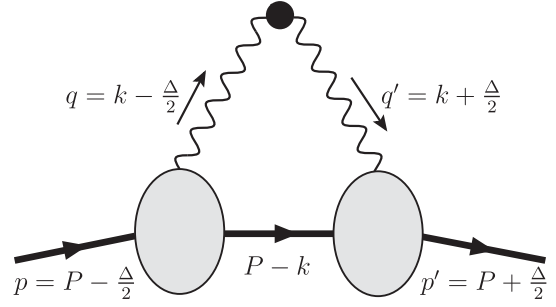


FIG. 1. The LO diagram contributing to the photon GFFs. The gray bubbles stand for the electromagnetic currents of the hadron, and the back dot on the top represents the vertex of the photon EMT operators.

charged constituents, namely, the current $J^{\mu} = e\bar{\psi}\gamma^{\mu}\psi$. Consequently, the computation of photon GFFs becomes the evaluations of photon bremsstrahlung from the hadron and the interaction between the photon and the EMT operator. Figure 1 presents the leading-order- α diagram of photon GFFs, where two photon exchanges are necessitated by the coupling between the EMT and the charged currents. In the following, we will show that the photon A-GFF can be expressed in terms of the electromagnetic (EM) form factor of the relativistic hadrons. We are interested in the leading-power contribution in the limit $P^+ \gg |\Delta_{\perp}|, m$ and work in the leading order of α . We outline the results below, with detailed derivations available in the Supplemental Material [111].

First, working in the infinite momentum frame: $\Delta^{\mu} = (0, 0, \Delta_{\perp})$, $P^{\mu} = (P^+, \frac{\Delta_{\perp}^2/4}{2P^+}, 0)$, we can find the A-GFF directly from the longitudinal component of the EMT matrix: $A_{\gamma}(t) = \frac{1}{2(P^+)^2} \langle p' | T_{\gamma}^{++} | p \rangle$. Then we calculate the A-GFF in Fig. 1 by putting the one-particle intermediate state on shell and cast the photon A-GFF into

$$A_{\gamma}(t) = 4\alpha \int_0^1 dx \int \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{q_{\perp} \cdot q'_{\perp}}{q^2 q'^2} F(q^2) F(q'^2). \quad (3)$$

Here F represents the charge form factor of a spinless hadron, parametrized from $\langle p_2 | J^{\mu} | p_1 \rangle = e(p_1 + p_2)^{\mu} \times F((p_1 - p_2)^2)$. We introduce the variable $q_{\perp} = k_{\perp} - \Delta_{\perp}/2$, $q'_{\perp} = k_{\perp} + \Delta_{\perp}/2$ for convenience, and $q^2 = -(q_{\perp}^2 + x^2 m^2)$ represents the virtuality of the photons, with x as the longitudinal momentum fraction. In evaluating Eq. (3), we have taken the small- x limit in the integrand, as the photon A-GFF is dominated by the soft photon with small virtuality. In this region, since $xm \sim |k_{\perp}| \ll P^+$, we retain the xm -terms in q^2 and q'^2 .

Alternatively, the photon A-GFF in the WW approximation can be obtained from the small- x WW photon GTMDs [112,113] $xf_{\gamma}(x, k_{\perp}, \Delta_{\perp})$ as follows:

$$A_{\gamma}(t) = \int_0^1 dx \int d^2 k_{\perp} xf_{\gamma}(x, k_{\perp}, \Delta_{\perp}). \quad (4)$$

One can show that the above result leads to the GFF consistent with that in Eq. (10).

The photon radius: Of particular interest is the small- t behavior of the A-GFF, as it determines various energy momentum radii [18,19,21] (see a recent review in [4]). For a fast-moving particle, we can define the photon light-front momentum density in the impact parameter $b_\perp$ space, through the two-dimensional (2D) inverse Fourier transform [20,21]:

$$\epsilon_\gamma(b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} A(-\Delta_\perp^2). \quad (5)$$

The associated mean-square transverse radius can then be computed by the slope of the A-GFF at $t = 0$ [21,67]:

$$\langle b_\perp^2 \rangle_\gamma \equiv \frac{\int d^2 b_\perp b_\perp^2 \epsilon_\gamma(b_\perp)}{\int d^2 b_\perp \epsilon_\gamma(b_\perp)} = \frac{4}{A_\gamma(0)} \left. \frac{dA_\gamma(t)}{dt} \right|_{t=0}. \quad (6)$$

Interestingly, the photon momentum radius from a charged particle is singular due to the long-range tail of the Coulomb field. To see this, let us first consider a pointlike charge, where $F(q^2) = 1$. The asymptotic behavior of photon A-GFF at small $|t|$ can be computed as follows:

$$A_\gamma(t) = \frac{\alpha}{\pi} \left[\text{UV} + \frac{t}{m^2} \left(\frac{3}{16} \frac{m\pi^2}{\sqrt{-t}} - \frac{1}{3} \right) + \dots \right], \quad (7)$$

where UV represents the ultraviolet divergent part due to the point-like charge input and one can find $\text{UV} = \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} + 1$ in the dimensional regularization. The rest part of the A-GFF is finite at $t = 0$. However, due to the presence of a nonanalytic term in Eq. (7), the slope dA_γ/dt diverges at $t = 0$, resulting in an infinite photon momentum radius. Since the small- t behavior of $A_\gamma(t)$ corresponds to the large- $b_\perp$ behavior of the light-front momentum density $\epsilon_\gamma(b_\perp)$, the infinite photon momentum radius $\langle b_\perp^2 \rangle_\gamma$ reflects the fact that the energy in the Coulomb field extends out to infinity. This nonanalytic singularity is inherent to any charged particle, irrespective of the internal structure, due to the long-range nature [114–116].

However, for charge-neutral particles, this singular behavior cancels out due to internal charge correlations. To illustrate this cancellation, we can consider a scalar dipole model by pairing the positive and negative charged particles at a distance of $R_\perp$ [109,117,118], as shown in Fig. 2. Summing up all four contributions gives

$$\begin{aligned} A_\gamma(t) = & 4\alpha \int_0^1 dx \int \frac{d^2 q_\perp d^2 q'_\perp}{(2\pi)^4} \frac{q_\perp \cdot q'_\perp}{(q_\perp^2 + m^2 x^2)(q'^2_\perp + m^2 x^2)} \\ & \times (2\pi)^2 \delta^{(2)}(\Delta_\perp + q_\perp - q'_\perp) \\ & \times [e^{-iq_\perp \cdot R_\perp/2} - e^{iq_\perp \cdot R_\perp/2}] F(-(q_\perp^2 + m^2 x^2)) \\ & \times [e^{iq'_\perp \cdot R_\perp/2} - e^{-iq'_\perp \cdot R_\perp/2}] F(-(q'^2_\perp + m^2 x^2)), \quad (8) \end{aligned}$$

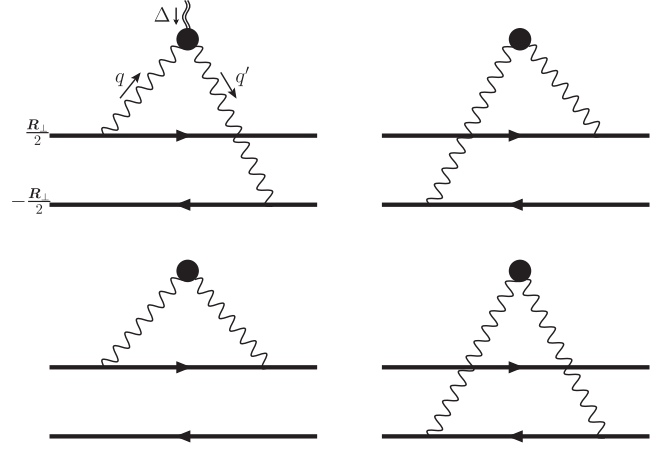


FIG. 2. Photon GFFs in a charge neutral dipole.

where the phase factors account for the relative positions of the charges. One can find that the opposite charges remove the singular behavior due to the long-range Coulomb fields, allowing for a finite photon momentum radius, see the demonstration in the Supplemental Material [111]. The above analysis can be extended to a spin-1/2 hadron [111].

Gluon Gffs and Radii. Now let us generalize our analysis to the gluon GFFs of relativistic hadrons. In CGC, the small- x gluons dominate the parton distribution at high energy. Similar to the QED case outlined in Eq. (4), we can obtain the relation between the gluon A-GFF and the unpolarized gluon GTMDs $A_g(t) = \int_0^1 dx \int d^2 k_\perp x \mathcal{G}_x(k_\perp, \Delta_\perp)$, which is in agreement with previous results regarding the gluon generalized parton distributions and GFFs at zero skewness [119].

There are two types of GTMDs in the CGC formalism, namely, the dipole and WW gluon GTMDs (see the definitions in Ref. [111]). As shown in Ref. [96], these GTMDs are related to the Fourier transforms of color dipole and quadrupole correlators, respectively.

$$\begin{aligned} x\mathcal{G}_x^{\text{WW}}(k_\perp, \Delta_\perp) = & \frac{2N_c}{\alpha_s} \int \frac{d^2 b_1 d^2 b_2}{(2\pi)^4} e^{-i\Delta_\perp \cdot \frac{b_1+b_2}{2} - ik_\perp \cdot (b_1-b_2)} \\ & \times \frac{1}{N_c} \langle \text{tr}[(i\partial_\perp^j U_{b_1}) U_{b_2}^\dagger (i\partial_\perp^j U_{b_2}) U_{b_1}^\dagger] \rangle_x, \quad (9) \end{aligned}$$

$$\begin{aligned} x\mathcal{G}_x^{\text{DP}}(k_\perp, \Delta_\perp) = & \frac{2N_c}{\alpha_s} \int \frac{d^2 b_1 d^2 b_2}{(2\pi)^4} e^{-i\Delta_\perp \cdot \frac{b_1+b_2}{2} - ik_\perp \cdot (b_1-b_2)} \\ & \times \frac{1}{N_c} \langle \text{tr}[(\partial_\perp^j U_{b_1})(\partial_\perp^j U_{b_2}^\dagger)] \rangle_x, \quad (10) \end{aligned}$$

where U_{b_i} represents the Wilson line extending from $x^- = -\infty$ to $x^- = +\infty$ at the transverse position b_i . Detailed derivations of the above relations can be found in Ref. [94] and references therein. Remarkably, after

integrating out transverse momentum $k_\perp$ and using the identities $(\partial_\perp^j U_{b_\perp})U_{b_\perp}^\dagger = -U_{b_\perp}(\partial_\perp^j U_{b_\perp}^\dagger)$ and $U_{b_\perp}U_{b_\perp}^\dagger = 1$, these two GTMDs reduce to the same gluon generalized parton distribution at zero skewness as well as the same gluon A-GFF.

Finally, one can arrive at an interesting general relation between the gluon A-GFF and the Laplacian of the dipole scattering amplitude as follows:

$$A_g(t) = \frac{2N_c}{\alpha_s} \int_0^1 dx \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \times \vec{\nabla}_{r_\perp}^2 [1 - S_x(b_\perp, r_\perp)]|_{r_\perp=0}. \quad (11)$$

Here, $S_x(b_\perp, r_\perp) = \frac{1}{N_c} \langle \text{tr}[U_{b_\perp+r_\perp/2} U_{b_\perp-r_\perp/2}^\dagger] \rangle_x$ represents the scattering amplitude between the color dipole with size $r_\perp$ and the target hadron at impact parameter $b_\perp$. In the CGC formalism, the dipole scattering amplitude usually follows the small- x evolution governed by the nonlinear small- x evolution equation [120–125] with certain initial conditions [86,87,126,127]. Also, phenomenological studies based on models and fitting of experimental data can provide useful information on $S_x(b_\perp, r_\perp)$ (see recent reviews [128,129]). Similarly, one can explore the relation to the BFKL intercept for $A_g(t)$ using the $b_\perp$ -dependent BFKL solution given in, e.g., [96,130,131] and reference therein.

Equation (11) has a wide range of implications. First, if we set $\epsilon_g(b_\perp) = \frac{2N_c}{\alpha_s} \int_0^1 dx \vec{\nabla}_{r_\perp}^2 [1 - S_x(b_\perp, r_\perp)]|_{r_\perp=0}$, we can interpret it as the gluon light-front momentum density, since it is related to the A-GFF via the Fourier transform $A_g(t) = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \epsilon_g(b_\perp)$. Therefore, the gluon transverse mean square radius [21] is given by

$$\langle b_\perp^2 \rangle_g = \frac{\int_0^1 dx \int d^2 b_\perp b_\perp^2 \vec{\nabla}_{r_\perp}^2 S_x(b_\perp, r_\perp)|_{r_\perp=0}}{\int_0^1 dx \int d^2 b_\perp \vec{\nabla}_{r_\perp}^2 S_x(b_\perp, r_\perp)|_{r_\perp=0}}. \quad (12)$$

As expected, the above formula is equivalent to the result computed from the slope of the gluon A-GFF at $t = 0$, as in the photon counterpart in Eq. (6).

Second, the A-GFF of a QCD dipole (onium state), which is akin to the dipole photon A-GFF in Eq. (8), can be obtained by substituting $S_x(b_\perp, r_\perp)$ in Eq. (11) with the onium-onium scattering amplitude [109].

Third, if one assumes the Gaussian form for the scattering amplitude $S_x(b_\perp, r_\perp) = \exp[-\frac{r_\perp^2}{4} Q_s^2(x, b_\perp)]$ [126] with the saturation momentum Q_s and neglects possible $\ln 1/r_\perp^2$ dependence when taking the Laplacian, one can obtain the following A-GFF and radius:

$$A_g(t) = \frac{2N_c}{\alpha_s} \int_0^1 dx \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} Q_s^2(x, b_\perp),$$

$$\langle b_\perp^2 \rangle_g = \frac{\int_0^1 dx \int d^2 b_\perp b_\perp^2 Q_s^2(x, b_\perp)}{\int_0^1 dx \int d^2 b_\perp Q_s^2(x, b_\perp)}. \quad (13)$$

The physical meaning above results becomes manifest when we further adopt the impact parameter dependent saturation (IP-Sat) model [127,132] for proton. Then, the saturation momentum $Q_s(x, b_\perp)$ can be parametrized in the form

$$Q_s^2(x, b_\perp) = \frac{2\pi^2 \alpha_s}{N_c} x g(x, \mu^2) T(b_\perp). \quad (14)$$

In the IP-Sat model, the x dependence is given by the collinear gluon distribution $g(x, \mu^2)$ with the scale $\mu^2 = \mu_0^2 + 4/r_\perp^2$ and the initial scale μ_0^2 given by data fits, while the proton profile function $T(b_\perp)$ accounts for the impact parameter dependence. Thus, one finds the gluon A-GFF $A_g(t) = A_g(0) \int d^2 b_\perp e^{-i\Delta_\perp \cdot b_\perp} T(b_\perp)$ with $A_g(0) = \int_0^1 dx x g(x, \mu^2)$. Since the dipole size $r_\perp$ is set to 0 (or $1/Q$ with Q the hard scattering scale) in Eq. (11), one may take the asymptotic limit and choose $\mu^2 \simeq Q^2$, thus finding the asymptotic gluon momentum fraction $\frac{4C_F}{4C_F+n_f}$ [3,133] for $A_g(0)$ with the quark flavor number n_f . In principle, we can numerically check that the low- x region with sufficiently large μ^2 contributes significantly. In practice, one may use a phenomenological model, such as the IP-Sat model, which provides a smooth prescription of the scattering matrix across the low- x to the large- x region. The above results demonstrate the relationship between the gluon EMT form factor and the gluon light-front momentum radius [21] $\langle b_\perp^2 \rangle_g = \int d^2 b_\perp b_\perp^2 T(b_\perp)$ with the proton shape profile, which has been an important topic extensively studied in the small- x formalism (e.g., [127,134–137]).

Additionally, it will be interesting to study the impact of the small- x evolution with impact parameter dependence on $Q_s^2(x, b_\perp)$ and the related A-GFF. It should be noted that the small- x evolution, such as that given by the BK equation, does not inherently lead to a finite radius [138,139]. Nonperturbative regularizations, particularly those related to QCD confinement, are necessary to ensure the finiteness of the gluon radius $\langle b_\perp^2 \rangle_g$ (see, e.g., [140,141]), as assumed in phenomenological models such as IP-Sat. We leave it for a future work.

Furthermore, exclusive diffractive processes, such as the vector meson production and the deeply virtual Compton scattering in ep scattering, provide gateways to constrain the proton density profile (e.g., [127,142–149]), thus offering important inputs to the gluon A-GFF and the transverse radius through Eqs. (11), (12). For instance, assuming a Gaussian profile $T(b_\perp) = \frac{1}{2\pi B_G} e^{-b_\perp^2/(2B_G)}$ [127] in the IP-Sat model, one finds that the gluon radius and the A-GFF of the proton (insensitive to γ^*p collisional energies

and x values) are given by $\sqrt{\langle b_\perp^2 \rangle_g} = \sqrt{2B_G}$ and $A_g(t) = A_g(0)e^{-B_G|t|/2}$, respectively. Based on the IP-Sat model, the fitting with the experimental data collected in exclusive diffractive processes at HERA leads to $B_G = 4.0 \pm 0.4 \text{ GeV}^{-2}$ [145], resulting in $\sqrt{\langle b_\perp^2 \rangle_g} = 0.56 \pm 0.03 \text{ fm}$. Exploring models [150–154] akin to IP-Sat and investigating geometric fluctuations [134–136] and color charge correlations [155–158] can offer further insights into gluon GFFs and radii.

In addition, in light of future EIC experiments, it is natural to extend the above analysis to high-energy electron-nucleus collisions. In particular, when the atomic mass number A is sufficiently large, one can use the well-known McLerran-Venugopalan model [85–87] and compute the corresponding gluon A-GFF and radius. Since gluons interact with the down and up quarks in the same way, it is reasonable to expect that the gluon densities $g(x)$ inside the proton and neutron are largely identical. Thus, one can write the saturation momentum of the target nucleus as $Q_s^2(x, b_\perp) = \frac{2\pi^2\alpha_s}{N_c} xg(x)T_A(b_\perp)$ with the nuclear thickness function $T_A(b_\perp)$. If one simply supposes that the nucleon density is uniform inside a large spherical nucleus with the geometric radius R_A [92,118,127], then one finds $T_A(b_\perp) = \frac{3A}{2\pi R_A^3} \sqrt{R_A^2 - b_\perp^2}$. This simple profile function then leads to $\langle b_\perp^2 \rangle_g = \frac{2}{5} R_A^2 = \frac{2}{3} \langle r^2 \rangle_g$ with the three-dimensional radius $\langle r^2 \rangle$ and $A_g(t) = A \int_0^1 dx xg(x)f_N(t)$ with $f_N(t) = 3 \frac{\sin R_A \Delta_\perp - R_A \Delta_\perp \cos R_A \Delta_\perp}{R_A^3 \Delta_\perp^3}$. More sophisticated theoretical inputs and EIC measurements, combined together, allow us to precisely pinpoint the nucleon distribution inside nuclei including properties like density fluctuations [129]. The WW method used in this work offers an interesting perspective at high energy, complementing previous studies of nuclear GFFs [18,159–162].

Last but not least, by studying the gluon mean square radius, one can even indirectly extract the neutron distribution information in medium and large nuclei. High-energy electron-nucleus collisions at EIC allow us to probe the proton and neutron densities collectively, while measurements of the electromagnetic and weak charge form factors are separately sensitive to the proton and neutron distributions, respectively. Let us write the nuclear profile function $T_A(b_\perp)$ as the sum of the transverse density distributions $\rho(b_\perp)$ for protons and neutrons:

$$T_A(b_\perp) = \rho_p(b_\perp) + \rho_n(b_\perp). \quad (15)$$

Thus, the transverse mean square radii of proton and neutron distributions can be cast into

$$\langle b_\perp^2 \rangle_{p,n} = \frac{\int d^2 b_\perp b_\perp^2 \rho_{p,n}(b_\perp)}{\int d^2 b_\perp \rho_{p,n}(b_\perp)}. \quad (16)$$

Eventually, one can derive a simple equation relating these radii to the nuclear gluon radius as follows:

$$\langle b_\perp^2 \rangle_g = \frac{Z}{A} \langle b_\perp^2 \rangle_p + \frac{A-Z}{A} \langle b_\perp^2 \rangle_n. \quad (17)$$

Here, the symbols A and Z represent the atomic mass and atomic numbers, respectively. Experimentally, the three radii in Eq. (17) can be measured separately. The nuclear gluon radius, which is related to the nucleon distribution in a nucleus, can be constrained by exclusive processes like diffractive vector meson production (see, e.g., [129,144,163,164]). Meanwhile, the proton mean square radius can be obtained from the charge form factor [165] $\langle b_\perp^2 \rangle_p = \frac{4}{F_{ch}(0)} \frac{dF_{ch}(t)}{dt} \big|_{t=0}$ in elastic electron scatterings. In addition, as the weak charge of the neutron is about 14 times of that of the proton, the measurement of the weak charge form factor and the parity violating effect in electron scattering can probe the neutron distribution (see [166] and references therein). Additionally, the mean square transverse radii in Eq. (17) are directly related to the conventional 3D radii through the following relation: $\langle b_\perp^2 \rangle_{n,p} = \frac{2}{3} \langle r^2 \rangle_{n,p}$ [167] for a large nucleus with spherical symmetry. In nuclear physics, the neutron skin thickness [166,168], defined as $S = \langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2}$, is a central focus in the studies of neutron-rich matter (see [169–171] for ^{208}Pb and the recent STAR measurement [172]). Therefore, using Eq. (17), EIC can open a gateway to constraining the neutron skin from the nuclear gluon radius and supplement to the direct measurements through the parity violating electron scatterings.

Summary. In summary, we have applied the WW method to compute the photon and gluon A-GFFs of fast-moving hadrons and have established a simple relation between the gluon A-GFF and the color-dipole scattering amplitude. With this relation, we can extract the gluon transverse radius and explore important information about the energy-momentum distribution of gluons inside hadrons. Furthermore, we suggest that the nuclear gluon radius can be written as the sum of the charge and weak charge radii of nuclei, providing a useful constraint on the neutron distributions. Our study represents only the initial step toward understanding the small- x effects in the gluon GFFs and corresponding radii with the eikonal approximation in the high-energy limit. It provides analytical insights and complementary information when compared to lattice simulations and experimental measurements. By incorporating sub-eikonal corrections using the theoretical framework developed in Refs. [173–179], we expect that the extension to other GFFs may become possible. With the advent of the upcoming EIC, we can gain a deeper understanding of the internal structure of hadrons and the related emergent phenomena.

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