

Ultrarelativistic gravity and extremely energetic cosmic ray

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Two main factors are taken into account when describing the behavior of energetic cosmic rays near black holes: the strong magnetic field and gravity, which determines the geodesic motion of the particles. We draw attention to the third factor: since cosmic ray particles have a nonzero spin, the spin-gravity coupling can play an important role in the ultrarelativistic regime. The reaction of the spinning particle on the gravitomagnetic components of the field of the fast-moving Schwarzschild mass is considered. Some examples and numerical estimations are presented.

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In [1], attention was paid to the properties of ultrarelativistic gravity in general relativity. It was shown that the gravitational field of a fast-moving Schwarzschild's mass bears an increasing resemblance to a gravitational plane-wave field, the greater the speed of the mass. It follows from this paper that the amplitude of the wave is proportional to the second power of the relativistic Lorentz γ -factor. Later the encounters between black holes were considered in the limit that the approach velocity tends to the speed of light [2,3]. In [4], the notion of gravitational field strength is critically examined and it was shown that this concept is strongly observer dependent. In [5], the impulsive tidal acceleration of a beam of ultrarelativistic particles in the gravitational field were studied. In the recent paper [6], it is stressed that the ultrarelativistic gravitational force acting on the two particles has properties usually associated with the strong force. In [7], it is summarized that various authors have pointed out that gravitational effects can be greatly amplified in the frame of an observer moving close to the speed of light.

To describe the gravitational field of a moving Schwarzschild's mass, in [1], the local components of the Riemann tensor were used

$$R_{(\alpha)(\beta)(\gamma)(\delta)} = \lambda^\pi_{(\alpha)} \lambda^\rho_{(\beta)} \lambda^\sigma_{(\gamma)} \lambda^\tau_{(\delta)} R_{\pi\rho\sigma\tau}, \quad (1)$$

where $\lambda^\mu_{(\nu)}$ are the orthogonal tetrads (in contrast to the indices of the global coordinate, the local indices are placed in the parenthesis).

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Later, in [8], the local components of Riemann's tensor were used to determine the gravitoelectric $E^{(i)}_{(k)}$ and the gravitomagnetic $B^{(i)}_{(k)}$ components of the gravitational field, in general case, for any metric. There are corresponding expressions

$$E^{(i)}_{(k)} = R^{(i)(4)}_{(k)(4)}, \quad (2)$$

$$B^{(i)}_{(k)} = -\frac{1}{2} R^{(i)(4)}_{(m)(n)} \epsilon^{(m)(n)}_{(k)}. \quad (3)$$

In particular, the gravitoelectric components $E^{(i)}_{(k)}$ describe the tidal acceleration in Schwarzschild's field according to the relation

$$\frac{D^2 l^{(i)}}{ds^2} = E^{(i)}_{(k)} l^{(k)}. \quad (4)$$

It was shown in [5] that in the rest frame of the fast-moving composite system, the tidal acceleration could be proportional to the second power of the relativistic Lorentz factor. The similar dependence takes place for another type of the tidal acceleration, which caused by the gravitomagnetic components in Kerr's field. These components arise due to the inner rotation of the Kerr mass.

Let us consider another case of the gravitomagnetic components, which arise for the observer that is moving relative to Schwarzschild's mass. The motion of the observer can be described by the orthogonal tetrads $\lambda^\mu_{(\nu)}$. In calculations, it is convenient to choose such orientation of the space local vectors: (1) The first vector is orthogonal to the plane that is determined by the instantaneous direction of the observer motion relative to the Schwarzschild mass and the radial direction; and (2) The second vector is directed

along the direction of motion of the observer. Then using the standard Schwarzschild coordinates we obtain the nonzero components of the gravitomagnetic field

$$B_{(2)}^{(1)} = B_{(1)}^{(2)} = \frac{3Mu_{\parallel}u_{\perp}}{r^3\sqrt{u_4^4-1}}\left(1-\frac{2M}{r}\right)^{-1/2}, \quad (5)$$

$$B_{(3)}^{(1)} = B_{(1)}^{(3)} = \frac{3Mu_{\perp}^2u^4}{r^3\sqrt{u_4^4-1}}\left(1-\frac{2M}{r}\right)^{1/2}, \quad (6)$$

where $u_{\parallel} \equiv dr/ds$ and $u_{\perp} \equiv r d\varphi/ds$ are the radial and tangential components of the observer's 4-velocity, τ is its proper time, and M is Schwarzschild's mass (units $c = G = 1$ are used) [9].

Let us analyze expressions (5) and (6) at different velocities of the observer relative to Schwarzschild's mass. Note that the gravitomagnetic components are nonzero only at $u_{\perp} \neq 0$. That is, here the situation is similar to the known one in the electrodynamic for the components of the magnetic field of a moving electric charge. Then it is easy to note that the components (5) and (6) significantly depend on the velocity of an observer relative to the Schwarzschild mass. Indeed, at $|u_{\perp}| \ll 1$, $|u_{\parallel}| \ll 1$, the common term $3M/r^3$ in expressions (5) and (6) is multiplied by the corresponding small values. Whereas in the ultrarelativistic case, for $|u_{\perp}| \gg 1$, this term is multiplied by the large (as compare to 1) values and then we have

$$B_{(2)}^{(1)} = B_{(1)}^{(2)} \sim \frac{3M}{r^3}\gamma, \quad B_{(3)}^{(1)} = B_{(1)}^{(3)} \sim \frac{3M}{r^3}\gamma^2, \quad (7)$$

where γ is the relativistic Lorentz factor as calculated by the particle velocity relative to the Schwarzschild mass.

Important that it is the gravitomagnetic, and not the gravitoelectric, components that affect the motion of the spinning particle from the point of view of the comoving observer [9,10]. Indeed, we have the relation

$$a_{(i)} = \frac{s_{(1)}}{m} R_{(i)(4)(2)(3)}, \quad (8)$$

where $a_{(i)}$ are the local components of the particle 3-acceleration relative to geodesic free fall as measured by the comoving observer; $s_{(1)}$ is the single nonzero component of the particle spin, i.e., here the first local space vector is oriented along the spin; m is the particle's mass.

In a specific case of particle motion in Schwarzschild's background, when the particle's spin is orthogonal to the plane determined by the direction of particle motion and the radial direction, by (8) we write

$$a_{(i)} = \frac{s_{(1)}}{m} B_{(i)}^{(1)}, \quad (9)$$

where the nonzero values of $B_{(i)}^{(1)}$ come from (5). According to (7), in the case of highly relativistic nonradial motion, the

acceleration components (9) depend on γ such that $a_{(2)} \sim \gamma$, $a_{(3)} \sim \gamma^2$. The component $a_{(1)}$ remain equal to zero at any velocity. The absolute value of the 3-acceleration is proportional to γ^2 . So, from the point of view of the observer comoving with a spinning particle in Schwarzschild's field, the spin-gravity acceleration is much greater at the ultrarelativistic particle's velocity than at the low velocity.

Can the ultrarelativistic spin-gravity acceleration play any significant role in some astrophysical processes? In order to get closer to the answer to this question, let us make some numerical estimates. First of all, note that the value of this acceleration rapidly decreases with the distance of the spinning particle from the source of the gravitational field. The second, in general, the fast-moving particle is for a very short time in the region not too far from the source of the field. Therefore, in terms of the value of the impact of the spin-gravity acceleration on the deviation of the particle motion from the geodesic, the most favorable situation is possible when the particle moves along the circular or quasicircular orbits. Of course, provided that the existence of such orbits is allowed by the equations of motion. It is important that the ultrarelativistic circular and quasicircular orbits of the spinning particles exist in Schwarzschild's field [9]. As an example, we point out the circular orbit of the spinning particle with the radial coordinate $r = 1.5r_g$, where r_g is the horizon radius. For realization of this orbit the spinning particle must possess the tangential velocity

$$|u_{\perp}| = \frac{3^{1/4}}{\sqrt{\varepsilon}}, \quad (10)$$

where $\varepsilon \equiv |s|/Mm$, and $|s|$ is the absolute value of the particle's spin. By the known condition for the spinning test particle we take into account the relation $\varepsilon \ll 1$. Then according to (10) we have $u_{\perp}^2 \gg 1$. As a result, for the relativistic Lorentz γ -factor of the spinning particle, as calculated by an observer which is at rest relative to Schwarzschild's mass, we have $\gamma^2 \gg 1$. That is, the necessary velocity for the motion in the circular orbit with $r = 1.5r_g$ is ultrarelativistic.

It is appropriate to compare the motion of the spinning particle in the circular orbit for $r = 1.5r_g$ with the geodesic motion of the spinless particle which starts from the same position in the tangential direction and with the same initial orbital velocity. It is known from the geodesic equations that in this case the spinless particle falls on the horizon surface within a finite proper time. While the spinning particle remains permanently in the circular orbit (certainly, in the ideal case only, when perturbations are absent). It means that in this case the spin-gravity coupling acts as a strong repulsive force which compensates the usual ("geodesic") attraction. Note that the absolute value of the acceleration $|a|$ in the circular orbit with $r = 1.5r_g = 3M$ is equal to

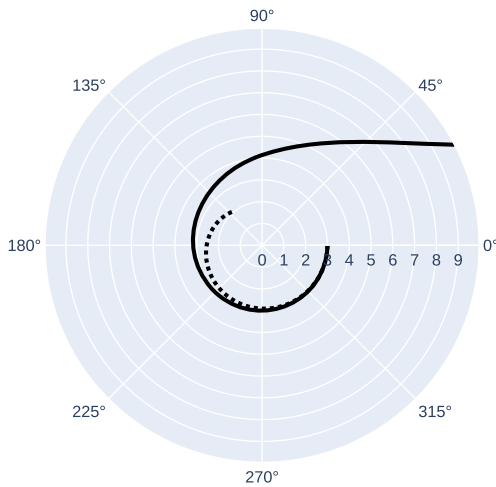


FIG. 1. Trajectories of the spinning (solid line) and the spinless particles (dotted line) for the case of the repulsive action of the spin-gravity coupling. The circle with value 2 corresponds to the horizon line.

$3^{-3/2}M^{-1}(1 + O(\epsilon))$. It is useful to compare the numerical value of this acceleration with the classical acceleration in the Newtonian theory of gravitation $a_{\text{class}} = M/r^2$. Then for $r = 1.5r_g$ we write $|a| = \sqrt{3}a_{\text{class}}(1 + O(\epsilon))$.

Figures 1 and 2 show typical cases of ultrarelativistic noncircular orbits of a spinning particle in Schwarzschild's field compared to the corresponding orbits of a spinless particle. Figure 1 describes the situation when both particles start from the position $r = 1.5r_g$ with the absolute value of the initial tangential velocity having the order defined by (10) ($u_{\perp} < 0$) and with the absolute value of the initial radial speed that satisfies the condition $|u_{\parallel}| \ll |u_{\perp}|$. In Fig. 2 initial conditions for the motion of particles differ from the case in Fig. 1 only in the sign of the tangential velocity ($u_{\perp} > 0$, for the same orientation of the particle spin). According to Fig. 1 spinless particle falls on the horizon surface, while the spinning particle moves away from the source of the gravitational field. That is, in this case, the spin-gravity coupling manifests itself as a repulsive action. While according to Fig. 2 a spinning particle falls to the horizon surface faster than a spinless particle. Here, the spin-gravity coupling is manifested as an additional attraction to the usual ("geodesic") attraction.

Drawings similar to Figs. 1 and 2 are also obtained for $r = 2r_g$ and other values $r > 1.5r_g$. Note that according to (7), the spin-gravity coupling decreases with increasing r as r^{-3} .

Do some particles in cosmic rays possess the sufficiently high γ for motions in the highly relativistic circular orbits, or on some their fragments, in the gravitational field of some black holes? By the numerical estimates for an electron in the gravitational field of a black hole with three of the Sun's mass the value ϵ is equal to 4×10^{-17} . Then the necessary value of γ for the realization of some

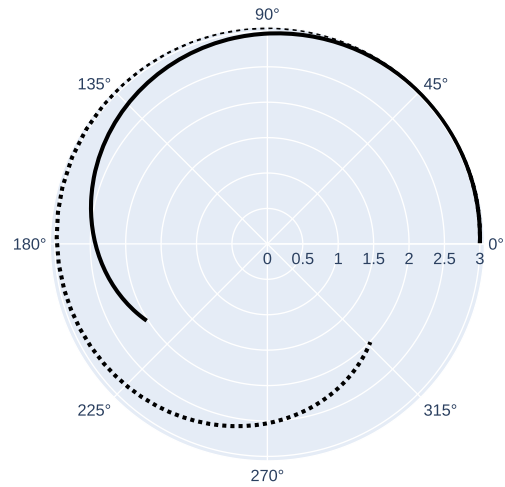


FIG. 2. Trajectories of the spinning (solid line) and the spinless particles (dotted line) for the case of the attractive action of the spin-gravity coupling.

ultrarelativistic significantly nongeodesic orbits by the electron near this black hole is of order 10^8 . This γ corresponds to the energy of the electron free motion of order 10^{14} eV. Analogously, for a proton in the field of such a black hole the corresponding energy is of order 10^{18} eV. For the massive black hole those values are greater: for example, when M is equal to 10^6 of the Sun's mass the corresponding value of the energy for an electron is of order 10^{17} eV and for a proton it is 10^{21} eV. Naturally, far from the black hole, if $r \gg 1.5r_g$, these values are greater because the necessary γ is proportional to $\sqrt{r}$.

In the context of further investigations of the possible role of the ultrarelativistic spin-gravity coupling in some astrophysical processes, it is appropriate to take into account the data which are presented in [11]. In this paper, the detection of an extremely energetic particle recorded by the surface detector array of the Telescope Array experiment is reported. A cosmic ray event with an energy of about 2.4×10^{20} eV is registered. The arrival direction of this cosmic ray points back to a void in the large-scale structure of the Universe. It is concluded in [11] that possible explanations include a large deflection by the foreground magnetic field, an unidentified source in the local extragalactic neighborhood, or an incomplete knowledge of particle physics. However, according to the above presented results in this letter, a priori, one cannot exclude another possibility: strong impact of the ultrarelativistic spin-gravity coupling on the trajectory of the extremely energetic spinning particle. We stress that the pointed above value 10^{21} eV for the proton near Schwarzschild's massive black hole is closer to 2.4×10^{20} eV. Therefore, we suppose that the black hole with M of order 10^6 could be the reason of the event which is described in [11].

Note that the influence of the spin-gravity coupling on the trajectory of a spinning particle can be significant only

during the time when the particle is in the spatial region near the black hole, limited by several values of r_g . In particular, this is illustrated by Fig. 1: after the particle approaches a value r of about $10r_g$, the influence of the spin-gravitational coupling becomes insignificant and the motion of the particle becomes practically geodesic.

So, traditionally, the two physical factors are taken into account for explanation of propagation of the extremely energetic cosmic ray near the black holes: magnetic field, and the ordinary (“geodesic”) gravity. Probably, in addition, it is appropriate to take into account the third factor: the ultrarelativistic spin-gravity coupling connected with the strong gravitomagnetic field. At the same time, according to current observational data, we cannot rule out the possibility of stronger than expected magnetic fields and even new physics.

Regarding other aspects of the existence of strong ultrarelativistic spin-gravitational coupling, we note the following. Probably, this coupling does not play any significant role in the astrophysical processes associated with the presence of accretion disks near black holes. Even though the spinning particles in the disk have orbital moment, the velocity of most of them is too small to feel the effect of the appreciable effect of the ultrarelativistic spin-gravity coupling. There is also no reason to think that this coupling has any influence on the formation of jets, since the influence of strong magnetic fields of black holes is decisive in this process.

The situation is somewhat different with the effect of electromagnetic synchrotron radiation of charged spinning particles, for example near Schwarzschild’s black hole. The intensity of this radiation is high, proportional to γ^4 [12]. Since real black holes have a strong magnetic field, its effect on the charged particle is decisive when assessing the origin of synchrotron radiation, which is registered in astrophysical observations. In order to reveal that some part of such radiation is caused not by a magnetic field, but by a strong spin-gravity coupling, it is necessary to analyze the polarization of the recorded synchrotron radiation in detail.

A potentially interesting case is the influence of a strong gravitational field on the neutrino trajectory. A neutrino with zero rest mass follows null geodesics, as a photon.

Whereas a neutrino with a nonzero mass can significantly deflects from the geodesic motion due to the influence of the ultrarelativistic spin-gravity coupling. In this context, in the following, the analysis of the data obtained by the IceCube Neutrino Observatory deserves attention.

Let us make a remark concerning the case when M of a black hole is not large, for example, it is of order 10 of the Sun’s mass. Then the space region near this black hole where ultrarelativistic spin-gravity coupling can be significant is much smaller than in the case of a black hole with M of order 10^6 of the Sun’s mass (this region is restricted approximately by $10r_g$). As a result, the time a particle stays in this region under the active action of the ultrarelativistic spin-gravity coupling is significantly shorter than in the case of a much more massive black hole. Accordingly, the deflection of the trajectory of the ultrarelativistic spinning particle from the null geodesic trajectory is much smaller. On the other hand, for a specific particle, the numerical value of the parameter ε , which is included in Eq. (10), is greater in the case of a black hole of the lower mass. Thus, the energy 10^{21} Ev for a proton passing near a lower-mass black hole is above the numerical estimates discussed on page 2 of the letter. Then it is of interest to observe possible deflection of the trajectory of an ultrarelativistic spinning particle from the null geodesic near black holes with different mass.

Note that any spinless ultrarelativistic particle follows a trajectory close to null geodesics, regardless of its mass. Whereas a spinning ultrarelativistic particle with a nonzero mass can significantly deflect from null geodesics when passing in the vicinity of a black hole. The value of this deflection depends significantly on the ratio of the particle’s spin to its mass and on the orientation of the spin (there is an effect of spatial separation of spinning particles with different spin orientation.) In the future, it is important to analyze in detail the results of astrophysical observations for spinning particles with different mass and spin orientation.

Probably, within the framework of relativistic astrophysics, an important direction of research will develop: ultrarelativistic astrophysics, which is connected with the study of ultrarelativistic spin-gravity coupling.

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