

Quantitative analysis of the gravitational wave spectrum sourced from a first-order chiral phase transition of QCD

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We investigate the cosmological first-order chiral phase transition of quantum chromodynamics (QCD), and for the first time calculate its parameters which can determine the gravitational wave spectrum. With the state-of-the-art calculation from the functional QCD method, we found that the large chemical potential of QCD phase transition results in very weak and fast first-order phase transitions at the temperature lower than $\mathcal{O}(10^2)$ MeV. These results further suggest that the gravitational wave (GW) signals of NANOGrav are very unlikely sourced from the chiral phase transition of QCD.

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Introduction. The space-based interferometers, such as μ Ares [1], Taiji [2,3], Tianqin [4,5], the Laser Interferometer Space Antenna (LISA) [6–9], the Big Bang Observer (BBO) [10,11] and the Deci-Hertz Interferometer Gravitational Wave Observatory (DECIGO) [12–14], offer a frequency window from 10^{-7} Hz to 10 Hz for detecting the stochastic gravitational waves background (SGWB). Moreover, the evidences of the SGWB at nanohz are detected by the pulsar timing arrays (PTA) collaborations, including the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) [15,16], the European Pulsar Timing Array (EPTA) [17,18], the Parkes Pulsar Timing Array (PPTA) [19,20], the International Pulsar Timing Array (IPTA) [21], and the Chinese Pulsar Timing Array (CPTA) [22]. The generation mechanism of SGWB is a highly debated topic with enormous models including supermassive black hole binaries (SMBHBs) [23–26], curvature perturbations [27,28], and new-physics models such as first-order phase transition (FOPT) [29–46], cosmic strings [47–53], and domain walls [54–62], etc. Among the studies, it has been

estimated that the SGWB can be induced by a strong FOPT at about $T_* \sim 1\text{--}10$ MeV [63–65], which makes the cosmic quantum chromodynamics (QCD) phase transition (PT) a competitive candidate. Moreover, the recent studies suggest that the large lepton asymmetries can induce the QCD FOPT in early Universe, and give the observed value of baryon asymmetry via the so called “sphaleron freeze-in” mechanism [66,67]. Thus, a strong QCD phase transition may be a simple solution for multiple problems in cosmology.

The nonperturbative nature of QCD makes it difficult to study its PT dynamics, especially when it comes to the quantitative information of thermodynamics quantities and the effective potential of QCD. Recently, the quantitative truncation scheme together with the computation of thermodynamics quantities are reached [68], and a novel method to determine the effective potential was proposed in Ref. [69]. These progresses make the quantitative analysis of the QCD PT viable through the Dyson-Schwinger equation (DSE) [68–76]. For the first time, we quantitatively determine the strength, the duration, and the percolation temperature T_* of the QCD PT. We then compute the gravitational waves (GWs) spectrum from QCD PT and found that the signal from QCD PT is unlikely to match with the NANOGrav measurements and the being constructed GW detectors in the frequency range from nanohertz to hertz.

Cosmic trajectories crossing the QCD PT. In the QCD epoch, the baryon number Y_B , the individual lepton flavor number Y_{L_f} , and the electric charge Q asymmetry can be

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considered as the conserved quantities during the QCD PT [77]. Thus, without any further assumptions, one can consider the conservation equations

$$Y_{L_f} = \frac{n_f + n_{\nu_f}}{s}, \quad (1a)$$

$$Y_B = \sum_i \frac{B_i n_i}{s}, \quad (1b)$$

$$Q = \sum_i \frac{Q_i n_i}{s}, \quad (1c)$$

with s the total entropy density of the Universe, n_i the net number density (i.e., the particle minus the antiparticle number density of the species i), $f = (e, \mu, \tau)$, $i = (e, \mu, \tau, u, d, s, c)$, B_i the baryon number of species i , Q_i the electric charge of species i , $Y_B = 8.7 \times 10^{-11}$ the observed baryon asymmetry [78], $Q = 0$ the charge neutrality for universe. Y_{L_f} is less constrained, allowing for a larger lepton asymmetry, provided it satisfies the big bang nucleosynthesis (BBN) and the cosmic microwave background (CMB) constraint $|Y_{L_e} + Y_{L_\mu} + Y_{L_\tau}| < 1.2 \times 10^{-2}$ [79–84]. Under this constraint, the individual Y_{L_f} will decrease once the neutrino oscillations start at $T_* \sim 10$ MeV [85–87]. Therefore, the large lepton asymmetries exist only for $T_* > 10$ MeV. For $T_* < 10$ MeV, the diminished Y_{L_f} will lower the lepton and the quark chemical potentials to satisfy the BBN constraint.

Under the conditions of the kinetic and the chemical equilibrium [77], the net leptons and the photons number densities are expressed by Fermi-Dirac distributions and Bose-Einstein distributions, respectively. In the QCD sector, the nonperturbative property prevents the QCD matter from being simply represented by the free particle distributions. Therefore, we apply the DSE to accomplish the complete calculations of the QCD phase diagram, the thermodynamic quantities and especially here the effective potential, which is for the first time obtained and applied to compute the gravitational wave signals.

This work focuses on the scenario ($Y_{L_e} = 0$, $Y_{L_\mu} = -Y_{L_\tau}$), which satisfies BBN constraints and can induce the QCD FOPT [66,88]. Figure 1 shows how the cosmic trajectory depends on Y_{L_μ} . The critical value $Y_{L_\mu} = -0.22$ triggers the FOPT, which is consistent with the previous study [67]. As Y_{L_μ} decreases from -0.14 to -0.22 , the cosmic trajectory quickly approaches the CEP. When $Y_{L_\mu} < -0.22$, the trajectory enters the FOPT region.

When the cosmic trajectory crosses the FOPT, there can be GW emission during the bubble nucleation. Before nucleation, the QCD effective potential, explained in the next section, can be used to study the process. After nucleation, when part of the Universe is converted to the true vacuum, the entropy is no longer conserved due to the

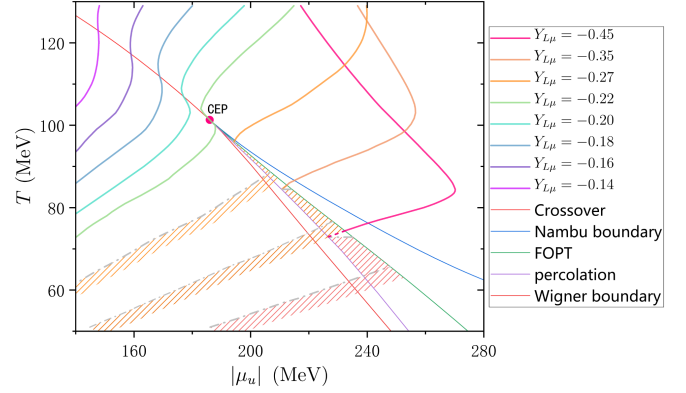


FIG. 1. Cosmic trajectories in the scenario $Y_{L_e} = 0$ and $Y_{L_\mu} = -Y_{L_\tau}$ in QCD phase diagram. The cosmic trajectories intersect the FOPT line at (T_c, μ_c) and the percolation line at (T_*, μ_*) . Solid line: the determined cosmic trajectory; dashed line: the supercooling stage for a cosmic trajectory before the percolation and after the FOPT; gray dot-dashed line: the upper bound for the possible cosmic trajectory; shadow region: an area contains possible cosmic trajectory.

non-equilibrium effect. The upper bound of the cosmic trajectory is determined under the condition that s/n_q is conserved after the PT as shown in Fig. 1. The evolution after the percolation stage is influenced by the dynamics of the bubble expansion, the increase in total entropy in the system due to nonequilibrium processes, and their interplay. The bubble expansion modifies the volume fraction of the true vacuum within the Hubble volume after percolation, which in turn affects extensive thermodynamic quantities. Meanwhile, the increase in total entropy, driven by nonequilibrium processes, causes the evolution to deviate from equilibrium. Both have significant impacts on conservation laws, making the determination of the cosmic trajectory challenging, with only the upper limit of the stage after percolation being determined. Nevertheless, our main focus is the GW signals generated during the percolation stage, which can be fully determined within the information before the percolation stage.

We illustrate the detailed evolution in Fig. 2 with $Y_{L_\mu} = -0.45$ along the path. During the $a - b$ period, the symmetric vacuum energy is lower. At this stage, the universe is filled with the symmetric vacuum, i.e., the Wigner vacuum in QCD. In the subsequent $b - c$ period, the energy of the Wigner vacuum exceeds that of Nambu vacuum, yet both are in local minima. This is then a supercooling period with the universe remaining in Wigner vacuum. At the point c , the Wigner vacuum initiates the decay into the true vacuum, leading to the nucleation and the percolation of bubbles. The $c - d$ remains unclear due to the absence of calculations for nonequilibrium effects. However, cosmic trajectories involving these processes are constrained within the shaded region. The dot-dashed line above point d mark the upper bound of the cosmic trajectory determined by the true vacuum under the conservation of s/n , while s/n

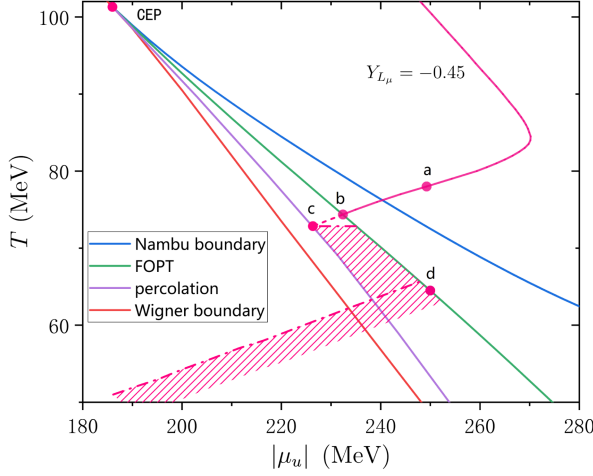


FIG. 2. The cosmic trajectory across the FOPT region in the scenario $Y_{L_e} = 0, Y_{L_\mu} = -Y_{L_e}$, where $Y_{L_\mu} = -0.45$. Solid line: the determined cosmic trajectory, such as $a - b$; dashed line: the supercooling stage for a cosmic trajectory before the percolation and after the FOPT, such as $b - c$; dot-dashed line: the upper bound for the possible cosmic trajectory; shadow region: an area contains possible cosmic trajectory.

generally increases due to the non-equilibrium effects. Near point d , the Gibbs phase equilibrium condition requires the cosmic trajectory aligns with the FOPT line. At point d , the Universe is filled with the Nambu vacuum, marking the end of the phase transition.

PT parameters. The QCD FOPT mechanism is expected to generate gravitational waves (GWs). To account for the causality tail in GW signals [89], we apply the GW spectrum modeled by NANOGrav. The GW spectrum is written as [63,90–96]

$$\Omega_b(f) = \mathcal{D}\tilde{\Omega}_b \left(\frac{\kappa_b \alpha_*}{1 + \alpha_*} \right)^2 (H_* R_*)^2 S(f/f_b), \quad (2)$$

$$\Omega_s(f) = \mathcal{D}\tilde{\Omega}_s \Upsilon(\tau_{sw}) \left(\frac{\kappa_s \alpha_*}{1 + \alpha_*} \right)^2 (H_* R_*) S(f/f_s), \quad (3)$$

with α_* the PT strength, $\beta/H_* = (8\pi)^{1/3} v_w / (H_* R_*)$ being the inverse PT duration [92] and v_w the bubble wall velocity. Both parameters requires the knowledge of the QCD effective potential, which we obtain by applying the homotopy method to the QCD theory [69]. The PT parameters are calculated on the percolation line, which corresponds to the percolation temperature T_* , i.e., the temperature when $\sim 34\%$ of the volume of the universe is converted into the true vacuum determined with the effective potential. These information allows us to obtain the GW spectrum.

The QCD effective potential comes from a generalized Legendre transformation to the Cornwall, Jackiw and Tomboulis (CJT) generating functional [69]. This

transformation converts the variable of the CJT effective potential, i.e., the quark propagator, to the self-energy, which is the key point to keep the effective potential to be bounded from below [97]. The new effective potential is written as [69]

$$\Gamma_\Sigma[\Sigma] = \text{TrLn}[S_0^{-1} + \Sigma]^{-1} - \text{Tr}[1] - \Gamma_2[(S + L)] + \text{Tr}[(S + L)\Sigma], \quad (4)$$

with S_0 the free quark propagator, S the dressed quark propagator, $\Sigma = \Sigma[(S + L)] \equiv \frac{\delta \Gamma_2[(S + L)]}{\delta(S + L)}$ the self-energy, L the external source, and Γ_2 the sum of all two-particle-irreducible diagrams as a functional of $S + L$.

Moreover, we apply the homotopy method, i.e., a simplified version of the functional integration, to determine the effective potential from the quantitative truncation scheme of the DSE [69]

$$\begin{aligned} \Gamma_\Sigma[M_q] &:= \Gamma_\Sigma[\Sigma_\chi] - \Gamma_\Sigma[\Sigma_0] \\ &= \int_0^\chi d\xi \text{Tr} \left\{ \frac{d\Sigma_\xi}{d\xi} [-(S_0^{-1} + \Sigma_\xi)^{-1} + S_\xi] \right\}, \end{aligned} \quad (5)$$

where M_q the dynamic quark mass that depends on the homotopy parameter and

$$\Sigma_\xi = \Sigma[S_\xi], \quad S_\xi = [(1 - \xi)S_1^{-1} + \xi S_2^{-1}]^{-1}, \quad (6)$$

with χ and ξ being the homotopy parameter, S_1 and S_2 being the Wigner and Nambu + solutions, and (Σ) the self energy whose explicit expression is placed in the supplement. In the functional space, a one-dimensional line parametrized by (χ) and (ξ) , on which the effective potential lies, is chosen.

We illustrate the effective potential Γ_Σ in Fig. 3, where the Wigner vacuum is the false vacuum, the Nambu+ is the true vacuum and the Wigner* is the unstable vacuum indicating the occurrence of FOPT. At the PT temperature T_c , the two vacuums are degenerate. Vacuum bubbles merge around the percolation temperature T_* . At that time, the false vacuum has an energy larger than the other. Since the homotopy method guarantees the functional relation between the effective potential and the equation of motion of QCD, the effective potential incorporates all the information from the nonperturbative DSE, which naturally includes the temperature effects and all the temperature-induced infrared effects.

Using the effective potential, we calculate the decay rate of the false vacuum using the PYTHON package [98]

$$\begin{aligned} \Gamma(T, \mu) &= \frac{\sqrt{|\lambda_-|}}{2\pi} \left| \frac{\det'(-\nabla^2 + V''_{\text{eff}}(\phi_b))}{\det(-\nabla^2 + V''_{\text{eff}}(\phi_F))} \right|^{-1/2} \\ &\times \left(\frac{S_3}{2\pi T} \right)^{\frac{3}{2}} \exp \left(-\frac{S_3}{T} \right), \end{aligned} \quad (7)$$

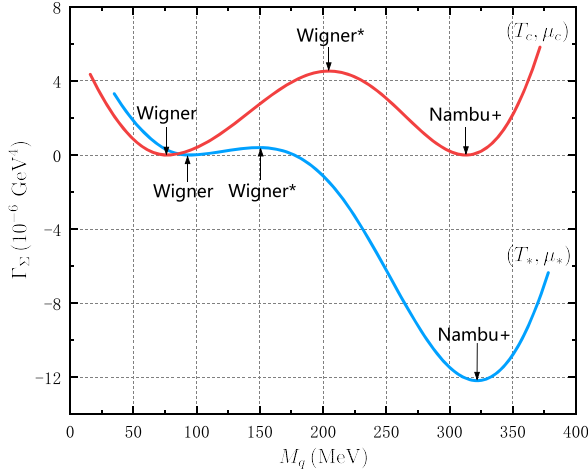


FIG. 3. The $N_f = 2$ effective potential evolving from $((T_c, \mu_c))$ to $((T_*, \mu_*))$ for the $Y_{L_\mu} = -0.45$ cosmic trajectory. The dynamic quark mass M_q is the order parameter of the effective potential and the effective potential is defined to be zero at the Wigner solution.

with

$$S_3 = 4\pi \int_0^\infty dr r^2 \left[\frac{1}{2} \left(\frac{d\phi_b}{dr} \right)^2 + V_{\text{eff}}(\phi_b) \right]. \quad (8)$$

Here, ϕ_b is the bounce solution, ϕ_F is false vacuum solution, λ_- is the negative eigenvalue of the functional determinant, S_3 is the 3-dimensional Euclidean actions for the $O(3)$ -symmetric bounce solutions, r is the bubble radius, $\phi = M_q$ is the dynamical quark mass, $V''_{\text{eff}} = d^2 V_{\text{eff}} / d\phi^2$, and $V_{\text{eff}} = \Gamma_\Sigma$ is the effective potential calculated from the DSE with the homotopy method [69]. The evolution of the decay rate and the percolation temperature with the evolution details in the Supplemental Material [99].

We calculate the bubble wall velocity and find it to be lower than both the Jouguet velocity and the speed of sound in the background radiation, which indicates the subsonic deflagration mode, with details putting in the Supplemental Material [99]. The GW spectrum parameter κ_s is determined as in Ref. [92]. Additionally, the local thermal equilibrium (LTE) effects could influence the bubble wall velocity [104,105]. They are not included here because α_* is small in our study, which makes the bubble wall velocity highly sensitive to the enthalpy ratio between the broken and symmetric phases (Ψ_n), and introduces significant uncertainty in determining v_w . Since v_w does not qualitatively affect our conclusions on the GW spectrum, LTE corrections are omitted.

The PT strength α_* depends on the available energy released from the PT, normalized to the energy densities of the background radiation

$$\alpha_* = \frac{\Delta\theta}{\rho_r} \bigg|_{(T_*, \mu_*)}, \quad (9)$$

with

$$\Delta\theta = \frac{1}{4} T \Delta s + \frac{1}{4} \mu_q \Delta n_q - \frac{1 + 1/c_{s,b}^2}{4} \Delta P, \\ c_{s,b}^2 = \frac{dp}{de} \bigg|_{s,b}, \quad \rho_r(T, \mu) = \sum_a \int \frac{d^3 p}{(2\pi)^3} E_a f_{F/B}(E_a, T, \mu). \quad (10)$$

Here, $E_a = \sqrt{p^2 + m_a^2}$, θ is the pseudotrace anomaly [106], s is the entropy density, n_q is the quark number density and ρ_r collects electron, muon, all neutrinos, photon and u/d quarks. The $c_{s,b}^2$ is the speed of sound for the Nambu phase, where the derivative is along the Isentropic trajectory, p_b is the respective pressure and e_b is the energy density. A critical value α_∞ that can determine whether the bubble walls can runaway is also considered [94,107], which also determines the parameter κ_b .

For the FOPT with finite chemical potential, the inverse PT duration β/H_* can be obtained as

$$\beta/H_* = 4\rho_r \frac{d}{d\rho_r} \left(\frac{S_3}{T} \right) \bigg|_{(T_*, \mu_*)}, \quad (11)$$

where the derivative lies along the cosmic trajectory. The details of above equations can be found in the Supplemental Material [99]. In summary, the QCD effective potential completely determines the parameters and the pattern of the GW spectrum as depicted in the next section.

Numerical results. After a complete computation for the GW spectrum, we find that neither α_* and β/H_* over the whole chiral PT can be matched with the NANOGrav measurements as shown in Fig. 4. The error bands in Figs. 4 and 5, coming from the computation of QCD sector with varying the chiral condensate by 10%, are negligible compared to the uncertainties in the gravitational wave models.

In detail, for α_* , its value is at least one order of magnitude smaller compared to the NANOGrav constraints. Such a small value is mainly due to the large energy density of the background radiation. The large energy density comes from the large chemical potentials of the leptons, which is a direct result of the conservation conditions in Eq. (1). In particular, the chemical potentials of the electron and electron neutrino are roughly same as the quark's, while those of muon neutrinos and tau neutrinos are more than three times than the quark's. For β/H_* , the value we obtained is three orders of magnitude larger than the one to match with the analysis from NANOGrav collaboration. This is mainly because the cosmic trajectory leans more toward the chemical potential, causing the derivative of S_3/T along the trajectory to be large. Meanwhile, in the Eq. (11), ρ_r outside of the derivative also maintains a large value due to the contribution of the nonvanishing chemical

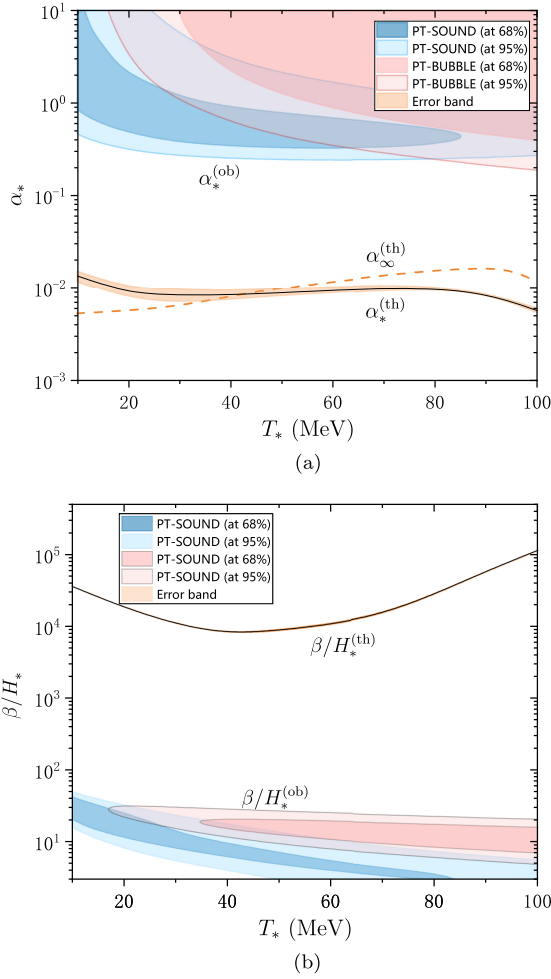


FIG. 4. (a) The PT strength α_* . (b) The inverse PT duration β/H_* . The temperature dependence of α_* and β/H_* is on the percolation line. The shaded region: the NANOGrav constraints for the PT-SOUND and the PT-BUBBLE model [63]. The superscripts of α_* and β/H_* are as follows: “th” denotes theory, and “ob” denotes observation.

potential along the percolation line. As a result, β/H_* remains significantly large.

Based on the obtained PT parameters α_* and β/H_* , we show our result of the GWs sourced from the QCD FOPT in Fig. 5. To be specific, for $T_* > 42.9$ MeV, where $\alpha < \alpha_\infty$, we only consider the PT-SOUND model from the NANOGrav. for $T_* < 42.9$ MeV, where $\alpha > \alpha_\infty$, both the bubble collision and the sound wave effects are taken into account for the GW spectrum. The quantitative computation here disapproves the QCD first-order chiral phase transition as a source of the SGWB to explain the signals detected by NANOGrav.

Conclusion. In this letter, we calculate the cosmic trajectories at large quark chemical potential, and for the first time quantitatively determine the PT parameters of the

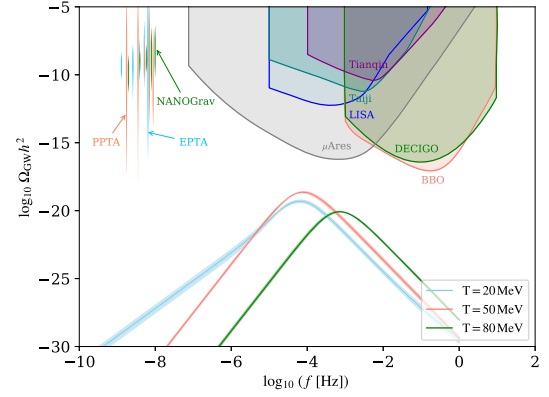


FIG. 5. The GW spectra of the QCD FOPT, where $T = [20, 50, 80]$ MeV correspond to points on the percolation line in the phase diagram. The violins plot the first five frequency bins of the datasets, including: EPTA [18], PPTA [20,108,109], and NANOGrav [16,63,110]. The sensitivity curves includes: μAres [1], Taiji [2,3], Tianqin [4,5], LISA [6–9], BBO [10,11], and DECIGO [12–14].

QCD FOPT, through their relation to the thermodynamic quantities and effective potential from the DSE.

We first verify that a large lepton asymmetry can induce the QCD FOPT. We consider the scenario ($Y_{L_e} = 0$, $Y_{L_\mu} = -Y_{L_\tau}$), and find that the FOPT occurs when $Y_{L_\mu} < -0.22$. This conclusion is essentially from the conservation laws during the Universe evolution without further assumptions. We further compute the PT parameters via the QCD effective potential, and then determine the GW spectrum through the sound shell model.

The obtained PT strength α_* is $\mathcal{O}(10^{-2})$, the inverse PT duration β/H_* is $\mathcal{O}(10^4)$. The values deviate significantly from the measurements of NANOGrav mainly due to the large energy density of the background radiation. This large energy density arises from the large chemical potentials of the leptons, which is a direct result of the conservation laws. We compare the GW spectra computed from PT parameters with violin plots and GW detector sensitivity curves. Our analysis shows that the SGWB signal from the QCD chiral FOPT is too weak for detection by NANOGrav or current and upcoming GW detectors.

Additionally, it may be worth exploring the scenario in Refs. [111–115], which provides an exit of electroweak (EW) supercooling with QCD phase transition. This involves a beyond-Standard-Model (BSM) scalar field that acts as the coefficient of the Higgs field’s quadratic term. The consequences are twofold: first, EW symmetry remains unbroken until the QCD FOPT occurs, resulting in a QCD phase transition with six massless quarks and a critical endpoint (CEP) at lower chemical potential; Second, the breaking of the Higgs and BSM fields releases significant vacuum energy, potentially exceeding the energy density of the thermal bath, which could produce detectable gravitational waves.

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