

Black hole effective theory for strongly interacting matterMatti Järvinen^{1,2,*} and Dorin Weissman^{1,†}¹*Asia Pacific Center for Theoretical Physics, Pohang, 37673, Korea*²*Department of Physics, Pohang University of Science and Technology, Pohang, 37673, Korea*

(Received 11 July 2024; accepted 18 December 2024; published 24 January 2025)

We establish a new tool for studying strongly coupled matter: an effective theory of black holes in gravity, which maps to a hydrodynamic description of field theories via the gauge-gravity duality. Our approach is inspired by previously known effective theories found in the limit of a high number of dimensions. We argue that the new approach can accurately describe phase transitions in a wide class of theories, such as the Yang-Mills and other nearly critical field theories. As an application to a previously unsolved problem, we analyze the interface between confining and deconfining phases in holographic Yang-Mills theory.

DOI: [10.1103/PhysRevD.111.L021903](https://doi.org/10.1103/PhysRevD.111.L021903)

Introduction. Analyzing phase transitions and related dynamical phenomena in strongly coupled field theories is a challenging problem with interesting applications. A prime example of a phase transition in a strongly interacting system is provided by the Yang-Mills theory, which has a first order confinement-deconfinement phase transition. In addition, full QCD may have a first order transition at finite baryon number density, which can potentially be probed directly in heavy-ion collisions in the future.

First order phase transitions may also take place in the early Universe. They produce gravitational waves, which are potentially observable. The standard model does not give rise to such phase transitions, but they could arise from strongly coupled extensions of the electroweak sector, strongly interacting theories at higher energies, or hidden sectors. Gravitational waves produced by the phase transitions near the electroweak scale may be observed by the LISA experiment [1], while transitions at much higher energies could explain the gravitational wave background recently observed by the NANOGrav experiment using pulsar timing [2].

Detailed studies of phase transitions and gravitational wave production require real-time dynamics, which are challenging to analyze directly in strongly coupled field theories. It is therefore useful to search for alternative approaches. A modern approach is to use the gauge-gravity duality, which maps the time evolution of the strongly coupled system to that of higher dimensional classical gravity. This however does not remove all issues: for

example, computing the time evolution in a generic $3 + 1$ dimensional setup in field theory, with dependence on all spatial coordinates, requires solving full five-dimensional classical evolution on the gravity side, which is numerically extremely costly (see, e.g., [3–5]).

The numerical analysis can be greatly simplified by considering gravity solutions in a high number of dimensions D , dual to conformal theories in $D - 1$ dimensions [6,7]. Finite temperature configurations are dual to black holes on the gravity side, and the dynamics of the black holes boils down to an effective theory having the same dimensionality as the field theory [8,9] (see also [10,11] and [12–17] for complementary approaches). This effective theory is a nonlinear hydrodynamic theory which describes the membranelike motion of the black hole horizon. The theory takes a nonrelativistic form with linear time derivatives, which makes simulations straightforward and much easier than directly simulating Einstein gravity [8,18–22]. The approach is different from the fluid-gravity correspondence [23] where one introduces an explicit derivative expansion: at large D , the hydrodynamic sector is decoupled from other modes [24], such that expanding in $1/D$ automatically leads to a hydrodynamic theory.

Going to conformal theories at a high number of dimensions seems to take one far away from the physically interesting $3 + 1$ dimensional examples. This is however not the case: there is a link between the large D approach and a class of lower-dimensional strongly coupled theories that include, in particular, the Yang-Mills theory (see [25,26]). Specifically, the large D gravity theory can be dimensionally reduced [27,28] to five-dimensional dilaton gravity with a potential that is similar to that used in the improved holographic QCD (IHQCD) model, which is known to give a precise description of the Yang-Mills theory via the holographic duality [29,30]. The same holds for spin models near criticality [31]. These latter models

*Contact author: matti.jarvinen@apctp.org†Contact author: dorin.weissman@apctp.org

belong to a wider class of theories which have a higher order phase transition at finite temperature [32].

The aim of this Letter is to combine the ideas discussed above to derive explicitly a large- D -inspired hydrodynamic theory that can be readily applied to carry out lightweight dynamical simulations of strongly coupled matter in low-dimensional systems. In order to achieve this we first find a generalization of the previously known effective theories. It works not only for theories obtained by dimensionally reducing high-dimensional gravity, but also for their generic deformations. Importantly, this generalization allows one to study the physics of phase transitions and interfaces. Among others, it covers geometries found in the IHQCD model and duals of near-critical spin models. As an example of an application to a previously unsolved problem, we construct the interface between the confined and deconfined phases of the IHQCD model by using the new framework.

The Large D inspired model. Our approach is inspired by the large D limit of general relativity [6,8,9,33]. The typical starting point in this approach is Einstein gravity with a cosmological constant in $D = d + n$ dimensions. Considering a particular class of solutions with a nontrivial metric dependence only on the first d dimensions [34], one can dimensionally reduce [27,28] the high-dimensional gravity action to the Einstein-dilaton action

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{4}{3} (\partial\phi)^2 + V(\phi) \right), \quad (1)$$

where we have specialized to $d = 5$, in which we will write our holographic model.

In the limit $D \rightarrow \infty$ (equivalent to $n \rightarrow \infty$), the dilaton potential following from the dimensional reduction is $V(\phi) = V_0 e^{4\phi/3}$. Interestingly, this potential is exactly the critical potential separating deconfining and confining setups [29,35]. The dimensional reduction when n is large but finite gives a specific correction to this potential. At this stage, our large D “inspired” model takes a departure from the ordinary expansion. Instead of fixing the potential at large n to that resulting from the dimensionally reduced pure gravity model, we consider a generic perturbation in $1/n$ around the $n \rightarrow \infty$ limit by taking

$$V(\phi) = V_0 e^{\frac{4}{3}\phi} \left(1 + \frac{1}{n} \delta V(\phi) + \mathcal{O}\left(\frac{1}{n^2}\right) \right). \quad (2)$$

The perturbation $\delta V(\phi)$ is left as an arbitrary function of ϕ . The constant V_0 is parametrized as

$$V_0 = \frac{n^2}{\ell^2}. \quad (3)$$

The explicit scaling with n^2 is necessary for the consistent expansion at large n . We see that the potential indeed agrees with the choices used for spin models in [31,32] at large values of ϕ , corresponding to low temperatures. Moreover,

the potential is close to that of the IHQCD model [29,30] at large ϕ .

Black hole effective theory. We can find nontrivial solutions to the Einstein-dilaton model (1) with potential (2) by constructing an effective theory in the $1/n$ -expansion in the vein of [6,8,9,33]. To this end we write an ansatz for the metric in Eddington-Finkelstein coordinates,

$$ds^2 = e^{2A} (-f dt^2 - 2 dt dr - 2 C_i dt dx^i + g_{ij} dx^i dx^j).$$

Including the first nontrivial corrections in $1/n$, the functions describing a generic evolving black hole are

$$A = -\frac{nr}{3\ell} + \frac{1}{n} A^{(1)}(\mathcal{R}, t, x^k), \quad (4)$$

$$f = 1 - m(t, x^k) \mathcal{R} + \frac{1}{n} f^{(1)}(\mathcal{R}, t, x^k), \quad (5)$$

$$g_{ij} = \frac{1}{n} \delta_{ij} + \frac{1}{n^2} \frac{p_i(t, x^k) p_j(t, x^k)}{m(t, x^k)} \mathcal{R}, \quad (6)$$

$$C_i = \frac{1}{n} p_i(t, x^k) \mathcal{R} + \frac{1}{n^2} C_i^{(1)}(\mathcal{R}, t, x^k), \quad (7)$$

where

$$\mathcal{R} \equiv e^{\frac{nr}{\ell}}. \quad (8)$$

The ansatz also includes the dilaton

$$\phi = \frac{nr}{2\ell} + \frac{1}{n} \phi^{(1)}(\mathcal{R}, t, x^k). \quad (9)$$

The space and time dependent functions m and p^i characterize the size and velocity of the black hole [8], and the corrections $A^{(1)}$, $f^{(1)}$, $C_i^{(1)}$, and $\phi^{(1)}$ are at this point arbitrary. To leading order in $1/n$, the black hole horizon is located at $\mathcal{R}_h = e^{\frac{nr_h}{\ell}} = 1/m$. The region outside it is $-\infty < r < r_h$, where $0 < \mathcal{R} < \mathcal{R}_h$.

Some comments are in order. The zeroth order solution with constant m is the same as the $n \rightarrow \infty$ solution (when $V = V_0 e^{4\phi/3}$ exactly) which matches with a well-studied string theory solution, the linear dilaton background [16,32,36,37]. Moreover, the dependence of the metric on r in (8) comes with an explicit factor of n . As in the large D literature, we consider the large n limit with $\mathcal{R}$ remaining finite. Therefore the expansion is also a near-horizon limit, as $r - r_h = \frac{\ell}{n} \log(m\mathcal{R})$ is small, giving rise to a membranelike picture. The coordinates x_i have also been rescaled to reflect the fact that the length scale of fluctuations in these directions is (before rescaling) of order $1/\sqrt{n}$. In the rescaled coordinates the expansions for g_{ij} and C_i start at order $1/n$. This reflects the usual scalings found at large D [6,8].

At this point we are ready to start solving the Einstein equations and the equation of motion for the dilaton as a series in $1/n$. We find explicit solutions for the corrections

$f^{(1)}$, $A^{(1)}$, $C^{(1)}$, and $\phi^{(1)}$ in terms of m , p_i , and the potential δV , and then impose regularity at the black hole horizon [34].

This leads to the following effective equations for m and p_i :

$$\begin{aligned} \partial_i m &= \ell \partial_i \partial^i m - \partial_i p^i, \\ \partial_i p_i &= \ell \partial_j \partial^j p_i - \partial_j \left(\frac{p^j p_i}{m} \right) + \frac{1}{2} \delta V' \left(-\frac{1}{2} \log m \right) \partial_i m, \end{aligned} \quad (10)$$

$$(11)$$

which constitute the main analytic result of this Letter.

The effective equations reduce exactly to those found in [8] when taking $\delta V(\phi) = 2\phi$ (for flat space solutions reduced over spherical extra dimensions), or $\delta V(\phi) = -2\phi$ [for anti-de Sitter (AdS) solutions reduced over flat extra dimensions] [34].

Thermodynamics and hydrodynamics. Before solving the effective equations, we derive thermodynamic relations. In this computation, we can simply set p_i to zero and m to a constant value, which needs to hold to a good approximation for the system to be near equilibrium. The temperature is the Hawking temperature of the black hole, which can be determined in a standard way by computing its surface gravity, proportional to df/dr at the horizon. Including the $\mathcal{O}(1/n)$ correction one finds [34]

$$T = \frac{n}{4\pi\ell} \left[1 + \frac{1}{n} \delta V \left(-\frac{1}{2} \log m \right) \right]. \quad (12)$$

The leading term is a constant, i.e., the black holes have the same temperature at any size in the $n \rightarrow \infty$ limit. Interestingly, after including the correction, the temperature is proportional to the potential (2),

$$T = \frac{\ell}{4\pi n} V(\phi) e^{-\frac{4}{3}\phi} \Big|_{\phi=-\frac{1}{2}\log m}, \quad (13)$$

with ϕ taking its zeroth order value at the horizon. The entropy density is given by the area of the black hole,

$$S = \frac{1}{4G_5} m, \quad (14)$$

where we took into account the scaling of the spatial coordinates implied by Eq. (6).

Let us comment on the perturbations of the black hole. Setting $m \mapsto m + e^{-i\omega t + iqx_1} \delta m$ and $p_i \mapsto e^{-i\omega t + iqx_1} \delta p_i$, where the amplitudes of the perturbations δm , δp_i are small, we solve the dispersion relations for the frequency ω and momentum q . Linearizing Eqs. (10) and (11), we find for perturbations δp_i perpendicular to the momentum

$$\omega = -i\ell q^2,$$

whereas the perturbations δm and δp_1 give

$$\omega = \pm \sqrt{-\frac{\delta V'(-\frac{1}{2}\log m)}{2}} q - i\ell q^2.$$

These modes are identified as the hydrodynamic modes, i.e., the dissipative shear modes and the sound modes, respectively. The speed of sound is related to the derivative of the potential. When the derivative becomes positive, there is a spinodal instability. Both these observations agree with the thermodynamic relations (12) and (14).

Domain walls. We now study domain wall solutions interpolating between two phases corresponding to black holes with different values of m . We focus on static solutions depending on only one spatial coordinate, $x \equiv x^1$. The first effective equation (10) tells us that $p' = \ell m''$. We choose the solution with zero integration constant, $p = \ell m'$, such that $p \rightarrow 0$ in regions where m goes to a constant. The remaining Eq. (11) is reduced to

$$m^{(3)} - \frac{2m'm''}{m} + \frac{(m')^3}{m^2} = -\frac{1}{2\ell^2} m' \delta V' \left(-\frac{1}{2} \log m \right). \quad (15)$$

From this equation, we can derive two conditions for the existence of a domain wall solution, interpolating between $m(x \rightarrow -\infty) = m_0$ and $m(x \rightarrow \infty) = m_1$. The first is obtained by taking the integral $\int_{-\infty}^x dx'$ of both sides of Eq. (15):

$$m'' - \frac{(m')^2}{m} = -\frac{1}{2\ell^2} \int_{m_0}^{m(x)} dm \delta V' \left(-\frac{1}{2} \log m \right), \quad (16)$$

while the second is obtained by taking $\int_{-\infty}^x dx' \frac{1}{m(x')}$:

$$\frac{m''}{m} - \frac{(m')^2}{2m^2} = \frac{1}{\ell^2} \left[\delta V \left(-\frac{1}{2} \log m(x) \right) - \delta V \left(-\frac{1}{2} \log m_0 \right) \right]. \quad (17)$$

From these two equations it follows that, for a domain wall solution satisfying $m'(\pm\infty) = 0 = m''(\pm\infty)$,

$$\int_{m_0}^{m_1} dm \delta V' \left(-\frac{1}{2} \log m \right) = 0 \quad (18)$$

and

$$\delta V \left(-\frac{1}{2} \log m_0 \right) - \delta V \left(-\frac{1}{2} \log m_1 \right) = 0. \quad (19)$$

Using the relations (12)–(14), we can immediately translate Eqs. (18) and (19), respectively, into the conditions

$$\Delta F = - \int S dT = 0, \quad \Delta T = 0, \quad (20)$$

meaning that a stable domain wall solution can exist between configurations with the same free energy and temperature, consistently with thermodynamics.

Note that in the derivation of Eq. (19), we assumed that m_0 and m_1 are both nonzero, which corresponds to a wall between two deconfined phases. A domain wall solution with one of the endpoints at zero (i.e., a wall between a confined and a deconfined phase [38,39]) does not have to

satisfy the condition on the temperature, but only the condition on the free energy. This reflects the fact that a regular solution with $m = 0$ exists at any temperature (see, e.g., [40,41]). The domain wall solution constructed below is in this latter class.

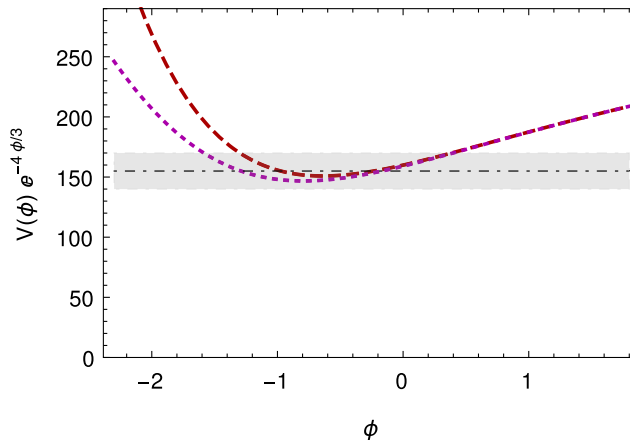
Application to Yang-Mills theory. As an application, we construct the domain wall between confined and deconfined phases in a holographic model for the Yang-Mills theory using our framework. While such a domain wall solution has been found previously in a specific six-dimensional holographic setup [38], it remains unsolved in five-dimensional models, in particular models such as IHQCD [29,30] which have been adjusted to match with lattice data.

We choose the potential V following IHQCD, considering two choices:

- (1) Using directly the IHQCD potential fitted using the full model (without invoking the expansion in $1/n$) to lattice data for thermodynamics and glueball spectrum in Yang-Mills theory. That is, we will use the potential V fitted in [42] (see also [43]), and treat its deviation from the purely exponential form as the perturbation δV in (2).
- (2) A potential which is otherwise identical to the first choice, but modified close to the boundary (small values of ϕ) such that it agrees better with the lattice data when the formulas (12) and (14) are used.

Explicitly, the potentials are

$$V_{\text{IHQCD}}(\phi) = 12 \left[\frac{1}{1 + \frac{c_{\text{UV}}}{e^\phi}} + V_1 e^\phi + \frac{V_2 e^{2\phi}}{1 + c_{\text{IR}} e^\phi} + V_{\text{IR}} \exp \left(-\frac{1}{c_{\text{IR}} e^\phi} \right) e^{4\phi/3} c_{\text{IR}}^{4/3} \times \sqrt{\log(1 + c_{\text{IR}} e^\phi)} \right],$$



where

$$V_1 = \frac{88}{27}, \quad V_2 = \frac{4619}{729}, \quad c_{\text{IR}} = 3, \quad V_{\text{IR}} = 2.05$$

for both choices. They differ only via the value of c_{UV} :

$$c_{\text{UV}} = 0 \quad (\text{choice 1}),$$

$$c_{\text{UV}} = 0.075 \quad (\text{choice 2}).$$

To apply our approach, we decompose the potentials as in (2):

$$V_{\text{IHQCD}}(\phi) = \hat{V}_0 e^{4\phi/3} [1 + \delta V_{\text{IHQCD}}(\phi)],$$

where $\hat{V}_0 = 155$ was chosen such that δV_{IHQCD} is small in the intermediate region where $e^\phi = \mathcal{O}(1)$, which is most important for the phase transition. This is the region, where $V(\phi) e^{-4\phi/3}$, which maps to the temperature via (13), is near its minimum (see Fig. 1, left panel). Note that we do not introduce a formal expansion parameter n here, but since $\hat{V}_0$ is numerically large and δV_{IHQCD} is numerically small in the relevant region (the gray band in Fig. 1 indicates where $\delta V < 0.1$), our approach is expected to work well.

The thermodynamics computed from (12) and (14) is compared to lattice data for Yang-Mills at a large number of colors [44] in Fig. 1 (right). We analyze the pressure P and the interaction measure $(\epsilon - 3P)/T^4$, where ϵ is the energy density, and fit Newton's constant G_5 independently for each potential. As expected, choice 1 which was fitted to lattice data in the IHQCD model without approximations, produces in our approach a fair description of the data only near $T = T_c$ where δV is small. Thanks to the adjustment close to the boundary, choice 2 gives a good fit in a larger range of temperatures.

We show the domain wall solutions obtained by solving Eqs. (16) and (17) for both potential choices in Fig. 2.

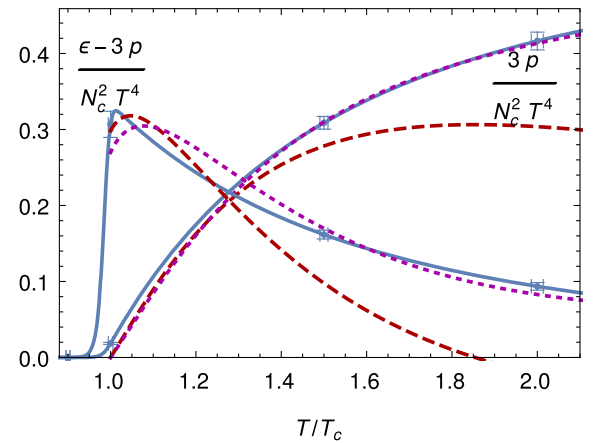


FIG. 1. The IHQCD potentials used in this Letter (left) and results from the large D black hole thermodynamics compared to lattice data [44] (right). The red dashed (magenta dotted) curves show the potentials and thermodynamics for our first choice (second choice) of potentials. The gray band in the left panel shows the region where the deviation from the constant $\hat{V}_0$ is small, $|\delta V| < 0.1$. In the right panel, the blue curves and error bars show the lattice results.

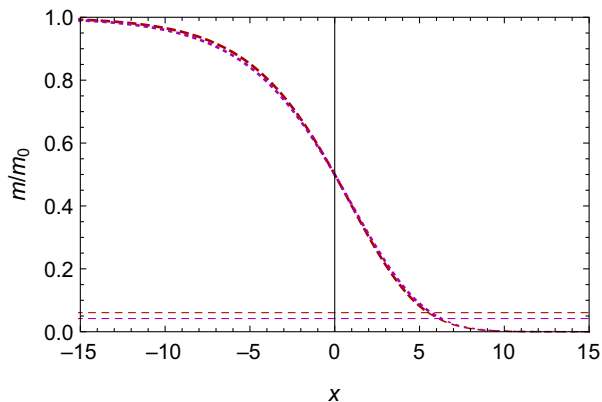


FIG. 2. The profiles of the domain walls shown as the red dashed (magenta dotted) curves for our first choice (second choice) of potentials. The curves almost overlap. The horizontal dashed lines indicate the value of m under which $|\delta V|$ becomes larger than 0.1.

Note that the thermodynamics of Fig. 1 (right) arise from the region of negative ϕ with $\delta V'(\phi) < 0$, where the black hole solutions are stable. The difference between the potentials can be used to estimate the accuracy of the $1/n$ expansion in this region: despite the difference in potentials at negative ϕ shown in Fig. 1 (left), the solutions in Fig. 2 are nearly identical.

At positive ϕ we do not have a direct handle to check the accuracy of the $1/n$ expansion, but we can see the potential exits the gray band of Fig. 1 (left) only in the tail of the domain wall (the part below the dashed lines in Fig. 2), which suggests that most of the domain wall is well approximated. Even though the range of ϕ where δV is small and the potential remains inside the gray band in Fig. 1 (left) appears to be limited, almost all of the domain wall solution is described by this region. This happens because in the effective theory ϕ is replaced by $-\frac{1}{2} \log m$, a slowly varying function.

Conclusions. We derived a novel generalized black hole effective theory which can be applied to describe strongly interacting matter in a wide class of theories via the gauge-gravity duality. We pointed out that the generalized theory can describe dynamics of phase transitions and interfaces.

In particular, we argued that it can be used to model Yang-Mills theory at finite temperature, near the phase transition. As an application of the theory, we found a new solution for the confinement-deconfinement domain wall in the IHQCD model.

Our approach has some limitations. Firstly, it does not give exact solutions to the full five-dimensional gravity theory. It only captures the hydrodynamic modes, and not the time dependence related to higher quasinormal modes

of the black hole. However, in our framework, similarly to the large D limit, the hydrodynamic modes are decoupled from the rest [6,24] so that only including the hydrodynamic sector is a good approximation. Secondly, owing to the nonrelativistic form of our approach it is unlikely to be a good approximation for the Yang-Mills theory (or quark-gluon plasma) at temperatures well above the critical temperature. As the temperature grows, so does the speed of sound, and the plasma is better approximated as a nearly conformal fluid.

The effective theory has several further applications, in particular to time-dependent scenarios. In the context of first order phase transitions in the early Universe, it would be interesting to simulate moving domain walls or expanding bubbles and their collisions, and how they produce gravitational waves. Note that the approach includes crucial ingredients for gravitational wave production, e.g., sound waves. Other related problems would be to study the interfaces with higher order phase transitions, and the evolution for an interface between thermal baths at different temperatures [45–48].

Another direction would be to improve and extend the approach further. First one could verify that the domain wall found in this Letter is indeed a good approximation to the exact wall by solving it directly in IHQCD, which is a challenging but doable computation. More generally, one should check how the energy-momentum tensor is extracted. Doing this properly typically requires matching of the effective theory with behavior near the boundary, as has been analyzed in the large D literature (see, e.g., [49,50]). One can also extend the approach systematically to higher order corrections in $1/n$, though there is no guarantee that these corrections can be solved analytically. Moreover, it might be possible to find similar effective theories in more general setups such as charged black holes [51,52], which means going to finite density on the field theory side.

Acknowledgments. We thank P. Betzios, U. Gürsoy, T. Ishii, R. Janik, N. Jokela, G. Policastro, E. Préau, J. Sonnenschein, J. Subils, and B. Withers for discussions. M. J. has been supported by an appointment to the JRG Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government and by the Korean Local Governments—Gyeongsangbuk-do Province and Pohang City—and by the National Research Foundation of Korea (NRF) funded by the Korean government (MSIT) (Grant No. 2021R1A2C1010834). D. W. was supported by an appointment to the YST Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government. This was also supported by the Korean Local Governments—Gyeongsangbuk-do Province and Pohang City.

- [1] P. A. Seoane *et al.* (LISA Collaboration), *Living Rev. Relativity* **26**, 2 (2023).
- [2] G. Agazie *et al.* (NANOGrav Collaboration), *Astrophys. J. Lett.* **951**, L8 (2023).
- [3] L. Bellantuono, R. A. Janik, J. Jankowski, and H. Soltanpanahi, *J. High Energy Phys.* **10** (2019) 146.
- [4] H. Bantilan, P. Figueras, and D. Mateos, *Phys. Rev. Lett.* **124**, 191601 (2020).
- [5] Y. Bea, J. Casallerrey-Solana, T. Giannakopoulos, A. Jansen, D. Mateos, M. Sanchez-Garitaonandia, and M. Zilhão, *J. High Energy Phys.* **09** (2022) 008; **03** (2023) 225(E).
- [6] R. Emparan, R. Suzuki, and K. Tanabe, *J. High Energy Phys.* **06** (2013) 009.
- [7] R. Emparan and C. P. Herzog, *Rev. Mod. Phys.* **92**, 045005 (2020).
- [8] R. Emparan, R. Suzuki, and K. Tanabe, *Phys. Rev. Lett.* **115**, 091102 (2015).
- [9] R. Emparan, T. Shiromizu, R. Suzuki, K. Tanabe, and T. Tanaka, *J. High Energy Phys.* **06** (2015) 159.
- [10] S. Bhattacharyya, A. De, S. Minwalla, R. Mohan, and A. Saha, *J. High Energy Phys.* **04** (2016) 076.
- [11] Y. Dandekar, S. Mazumdar, S. Minwalla, and A. Saha, *J. High Energy Phys.* **12** (2016) 140.
- [12] R. A. Janik, M. Jarvinen, and J. Sonnenschein, *J. High Energy Phys.* **09** (2021) 129.
- [13] F. R. Ares, O. Henriksson, M. Hindmarsh, C. Hoyos, and N. Jokela, *Phys. Rev. Lett.* **128**, 131101 (2022).
- [14] F. R. Ares, O. Henriksson, M. Hindmarsh, C. Hoyos, and N. Jokela, *Phys. Rev. D* **105**, 066020 (2022).
- [15] R. A. Janik, M. Jarvinen, H. Soltanpanahi, and J. Sonnenschein, *Phys. Rev. Lett.* **129**, 081601 (2022).
- [16] S. Fichet, E. Megias, and M. Quiros, *Phys. Rev. D* **109**, 106011 (2024).
- [17] S. Fichet, E. Megias, and M. Quiros, *J. High Energy Phys.* **08** (2024) 077.
- [18] T. Andrade, R. Emparan, D. Licht, and R. Luna, *J. High Energy Phys.* **04** (2019) 121.
- [19] R. Emparan, D. Licht, R. Suzuki, M. Tomašević, and B. Way, *J. High Energy Phys.* **02** (2022) 090.
- [20] D. Licht, R. Suzuki, and B. Way, *J. High Energy Phys.* **12** (2022) 146.
- [21] R. Luna and M. Sanchez-Garitaonandia, *J. High Energy Phys.* **02** (2023) 147.
- [22] R. Emparan, R. Luna, R. Suzuki, M. Tomašević, and B. Way, *J. High Energy Phys.* **05** (2023) 182.
- [23] V. E. Hubeny, S. Minwalla, and M. Rangamani, in *Theoretical Advanced Study Institute in Elementary Particle Physics: String theory and its Applications: From MeV to the Planck Scale* (World Scientific, Singapore, 2012), pp. 348–383, [arXiv:1107.5780](https://arxiv.org/abs/1107.5780).
- [24] R. Emparan, R. Suzuki, and K. Tanabe, *J. High Energy Phys.* **07** (2014) 113.
- [25] P. Betzios, U. Gürsoy, M. Järvinen, and G. Policastro, *Phys. Rev. D* **101**, 086026 (2020).
- [26] U. Gürsoy, M. Järvinen, G. Policastro, and N. Zinnato, *J. High Energy Phys.* **06** (2022) 018.
- [27] I. Kanitscheider, K. Skenderis, and M. Taylor, *J. High Energy Phys.* **09** (2008) 094.
- [28] B. Gouteraux, J. Smolic, M. Smolic, K. Skenderis, and M. Taylor, *J. High Energy Phys.* **01** (2012) 089.
- [29] U. Gürsoy and E. Kiritsis, *J. High Energy Phys.* **02** (2008) 032.
- [30] U. Gürsoy, E. Kiritsis, and F. Nitti, *J. High Energy Phys.* **02** (2008) 019.
- [31] U. Gürsoy, *J. High Energy Phys.* **12** (2010) 062.
- [32] U. Gürsoy, *J. High Energy Phys.* **01** (2011) 086.
- [33] T. Andrade, C. Pantelidou, and B. Withers, *J. High Energy Phys.* **09** (2018) 138.
- [34] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevD.111.L021903> contains a brief review of the dimensional reduction with explicit expressions for the potential in Sec. I, as well as some technical subtleties regarding the solutions of the Einstein equations and a derivation of the thermodynamics in Sec. II.
- [35] S. Barbosa, S. Fichet, E. Megias, and M. Quiros, [arXiv:2406.02899](https://arxiv.org/abs/2406.02899).
- [36] O. Aharony, M. Berkooz, D. Kutasov, and N. Seiberg, *J. High Energy Phys.* **10** (1998) 004.
- [37] G. Bertoldi and C. Hoyos-Badajoz, *J. High Energy Phys.* **08** (2009) 078.
- [38] O. Aharony, S. Minwalla, and T. Wiseman, *Classical Quantum Gravity* **23**, 2171 (2006).
- [39] P. Figueras and S. Tunyasuvunakool, *J. High Energy Phys.* **06** (2014) 025.
- [40] U. Gürsoy, E. Kiritsis, L. Mazzanti, and F. Nitti, *Phys. Rev. Lett.* **101**, 181601 (2008).
- [41] U. Gürsoy, E. Kiritsis, L. Mazzanti, and F. Nitti, *J. High Energy Phys.* **05** (2009) 033.
- [42] N. Jokela, M. Järvinen, and J. Remes, *J. High Energy Phys.* **03** (2019) 041.
- [43] U. Gürsoy, E. Kiritsis, L. Mazzanti, and F. Nitti, *Nucl. Phys.* **B820**, 148 (2009).
- [44] M. Panero, *Phys. Rev. Lett.* **103**, 232001 (2009).
- [45] M. J. Bhaseen, B. Doyon, A. Lucas, and K. Schalm, *Nat. Phys.* **11**, 5 (2015).
- [46] R. Pourhasan, *J. High Energy Phys.* **02** (2016) 005.
- [47] C. Ecker, J. Erdmenger, and W. van der Schee, *SciPost Phys.* **11**, 047 (2021).
- [48] C. Bachas, Z. Chen, and V. Papadopoulos, *J. High Energy Phys.* **11** (2021) 095.
- [49] R. Suzuki and K. Tanabe, *J. High Energy Phys.* **10** (2015) 107.
- [50] R. Emparan, R. Luna, M. Martínez, R. Suzuki, and K. Tanabe, *J. High Energy Phys.* **05** (2018) 104.
- [51] S. Bhattacharyya, M. Mandlik, S. Minwalla, and S. Thakur, *J. High Energy Phys.* **04** (2016) 128.
- [52] R. Emparan, K. Izumi, R. Luna, R. Suzuki, and K. Tanabe, *J. High Energy Phys.* **06** (2016) 117.