

Role of the curvaton after the last Planck dataset

Gongjun Choi^{*,†}, Wenqi Ke[‡], and Keith A. Olive[‡]

William I. Fine Theoretical Physics Institute, School of Physics and Astronomy, *University of Minnesota*,
Minneapolis, Minnesota 55455, USA



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The expected improvements in the precision of inflationary physics observables including the scalar spectral index n_s and the tensor-to-scalar ratio r will reveal more than just the viability of a particular model of inflation. In the presence of a curvaton field χ , supposedly dead models of inflation can be resurrected, as these observables are affected by curvaton perturbations. For currently successful models, improved constraints will enable us to constrain the properties of extra decaying scalar degrees of freedom produced during inflation. In this work, we demonstrate these diverse uses of a curvaton field with the most recent constraints on (n_s, r) and two exemplary inflation models, a Starobinsky-like model and a model of new inflation. The specific models used as examples can be derived in the context of supergravity. Our analysis invokes three free parameters: the curvaton mass m_χ , its decay rate Γ_χ , and the reheating temperature T_{RH} produced by inflaton decays. We systematically analyze possible postinflationary era scenarios of a curvaton field. By projecting the most recent cosmic microwave background (CMB) data on (n_s, r) into this parameter space, we can either set constraints on the curvaton parameters from successful models of inflation (so that the success is not spoiled) or determine the parameters that are able to save a model for which n_s is predicted to be below the experimental data. We emphasize that the initial value of $\langle \chi^2 \rangle \propto H^4/m_\chi^2$ produced during inflation is determined from a stochastic approach and thus not a free parameter in our analysis. We also investigate the production of local non-Gaussianity $f_{\text{NL}}^{(\text{loc})}$ and apply current CMB constraints to the parameter space. Intriguingly, we find that a large value of $f_{\text{NL}}^{(\text{loc})}$ of $\mathcal{O}(1)$ can be produced for both of the two representative inflation models.

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I. INTRODUCTION

Every model of inflation is characterized by its prediction of the cosmic microwave background (CMB) anisotropy spectrum and, in particular, the spectral index n_s of scalar perturbations and the tensor-to-scalar ratio r . As a consequence, constraints on n_s and r are used to triage models of inflation. For instance, the original Starobinsky model [1] predicts $r \sim 0.004$ for $55e$ -folds, consistent with experimental upper limit of $r < 0.036$ from a combination of Planck and BICEP/Keck [2–4]. In contrast, models of chaotic inflation [5] characterized by a quartic potential predict $r = 0.27$ and are no longer considered viable. As such, one may be tempted to rule out parameter space of a

model that violates the most recent constraints on r and $0.956 < n_s < 0.973$ at 95% confidence level (CL).

Such exclusions are justified provided that the observables n_s and r receive contributions from a single inflaton perturbation during inflation. This conclusion, however, can be hasty in cases for which n_s and r have contributions from the perturbations in other fields. The presence of a scalar spectator field χ during inflation can affect the predictions of n_s and r if its effective mass is smaller than the Hubble scale during inflation, $m_\chi \ll H_1$, and it decays after inflaton reheating. The isocurvature perturbations in χ and a fraction of the total energy contributed by χ give rise to contributions to n_s and r . Thus, whenever inflationary models are extended to include the aforementioned light spectator scalar field, or curvaton [6–8], there can be a change to viable parameter space of models consistent with constraints from CMB observables.

When constraints on n_s and r cannot be explained by a single field inflation model, the curvaton becomes particularly useful: the excluded parameter space can be resurrected with the aid of the curvaton. However, even for inflation models that can account correctly for n_s and r , it is still informative to determine the modification to n_s and r in

* Contact author: choi0988@umn.edu

† Contact author: wke@umn.edu

‡ Contact author: olive@umn.edu

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the presence of χ and constrain the viable parameter space of potential candidates for curvaton fields.

In some sense, introducing the curvaton seems to complicate our understanding for the early Universe physics because the curvaton makes the landscape of working scenarios richer. Put another way, for a given set of observables (n_s, r) , it is not possible to constrain the curvaton sector unless we clearly specify the inflaton sector. In this case, an additional observable is particularly useful. Namely, the prediction of the local-type non-Gaussianity $f_{\text{NL}}^{(\text{loc})}$. In single field inflationary models, $f_{\text{NL}}^{(\text{loc})} \sim \mathcal{O}(10^{-2})$ [9,10], and the ambiguity may be eliminated when a nonzero value of $f_{\text{NL}}^{(\text{loc})}$ is detected, as a curvaton can produce a significant amount of non-Gaussianity in the primordial density perturbation [11–13]. The current constraint on $f_{\text{NL}}^{(\text{loc})}$ from the Planck 2018 dataset is $f_{\text{NL}}^{(\text{loc})} = -0.9 \pm 5.1$ [14] at 68% CL.

In what follows, we first review the CMB observables in single inflationary models and how they are affected by the presence of a curvaton. In Sec. III, we outline the coupled set of equations of motion for the inflaton, curvaton, and radiation produced by both inflaton and curvaton decay. Since we assume $m_\chi \ll H_1$ during inflation, we use the quantum fluctuations in χ produced during inflation to determine the initial conditions for the curvaton. As a consequence, we are able to describe the possible scenarios with three parameters: the curvaton mass m_χ , decay rate Γ_χ , and the inflaton reheating temperature T_{RH} (or, equivalently, the inflaton decay rate Γ_ϕ). We assume the scale of inflation is fixed by the normalization of the CMB anisotropy spectrum. These scenarios are described in detail in Sec. IV and are distinguished by the order of events leading to curvaton oscillations and decays relative to inflaton decays. These results are applied to two very different models of inflation in Sec. V. The two examples we consider are a Starobinsky-like model [1] and a model reminiscent of new inflation [15]. The former is consistent with current CMB results, while the latter is not. There are of course many other possibilities one can consider. The general results found in Sec. IV can be applied to any model. The two models chosen lead to very different results. For consistency, both can be derived in the context of no-scale supergravity [16] as discussed in Appendix A, but this is by no means critical to our analysis. Our conclusions are found in Sec. VI.

II. COSMOLOGICAL OBSERVABLES

One of the most important attributes of inflationary theory is the fact that we can identify four observables that can test the theory's viability. One of the key predictions of inflation is the production of curvature perturbations. The normalization of the power spectrum $P_\zeta(k)$ of the curvature perturbation is fixed by the observations of the CMB anisotropies. Expanding the scalar power spectrum as

$$\ln P_\zeta(k) = \ln A_s + (n_s - 1) \ln(k/k_*) + \cdots, \quad (1)$$

Planck [2] has determined $A_s \simeq (2.10 \pm 0.03) \times 10^{-9}$ at the CMB pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$. For a given model of inflation, i.e., scalar potential for the inflaton, this measurement amounts to fixing the scale of inflation or the Hubble parameter during inflation H_1 .

The next term in the expansion of the power spectrum is the scalar tilt n_s . From our understanding of large scale structure, it was expected that the power spectrum is approximately flat with $n_s \approx 1$ [17,18] and first calculated for the Starobinsky model [1] by Mukhanov and Chibisov [19] who found a slightly red tilt, i.e., $n_s < 1$. This too has been measured to high precision by Planck [2] to be $n_s = 0.9649 \pm 0.0042$ and has played an important role in discerning between different inflation models.

In addition to the scalar power spectrum, inflation models predict a spectrum of tensor perturbations. The ratio of the tensor-to-scalar perturbations r , at the same pivot k_* , has not been measured, yet a strong upper limit of $r < 0.036$ has been set [3,4]. This upper limit has also played an important role in constraining models of inflation as discussed above. In principle, the tensor tilt n_T is also a predicted observable, but to date, as the value of r has not been measured, there is clearly no measurement or limit on n_T .

Finally, there is the possibility (particularly in multifield models of inflation) that the perturbation spectrum is not perfectly Gaussian. The degree of non-Gaussianity can be parametrized by the coefficient of the CMB bispectrum, $f_{\text{NL}}^{(\text{loc})}$. Planck has not seen evidence of non-Gaussianity and limits $f_{\text{NL}}^{(\text{loc})} = -1 \pm 5$ [14].

The inflationary observables, can be easily computed in single field models from the inflaton potential. The slow-roll parameters are defined as

$$\epsilon_\varphi \equiv \frac{1}{2} M_P^2 \left(\frac{V_\varphi}{V} \right)^2, \quad \eta_\varphi \equiv M_P^2 \left(\frac{V_{\varphi\varphi}}{V} \right), \quad (2)$$

where V is the total scalar potential and the subscripts denote the derivatives with respect to the subscript fields. For single field inflation models, V is just the inflaton potential.¹ From these, we can calculate

$$A_s \simeq \frac{V_*}{24\pi^2 \epsilon_* M_P^4}, \quad (3)$$

$$n_s = n_* \simeq 1 - 6\epsilon_* + 2\eta_*, \quad (4)$$

and

¹These definitions are general so that they may be applied to multiple fields such as the inflaton ϕ and curvaton χ with $\varphi \equiv \phi, \chi$.

$$r = r_* \simeq 16\epsilon_* \quad (5)$$

In each case the subscript $*$ refers to evaluating the slow-roll parameters for a single field model with the potential (and its derivatives) evaluated at the field value ϕ_* .

Inflation ends when the accelerated expansion of the Universe stops; that is, when $\ddot{a} = 0$ and $a = a_{\text{end}}$, where a is the cosmological scale factor. Inflation oscillations (and the matter-dominated era) begin at $a = a_{\text{end}}$. In terms of the (potential) slow-roll parameters, we have $\epsilon_\phi \simeq (1 + \sqrt{1 - \eta_\phi/2})^2$. The number of e -folds between the CMB pivot scale and the end of inflation can be computed by

$$N_* \simeq -\frac{1}{M_P^2} \int_{\phi_*}^{\phi_{\text{end}}} \frac{V}{V_\phi} d\phi. \quad (6)$$

Thus, once the inflation model is specified, for a given number of e -folds, the three observables in Eqs. (3)–(5) are calculable ($f_{\text{NL}}^{(\text{loc})}$ is negligible in this case). The relation between N_* and n_s can be improved once the inflation model is extended to include reheating as discussed in more detail below.

The presence of a massive (unstable) scalar field that comes to dominate (or close to dominating) the energy density may affect the above predictions of the inflationary model. Such a field, a curvaton, picks up isocurvature fluctuations that (upon decay) alter the CMB observables [6–8, 20–39]. In the remainder of the section, we assume the presence of a massive scalar field χ with potential $V(\chi)$.

The nonadiabatic part of the perturbation in χ is quantified by [30]

$$S_\chi \equiv 3(\zeta_\chi - \zeta_\phi), \quad (7)$$

where ζ_ϕ and ζ_χ are curvature perturbations in ϕ and χ , respectively. Then in the approximation of $\chi_{\text{osc}} \simeq \chi_{\text{inf}} + \delta\chi_*$, where χ_{inf} is the initial background field value of χ at the end of inflation (discussed in more detail in the next section), we can write [30]

$$S_\chi \simeq 2 \frac{\delta\chi_*}{\chi_{\text{inf}}}. \quad (8)$$

Then for $\delta\chi_* = H_{\text{end}}/2\pi^2$ and assuming $m_\chi \ll H_{\text{end}}$, $\chi_{\text{inf}} = \sqrt{3/8\pi^2 H_{\text{end}}^2}/m_\chi$ (see below), we have

²The value of the Hubble parameter at the end of inflation H_{end} is slightly below its value during inflation H_1 , though we do not keep this distinction here.

$$\begin{aligned} S_\chi &\simeq 2 \left(\frac{H_{\text{end}}}{2\pi} \right) \sqrt{\frac{8\pi^2}{3}} \frac{m_\chi}{H_{\text{end}}^2} \\ &\simeq 2 \sqrt{\frac{2}{3}} \frac{m_\chi}{H_{\text{end}}}. \end{aligned} \quad (9)$$

From (9), we obtain for the curvaton power spectrum

$$P_{S_\chi} \simeq \frac{8}{3} \left(\frac{m_\chi}{H_{\text{end}}} \right)^2. \quad (10)$$

When the curvaton decays, the isocurvature perturbations are transferred to curvature perturbation in the radiation with the total perturbation given by [30]

$$\zeta = R_\chi \zeta_\chi + (1 - R_\chi) \zeta_\phi, \quad (11)$$

with

$$R_\chi = \frac{3\rho_\chi}{4\rho_R + 3\rho_\chi} \Big|_{d\chi}, \quad (12)$$

evaluated when χ decays ($d\chi$). Here, ρ_R is the existing radiation density from inflaton decays. It will be useful to compute R_χ below from the ratio of the curvaton to the radiation density, and we define the factor $x \equiv (\rho_\chi/\rho_r)|_{d\chi}$ so that

$$R_\chi = \frac{3x}{4 + 3x}, \quad (13)$$

and the cases $x < 1$ ($x > 1$) will represent radiation (matter) domination scenarios at the time of curvaton decay. Then, from (7) and (11)

$$\zeta = \zeta_\phi + \frac{1}{3} R_\chi S_\chi, \quad (14)$$

and since the fluctuations in ϕ and χ are uncorrelated, we can write

$$P_\zeta = P_{\zeta_\phi} + \frac{R_\chi^2}{9} P_{S_\chi}. \quad (15)$$

As before, the total spectrum is normalized by $P_\zeta(k_*) = A_s$. If we define the ratio of the curvaton contribution to the power spectrum relative to the total as

$$\omega_\chi \equiv \frac{R_\chi^2 P_{S_\chi}}{9 P_\zeta}, \quad (16)$$

we can rewrite the CMB observables as

$$n_s = n_* + \omega_\chi (4\epsilon_* + 2\eta_\chi - 2\eta_*). \quad (17)$$

The contribution from the curvaton to the spectral index is

$$\delta n_s = \omega_\chi (4\epsilon_* + 2\eta_\chi - 2\eta_*) = \frac{16R_\chi^2}{27A_s} \left(\frac{m_\chi}{H_{\text{end}}} \right)^2 (2\epsilon_* + \eta_\chi - \eta_*), \quad (18)$$

where we used (10) and (16) for the second equality.

In addition, ω_χ in (16) enters in the modified tensor-to-scalar ratio

$$r = r_*(1 - \omega_\chi), \quad (19)$$

and the local-type non-Gaussianity parameter is

$$f_{\text{NL}}^{(\text{loc})} = \left(\frac{5}{4R_\chi} - \frac{5}{3} - \frac{5}{6}R_\chi \right) \omega_\chi^2. \quad (20)$$

III. COSMOLOGICAL EVOLUTION OF THE INFLATON AND CURVATON

Given an inflationary model, we will assume a very simple form for the curvaton potential,

$$V(\chi) = \frac{1}{2} m_\chi^2 \chi^2. \quad (21)$$

In all of our analyses below, m_χ will refer to the effective mass of the curvaton, which includes all contributions from self-interactions or interactions with other fields. For simplicity, we assume, however, that m_χ is constant. As the curvaton is required to decay (if stable, χ may comprise dark matter,³ but cannot affect the inflationary observables and is subject to constraints from isocurvature fluctuations), we must also couple the curvaton to Standard Model fields, but we are only interested in its lifetime or decay rate Γ_χ . In addition, the cosmological history will depend on the reheating temperature T_{RH} , which is determined by the inflaton couplings to Standard Model fields and can be parametrized by the inflaton decay rate Γ_ϕ . Thus, our study can be cast in terms of three parameters, which we take to be *a priori* as free: m_χ , Γ_χ , and T_{RH} .

In principle, to study the evolution of the curvaton, we must also specify its initial field value χ_i . During inflation, all scalar fields are subject to quantum fluctuations [52,53], which grow linearly in time up to an asymptotic value determined by H_I and the mass of the scalar. For $m_\chi \ll H_I$, the fluctuations characterized by $\langle \chi^2 \rangle$ can be much larger than H_I^2 and the long wavelength modes of these fluctuations that obey the classical equations of motion [54] are indistinguishable from a homogeneous scalar field.

³The cosmological evolution of stable spectator fields have been studied in [40–51].

Typically, when $\phi = \phi(a_{\text{end}}) = \phi_{\text{end}}$, the energy density in the inflaton given by $\rho_\phi(a_{\text{end}}) = \rho_{\text{end}} = \frac{3}{2} V(\phi_{\text{end}})$ is somewhat less than the inflaton energy density during inflation; hence the Hubble parameter, $H_{\text{end}} \lesssim H_1$. Therefore, we will assume that the initial value of χ is given by [25,36,51]

$$\chi_{\text{inf}}^2 = \langle \chi^2 \rangle \simeq \frac{3}{8\pi^2} \left(\frac{H_{\text{end}}}{m_\chi} \right)^2 H_{\text{end}}^2. \quad (22)$$

While one can choose to (arbitrarily) take initial field values larger than that given in (22), the quantum fluctuations produced during inflation provide a lower bound to the initial scalar field value. In all that follows, we assume the relation between χ_{inf} and m_χ in (22) and the value of H_{end} specified by the choice of the model of inflation.

The three parameters (m_χ , Γ_χ , and T_{RH}) will determine the order of events when χ begins to oscillate, denoted as a_χ , when χ decays at $a_{d\chi}$ and reheating at a_{RH} . We assume that all three events take place after the end of inflation, i.e., $a_\chi, a_{d\chi}, a_{\text{RH}} > a_{\text{end}}$. We will also assume that $a_\chi < a_{d\chi}$.⁴

The equations of motion for the two scalars are

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0, \quad (23)$$

$$\ddot{\chi} + 3H\dot{\chi} + \Gamma_\chi\dot{\chi} + m_\chi^2\chi = 0. \quad (24)$$

In terms of the energy densities, we can write

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi\rho_\phi = 0, \quad (25)$$

$$\dot{\rho}_\chi + 3H\rho_\chi + \Gamma_\chi\rho_\chi = 0, \quad (26)$$

$$\dot{\rho}_\text{R} + 4H\rho_\text{R} = \Gamma_\phi\rho_\phi + \Gamma_\chi\rho_\chi, \quad (27)$$

together with

$$\rho_\phi + \rho_\chi + \rho_\text{R} = 3H^2 M_P^2, \quad (28)$$

where ρ_R is the energy density of radiation produced by inflaton (or curvaton) decay, $H = \dot{a}/a$, and $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. We have also assumed that the equation of state for the inflaton and curvaton (postinflation) during their oscillatory phase is pressureless ($P_\phi/\rho_\phi = 0$, $\phi = \phi, \chi$), which is valid if the inflaton oscillations are governed by a quadratic term about its potential minimum.

⁴This amounts to assuming a perturbative decay of χ . If Γ_χ is parametrized by a Yukawa-like interaction with $\Gamma_\chi = (y_\chi^2/8\pi)m_\chi$, $a_\chi < a_{d\chi}$ is equivalent to assuming $\Gamma_\chi < m_\chi$.

Prior to inflaton reheating, the expansion rate is dominated by the energy density in inflaton oscillations and the solution to (25) for $\Gamma_\phi \ll H$ is

$$\rho_\phi(a) = \rho_{\text{end}} \left(\frac{a_{\text{end}}}{a} \right)^3. \quad (29)$$

We also assume that the inflationary sector is coupled to the Standard Model so as to account for reheating. As the inflaton starts to decay, the radiation density quickly rises to a maximum at $a = a_{\text{max}} = (8/3)^{2/3} a_{\text{end}}$ [55,56] and the radiation density prior to reheating evolves as

$$\rho_R(a) = \frac{2\sqrt{3}}{5} \Gamma_\phi M_P \sqrt{\rho_{\text{end}}} \left(\frac{a_{\text{end}}}{a} \right)^4 \left[\left(\frac{a}{a_{\text{end}}} \right)^{5/2} - 1 \right], \quad (30)$$

for $a_{\text{end}} < a < a_{\text{RH}}$ where for now we have ignored any possible contribution to ρ_R from χ decays.

We define the reheating temperature as the temperature of the radiation bath (produced by inflaton decays) when the energy density of the radiation and the energy density left in the inflaton condensate are equal at $a = a_{\text{RH}}$, i.e., $\rho_\phi(a_{\text{RH}}) = \rho_R(a_{\text{RH}})$. Using (29) and (30), one finds

$$\left(\frac{a_{\text{RH}}}{a_{\text{end}}} \right)^{3/2} \simeq \frac{5\sqrt{\rho_{\text{end}}}}{2\sqrt{3}\Gamma_\phi M_P}, \quad (31)$$

and therefore

$$\rho_{\text{RH}} = \rho_\phi(a_{\text{RH}}) = \rho_{\text{end}} \left(\frac{a_{\text{end}}}{a_{\text{RH}}} \right)^3 = \frac{12}{25} (\Gamma_\phi M_P)^2 \equiv \alpha T_{\text{RH}}^4, \quad (32)$$

where $\alpha = g_{\text{RH}} \pi^2/30$ and $g_{\text{RH}} = 427/4$ is the number of Standard Model degrees of freedom at sufficiently high temperature. For $a > a_{\text{RH}}$, we can write

$$\rho_R(a) = \rho_{\text{RH}} \left(\frac{a_{\text{RH}}}{a} \right)^4. \quad (33)$$

Finally, for $a > a_\chi$, the energy density in χ is given by

$$\rho_\chi(a) = \frac{1}{2} m_\chi^2 \chi_{\text{inf}}^2 \left(\frac{a_\chi}{a} \right)^3 = \frac{3}{16\pi^2} H_{\text{end}}^4 \left(\frac{a_\chi}{a} \right)^3, \quad (34)$$

where the second equality comes from (22). For $a_{\text{end}} < a < a_\chi$, ρ_χ remains constant with $\rho_\chi = \frac{3}{16\pi^2} H_{\text{end}}^4$. χ oscillations begin at $a = a_\chi$ defined by $\frac{3}{2}H = m_\chi$ when the total energy density is dominated by matter and by $2H = m_\chi$ when the energy density is dominated by radiation. The determination of a_χ will thus depend on whether or not the inflaton has decayed. Similarly, χ decays occur when $a = a_{d\chi}$ defined by $\frac{3}{2}H = \Gamma_\chi$ or $2H = \Gamma_\chi$ for matter and radiation domination (MD and RD)

respectively.⁵ Like a_χ , the determination of $a_{d\chi}$ will depend on Γ_ϕ .

Given the above, there are three logical possibilities:

- (1) $a_{\text{end}} < a_\chi < a_{d\chi} < a_{\text{RH}}$.
- (2) $a_{\text{end}} < a_\chi < a_{\text{RH}} < a_{d\chi}$.
- (3) $a_{\text{end}} < a_{\text{RH}} < a_\chi < a_{d\chi}$.

In the first case, though we will derive the ratios a_χ/a_{end} and $a_{d\chi}/a_{\text{end}}$ for completeness, this case is not particularly interesting, as χ decays as a subdominant species before reheating. In order to avoid a second period of inflation, we enforce $\rho_\chi(a_\chi) < \rho_\phi(a_\chi)$. When χ starts to oscillate, the energy of the Universe is still dominated by the inflaton, and the ratio ρ_ϕ/ρ_χ remains constant until $a = a_{d\chi}$. For $a_{d\chi} < a < a_{\text{RH}}$, the ratio $\rho_{r\chi}/\rho_\phi$ decreases until a_{RH} with $\rho_{r\chi}$ the energy density of radiation from χ decay. This means the matter domination era continues for $a_{\text{end}} < a < a_{\text{RH}}$ for case 1. As χ or its decay products never come to dominate the energy of Universe until a_{RH} , χ cannot make a significant contribution to n_s and r . In the latter two cases, we will further distinguish whether or not χ decays during a matter- (χ -dominated) or radiation-dominated era.

IV. SCENARIOS

In this section, we will consider the three evolutionary scenarios classified by different values of three free parameters,

$$\text{Free parameters: } m_\chi, \Gamma_\chi, T_{\text{RH}}. \quad (35)$$

Every choice of $(m_\chi, \Gamma_\chi, T_{\text{RH}})$ will correspond to a specific evolutionary scenario and thus to unique predictions for the inflationary observables.

A. Case I

As noted earlier, bearing in mind standard cold inflationary models, we assume that the Universe is dominated by the inflaton when χ oscillations begin. Then, the expansion rate for $a_{\text{end}} < a < a_\chi$ is given by $H(a)^2 = \rho_\phi(a)/3M_P^2$. Oscillations of χ begin when $\frac{3}{2}H = m_\chi$. Similarly, χ decays occur when $\frac{3}{2}H = \Gamma_\chi$. From these, the scale factors when χ oscillations begin and when χ decays are easily obtained,

$$\left(\frac{a_\chi}{a_{\text{end}}} \right)^3 = \frac{3}{4} \frac{\rho_{\text{end}}}{m_\chi^2 M_P^2}, \quad (36)$$

and

⁵The coefficient of H relative to m_χ or Γ_χ depends on the ratio of the radiation to matter density z , which tends to $\frac{3}{2}$ and 2 at large and small z , respectively. See Appendix B for the functional form of this coefficient.

$$\left(\frac{a_{d\chi}}{a_{\text{end}}}\right)^3 = \frac{3}{4} \frac{\rho_{\text{end}}}{\Gamma_\chi^2 M_P^2}. \quad (37)$$

Demanding the absence of the second inflationary phase driven by χ will require that the energy density of χ does not dominate the total energy before χ begins its oscillations. From (29), (34), and (36), this leads to

$$\frac{3}{16\pi^2} H_{\text{end}}^4 < \frac{4}{3} m_\chi^2 M_P^2 < \rho_{\text{end}}. \quad (38)$$

The first of these inequalities comes from $\rho_\chi(a_\chi) < \rho_\phi(a_\chi)$. The second inequality in Eq. (38) is then derived simply from $a_{\text{end}} < a_\chi$. A violation of this inequality, namely, $m_\chi < 3H_{\text{end}}^2/8\pi M_P$, would lead to a second period of inflation driven by χ . Clearly we must have $H_{\text{end}}^4 < 16\pi^2 \rho_{\text{end}}/3$ to allow a finite mass range in m_χ .⁶

The energy density of χ at the time of decay is

$$\begin{aligned} \rho_\chi|_{d\chi} &= \frac{1}{2} m_\chi^2 \chi_{\text{inf}}^2 \left(\frac{a_\chi}{a_{d\chi}}\right)^3 = \frac{3}{16\pi^2} H_{\text{end}}^4 \left(\frac{a_\chi}{a_{d\chi}}\right)^3 \\ &= \frac{3}{16\pi^2} H_{\text{end}}^4 \frac{\Gamma_\chi^2}{m_\chi^2}, \end{aligned} \quad (39)$$

which can be compared with the energy density in inflaton oscillations at the same time

$$\rho_\phi|_{d\chi} = \rho_{\text{end}} \left(\frac{a_{\text{end}}}{a_{d\chi}}\right)^3 = \frac{4}{3} \Gamma_\chi^2 M_P^2. \quad (40)$$

Since $a_\chi < a_{d\chi}$, we must have $\Gamma_\chi < m_\chi$ and for $a_{d\chi} < a_{\text{RH}}$, we must have $\Gamma_\chi > \sqrt{3\alpha} T_{\text{RH}}^2/2M_P$. Putting these two constraints together, this case must satisfy

$$\frac{5\sqrt{\alpha} T_{\text{RH}}^2}{\sqrt{12} M_P} = \frac{3}{5} \Gamma_\phi < \Gamma_\chi < m_\chi. \quad (41)$$

However, as noted earlier, since neither χ nor its decay products provide a significant contribution to the total energy ($\omega_\chi \ll 1$), this case will not be able to affect the inflationary observables.

B. Case II

In this case, the ratio a_χ/a_{end} is again given by Eq. (36). When χ decays, the Universe could be in either a RD era or a MD era.

⁶Throughout, by m_χ , we mean the effective mass of m_χ . In the absence of self-interactions or couplings to the inflaton $m_{\chi,\text{eff}} = m_\chi$. Including these interactions would increase $m_{\chi,\text{eff}}$ and therefore decrease $\langle \chi_{\text{inf}}^2 \rangle$ and ρ_χ . Thus, bare masses in violation of the left side of the inequality in (38) simply require interaction which force the effective mass to satisfy (38).

Case II-RD. If χ decays occur in a RD background with $H(a > a_{\text{RH}})^2 = (\rho_{\text{RH}}/3M_P^2)(a_{\text{RH}}/a)^4$ one finds

$$\left(\frac{a_{d\chi}}{a_{\text{end}}}\right)^3 = \left(\frac{4}{3}\right)^{\frac{3}{4}} \frac{\rho_{\text{end}}}{\alpha^{\frac{1}{4}} T_{\text{RH}} (\Gamma_\chi M_P)^{\frac{3}{2}}}. \quad (42)$$

At $a = a_{d\chi}$, the energy density in χ is

$$\begin{aligned} \rho_\chi|_{d\chi} &= \frac{1}{2} m_\chi^2 \chi_{\text{inf}}^2 \left(\frac{a_\chi}{a_{d\chi}}\right)^3 = \frac{3}{16\pi^2} H_{\text{end}}^4 \left(\frac{a_\chi}{a_{d\chi}}\right)^3 \\ &= \frac{3}{16\pi^2} \left(\frac{3}{4}\right)^{\frac{7}{4}} H_{\text{end}}^4 \frac{\alpha^{\frac{1}{4}} T_{\text{RH}} \Gamma_\chi^{\frac{3}{2}}}{m_\chi^2 M_P^{\frac{1}{2}}}, \end{aligned} \quad (43)$$

which can be compared to the energy density in radiation at the same time

$$\rho_r|_{d\chi} = \rho_{\text{RH}} \left(\frac{a_{\text{RH}}}{a_{d\chi}}\right)^4 = \frac{3}{4} \Gamma_\chi^2 M_P^2. \quad (44)$$

From Eq. (44), we also see that χ decays occur before big bang nucleosynthesis (BBN) if $\Gamma_\chi \gtrsim 10^{-22}$ GeV.

To compute R_χ , from (43) and (44) we first compute

$$x = \frac{\rho_\chi}{\rho_r}|_{d\chi} = \frac{3}{16\pi^2} \left(\frac{3}{4}\right)^{\frac{3}{4}} \frac{H_{\text{end}}^4 \alpha^{\frac{1}{4}} T_{\text{RH}}}{m_\chi^2 \Gamma_\chi^{\frac{1}{2}} M_P^{\frac{5}{2}}}. \quad (45)$$

These results hold so long as

$$(i): \frac{3}{4} \Gamma_\chi M_P < \frac{1}{2} \sqrt{3\alpha} T_{\text{RH}}^2 < m_\chi M_P < \frac{1}{2} \sqrt{3\rho_{\text{end}}}, \quad (46)$$

from $a_{\text{end}} < a_\chi < a_{\text{RH}} < a_{d\chi}$ and

$$(ii\text{-RD}): \Gamma_\chi > \frac{27\sqrt{3}}{2048\pi^4} \frac{H_{\text{end}}^8 \sqrt{\alpha} T_{\text{RH}}^2}{m_\chi^4 M_P^5}. \quad (47)$$

from $\rho_\chi(a_{d\chi}) < \rho_r(a_{d\chi})$, i.e., $x < 1$.

Case II-MD. For sufficiently small Γ_χ , χ oscillations will come to dominate over the radiation from inflaton decays, leading to a second period of matter domination. Matter radiation equality occurs at a_{eq} when

$$\begin{aligned} \rho_\chi(a_{\text{eq}}) &= \frac{3}{16\pi^2} H_{\text{end}}^4 \left(\frac{a_\chi}{a_{\text{eq}}}\right)^3 \\ &= \alpha T_{\text{RH}}^4 \left(\frac{a_{\text{RH}}}{a_{\text{eq}}}\right)^4 = \rho_r(a_{\text{eq}}), \end{aligned} \quad (48)$$

leading to

$$\frac{a_{\text{RH}}}{a_{\text{eq}}} = \frac{9}{64\pi^2} \frac{H_{\text{end}}^4}{m_\chi^2 M_P^2}. \quad (49)$$

When χ decays in a MD period, we must recompute $a_{d\chi}$ and we obtain

$$\left(\frac{a_{d\chi}}{a_{\text{end}}}\right)^3 = \frac{81}{256\pi^2} \frac{H_{\text{end}}^6}{m_\chi^2 \Gamma_\chi^2 M_P^2}. \quad (50)$$

Then $a_{d\chi} > a_{\text{eq}}$ is equivalent to

$$\text{(ii-MD): } \Gamma_\chi < \frac{81\sqrt{3}}{8192\pi^4} \frac{H_{\text{end}}^8 \sqrt{\alpha} T_{\text{RH}}^2}{m_\chi^4 M_P^5}. \quad (51)$$

Recomputing x , we find

$$x = \left(\frac{3}{4}\right) \left(\frac{9}{64\pi^2}\right)^4 \frac{\alpha T_{\text{RH}}^4 H_{\text{end}}^{16}}{m_\chi^8 \Gamma_\chi^2 M_P^{10}}. \quad (52)$$

When χ decays in a MD era, there is entropy production quantified by $s_f/s_i = x^{\frac{3}{4}}$ and a new reheating temperature determined by Γ_χ . For successful BBN, we again require $\Gamma_\chi \gtrsim 10^{-22}$ GeV. The amount of entropy production depends on T_{RH} and H_{end} and m_χ through x in (52).

In sum, (46) applies to both of the cases II-RD and II-MD. The two cases differ in the evaluation of x in (45) and (52) and the conditions on Γ_χ (47) and (51).⁷

C. Case III

Case III differs from Case II in that reheating occurs earlier than the onset of χ oscillations, i.e., when $a_\chi > a_{\text{RH}}$ or when $\frac{4}{5}\Gamma_\phi > m_\chi$. To determine when oscillations begin, we must use the expression of Hubble expansion rate in a RD era and we find

$$\left(\frac{a_\chi}{a_{\text{RH}}}\right)^4 = \frac{4}{3} \frac{\alpha T_{\text{RH}}^4}{m_\chi^2 M_P^2}. \quad (53)$$

Case III-RD. If the Universe is in a RD era when χ decays

$$\left(\frac{a_{d\chi}}{a_{\text{RH}}}\right)^4 = \frac{4}{3} \frac{\alpha T_{\text{RH}}^4}{\Gamma_\chi^2 M_P^2}. \quad (54)$$

At $a_{d\chi}$, we have

⁷More accurately, as $a_{d\chi} \rightarrow a_{\text{eq}}$, the expressions should be subject to the rescalings $\Gamma_\chi \rightarrow 2\Gamma_\chi/\beta_x$ and $\Gamma_\chi \rightarrow 3\Gamma_\chi/2\beta_x$, for radiation- and matter-dominated eras, respectively, where $\beta_x = \frac{3}{4}(\sqrt{2} + 1)$. This is because at a_{eq} , $Ht = 1/\beta$ rather than $(1/2)$ or $(2/3)$. Similarly, as $a_\chi \rightarrow a_{\text{RH}}$, the expressions in case II and case III should be rescaled by $m_\chi \rightarrow 3m_\chi/2\beta_y$ and $m_\chi \rightarrow 2m_\chi/\beta_y$. Away from $a_{d\chi} = a_{\text{eq}}$ or $a_\chi = a_{\text{RH}}$, the same rescalings apply, with the functional form of β_x and β_y given in Appendix B.

$$\rho_\chi(a_{d\chi}) = \frac{3}{16\pi^2} H_{\text{end}}^4 \left(\frac{a_\chi}{a_{d\chi}}\right)^3 = \frac{3}{16\pi^2} H_{\text{end}}^4 \left(\frac{\Gamma_\chi}{m_\chi}\right)^{\frac{3}{2}}. \quad (55)$$

By comparing (55) to $\rho_r(a_{d\chi})$, which is still given by Eq. (44), one obtains

$$x = \frac{1}{4\pi^2} \frac{H_{\text{end}}^4}{m_\chi^{\frac{3}{2}} \Gamma_\chi^{\frac{1}{2}} M_P^2}. \quad (56)$$

These results hold so long as

$$\text{(i): } \Gamma_\chi < m_\chi < \frac{2}{\sqrt{3}} \frac{\sqrt{\alpha} T_{\text{RH}}^2}{M_P} < \frac{2}{\sqrt{3}} \frac{\sqrt{\rho_{\text{end}}}}{M_P}, \quad (57)$$

from the defining condition for case III, i.e., $a_{\text{end}} < a_{\text{RH}} < a_\chi < a_{d\chi}$ and

$$\text{(ii-RD): } \frac{H_{\text{end}}^8}{16\pi^4 m_\chi^3 M_P^4} < \Gamma_\chi, \quad (58)$$

from $\rho_\chi(a_{d\chi}) < \rho_r(a_{d\chi})$, i.e., $x < 1$.⁸ As in Case II, χ decays before BBN imply $\Gamma_\chi \gtrsim 10^{-22}$ GeV.

Case III-MD. Once again, for sufficiently small Γ_χ (late χ decay), it is also possible for χ to decay in a MD era. This occurs if $a_\chi < a_{\text{eq}} < a_{d\chi}$ is satisfied. From (53) and $\rho_R(a_{\text{eq}}) = \rho_\chi(a_{\text{eq}})$, one can find

$$\frac{a_{\text{RH}}}{a_{\text{eq}}} = \frac{3}{16\pi^2} \left(\frac{4}{3}\right)^{\frac{3}{4}} \frac{H_{\text{end}}^4}{\alpha^{\frac{1}{4}} T_{\text{RH}} m_\chi^{\frac{3}{2}} M_P^{\frac{3}{2}}}. \quad (59)$$

Recomputing $a_{d\chi}$ from (53) and $(3/2)H(a_{d\chi}) = \Gamma_\chi$, one finds

$$\left(\frac{a_{d\chi}}{a_{\text{RH}}}\right)^3 = \frac{3}{16\pi^2} \left(\frac{3}{4}\right)^{\frac{1}{4}} \frac{H_{\text{end}}^4 \alpha^{\frac{3}{4}} T_{\text{RH}}^3}{m_\chi^{\frac{3}{2}} \Gamma_\chi^2 M_P^{\frac{7}{2}}}. \quad (60)$$

The condition $a_{\text{eq}} < a_{d\chi}$ is equivalent to

$$\text{(ii-MD): } \Gamma_\chi < \frac{3H_{\text{end}}^8}{64\pi^4 m_\chi^3 M_P^4}. \quad (61)$$

Then, the comparison of $\rho_R(a_{d\chi})$ to $\rho_\chi(a_{d\chi})$ yields

$$x = \left(\frac{9}{4096\pi^8} \frac{H_{\text{end}}^{16}}{m_\chi^6 \Gamma_\chi^2 M_P^8}\right)^{\frac{1}{3}}. \quad (62)$$

As in case II-MD, there is entropy production and a new reheating temperature.

In sum, (57) applies to both of the cases III-RD and III-MD. Two cases differ in the evaluation of x in (56) and (62)

⁸The same condition can be obtained from $a_{\text{eq}} > a_{d\chi}$.

and the conditions (58) and (61). Notice that expressions of x of both cases are independent of T_{RH} .

V. APPLICATION TO MODELS

The discussion in the previous section is model independent. As we discussed above, the parameter space under consideration is described by T_{RH} , m_χ , and Γ_χ . Once specified, each set of these three parameters will correspond to a model with an evolution described by cases I, II, or III (and whether or not curvaton decay occurs in a RD or MD universe). However, the precise boundaries of these cases also depends on H_{end} and therefore is not completely model independent.

Furthermore, the derived constraints on the curvaton scenario, in particular, its mass range and decay rate, depend on the specific inflation model. While R_χ and ω_χ only carry the dependence on H_{end} , the observables n_s and r depend on the inflation slow-roll parameters, and even the differences δn_s and $\delta r = -r_* \omega_\chi$ depend explicitly on ϵ_* and η_* .

Here, we illustrate the curvaton constraints in two well-motivated examples: the first is a Starobinsky-like model [1], which is in good agreement with CMB bounds, and the second is a new inflation model [15], which predicts a value of n_s significantly below the measured value. In the former model, the curvaton is constrained so that it does not upset the already good agreement with experiment. Nevertheless, for certain parameters, the already good agreement is actually improved. For the latter, the contribution from the curvaton becomes important, as it is capable of raising the prediction in n_s up to the experimental value.

The Starobinsky model, when derived directly from a $R + R^2$ theory of gravity, has a coupling to the Higgs boson in the Einstein frame. As a consequence, there is a well-defined reheating temperature (so long as the Higgs boson is not conformally coupled to curvature) of $T_{\text{RH}} \simeq 3 \times 10^9$ GeV (see, for example, [57]). We could apply our results to this specific value, but we note that a model derived from no-scale supergravity [16] will lead to the exact same potential (see, for example, [58]), but with an unspecified reheating temperature. This allows us to use more general aspects of our analysis. Since we are also able to derive a model of new inflation in the same context, we use a supergravity origin for this as well. These derivations are given in Appendix A. However, we stress that in no way are the results that follow directly tied to supergravity and are based only on the properties of the scalar potential. Of course, choosing these two models is not exhaustive but is representative of two very different conclusions in the absence of a curvaton.

The new inflation model we employ shares many of the characteristics of the original new inflation model [15] but avoids many of the issues that plagued the model when derived from a nonsupersymmetric SU(5) theory. In this model, the inflaton evolves from a very small field value

($\phi \ll M_P$) to a minimum at $\phi \approx M_P$. The scalar potential in this case is

$$V_n(z) \simeq M^2 M_P^2 (z^3 - 1)^2, \quad (63)$$

where $z = \phi/M_P$ is the canonical inflaton field value normalized by the Planck mass. This potential is chosen as is qualitatively similar to the original new inflation model [15] and is discussed further in Appendix A. The inflaton mass for this potential is $m_\phi = \sqrt{18}M$. For $N_* = 55$, the new inflation model gives an extremely small tensor-to-scalar ratio $r \simeq 2.4 \times 10^{-8}$, whereas the scalar spectral index is $n_s \simeq 0.928$, significantly lower than the CMB determination. For this reason, this type of model of inflation was considered excluded.

In the Starobinsky model, the inflaton evolves from a plateau at large field values to a minimum at $\phi = 0$. The potential in this case is

$$V_S(z) = \frac{3}{4} M^2 M_P^2 \left(1 - e^{-\sqrt{\frac{2}{3}}z}\right)^2, \quad (64)$$

where again $z = \phi/M_P$ is the inflaton normalized by the Planck scale. In this case, the inflaton mass is $m_\phi = M$. For $N_* = 55$, we have $r \simeq 3.5 \times 10^{-3}$ and $n_s \simeq 0.965$, both in good agreement with CMB observations. Here, we emphasize that we will allow for an arbitrary reheating temperature.

To see the dependence of the cosmological observables on the reheating temperature, recall that the number of e -folds can be computed by [59,60]

$$N_* = \ln \left[\frac{1}{\sqrt{3}} \left(\frac{\pi^2}{30} \right)^{1/4} \left(\frac{43}{11} \right)^{1/3} \frac{T_0}{H_0} \right] - \ln \left(\frac{k_*}{a_0 H_0} \right) - \frac{1}{12} \ln g_{\text{RH}} + \frac{1}{4} \ln \left(\frac{V_*^2}{M_{\text{Pl}}^4 \rho_{\text{end}}} \right) + \ln R_{\text{rad}}, \quad (65)$$

where $H_0 = 67.36 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the present day Hubble parameter, and we take $a_0 = 1$. The last term above is the reheating parameter, which we approximate as

$$\ln R_{\text{rad}} \simeq \frac{1}{6} \ln \left(\frac{\Gamma_\phi}{H_{\text{end}}} \right). \quad (66)$$

The reheating temperature is related to the inflaton decay rate through Eq. (32). For concreteness, we consider reheating through a Yukawa-like coupling y , so the inflaton decay rate is given by

$$\Gamma_\phi = \frac{y_\phi^2}{8\pi} m_\phi, \quad (67)$$

where m_ϕ is the inflaton mass.

Insisting on a perturbative coupling forbids very high values of T_{RH} . The prediction of n_s in a given model is

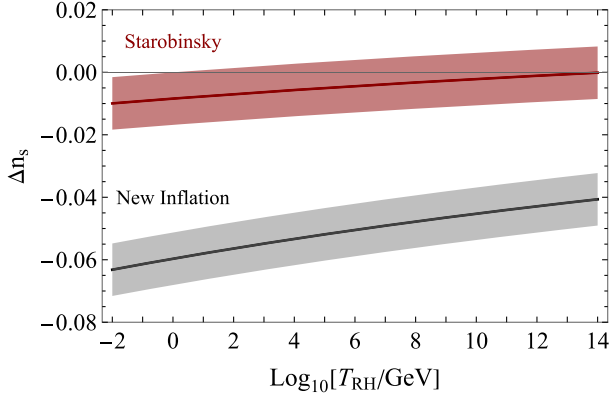


FIG. 1. The difference, $\Delta n_s = n_* - n_{s,\text{CMB}}$, between the model prediction of n_s from new inflation and the Starobinsky model and the value determined from CMB observations [61]. We take $n_{s,\text{CMB}} = 0.9649$ (solid lines) with $\pm 2\sigma = 0.0084$ standard deviations (shaded regions).

obtained by (4), and its dependence on T_{RH} enters through the determination of N_* [through Eqs. (65) and (66)]. In Fig. 1, we show as a function of T_{RH} the deviation Δn_s of the predicted n_s of the Starobinsky model and new inflation model without introducing the curvaton from the observed $n_{s,\text{CMB}}$ from the CMB temperature anisotropy power spectrum [61]. The solid lines show the difference between n_s and the Planck central value of $n_{s,\text{CMB}} = 0.9649$ and the band corresponds to the difference between n_s and the $2\sigma = 0.0084$ range. For all T_{RH} below the Planck scale, the new inflation model prediction of n_s is always below experimental constraints $0.958 < n_s < 0.975$, with the discrepancy being of order 0.05. In contrast, the Starobinsky model alone can successfully account for the observed n_s within 2σ for any $T_{\text{RH}} \gtrsim 4$ GeV and, for sufficiently high T_{RH} , even the central Planck value can be attained. Therefore, if the inflaton sector of these models is extended by including the curvaton, from (18), we expect that the new inflation model requires a much more weakly coupled curvaton (smaller Γ_χ) than the Starobinsky model for a fixed T_{RH} and m_χ , since smaller Γ_χ corresponds to a later decay of χ and is expected to yield a larger value of R_χ .

From Eq. (18), we can compute the change in the tilt δn_s and, from Eq. (19), the change in the tensor-to-scalar ratio δr . Notice first that the shift in r is always toward lower r . The shift in n_s is more model dependent. However, $\eta_\chi = M_P^2(m_\chi^2/V)$. For the Starobinsky model, this gives $\eta_\chi = \frac{4}{3}m_\chi^2/m_\phi^2$ and for the new inflation model, $\eta_\chi = 18m_\chi^2/m_\phi^2$. In both cases, the contribution is positive and presumably small (for $m_\chi \ll m_\phi$). For $N_* = 55$, $\epsilon_* \simeq 0.00022$ and $\eta_* = -0.0169$ for the Starobinsky model and $\epsilon_* \simeq 1.5 \times 10^{-9}$ and $\eta_* = -0.036$ for the new inflation model. In both cases, δn_s is always positive and it may be possible to find a set of parameters such that $\delta n_s + \Delta n_s = n_s - n_{s,\text{CMB}} = 0$.

Given the classifications of the different cosmological scenarios from a_{end} to $a_{d\chi}$ in Sec. IV, for each T_{RH} , one can specify a pair of curvaton parameters (m_χ, Γ_χ) that enables the net n_s in (17) to perfectly fit the observed scalar spectral index from the CMB. In Fig. 2, for two specific reheating temperatures in each of the two inflationary models considered, we show the result of probing the (m_χ, Γ_χ) parameter space in which the curvaton provides a solution to $\delta n_s + \Delta n_s = 0$, that is where the curvaton contribution to n_s can bridge a deficit in the model. Recall that since δn_s is always positive, such a solution only exists if $\Delta n_s < 0$.

The upper two panels in Fig. 2 show results for the Starobinsky model for fixed $T_{\text{RH}} = 10^5$ GeV (left) and 10^{13} GeV (right). The lower panels show the results for the new inflation model with $T_{\text{RH}} = 10^9$ GeV (left) and 10^{13} GeV (right). The background shaded regions distinguish case II-RD (blue), II-MD (red), III-RD (cyan), and III-MD (yellow). Case I corresponding to $a_{d\chi} < a_{\text{RH}}$ is found in the unshaded region at high Γ_χ . The unshaded regions at small m_χ are not viable because a second period of inflation ensues. We also delineate the region where χ decays before the onset of oscillations ($a_\chi > a_{d\chi}$).

As discussed earlier, we take $\Gamma_\chi \gtrsim 10^{-22}$ GeV to insure that χ decay in a RD era occurs before the temperature of the primordial plasma reaches $T \simeq 10$ MeV and that the new reheating temperature is sufficiently high when χ decays in a MD era. This guarantees that light element production during BBN is not spoiled by late time χ decay. Since case III requires early inflaton reheating (to ensure $a_{\text{RH}} < a_\chi$), this case is only realized for sufficiently high T_{RH} . This explains why the yellow and cyan shaded background appear only for higher T_{RH} (seen in the right panels).

We also noted earlier that, when χ decays and dominates the energy density, entropy is produced during this second period of reheating. The amount of entropy production is determined by the value of x , that is the ratio ρ_χ/ρ_r and $s_f/s_i = x^{\frac{3}{4}}$. For very large x , even if BBN proceeds normally, a large amount of entropy production can over-dilute any existing baryon asymmetry. If we assume a maximal asymmetry of $n_B/s \sim 10^{-2}$, then the largest allowed value of x is $\sim 5 \times 10^{10}$. This constant value of x appears as a thick black diagonal line and is shown on the upper right panel in Fig. 2. All points below this line are excluded as the entropy production becomes excessive. For the new inflation model, $x < 5 \times 10^{10}$ holds for all of the displayed blue- and cyan-shaded regions corresponding to II, III-MD.

The regions in the (m_χ, Γ_χ) plane with acceptable values of n_s are shown by the gray bands in Fig. 2. However, the meaning of the two bands is different for the Starobinsky and new inflation models. For the Starobinsky model, the left/upper side of the band corresponds to $\Delta n_s + \delta n_s = 0$, whereas the right/lower edge of the band corresponds to

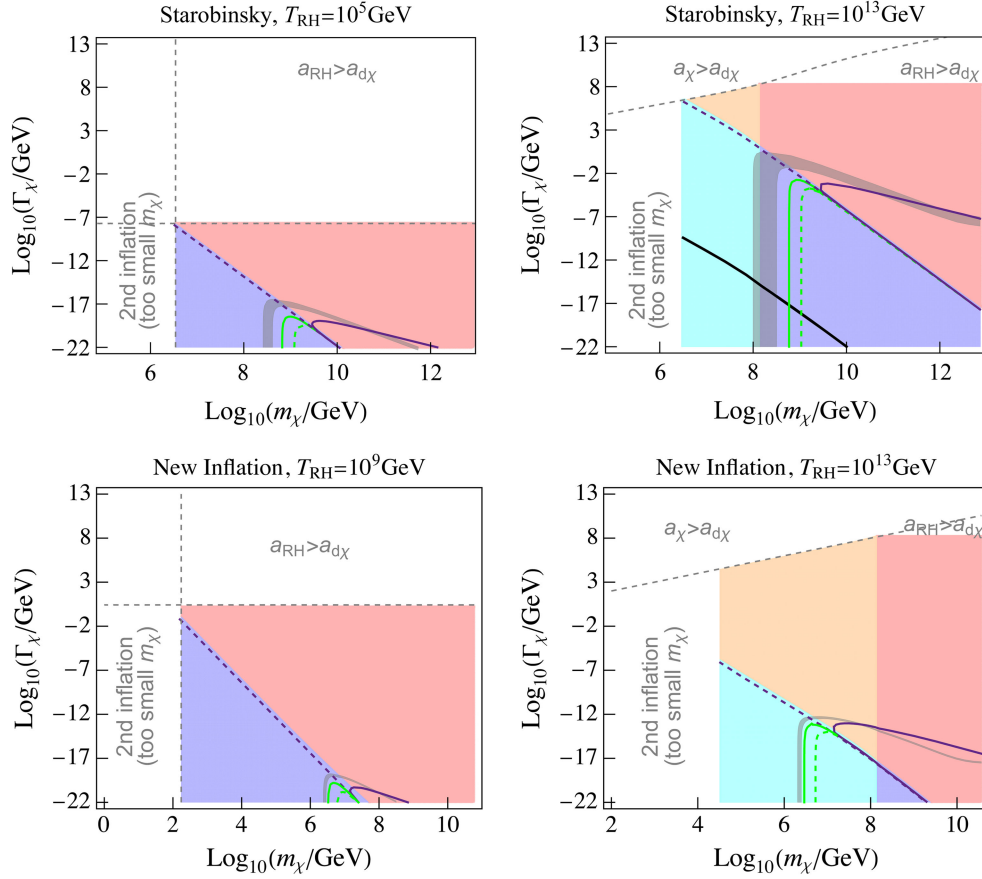


FIG. 2. The m_χ vs Γ_χ parameter space for a curvaton in the Starobinsky model (upper) and the new inflation model (lower) for $T_{\text{RH}} = 10^5, 10^9$ GeV (left) and $T_{\text{RH}} = 10^{13}$ GeV (right). The background shading corresponds to cases II-RD (red), II-MD (blue), III-RD (yellow), and III-MD (cyan). The unshaded regions correspond to an exclusion due to a second period of inflation (low m_χ), case I (high Γ_χ), or $a_{d\chi} < a_\chi$ (also high Γ_χ at high T_{RH}). Within the gray bands shown for the Starobinsky model, $\Delta n_s + \delta n_s = 0$ including -2σ deviations. All of the plane to the left and above this band is acceptable within $+2\sigma$. For the new inflation model, the band corresponds to $\Delta n_s + \delta n_s = 0$ with $\pm 2\sigma$ deviations. The set of parameters $m_\chi, \Gamma_\chi, T_{\text{RH}}$ along the purple (green) solid and dashed lines yields $f_{\text{NL}} = 9$ and $f_{\text{NL}} = 0$ ($f_{\text{NL}} = -1$ and $f_{\text{NL}} = -11$), respectively. The black line seen in the upper right panel corresponds to $x = 5 \times 10^{10}$, below which there is an excess of entropy production when χ decays.

$\Delta n_s + \delta n_s = 0.0084$, corresponding to the 2σ lower bound on the experimental value of n_s . Since the Starobinsky model already matches the observation quite well, the model is within the 2σ upper bound for all points above and to the left of the gray band. Only the points in the lower left corner of the plane are excluded, as they provide a correction δn_s in excess of the predicted value of Δn_s shown in Fig. 1. This can perhaps be better understood from Fig. 3, which shows the behavior of δn_s as a function of m_χ for fixed $\Gamma_\chi = 10^{-12}$ GeV for two reheating temperatures. Here we see clearly that for $T_{\text{RH}} = 10^5$ GeV the curvaton correction δn_s is small for all masses considered and therefore consistent with 2σ of the observation. In contrast, for $T_{\text{RH}} = 10^{13}$ GeV, the curvaton correction is large for $m_\chi \gtrsim 3 \times 10^8$ GeV. Similarly, in Fig. 4 we show a vertical cut on the (m_χ, Γ_χ) plane with fixed $m_\chi = 10^9$ GeV as a function of Γ_χ . In this case, we see that the correction to n_s is excessive at small Γ_χ .

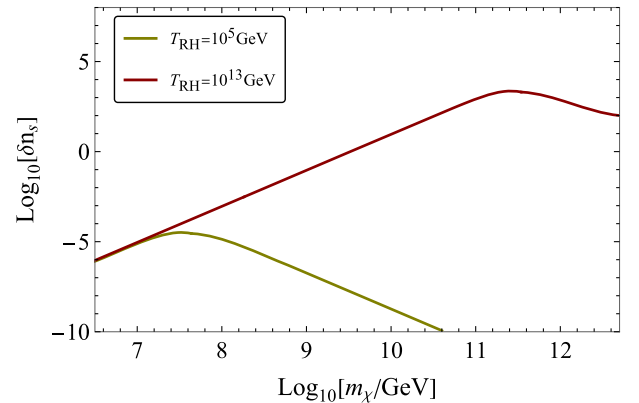


FIG. 3. δn_s as a function of m_χ for the Starobinsky model, with fixed $\Gamma_\chi = 10^{-12}$ GeV for $T_{\text{RH}} = 10^5$ GeV (red) and 10^{13} GeV (green).

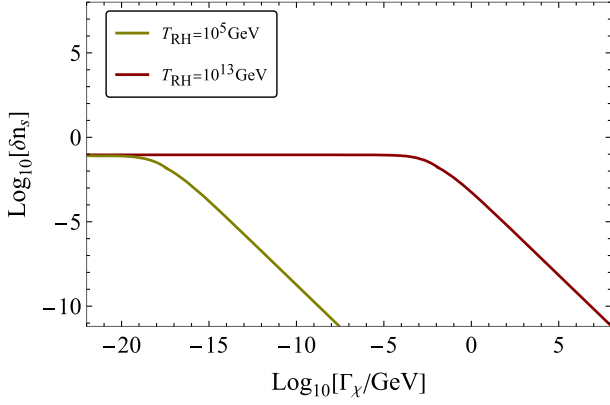


FIG. 4. δn_s as a function of Γ_χ for the Starobinsky model, with fixed $m_\chi = 10^9$ GeV for $T_{\text{RH}} = 10^5$ GeV (red) and 10^{13} GeV (green).

For the new inflation model, the predicted value of n_s is always below the observed value and a curvaton correction is needed to keep this model viable. In this case, therefore, the band corresponds to the $\pm 2\sigma$ variation in Δn_s shown in Fig. 1. The allowed region for $T_{\text{RH}} = 10^9$ GeV shown in the lower left panel of Fig. 2 is found at very low values of Γ_χ and masses of order 10^6 – 10^9 GeV. The ranges in Γ_χ and m_χ are larger for $T_{\text{RH}} = 10^{13}$ GeV, as seen in the lower right panel of Fig. 2.

By comparing the gray bands in the upper and lower panels, we confirm our expectation that the larger $|\Delta n_s|$ contributed by the curvaton in the new inflation model requires larger R_χ and thus a later decay of χ (i.e., smaller Γ_χ) for a given value of m_χ .⁹

We also show in Fig. 2 contours of constant $f_{\text{NL}}^{(\text{loc})}$ using purple (green) solid and dashed lines corresponding to $f_{\text{NL}}^{(\text{loc})} = 9$ and 0 ($f_{\text{NL}}^{(\text{loc})} = -1$ and -11), respectively. The contour with $f_{\text{NL}}^{(\text{loc})} = 0$ runs close to (but slightly below) the MD/RD boundary with $x = 1$. From Eqs. (13) and (20) we find that $f_{\text{NL}}^{(\text{loc})} = 0$ corresponds to $x \simeq 1.85$. Given the current constraint $f_{\text{NL}}^{(\text{loc})} = -0.9 \pm 5.1$ [14] at 68% CL, the regions in (m_χ, Γ_χ) space surrounded by the purple solid and green dashed lines are excluded at the 2σ level. After applying this exclusion, we still see that a significant fraction of each gray band remains viable, enabling a better fit to the observed n_s . Intriguingly, when χ decays in a RD era, it can produce $f_{\text{NL}}^{(\text{loc})}$ as large as $\mathcal{O}(1)$ in the regions near the intersection between the gray bands and the purple solid lines. If a large value of $f_{\text{NL}}^{(\text{loc})} = \mathcal{O}(1)$ is

⁹Note that H_{end} of the Starobinsky model is 2 orders of magnitude larger than that of the new inflation model. This makes P_{S_χ} in (10) of the two models comparable for the region of m_χ belonging to the gray bands. Thus, qualitatively $n_s - n_*$ in (17) is mostly sensitive to R_χ .

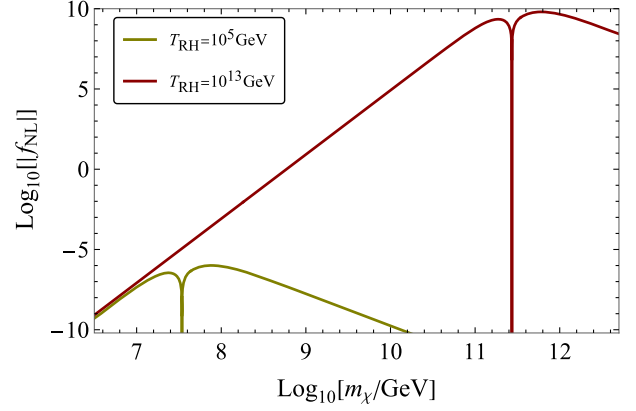


FIG. 5. f_{NL} as a function of m_χ for the Starobinsky model, with fixed $\Gamma_\chi = 10^{-12}$ GeV for $T_{\text{RH}} = 10^5$ GeV (red) and 10^{13} GeV (green).

detected in a future survey, such regions will become of particular interest. On the other hand, the parts of the gray bands residing in the blue or cyan regions are characterized by $|f_{\text{NL}}^{(\text{loc})}| < 1$ and $f_{\text{NL}}^{(\text{loc})} < 0$. The associated curvaton decays in a MD era and is characterized by smaller values of m_χ and Γ_χ . Since $f_{\text{NL}}^{(\text{loc})}$ in (20) becomes negative for $x \geq 1.85$, the curvaton decaying in a MD era tends to produce a negative $f_{\text{NL}}^{(\text{loc})}$. Although the absolute magnitude $|f_{\text{NL}}^{(\text{loc})}|$ itself is small, the sign of $f_{\text{NL}}^{(\text{loc})}$ can be still invoked to distinguish a curvaton that decays in a MD era from one that decays in a RD era.

The behavior of $f_{\text{NL}}^{(\text{loc})}$ across the (m_χ, Γ_χ) plane is more clearly seen in Figs. 5 and 6. In the former, we show $f_{\text{NL}}^{(\text{loc})}$ as a function of m_χ for fixed $\Gamma_\chi = 10^{-12}$ GeV. For $T_{\text{RH}} = 10^5$ GeV, the value of $f_{\text{NL}}^{(\text{loc})}$ is always small and remains within the Planck bound. However, for $T_{\text{RH}} = 10^{13}$ GeV, we see a rapid increase in $f_{\text{NL}}^{(\text{loc})}$

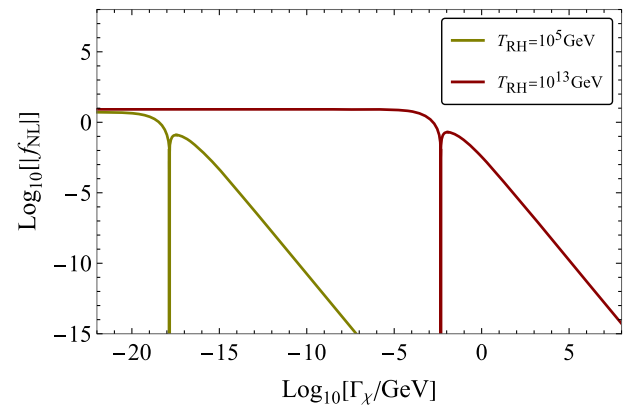


FIG. 6. f_{NL} as a function of Γ_χ for the Starobinsky model, with fixed $m_\chi = 10^9$ GeV for $T_{\text{RH}} = 10^5$ GeV (red) and 10^{13} GeV (green).

excluding masses $m_\chi \gtrsim 10^9$ GeV. Note the downward spike in $f_{\text{NL}}^{(\text{loc})}$ occurs when $x = 1.85$ when $f_{\text{NL}}^{(\text{loc})}$ changes sign (we are plotting the log of the absolute value). The analogous cut showing $f_{\text{NL}}^{(\text{loc})}$ as a function of Γ_χ for fixed $m_\chi = 10^9$ GeV is shown in Fig. 6.

Note that, for the parts of gray bands residing in blue or cyan backgrounds, δn_s and $f_{\text{NL}}^{(\text{loc})}$ are insensitive to a change in Γ_χ for fixed m_χ . This is attributable to the fact that R_χ is almost constant when $x \gg 1$. It is then difficult to infer Γ_χ given the insensitivity of δn_s and $f_{\text{NL}}^{(\text{loc})}$ to Γ_χ .

We would also like to point out that with the minimal cosmological assumptions, the primordial (inflationary) gravitational wave spectrum can be used for exploring curvatons residing in the intersections between gray bands and blue- or cyan-shaded regions. As a result of the entropy released from the late time decay of χ in a MD era, when compared to the modes reentering the horizon after χ decay, the primordial gravitational wave spectrum of modes that reentered the horizon before χ decay is suppressed by $x = (s_f/s_i)^{4/3}$ [62–68]. In addition, the frequency f_{pGW} of the primordial gravitational wave spectrum at which the suppression occurs is given by

$$f_{\text{pGW}} \simeq 2.65 \text{ Hz} \left(\frac{g_*}{106.75} \right)^{1/2} \left(\frac{g_{*,s}}{106.75} \right)^{-1/3} \left(\frac{T_{d\chi}}{10^8 \text{ GeV}} \right), \quad (68)$$

where $T_{d\chi}$ is the temperature of the thermal bath when χ decays and obtained from $(3/2)H \simeq \Gamma_\chi$, and g_* and $g_{*,s}$ are the effective relativistic degrees of freedom for the energy density and entropy density. For $\Gamma_\chi \lesssim 10^{-2}$ GeV corresponding to $f_{\text{pGW}} \leq \mathcal{O}(1)$ Hz, such a suppression might be probed by the future spaced-based interferometers like LISA [69], DECIGO [70,71], BBO [72–74], and future pulsar timing arrays like SKA [75–77]. Although this signature might be difficult to be seen for the new inflation model case due to the extremely small tensor-to-scalar ratio $r \simeq 2.4 \times 10^{-8}$, it might be tested for the Starobinsky model featured by the relatively high $r \simeq 3.5 \times 10^{-3}$.

Before concluding this section, we would like to note that the effect of the curvaton is not limited to either being constrained (as in the Starobinsky model) or increasing n_s (as in new inflation). It is well known that the prediction of r in chaotic inflation [5] is in excess of the current experimental limits on r [3,4]. However, from Eq. (19), we see that if the curvaton provides a significant contribution to the energy density before it decays, $\omega_\chi \lesssim 1$, and δr may be large enough to bring the calculated value into agreement with experiment [36,39,78–82]. In this case, n_s is affected, and we must check whether or not consistency is maintained. For example, for quartic chaotic inflation, $N_* = 55$, $r \simeq 0.29$, and $n_s = .945$. Increasing N_* to $\simeq 444$ will sufficiently lower r to, say, 0.036, but at the expense of

increasing n_s to 0.993. The inclusion of a curvaton can bring r to 0.036 and requires only $N_* > 47$ to get $n_s > 0.956$. To avoid solutions with small Γ_χ in conflict with BBN, we find solutions with $T_{\text{RH}} \gtrsim 10^5$ GeV which can be achieved, e.g., with $m_\chi = 10^9$ and $\Gamma_\chi \gtrsim 2.2 \times 10^{-21}$ GeV. In this case, $N_* = 50.9$, $n_s = 0.959$, $r = 0.036$. Similarly for the quadratic model, for $N_* = 55$, the predicted values of n_s and r are 0.964 and 0.14, respectively. The curvaton would allow $r = 0.036$ and $n_s < 0.973$ but only if $N_* < 44$, in which case, $T_{\text{RH}} < 7$ MeV and is close to being in violation of BBN constraints.

VI. DISCUSSION

The theory of inflation is an important element of what has become known as the standard cosmological model. Generic predictions include flatness ($\Omega = 1$) and the production of density perturbations that seed the formation of structure in the Universe. Specific models of inflation also make predictions on the tilt of the anisotropy spectrum n_s , the scalar-to-tensor ratio r , and the degree of non-Gaussianity in the spectrum f_{NL} . While $n_s < 1$ is a common prediction in models, its precise value may vary. Similarly, the range of predicted values of r is greatly model dependent. In contrast, very small f_{NL} is a common feature of single field models.

The precision at which n_s has been measured [2,61] and the improvement in the limit on r [3,4] enables one to discriminate between different models of inflation. For example, models of new inflation (such as that studied here) predict a value of n_s far outside the experimental range. Models of chaotic inflation predict a value of r in excess of the observed upper limit, whereas the Starobinsky model [1] predicts a value of n_s in very good agreement with experiment and a value of $r = 0.0035$, which is about a factor of 10 below the current upper limit. This may be tested in the next round of precision CMB experiments. The improved limits in non-Gaussianity [14] have given further credence to single field inflation models. Thus, a single field model such as the Starobinsky model remains on solid footing as a candidate for this important part of our cosmological history.

However, the physics of the early Universe may be more complicated than that described by a single field inflationary model. The presence of a massive, unstable, scalar field may dramatically alter the predictions of a given model. This scalar, the curvaton, may, upon decay, transfer its isocurvature fluctuations (produced during inflation) to adiabatic fluctuations in the radiation produced by its decay.

Here, we have followed the formalism in [30] to compute δn_s and δr . For the two models of inflation considered here, $\delta n_s > 0$. For the Starobinsky model, we require the correction to be small (particularly when the reheating temperature is large). In contrast, as a single field inflation model, the new inflation model considered here is excluded,

but with a correction $\delta n_s \sim 0.05$ may again be a viable candidate for inflation.

Within the context of these two models, we have considered the parameters m_χ and Γ_χ , which define the curvaton sector, and we have delineated the allowed parameter space that depends on the inflaton reheating temperature T_{RH} . For the Starobinsky model, since a correction is not absolutely necessary, there are a wide range of acceptable parameters, particularly those with large decay rates. In this case, the curvaton decays before it has a chance to dominate the energy density of the Universe and leave an imprint on the CMB spectrum. Nevertheless, we have also identified the range of parameters (seen in Fig. 2) for which the curvaton corrections spoil the good agreement of the model. This is seen at relatively large curvaton masses and long lifetimes.

In contrast to the Starobinsky model, the model of new inflation needs a curvaton to regain viability. Depending on the reheating temperature, there is only a narrow band of parameters where this is possible. For a somewhat low reheating temperature of 10^9 GeV, the curvaton decay rate must be very small $\Gamma_\chi < 10^{-18.5}$ GeV with a narrow range in curvaton masses. The parameter range increases at $T_{\text{RH}} = 10^{13}$ GeV, but still $\Gamma_\chi \lesssim 10^{-12}$ GeV is necessary.

Finally, curvaton models are able to predict much larger amounts of non-Gaussianities than single field models. Though $f_{\text{NL}} = 0$ is possible (parameters for this are found near the MD-RD boundaries for both cases II and III), most of the regions where f_{NL} exceeds the current Planck bounds occur where δn_s is also excessive. Nevertheless, there are some sets of parameters that provide an acceptable correction to n_s and yet a value of f_{NL} which is large (and positive).

Clearly the questions concerning the existence of a curvaton or a particular model of inflation require new observations. More precision in n_s may point to the existence of a curvaton even for the Starobinsky model. Of course, nonzero determinations of either r or f_{NL} would be greatly welcomed.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: INFLATON POTENTIALS FROM SUPERGRAVITY

Both of the models of inflation we consider can be derived in the context of no-scale supergravity [16] defined by the common Kähler potential

$$K = -3 \ln \left(T + T^\dagger - \frac{1}{3} |\phi|^2 \right) \quad (\text{A1})$$

where T is the volume modulus chiral superfield and ϕ is the chiral superfield for the inflaton. The inflaton field is identified with the real part of the scalar component of ϕ . Above, we have used ϕ for the scalar component of the chiral superfield for notational simplicity. The two models are distinguished by the following different superpotentials [83,84]:

$$W = M \left(\phi - \frac{\phi^3}{9M_P^2} - \frac{\phi^4}{\sqrt{2}M_P^3} \right) \quad \text{new inflation} \\ \times (\text{Ellis } et al.), \quad (\text{A2})$$

$$W = M \left(\frac{\phi^2}{2} - \frac{\phi^3}{3\sqrt{3}M_P} \right) \quad \text{Starobinsky} \\ \times (\text{Ellis } et al.). \quad (\text{A3})$$

Because of the nontrivial field-space manifold, a canonically normalized inflaton φ can be defined by

$$\varphi = \sqrt{3} \tanh \left(\frac{\phi}{\sqrt{6}M_P} \right). \quad (\text{A4})$$

For the new inflation model, along the direction $\phi = \phi^\dagger$, the scalar potential in supergravity leads to¹⁰

$$V_n(x) = M^2 M_P^2 \left(1 - 6\sqrt{6} \sinh \left(\frac{z}{\sqrt{6}} \right)^2 \tanh \left(\frac{z}{\sqrt{6}} \right) \right)^2 \quad (\text{A5})$$

$$\simeq M^2 M_P^2 (z^3 - 1)^2, \quad (\text{A6})$$

where $z = \phi/M_P$ is the inflaton field value normalized by the Planck mass.¹¹

The Starobinsky potential is given by

$$V_S(\phi) = \frac{3}{4} M^2 M_P^2 \left(1 - e^{-\sqrt{\frac{3}{2}} \frac{\phi}{M_P}} \right)^2, \quad (\text{A7})$$

¹⁰This superpotential differs slightly from that in [83], as a different basis is being considered here. Here, T is assumed fixed, whereas in [83], it was K that was assumed fixed. The resulting potentials are very similar and Eq. (A6) was derived in the latter case. Equation (A2) provides some higher order corrections to V in (A5) which do not affect the inflationary dynamics.

¹¹A similar model can be derived in the context of minimal supergravity [with $K = |\phi|^2$ and $W = (1 - \phi)^2$ [85]]. Though it is difficult to obtain light scalar fields during inflation in minimal supergravity [86], this problem can be avoided in no-scale supergravity [87]. We do not attempt to construct a full curvaton model in supergravity here.

though there are many other forms of the superpotential (A3) that lead to the same scalar potential [88,89].

APPENDIX B: CORRECTIONS TO Ht

In Sec. IV, we have defined the moment when χ oscillations start and when χ decays, by the equalities $\bar{\beta}H = m_\chi$ and $\bar{\beta}H = \Gamma_\chi$, where $\bar{\beta} = \frac{3}{2}$ or 2 for matter- or radiation-dominated scenarios. In reality, these relations receive a correction when the matter and radiation energy densities are comparable.

To derive the exact relations between H and m_χ , Γ_χ , let us define $Ht = 1/\beta_x$ near a_{eq} and $Ht = 1/\beta_y$ near a_{RH} . In the former case, the total energy density is composed of radiation and χ , with

$$\rho_{\text{tot}} = \frac{\rho_{\text{eq}}}{2} \left[\left(\frac{a_{\text{eq}}}{a} \right)^4 + \left(\frac{a_{\text{eq}}}{a} \right)^3 \right], \quad (\text{B1})$$

which implies for the Hubble constant

$$H \equiv \frac{\dot{a}}{a} = \frac{\sqrt{\rho_{\text{eq}}}}{\sqrt{6}M_P} \left(\frac{a_{\text{eq}}}{a} \right)^2 \sqrt{1 + \frac{a}{a_{\text{eq}}}}. \quad (\text{B2})$$

Integrating the above expression and using the relation $x = a_{d\chi}/a_{\text{eq}}$, we obtain

$$t_{\text{eq}} = 2M_P \sqrt{\frac{2}{3\rho_{\text{eq}}}} \left(2 - 2\sqrt{1+x} + x\sqrt{1+x} \right) \equiv f[x(t)]. \quad (\text{B3})$$

The time derivative of the function f yields

$$1 = \frac{\sqrt{6}M_P x}{\sqrt{\rho_{\text{eq}}(1+x)}} \frac{\dot{a}_{d\chi}}{a_{\text{eq}}}, \quad (\text{B4})$$

which eventually leads to the expression for β_x ,

$$\beta_x = \frac{3x^2}{2(x^2 - x - 2 + 2\sqrt{1+x})}. \quad (\text{B5})$$

One recovers $\beta_x = 2$ ($\beta_x = 3/2$) for $x \ll 1$ ($x \gg 1$). At the boundary of the MD and RD era (i.e., $a_{d\chi} = a_{\text{eq}}$ or $x = 1$), we have $\frac{3}{4}(1 + \sqrt{2})$.

On the other hand, before a_{RH} , the radiation redshifts as $a^{-3/2}$, whereas the matter energy density redshifts as a^{-3} . Introducing $y \equiv a_\chi/a_{\text{RH}}$, we obtain analogously

$$\beta_y = \frac{3y^{3/2}}{4(1 + y^{3/2} - \sqrt{1 + y^{3/2}})}, \quad (\text{B6})$$

which becomes $3/2$ for $y \ll 1$, whereas at the boundary of case II and III (i.e., $y = 1$), we have $\beta_y = \frac{3}{8}(2 + \sqrt{2})$. After reheating, the inflaton energy density decreases exponentially, thus the functional form of β_y depends on the inflaton decay rate. Given that β_y converges very rapidly to 2, we approximate it as

$$\beta_y = \frac{1}{8}(3\sqrt{2} - 10)e^{1-y} + 2. \quad (\text{B7})$$

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