

All-loop renormalization group invariants for the MSSM

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For the minimal supersymmetric Standard Model, from the gauge couplings, Yukawa couplings, and the coefficient μ in the part of the superpotential quadratic in the Higgs superfields, we construct combinations which (for certain renormalization prescriptions) do not depend on the renormalization point in all loops. In other words, these combinations are the renormalization group invariants. Similar invariants are also constructed for the next-to-minimal supersymmetric Standard Model. The derivation is based on the nonrenormalization of the superpotential and the Novikov-Shifman-Vainshtein-Zakharov equations. We argue that the scale invariance of the considered combinations takes place in the class of the higher derivatives+minimal subtractions of logarithms scheme. This fact has been verified in the lowest orders, up to and including the one in which the dependence on the renormalization prescription becomes essential. It is also demonstrated that in the $\overline{\text{DR}}$ scheme the renormalization group invariance does not take place starting from the approximation, where the scheme dependence manifests itself.

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I. INTRODUCTION

The detailed analysis of quantum corrections can tell a lot about the world around us. For instance, the renormalization group running of gauge couplings in the supersymmetric extensions of the Standard Model [1–3] is a strong indication in favor of supersymmetry [4,5] and grand unification [6]. Another interesting prediction of the grand unified theories is the unification of the Yukawa couplings [7–9], which leads to certain relations between masses of elementary particles. However, it is very difficult to match such predictions with the experimental data [10] because a theory describing physics beyond the Standard Model is not exactly known. At present, one of the most promising candidates for this is supersymmetric extensions of the Standard Model. The simplest of them is the minimal supersymmetric Standard Model (MSSM), which is a theory invariant under the $SU(3) \times SU(2) \times U(1)$ gauge transformations with softly broken supersymmetry, see, e.g., [6]. However, the soft terms do not affect the renormalization of the rigid theory and, in particular, such parameters as the gauge and Yukawa couplings. That is why a lot of MSSM quantum properties can be investigated using the results obtained for $\mathcal{N} = 1$ supersymmetric theories. For instance, the renormalization group running of the gauge couplings can be described with the help of

Novikov-Shifman-Vainshtein-Zakharov (NSVZ) β functions [11–14] in all orders, which for the MSSM were first written in [15]. Various phenomenological applications of these exact relations can be found in, e.g., [16–19].

The existence of the exact equations relating the β functions to the anomalous dimensions of the chiral matter superfields can lead to some interesting consequences. For instance, according to [20], in $\mathcal{N} = 1$ supersymmetric QCD interacting with the Abelian gauge superfield in a supersymmetric way the running of the strong gauge coupling α_s is related to the running of the electromagnetic gauge coupling α . Assuming that there are N_f identical flavors consisting of the chiral superfields ϕ_a and $\tilde{\phi}_a$ in the irreducible representations R and $\bar{R}$ of the non-Abelian gauge group G with opposite $U(1)$ charges, this relation can be presented in the form of the renormalization group invariant (RGI),

$$\left(\frac{\alpha_s}{\mu^3}\right)^{C_2} \exp\left(\frac{2\pi}{\alpha_s} - \frac{T(R)}{\dim R} \cdot \frac{2\pi}{\alpha}\right) = \text{RGI}. \quad (1)$$

Here $\text{tr}(T^A T^B) = T(R)\delta^{AB}$, where T^A are the generators in the representation R (of the dimension $\dim R$). The Casimir C_2 is defined by the equation $f^{ACD}f^{BCD} = C_2\delta^{AB}$, where f^{ABC} are the structure constants.

Equation (1) is obtained by eliminating the anomalous dimension of the matter superfields from the NSVZ equations.¹ Therefore, exactly as with the NSVZ relations,

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¹If the flavors have different electric charges, then it is possible to relate the β function of $\mathcal{N} = 1$ supersymmetric QCD to the Adler D function [21] using the NSVZ-like equation for it [22,23].

it is satisfied only for certain renormalization prescriptions and should be valid in an arbitrary NSVZ scheme. Such schemes can naturally be constructed with the help of the higher covariant derivative regularization [24–26] formulated in $\mathcal{N} = 1$ superspace [27,28]. According to [29,30], the NSVZ equations are valid for the renormalization group functions (RGFs) defined in terms of the bare couplings for an arbitrary renormalization prescription supplementing the higher covariant derivative regularization and for (standard) RGFs defined in terms of the renormalized couplings in the higher derivatives + minimal subtractions of logarithms (HD + MSL) scheme.² In the latter case, for a theory regularized by higher covariant derivatives the renormalization is made with the help of minimal subtractions of logarithms [29,35]. In the $\overline{\text{DR}}$ scheme (when a theory is regularized by dimensional reduction [36] and divergences are removed by modified minimal subtraction [37]), Eq. (1) is not valid [20] exactly as the NSVZ equation(s) [38–41].

However, it would be interesting to consider a more realistic theory like the MSSM, which is the simplest supersymmetric extension of the Standard Model. Although in this model supersymmetry is softly broken, the renormalization of the parameters entering the rigid part of its action occurs exactly as for the supersymmetric theories. The renormalization of the soft terms can be related to the renormalization of the rigid theory [42–44]. For example, the running of the gaugino masses is determined by all-loop RGIs constructed in [42].

In this paper, for the MSSM we construct two independent RGIs analogous to (1). For this purpose the anomalous dimensions of the chiral matter superfields are eliminated from the all-loop equations describing the renormalization group running of the gauge couplings, Yukawa couplings, and the μ parameter. One of the resulting expressions does not contain the renormalization point μ . Another similar expression explicitly depends on μ , but is independent of the μ parameter in the MSSM superpotential.

The paper is organized as follows. In Sec. II we recall some basic information about the MSSM and write down the all-loop exact equations describing the renormalization group evolution of some its parameters. In Sec. III we eliminate the anomalous dimensions of the chiral matter superfields from these equations and obtain two relations restricting the running of the gauge and Yukawa couplings in all orders in the form of RGIs analogous to (1). Modifications of the results to the case of the next-to-minimal supersymmetric Standard Model (NMSSM) are discussed in Sec. IV. The renormalization group invariance of the constructed expressions is verified by the explicit three-loop calculation in Sec. V, where we

demonstrate that it is really valid in the HD + MSL scheme, but is not valid in the $\overline{\text{DR}}$ one. The results are briefly discussed in the Conclusion, and Appendix A contains technical details of the calculations. The explicit expressions for RGFs in the HD + MSL and $\overline{\text{DR}}$ schemes are presented in Appendixes B and C, respectively.

II. EXACT EQUATIONS DESCRIBING THE RENORMALIZATION OF THE MSSM PARAMETERS

The MSSM is an extension of the Standard Model, which is a gauge theory with softly broken supersymmetry and the gauge group

$$G = SU(3) \times SU(2) \times U(1)_Y. \quad (2)$$

This implies that it contains three gauge couplings

$$\alpha_3 = \frac{e_3^2}{4\pi}; \quad \alpha_2 = \frac{e_2^2}{4\pi}; \quad \alpha_1 = \frac{5}{3} \cdot \frac{e_1^2}{4\pi} \quad (3)$$

corresponding to the subgroups $SU(3)$, $SU(2)$, and $U(1)$, respectively. (The factor $5/3$ in the coupling constant α_1 is introduced so that the unification of couplings has the form $\alpha_1 = \alpha_2 = \alpha_3$.) It is convenient to formulate the theory with the help of $\mathcal{N} = 1$ superfields using the spurion technique [45] for the soft terms. In this case quarks, leptons, and Higgs doublets are components of the chiral superfields listed in Table I, where, in particular, the superfields including the (charge conjugated) left quarks and leptons can be written in the form

$$Q_I = \begin{pmatrix} \tilde{U} \\ \tilde{D} \end{pmatrix}_I; \quad L_I = \begin{pmatrix} \tilde{N} \\ \tilde{E} \end{pmatrix}_I. \quad (4)$$

In Table I we also present quantum numbers of all chiral matter superfields with respect to the gauge group [representations for $SU(3)$ and $SU(2)$ and hypercharges for $U(1)$]. In our notation, the indices I, J range from 1 to 3 and numerate generations.

The MSSM action also contains dimensionless Yukawa couplings $(Y_U)_{IJ}$, $(Y_D)_{IJ}$, and $(Y_E)_{IJ}$ (which are 3×3 matrices) inside

$$\Delta S = \frac{1}{2} \int d^4x d^2\theta W + \text{c.c.}, \quad (5)$$

TABLE I. The MSSM chiral matter superfields and their quantum numbers with respect to the various parts of the gauge group.

Superfield	Q_I	U_I	D_I	L_I	E_I	H_u	H_d
$SU(3)$	$\bar{3}$	3	3	1	1	1	1
$SU(2)$	2	1	1	2	1	2	2
$U(1)_Y$	$-1/6$	$2/3$	$-1/3$	$1/2$	-1	$-1/2$	$1/2$

²This statement has been verified by some explicit multiloop calculations in such orders of the perturbation theory where the scheme dependence becomes essential [31–34].

where the superpotential is given by the expression

$$W = (Y_U)_{IJ}(\tilde{U} \tilde{D})_I^a \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{u1} \\ H_{u2} \end{pmatrix} U_{aJ} + (Y_D)_{IJ}(\tilde{U} \tilde{D})_I^a \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix} \\ \times D_{aJ} + (Y_E)_{IJ}(\tilde{N} \tilde{E})_I \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix} E_J + \mu(H_{u1} H_{u2}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix}. \quad (6)$$

Moreover, the superpotential contains a term with the parameter μ , which has the dimension of mass. The color index a in Eq. (6) ranges from 1 to 3.

The renormalization group running of the gauge couplings in the MSSM can be described exactly in all loops with the help of the NSVZ β functions [15]. They can be written in the form [46]

$$\frac{\beta_1}{\alpha_1^2} = -\frac{3}{5} \cdot \frac{1}{2\pi} \left[-11 + \text{tr} \left(\frac{1}{6} \gamma_Q + \frac{4}{3} \gamma_U + \frac{1}{3} \gamma_D + \frac{1}{2} \gamma_L + \gamma_E \right) + \frac{1}{2} \gamma_{H_u} + \frac{1}{2} \gamma_{H_d} \right]; \\ \frac{\beta_2}{\alpha_2^2} = -\frac{1}{2\pi(1 - \alpha_2/\pi)} \left[-1 + \text{tr} \left(\frac{3}{2} \gamma_Q + \frac{1}{2} \gamma_L \right) + \frac{1}{2} \gamma_{H_u} + \frac{1}{2} \gamma_{H_d} \right]; \\ \frac{\beta_3}{\alpha_3^2} = -\frac{1}{2\pi(1 - 3\alpha_3/2\pi)} \left[3 + \text{tr} \left(\gamma_Q + \frac{1}{2} \gamma_U + \frac{1}{2} \gamma_D \right) \right] \quad (7)$$

and relate three gauge β functions of the theory to the anomalous dimensions of the chiral matter superfields. They are defined by the equations

$$\beta_i(\alpha, Y) = \frac{d\alpha_i}{d \ln \mu} \Big|_{\alpha_0, Y_0 = \text{const}}; \quad \gamma_i(\alpha, Y) = \frac{d \ln Z_i}{d \ln \mu} \Big|_{\alpha_0, Y_0 = \text{const}}, \quad (8)$$

where the subscript 0 denotes the bare values.

RGFs describing the renormalization of the Yukawa couplings and of the parameter μ can also be related to the anomalous dimensions of the matter superfields due to the nonrenormalization of the superpotential [47]. Taking into account that the superpotential does not receive divergent quantum corrections, we see that there are renormalization prescriptions for which

$$Y_{0U} = (Z_{H_u})^{-1/2} ((Z_Q)^T)^{-1/2} Y_U (Z_U)^{-1/2}; \quad Y_{0D} = (Z_{H_d})^{-1/2} ((Z_Q)^T)^{-1/2} Y_D (Z_D)^{-1/2}; \\ Y_{0E} = (Z_{H_d})^{-1/2} ((Z_L)^T)^{-1/2} Y_E (Z_E)^{-1/2}; \quad \mu_0 = (Z_{H_u})^{-1/2} (Z_{H_d})^{-1/2} \mu, \quad (9)$$

so that

$$\frac{dY_U}{d \ln \mu} = \frac{1}{2} (\gamma_{H_u} Y_U + (\gamma_Q)^T Y_U + Y_U \gamma_U); \quad \frac{dY_D}{d \ln \mu} = \frac{1}{2} (\gamma_{H_d} Y_D + (\gamma_Q)^T Y_D + Y_D \gamma_D); \\ \frac{dY_E}{d \ln \mu} = \frac{1}{2} (\gamma_{H_d} Y_E + (\gamma_L)^T Y_E + Y_E \gamma_E); \quad \frac{d\mu}{d \ln \mu} = \frac{1}{2} (\gamma_{H_u} + \gamma_{H_d}) \mu. \quad (10)$$

It is important that Eq. (9) and, therefore, Eq. (10) are valid in the HD + MSL scheme because in this scheme all renormalization constants contain only powers of $\ln \Lambda/\mu$, where Λ is the dimensionful regularization parameter. Taking into account that the NSVZ relations are also satisfied for this renormalization prescription, we see that all exact equations describing the renormalization of the MSSM parameters are satisfied in the HD + MSL scheme, the use of which is always assumed in what follows. Note that in the $\overline{\text{DR}}$ scheme the NSVZ equations (7) are not valid

starting from the three-loop approximation for the β functions (and the two-loop approximation for the anomalous dimensions), see [48] for details.

III. ALL-LOOP RGIS FOR THE MSSM

The renormalization group equations (10) for the Yukawa couplings can be multiplied by the corresponding inverse matrices. After that, it is possible to calculate traces of the resulting equations using the formula

$$\mathrm{tr} \left[M^{-1} \frac{dM}{d \ln \mu} \right] = \frac{d}{d \ln \mu} \mathrm{tr} \ln M = \frac{d}{d \ln \mu} \ln \det M, \quad (11)$$

which can easily be proved if we present the matrix M in the form $M = 1 + m$,

$$\begin{aligned} \frac{d}{d \ln \mu} \mathrm{tr} \ln M &= \mathrm{tr} \frac{d}{d \ln \mu} \ln(1 + m) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \mathrm{tr} \frac{d}{d \ln \mu} (m^n) \\ &= \sum_{n=1}^{\infty} (-1)^{n+1} \mathrm{tr} \left(m^{n-1} \frac{dm}{d \ln \mu} \right) = \mathrm{tr} \left(\frac{1}{1 + m} \cdot \frac{dm}{d \ln \mu} \right) = \mathrm{tr} \left(M^{-1} \frac{dM}{d \ln \mu} \right). \end{aligned} \quad (12)$$

Then (taking into account that the indices numerating generations range from 1 to 3) we see that the equations describing how the determinants of the Yukawa matrices depend on the renormalization point μ are written as

$$\begin{aligned} \gamma_{\det Y_U} &\equiv \frac{d \ln \det Y_U}{d \ln \mu} = \mathrm{tr} \left[(Y_U)^{-1} \frac{dY_U}{d \ln \mu} \right] = \frac{1}{2} (3\gamma_{H_u} + \mathrm{tr}(\gamma_Q + \gamma_U)); \\ \gamma_{\det Y_D} &\equiv \frac{d \ln \det Y_D}{d \ln \mu} = \mathrm{tr} \left[(Y_D)^{-1} \frac{dY_D}{d \ln \mu} \right] = \frac{1}{2} (3\gamma_{H_d} + \mathrm{tr}(\gamma_Q + \gamma_D)); \\ \gamma_{\det Y_E} &\equiv \frac{d \ln \det Y_E}{d \ln \mu} = \mathrm{tr} \left[(Y_E)^{-1} \frac{dY_E}{d \ln \mu} \right] = \frac{1}{2} (3\gamma_{H_d} + \mathrm{tr}(\gamma_L + \gamma_E)). \end{aligned} \quad (13)$$

We will also need the last equation in (10), which is convenient to present in the form

$$\gamma_{\mu} \equiv \frac{d \ln \mu}{d \ln \mu} = \frac{1}{2} (\gamma_{H_u} + \gamma_{H_d}). \quad (14)$$

It is also expedient to rewrite Eq. (7) as

$$\begin{aligned} \frac{d}{d \ln \mu} \left(\frac{1}{\alpha_1} \right) &= \frac{3}{5} \cdot \frac{1}{2\pi} \left[-11 + \mathrm{tr} \left(\frac{1}{6} \gamma_Q + \frac{4}{3} \gamma_U + \frac{1}{3} \gamma_D + \frac{1}{2} \gamma_L + \gamma_E \right) + \frac{1}{2} \gamma_{H_u} + \frac{1}{2} \gamma_{H_d} \right]; \\ \frac{d}{d \ln \mu} \left(\frac{1}{\alpha_2} + \frac{1}{\pi} \ln \alpha_2 \right) &= \frac{1}{2\pi} \left[-1 + \mathrm{tr} \left(\frac{3}{2} \gamma_Q + \frac{1}{2} \gamma_L \right) + \frac{1}{2} \gamma_{H_u} + \frac{1}{2} \gamma_{H_d} \right]; \\ \frac{d}{d \ln \mu} \left(\frac{1}{\alpha_3} + \frac{3}{2\pi} \ln \alpha_3 \right) &= \frac{1}{2\pi} \left[3 + \mathrm{tr} \left(\gamma_Q + \frac{1}{2} \gamma_U + \frac{1}{2} \gamma_D \right) \right]. \end{aligned} \quad (15)$$

The anomalous dimensions of the chiral matter superfields and μ can be eliminated from Eqs. (13)–(15), thereby obtaining a differential equation that contains only derivatives of the gauge and Yukawa couplings. Integrating it we obtain the expression RGI_2 in (16), which vanishes after differentiating with respect to the renormalization point μ . Alternatively, it is possible to construct the expression RGI_1 which does not explicitly depend on the scale μ , but contains the parameter μ . The details of the calculation are presented in Appendix A. Two resulting renormalization group invariants can be written in the form

$$\begin{aligned} \mathrm{RGI}_1 &= \frac{\mu^{9/2} (\alpha_3)^3 (\alpha_2)^{1/2}}{(\det Y_E)^{1/2} (\det Y_U)^{5/3} (\det Y_D)^{7/6}} \exp \left(\frac{2\pi}{\alpha_3} + \frac{\pi}{2\alpha_2} + \frac{5\pi}{6\alpha_1} \right); \\ \mathrm{RGI}_2 &= \frac{(\alpha_3)^3 \det Y_E (\det Y_U)^{1/3}}{\mu^9 \alpha_2 (\det Y_D)^{2/3}} \exp \left(\frac{2\pi}{\alpha_3} - \frac{\pi}{\alpha_2} - \frac{5\pi}{3\alpha_1} \right). \end{aligned} \quad (16)$$

Although their existence follows from the particular structure of the MSSM, similar invariants could exist in other theories for which the anomalous dimensions of all chiral matter superfields can be eliminated from the NSVZ equations.

Instead of the renormalization group invariants ($\mathrm{RGI}_1, \mathrm{RGI}_2$) it is possible to use the equivalent set ($\mathrm{RGI}_3, \mathrm{RGI}_4$), where the expressions

$$\begin{aligned} \text{RGI}_3 &\equiv \left(\frac{\text{RGI}_1}{\text{RGI}_2} \right)^{2/3} = \frac{\mu^3 \mu^6 \alpha_2}{(\det Y_E)(\det Y_U)^{4/3}(\det Y_D)^{1/3}} \exp\left(\frac{\pi}{\alpha_2} + \frac{5\pi}{3\alpha_1}\right); \\ \text{RGI}_4 &\equiv (\text{RGI}_1)^{2/3}(\text{RGI}_2)^{1/3} = \frac{\mu^3 (\alpha_3)^3}{\mu^3 \det Y_U \det Y_D} \exp\left(\frac{2\pi}{\alpha_3}\right) \end{aligned} \quad (17)$$

also have a rather simple form.

IV. RGIS FOR NMSSM

The parameter μ in the superpotential (6) should be of the order of the electroweak scale, which is impossible to explain in the MSSM. The μ problem can be solved in the NMSSM [49,50], which contains an additional chiral matter superfield S . This superfield is a singlet with respect to $SU(3) \times SU(2) \times U(1)$. Then it is possible to replace the μ term

$$\Delta W_{\text{MSSM}} = \mu (H_{u1} H_{u2}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix} \quad (18)$$

by the gauge invariant expression

$$\Delta W_{\text{NMSSM}} = \lambda S (H_{u1} H_{u2}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix} + \frac{\kappa}{3} S^3, \quad (19)$$

in which λ and κ are new dimensionless couplings. In this case, the effective value of μ is equal to the vacuum expectation value of (the lowest component of) S multiplied by λ and can have an order of the electroweak scale. Because of the nonrenormalization of the superpotential, the anomalous dimensions of λ and κ satisfy the all-loop equations

$$\gamma_\kappa \equiv \frac{d \ln \kappa}{d \ln \mu} = \frac{3}{2} \gamma_S; \quad \gamma_\lambda \equiv \frac{d \ln \lambda}{d \ln \mu} = \frac{1}{2} (\gamma_S + \gamma_{H_u} + \gamma_{H_d}), \quad (20)$$

where the derivatives with respect to $\ln \mu$ should be taken at fixed values of all bare couplings. From Eq. (20) we see that the sum which for the MSSM gives the anomalous dimension γ_μ can be expressed in terms of γ_λ and γ_κ ,

$$\frac{1}{2} (\gamma_{H_u} + \gamma_{H_d}) = \gamma_\lambda - \frac{1}{3} \gamma_\kappa. \quad (21)$$

Since S is a singlet with respect to the gauge group, the NSVZ relations for the NMSSM are the same as for the MSSM (although the anomalous dimensions are different). Evidently, the equations describing the running of the Yukawa couplings Y_E , Y_U , and Y_D also remain unchanged. That is why RGIs for the NMSSM can be obtained from the ones for the MSSM after the replacement

$$\mu \rightarrow \lambda \kappa^{-1/3}. \quad (22)$$

The expression RGI_2 does not depend on μ and, therefore, is also RGI for the NMSSM. The other RGIs after the replacement (22) give

$$\begin{aligned} \text{RGI}_1 &\rightarrow \widetilde{\text{RGI}}_1 = \frac{\lambda^{9/2} (\alpha_3)^3 (\alpha_2)^{1/2}}{\kappa^{3/2} (\det Y_E)^{1/2} (\det Y_U)^{5/3} (\det Y_D)^{7/6}} \exp\left(\frac{2\pi}{\alpha_3} + \frac{\pi}{2\alpha_2} + \frac{5\pi}{6\alpha_1}\right); \\ \text{RGI}_3 &\rightarrow \widetilde{\text{RGI}}_3 = \frac{\lambda^3 \mu^6 \alpha_2}{\kappa (\det Y_E)(\det Y_U)^{4/3}(\det Y_D)^{1/3}} \exp\left(\frac{\pi}{\alpha_2} + \frac{5\pi}{3\alpha_1}\right); \\ \text{RGI}_4 &\rightarrow \widetilde{\text{RGI}}_4 = \frac{\lambda^3 (\alpha_3)^3}{\kappa \mu^3 \det Y_U \det Y_D} \exp\left(\frac{2\pi}{\alpha_3}\right). \end{aligned} \quad (23)$$

Thus, for the NMSSM there are also two independent invariants. As for the MSSM, it is possible to use two convenient choices, $(\widetilde{\text{RGI}}_1, \widetilde{\text{RGI}}_2)$ or $(\widetilde{\text{RGI}}_3, \widetilde{\text{RGI}}_4)$.

V. THE THREE-LOOP CHECK

To verify the renormalization group invariance of the expressions constructed in Sec. III, we will use the three-loop results for MSSM β functions and the two-loop results for the anomalous dimensions of the chiral matter superfields. In the $\overline{\text{HD}} + \overline{\text{MSL}}$ and $\overline{\text{DR}}$ schemes, they have been

calculated in [48,51], respectively. It is easier to check the invariance of the set $(\text{RGI}_3, \text{RGI}_4)$, which is what we will do. Differentiating $\ln(\text{RGI}_3)$ and $\ln(\text{RGI}_4)$ with respect to $\ln \mu$ we obtain the exact equations

$$\begin{aligned} 0 &= \left(\frac{1}{\alpha_2} - \frac{\pi}{\alpha_2^2} \right) \beta_2 - \frac{5\pi}{3\alpha_1^2} \beta_1 + 6 + 3\gamma_\mu \\ &\quad - \gamma_{\det Y_E} - \frac{4}{3} \gamma_{\det Y_U} - \frac{1}{3} \gamma_{\det Y_D}; \end{aligned} \quad (24)$$

$$0 = \left(\frac{3}{\alpha_3} - \frac{2\pi}{\alpha_3^2} \right) \beta_3 - 3 + 3\gamma_\mu - \gamma_{\det Y_U} - \gamma_{\det Y_D}. \quad (25)$$

The expressions entering these equations in the HD + MSL and $\overline{\text{DR}}$ schemes are presented in Appendixes B and C, respectively. In the HD + MSL scheme, they depend on the regularization parameters

$$\begin{aligned} A &\equiv \int_0^\infty dx \ln x \frac{d}{dx} \frac{1}{R(x)}; & a_{\varphi,3} &\equiv \frac{M_{\varphi,3}}{\Lambda}; & a_{\varphi,2} &\equiv \frac{M_{\varphi,2}}{\Lambda}; \\ B &\equiv \int_0^\infty dx \ln x \frac{d}{dx} \frac{1}{F^2(x)}; & a_3 &\equiv \frac{M_3}{\Lambda}; & a_2 &\equiv \frac{M_2}{\Lambda}; & a_1 &\equiv \frac{M_1}{\Lambda}. \end{aligned} \quad (26)$$

Here the functions $R(x)$ and $F(x)$ are used for introducing higher covariant derivatives in the gauge and matter parts of the action, respectively.³ They satisfy the conditions $R(0) = 1$, $F(0) = 1$ and should rapidly increase in the limit $x \rightarrow \infty$. Because of the presence of these functions in the classical action, loop integrals will also contain them. Therefore, the results for higher order contributions to RGFs will depend on the choice of these functions. For supersymmetric theories regularized by higher covariant derivatives, the loop integrals giving the two-loop anomalous dimensions and the three-loop β functions have been calculated in [52]. According to this calculation, these contributions to RGFs depend on the functions $R(x)$ and $F(x)$ only via the parameters A and B defined by Eq. (26), see Eqs. (B1)–(B7) for the case of the MSSM.

After introducing higher derivatives in the classical action, divergences remain only in the one-loop approximation. These remaining divergences are regularized by inserting the Pauli-Villars determinants into the generating functional [26]. According to [53,54], in the case of a simple gauge group it is necessary to use two sets of the

Pauli-Villars superfields. One of them has the mass M_φ and cancels one-loop divergences coming from the quantum gauge superfield and ghosts, while the other has the mass M and cancels the one-loop divergences coming from the matter superfields. For theories with multiple gauge couplings, such sets should be introduced for each simple subgroup of the gauge group. For each $U(1)$ subgroup, it is necessary to introduce only the latter Pauli-Villars superfield. In particular, for the MSSM, we need the Pauli-Villars superfields with the mass M_1 for the subgroup $U(1)$. For the subgroups $SU(2)$ and $SU(3)$, the Pauli-Villars masses are $M_{\varphi,2}$, M_2 and $M_{\varphi,3}$, M_3 , respectively. Because of the arbitrariness in the choice of the regulators R , F and the parameters $a_{\varphi,K}$ and a_K , there is a class of the HD + MSL scheme, each of them being an NSVZ scheme.

Substituting the expressions for RGFs in the HD + MSL scheme (presented in Appendix B) into Eqs. (24) and (25), we see that in the considered approximation these equations are valid independent of the values of the regularization parameters (26),

$$\left[\left(\frac{1}{\alpha_2} - \frac{\pi}{\alpha_2^2} \right) \beta_2 - \frac{5\pi}{3\alpha_1^2} \beta_1 + 6 + 3\gamma_\mu - \gamma_{\det Y_E} - \frac{4}{3} \gamma_{\det Y_U} - \frac{1}{3} \gamma_{\det Y_D} \right]_{\text{HD+MSL}} = O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \quad (27)$$

$$\left[\left(\frac{3}{\alpha_3} - \frac{2\pi}{\alpha_3^2} \right) \beta_3 - 3 + 3\gamma_\mu - \gamma_{\det Y_U} - \gamma_{\det Y_D} \right]_{\text{HD+MSL}} = O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6). \quad (28)$$

Therefore, the expressions (17) [and, therefore, the expressions (16)] in the HD + MSL scheme(s) are also independent of μ in the considered order.

However, substituting RGFs obtained in the $\overline{\text{DR}}$ scheme we see that Eqs. (24) and (25) are not satisfied,

$$\begin{aligned} &\left[\left(\frac{1}{\alpha_2} - \frac{\pi}{\alpha_2^2} \right) \beta_2 - \frac{5\pi}{3\alpha_1^2} \beta_1 + 6 + 3\gamma_\mu - \gamma_{\det Y_E} - \frac{4}{3} \gamma_{\det Y_U} - \frac{1}{3} \gamma_{\det Y_D} \right]_{\overline{\text{DR}}} \\ &= \frac{1}{2\pi^2} \left(\frac{1243\alpha_1^2}{400} + \frac{17\alpha_2^2}{16} - 5\alpha_3^2 \right) + \frac{1}{16\pi^3} \text{tr}(Y_U Y_U^+) \left(\frac{143\alpha_1}{180} + \frac{11\alpha_2}{4} + \frac{44\alpha_3}{9} \right) \\ &\quad + \frac{1}{16\pi^3} \text{tr}(Y_D Y_D^+) \left(\frac{14\alpha_1}{45} + 2\alpha_2 + \frac{32\alpha_3}{9} \right) + \frac{1}{16\pi^3} \text{tr}(Y_E Y_E^+) \left(\frac{9\alpha_1}{10} + \frac{3\alpha_2}{2} \right) \end{aligned}$$

³The detailed description of the higher covariant derivative regularization for the MSSM can be found in [46].

$$\begin{aligned}
& -\frac{1}{(16\pi^2)^2} \left[11 \operatorname{tr}((Y_U Y_U^\dagger)^2) + 8 \operatorname{tr}((Y_D Y_D^\dagger)^2) + 6 \operatorname{tr}((Y_E Y_E^\dagger)^2) + \frac{19}{3} \operatorname{tr}(Y_D Y_D^\dagger Y_U Y_U^\dagger) \right. \\
& + 11 (\operatorname{tr}(Y_U Y_U^\dagger))^2 + 8 (\operatorname{tr}(Y_D Y_D^\dagger))^2 + 2 (\operatorname{tr}(Y_E Y_E^\dagger))^2 + \frac{26}{3} \operatorname{tr}(Y_E Y_E^\dagger) \operatorname{tr}(Y_D Y_D^\dagger) \left. \right] \\
& + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6) \neq O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6);
\end{aligned} \tag{29}$$

$$\begin{aligned}
& \left[\left(\frac{3}{\alpha_3} - \frac{2\pi}{\alpha_3^2} \right) \beta_3 - 3 + 3\gamma_\mu - \gamma_{\det Y_U} - \gamma_{\det Y_D} \right]_{\overline{\text{DR}}} = \frac{1}{2\pi^2} \left(\frac{363\alpha_1^2}{400} + \frac{9\alpha_2^2}{16} - \frac{21\alpha_3^2}{8} \right) \\
& + \frac{1}{16\pi^3} \operatorname{tr}(Y_U Y_U^\dagger) \left(\frac{13\alpha_1}{30} + \frac{3\alpha_2}{2} + \frac{8\alpha_3}{3} \right) + \frac{1}{16\pi^3} \operatorname{tr}(Y_D Y_D^\dagger) \left(\frac{7\alpha_1}{30} + \frac{3\alpha_2}{2} + \frac{8\alpha_3}{3} \right) \\
& - \frac{1}{(16\pi^2)^2} \left[6 \operatorname{tr}((Y_U Y_U^\dagger)^2) + 6 \operatorname{tr}((Y_D Y_D^\dagger)^2) + 6 (\operatorname{tr}(Y_U Y_U^\dagger))^2 + 6 (\operatorname{tr}(Y_D Y_D^\dagger))^2 \right. \\
& + 2 \operatorname{tr}(Y_E Y_E^\dagger) \operatorname{tr}(Y_D Y_D^\dagger) + 4 \operatorname{tr}(Y_D Y_D^\dagger Y_U Y_U^\dagger) \left. \right] + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6) \\
& \neq O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6).
\end{aligned} \tag{30}$$

Certainly, this occurs in such an approximation where the scheme dependence becomes essential.

VI. CONCLUSION

In this paper we have demonstrated that for the MSSM it is possible to construct two independent scale invariant expressions from the gauge couplings, Yukawa couplings, μ parameter, and the renormalization point μ . One of them can be chosen independent of the parameter μ . The derivation is based on the all-loop exact equations following from the NSVZ β functions and the nonrenormalization of the superpotential. That is why the renormalization group invariance takes place only for certain renormalization prescriptions in which these equations are satisfied. In particular, it should be valid in all loops in the HD + MSL scheme(s), when a theory is regularized by higher covariant derivatives and the renormalization is made with the help of minimal subtractions of logarithms. This statement has

been verified in the three-loop approximation for the β functions and two-loop approximation for the anomalous dimensions (of the Yukawa couplings and the μ parameter). In the $\overline{\text{DR}}$ scheme, the expressions constructed in this paper are not RGIs starting from such orders of the perturbation theory where the scheme dependence of RGFs becomes essential. The results can also be extended to the case of the NMSSM.

DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: DETAILED DERIVATION OF RGIS FOR THE MSSM

We start with the system of all-order exact Eqs. (13)–(15) and eliminate from them the expressions γ_{H_u} and γ_{H_d} . As a result, we obtain the system of differential equations

$$\begin{aligned}
& \frac{d}{d \ln \mu} \left(\frac{2\pi}{\alpha_3} + 3 \ln \alpha_3 \right) = 3 + \operatorname{tr} \left(\gamma_Q + \frac{1}{2} \gamma_U + \frac{1}{2} \gamma_D \right); \\
& \frac{d}{d \ln \mu} \left(\frac{2\pi}{\alpha_2} + 2 \ln \alpha_2 - \ln \mu \right) = -1 + \operatorname{tr} \left(\frac{3}{2} \gamma_Q + \frac{1}{2} \gamma_L \right); \\
& \frac{d}{d \ln \mu} \left(\frac{5}{3} \cdot \frac{2\pi}{\alpha_1} - \ln \mu \right) = -11 + \operatorname{tr} \left(\frac{1}{6} \gamma_Q + \frac{4}{3} \gamma_U + \frac{1}{3} \gamma_D + \frac{1}{2} \gamma_L + \gamma_E \right); \\
& \frac{d}{d \ln \mu} (\ln \det Y_D + \ln \det Y_U - 3 \ln \mu) = \frac{1}{2} \operatorname{tr} (2\gamma_Q + \gamma_U + \gamma_D); \\
& \frac{d}{d \ln \mu} (\ln \det Y_E - \ln \det Y_D) = \frac{1}{2} \operatorname{tr} (\gamma_L + \gamma_E - \gamma_Q - \gamma_D).
\end{aligned} \tag{A1}$$

After that, we eliminate the expressions $\text{tr}(\gamma_L)$ and $\text{tr}(\gamma_E)$. The resulting equations contain the anomalous dimensions γ_Q , γ_D , and γ_U only in the combination $\text{tr}(2\gamma_Q + \gamma_U + \gamma_D)$,

$$\begin{aligned} \frac{d}{d \ln \mu} \left(\frac{2\pi}{\alpha_3} + 3 \ln \alpha_3 \right) &= 3 + \frac{1}{2} \text{tr}(2\gamma_Q + \gamma_U + \gamma_D); \\ \frac{d}{d \ln \mu} \left(\frac{2\pi}{\alpha_2} + 2 \ln \alpha_2 + \frac{5}{3} \cdot \frac{2\pi}{\alpha_1} - 2 \ln \mu - 2 \ln \det Y_E + 2 \ln \det Y_D \right) &= -12 + \frac{4}{3} \text{tr}(2\gamma_Q + \gamma_U + \gamma_D); \\ \frac{d}{d \ln \mu} (\ln \det Y_D + \ln \det Y_U - 3 \ln \mu) &= \frac{1}{2} \text{tr}(2\gamma_Q + \gamma_U + \gamma_D). \end{aligned} \quad (\text{A2})$$

The appearance of this combination on the right-hand sides of all these equations seems to be a particular feature of the MSSM. Because of this feature, it is possible to eliminate all anomalous dimensions of the MSSM chiral matter superfields and obtain the system of two differential equations that do not contain them,

$$\begin{aligned} \frac{d}{d \ln \mu} \left(\frac{8}{3} \cdot \frac{2\pi}{\alpha_3} + 8 \ln \alpha_3 - \frac{2\pi}{\alpha_2} - 2 \ln \alpha_2 - \frac{5}{3} \cdot \frac{2\pi}{\alpha_1} + 2 \ln \mu + 2 \ln \det Y_E - 2 \ln \det Y_D - 20 \ln \mu \right) &= 0; \\ \frac{d}{d \ln \mu} \left(\frac{2\pi}{\alpha_3} + 3 \ln \alpha_3 - \ln \det Y_D - \ln \det Y_U + 3 \ln \mu - 3 \ln \mu \right) &= 0. \end{aligned} \quad (\text{A3})$$

Next, there are two options. Namely, it is possible either to construct an equation that does not explicitly depend on the scale μ or does not contain the MSSM parameter μ . In the former case, we obtain the differential equation

$$\frac{d}{d \ln \mu} \left(\frac{2\pi}{\alpha_3} + 3 \ln \alpha_3 + \frac{\pi}{2\alpha_2} + \frac{1}{2} \ln \alpha_2 + \frac{5\pi}{6\alpha_1} - \frac{1}{2} \ln \det Y_E - \frac{7}{6} \ln \det Y_D - \frac{5}{3} \ln \det Y_U + \frac{9}{2} \ln \mu \right) = 0, \quad (\text{A4})$$

from which we conclude that the expression

$$\frac{\mu^{9/2} (\alpha_3)^3 (\alpha_2)^{1/2}}{(\det Y_E)^{1/2} (\det Y_U)^{5/3} (\det Y_D)^{7/6}} \exp \left(\frac{2\pi}{\alpha_3} + \frac{\pi}{2\alpha_2} + \frac{5\pi}{6\alpha_1} \right) \equiv \text{RGI}_1 \quad (\text{A5})$$

is independent of μ .

Alternatively, it is possible to eliminate the parameter μ present in the MSSM superpotential. The resulting differential equation can be written in the form

$$\frac{d}{d \ln \mu} \left(\frac{2\pi}{\alpha_3} + 3 \ln \alpha_3 - \frac{\pi}{\alpha_2} - \ln \alpha_2 - \frac{5\pi}{3\alpha_1} + \ln \det Y_E - \frac{2}{3} \ln \det Y_D + \frac{1}{3} \ln \det Y_U - 9 \ln \mu \right) = 0. \quad (\text{A6})$$

Integrating it, we obtain the second RGI

$$\frac{(\alpha_3)^3 \det Y_E (\det Y_U)^{1/3}}{\mu^9 \alpha_2 (\det Y_D)^{2/3}} \exp \left(\frac{2\pi}{\alpha_3} - \frac{\pi}{\alpha_2} - \frac{5\pi}{3\alpha_1} \right) \equiv \text{RGI}_2, \quad (\text{A7})$$

which, however, explicitly depends on the renormalization point μ .

APPENDIX B: EXPRESSIONS FOR RGFs IN THE HD + MSL SCHEME

The expressions for all MSSM β functions in the three-loop approximation and all anomalous dimensions of the chiral matter superfields in the two-loop approximation for

an arbitrary renormalization prescription supplementing the higher covariant derivative regularization have been calculated in [51]. In particular, it is easy to obtain the results in the HD + MSL scheme setting all finite constants to 0 (or, equivalently, replacing $\alpha_{0i} \rightarrow \alpha_i$, $Y_{0i} \rightarrow Y_i$ in RGFs defined in terms of the bare couplings). After that, the expressions for the three-loop MSSM β functions in the HD + MSL scheme take the form

$$\begin{aligned}
\frac{\beta_1(\alpha, Y)}{\alpha_1^2} = & -\frac{1}{2\pi} \cdot \frac{3}{5} \left\{ -11 - \frac{199\alpha_1}{60\pi} - \frac{9\alpha_2}{4\pi} - \frac{22\alpha_3}{3\pi} + \frac{1}{8\pi^2} \text{tr} \left(\frac{13}{3} Y_U Y_U^+ + \frac{7}{3} Y_D Y_D^+ + 3 Y_E Y_E^+ \right) \right. \\
& + \frac{1}{2\pi^2} \left[\frac{5131\alpha_1^2}{3600} + \frac{27\alpha_2^2}{16} + \frac{88\alpha_3^2}{9} + \frac{23\alpha_1\alpha_2}{40} + \frac{137\alpha_1\alpha_3}{45} + \alpha_2\alpha_3 + \frac{2189\alpha_1^2}{100} \left(\ln a_1 + 1 + \frac{A}{2} \right) \right. \\
& + \frac{9\alpha_2^2}{4} \left(7 \ln a_2 - 6 \ln a_{\varphi,2} + 1 + \frac{A}{2} \right) - 22\alpha_3^2 \left(3 \ln a_{\varphi,3} - 2 \ln a_3 + 1 + \frac{A}{2} \right) \left. \right] + \frac{1}{8\pi^3} \text{tr}(Y_U Y_U^+) \\
& \times \left[2\alpha_2 + 2\alpha_3 + (B-A) \left(\frac{169\alpha_1}{180} + \frac{13\alpha_2}{4} + \frac{52\alpha_3}{9} \right) \right] + \frac{1}{8\pi^3} \text{tr}(Y_D Y_D^+) \left[\frac{\alpha_2}{2} + 2\alpha_3 + (B-A) \right. \\
& \times \left(\frac{49\alpha_1}{180} + \frac{7\alpha_2}{4} + \frac{28\alpha_3}{9} \right) \left. \right] + \frac{1}{8\pi^3} \text{tr}(Y_E Y_E^+) \left[\frac{3\alpha_2}{2} + (B-A) \left(\frac{27\alpha_1}{20} + \frac{9\alpha_2}{4} \right) \right] - \frac{1}{(8\pi^2)^2} \left[\frac{15}{4} \right. \\
& \times \text{tr}((Y_U Y_U^+)^2) + \frac{11}{4} \text{tr}((Y_D Y_D^+)^2) + \frac{9}{4} \text{tr}((Y_E Y_E^+)^2) + \frac{19}{6} \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + \frac{17}{4} (\text{tr}(Y_U Y_U^+))^2 \\
& + \frac{5}{4} (\text{tr}(Y_D Y_D^+))^2 + \frac{5}{4} (\text{tr}(Y_E Y_E^+))^2 + \frac{25}{6} \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) \left. \right] \left. \right\} + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \quad (\text{B1})
\end{aligned}$$

$$\begin{aligned}
\frac{\beta_2(\alpha, Y)}{\alpha_2^2} = & -\frac{1}{2\pi} \left\{ -1 - \frac{9\alpha_1}{20\pi} - \frac{25\alpha_2}{4\pi} - \frac{6\alpha_3}{\pi} + \frac{1}{8\pi^2} \text{tr}(3 Y_U Y_U^+ + 3 Y_D Y_D^+ + Y_E Y_E^+) \right. \\
& + \frac{1}{2\pi^2} \left[\frac{23\alpha_1^2}{400} - \frac{137\alpha_2^2}{16} + 8\alpha_3^2 - \frac{9\alpha_1\alpha_2}{40} + \frac{\alpha_1\alpha_3}{5} - 3\alpha_2\alpha_3 + \frac{297\alpha_1^2}{100} \left(\ln a_1 + 1 + \frac{A}{2} \right) + \frac{21\alpha_2^2}{4} \right. \\
& \times \left(7 \ln a_2 - 6 \ln a_{\varphi,2} + 1 + \frac{A}{2} \right) - 18\alpha_3^2 \left(3 \ln a_{\varphi,3} - 2 \ln a_3 + 1 + \frac{A}{2} \right) \left. \right] + \frac{1}{8\pi^3} \text{tr}(Y_U Y_U^+) \\
& \times \left[\frac{2\alpha_1}{5} + 3\alpha_2 + 2\alpha_3 + (B-A) \left(\frac{13\alpha_1}{20} + \frac{9\alpha_2}{4} + 4\alpha_3 \right) \right] + \frac{1}{8\pi^3} \text{tr}(Y_D Y_D^+) \left[\frac{\alpha_1}{10} + 3\alpha_2 + 2\alpha_3 \right. \\
& + (B-A) \left(\frac{7\alpha_1}{20} + \frac{9\alpha_2}{4} + 4\alpha_3 \right) \left. \right] + \frac{1}{8\pi^3} \text{tr}(Y_E Y_E^+) \left[\frac{3\alpha_1}{10} + \alpha_2 + (B-A) \left(\frac{9\alpha_1}{20} + \frac{3\alpha_2}{4} \right) \right] \\
& - \frac{1}{(8\pi^2)^2} \left[\frac{15}{4} \text{tr}((Y_U Y_U^+)^2) + \frac{15}{4} \text{tr}((Y_D Y_D^+)^2) + \frac{5}{4} \text{tr}((Y_E Y_E^+)^2) + \frac{9}{4} (\text{tr}(Y_U Y_U^+))^2 \right. \\
& + \frac{9}{4} (\text{tr}(Y_D Y_D^+))^2 + \frac{1}{4} (\text{tr}(Y_E Y_E^+))^2 + \frac{3}{2} \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) + \frac{3}{2} \text{tr}(Y_D Y_D^+ Y_U Y_U^+) \left. \right] \left. \right\} \\
& + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \quad (\text{B2})
\end{aligned}$$

$$\begin{aligned}
\frac{\beta_3(\alpha, Y)}{\alpha_3^2} = & -\frac{1}{2\pi} \left\{ 3 - \frac{11\alpha_1}{20\pi} - \frac{9\alpha_2}{4\pi} - \frac{7\alpha_3}{2\pi} + \frac{1}{8\pi^2} \text{tr}(2 Y_U Y_U^+ + 2 Y_D Y_D^+) + \frac{1}{2\pi^2} \left[\frac{137\alpha_1^2}{1200} \right. \right. \\
& + \frac{27\alpha_2^2}{16} + \frac{\alpha_3^2}{6} + \frac{3\alpha_1\alpha_2}{40} - \frac{11\alpha_1\alpha_3}{60} - \frac{3\alpha_2\alpha_3}{4} + \frac{363\alpha_1^2}{100} \left(\ln a_1 + 1 + \frac{A}{2} \right) + \frac{9\alpha_2^2}{4} \left(-6 \ln a_{\varphi,2} \right. \\
& + 7 \ln a_2 + 1 + \frac{A}{2} \left. \right) - 24\alpha_3^2 \left(3 \ln a_{\varphi,3} - 2 \ln a_3 + 1 + \frac{A}{2} \right) \left. \right] + \frac{1}{8\pi^3} \text{tr}(Y_U Y_U^+) \left[\frac{3\alpha_1}{20} + \frac{3\alpha_2}{4} \right. \\
& + 3\alpha_3 + (B-A) \left(\frac{13\alpha_1}{30} + \frac{3\alpha_2}{2} + \frac{8\alpha_3}{3} \right) \left. \right] + \frac{1}{8\pi^3} \text{tr}(Y_D Y_D^+) \left[\frac{3\alpha_1}{20} + \frac{3\alpha_2}{4} + 3\alpha_3 + (B-A) \right. \\
& \times \left(\frac{7\alpha_1}{30} + \frac{3\alpha_2}{2} + \frac{8\alpha_3}{3} \right) \left. \right] - \frac{1}{(8\pi^2)^2} \left[\frac{3}{2} \text{tr}((Y_U Y_U^+)^2) + \frac{3}{2} \text{tr}((Y_D Y_D^+)^2) + 3 (\text{tr}(Y_U Y_U^+))^2 \right. \\
& + 3 (\text{tr}(Y_D Y_D^+))^2 + \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) + \text{tr}(Y_D Y_D^+ Y_U Y_U^+) \left. \right] \left. \right\} + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6). \quad (\text{B3})
\end{aligned}$$

Similarly, starting from the expressions for the anomalous dimensions of the chiral matter superfields presented in [51], it is possible to construct expressions for the anomalous dimensions entering Eqs. (24) and (25) in the two-loop approximation,

$$\begin{aligned}
\gamma_{\det Y_U}(\alpha, Y) &= \frac{1}{2} (3\gamma_{H_u}(\alpha, Y) + \text{tr} \gamma_Q(\alpha, Y) + \text{tr} \gamma_U(\alpha, Y)) \\
&= -\frac{13\alpha_1}{20\pi} - \frac{9\alpha_2}{4\pi} - \frac{4\alpha_3}{\pi} + \frac{1}{16\pi^2} \text{tr}(12Y_U Y_U^+ + Y_D Y_D^+) + \frac{1}{2\pi^2} \left[\frac{169\alpha_1^2}{1200} + \frac{27\alpha_2^2}{16} + \frac{16\alpha_3^2}{3} \right. \\
&\quad + \frac{3\alpha_1\alpha_2}{8} + \frac{17\alpha_1\alpha_3}{15} + 3\alpha_2\alpha_3 - \frac{27\alpha_2^2}{2} \left(\ln a_{\varphi,2} + 1 + \frac{A}{2} \right) - 36\alpha_3^2 \left(\ln a_{\varphi,3} + 1 + \frac{A}{2} \right) \\
&\quad + \frac{429\alpha_1^2}{100} \left(\ln a_1 + 1 + \frac{A}{2} \right) + \frac{63\alpha_2^2}{4} \left(\ln a_2 + 1 + \frac{A}{2} \right) + 24\alpha_3^2 \left(\ln a_3 + 1 + \frac{A}{2} \right) \left. \right] + \frac{1}{16\pi^3} \\
&\quad \times \text{tr}(Y_U Y_U^+) \left[\frac{7\alpha_1}{10} + \frac{3\alpha_2}{2} + 12\alpha_3 + (B-A) \left(\frac{13\alpha_1}{5} + 9\alpha_2 + 16\alpha_3 \right) \right] + \frac{1}{16\pi^3} \text{tr}(Y_D Y_D^+) \\
&\quad \times \left[\frac{\alpha_1}{10} + (B-A) \left(\frac{7\alpha_1}{60} + \frac{3\alpha_2}{4} + \frac{4\alpha_3}{3} \right) \right] - \frac{1}{(16\pi^2)^2} \left[31 \text{tr}((Y_U Y_U^+)^2) + 2 \text{tr}((Y_D Y_D^+)^2) \right. \\
&\quad + 11 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + 9(\text{tr}(Y_U Y_U^+))^2 + 3(\text{tr}(Y_D Y_D^+))^2 + \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) \left. \right] \\
&\quad + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \tag{B4}
\end{aligned}$$

$$\begin{aligned}
\gamma_{\det Y_D}(\alpha, Y) &= \frac{1}{2} (3\gamma_{H_d}(\alpha, Y) + \text{tr} \gamma_Q(\alpha, Y) + \text{tr} \gamma_D(\alpha, Y)) \\
&= -\frac{7\alpha_1}{20\pi} - \frac{9\alpha_2}{4\pi} - \frac{4\alpha_3}{\pi} + \frac{1}{16\pi^2} \text{tr}(Y_U Y_U^+ + 12Y_D Y_D^+ + 3Y_E Y_E^+) + \frac{1}{2\pi^2} \left[\frac{49\alpha_1^2}{1200} + \frac{27\alpha_2^2}{16} \right. \\
&\quad + \frac{16\alpha_3^2}{3} + \frac{3\alpha_1\alpha_2}{8} + \frac{\alpha_1\alpha_3}{3} + 3\alpha_2\alpha_3 - \frac{27\alpha_2^2}{2} \left(\ln a_{\varphi,2} + 1 + \frac{A}{2} \right) - 36\alpha_3^2 \left(\ln a_{\varphi,3} + 1 + \frac{A}{2} \right) \\
&\quad + \frac{231\alpha_1^2}{100} \left(\ln a_1 + 1 + \frac{A}{2} \right) + \frac{63\alpha_2^2}{4} \left(\ln a_2 + 1 + \frac{A}{2} \right) + 24\alpha_3^2 \left(\ln a_3 + 1 + \frac{A}{2} \right) \left. \right] + \frac{1}{16\pi^3} \\
&\quad \times \text{tr}(Y_U Y_U^+) \left[\frac{\alpha_1}{5} + (B-A) \left(\frac{13\alpha_1}{60} + \frac{3\alpha_2}{4} + \frac{4\alpha_3}{3} \right) \right] + \frac{1}{16\pi^3} \text{tr}(Y_D Y_D^+) \left[-\frac{\alpha_1}{10} + \frac{3\alpha_2}{2} \right. \\
&\quad + 12\alpha_3 + (B-A) \left(\frac{7\alpha_1}{5} + 9\alpha_2 + 16\alpha_3 \right) \left. \right] + \frac{1}{16\pi^3} \text{tr}(Y_E Y_E^+) \left[\frac{9\alpha_1}{10} + (B-A) \left(\frac{27\alpha_1}{20} \right. \right. \\
&\quad + \left. \left. \frac{9\alpha_2}{4} \right) \right] - \frac{1}{(16\pi^2)^2} \left[2 \text{tr}((Y_U Y_U^+)^2) + 31 \text{tr}((Y_D Y_D^+)^2) + 9 \text{tr}((Y_E Y_E^+)^2) \right. \\
&\quad + 11 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + 3(\text{tr}(Y_U Y_U^+))^2 + 9(\text{tr}(Y_D Y_D^+))^2 + 3 \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) \left. \right] \\
&\quad + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \tag{B5}
\end{aligned}$$

$$\begin{aligned}
\gamma_{\det Y_E}(\alpha, Y) &= \frac{1}{2} (3\gamma_{H_d}(\alpha, Y) + \text{tr}\gamma_L(\alpha, Y) + \text{tr}\gamma_E(\alpha, Y)) \\
&= -\frac{27\alpha_1}{20\pi} - \frac{9\alpha_2}{4\pi} + \frac{1}{16\pi^2} \text{tr}(9Y_D Y_D^+ + 6Y_E Y_E^+) + \frac{1}{2\pi^2} \left[\frac{243\alpha_1^2}{400} + \frac{27\alpha_2^2}{16} + \frac{27\alpha_1\alpha_2}{40} \right. \\
&\quad \left. - \frac{27\alpha_2^2}{2} \left(\ln a_{\varphi,2} + 1 + \frac{A}{2} \right) + \frac{891\alpha_1^2}{100} \left(\ln a_1 + 1 + \frac{A}{2} \right) + \frac{63\alpha_2^2}{4} \left(\ln a_2 + 1 + \frac{A}{2} \right) \right] + \frac{1}{16\pi^3} \\
&\quad \times \text{tr}(Y_D Y_D^+) \left[-\frac{3\alpha_1}{10} + 12\alpha_3 + (B-A) \left(\frac{21\alpha_1}{20} + \frac{27\alpha_2}{4} + 12\alpha_3 \right) \right] + \frac{1}{16\pi^3} \text{tr}(Y_E Y_E^+) \\
&\quad \times \left[\frac{9\alpha_1}{10} + \frac{3\alpha_2}{2} + (B-A) \left(\frac{27\alpha_1}{10} + \frac{9\alpha_2}{2} \right) \right] - \frac{1}{(16\pi^2)^2} \left[27 \text{tr}((Y_D Y_D^+)^2) + 13 \text{tr}((Y_E Y_E^+)^2) \right. \\
&\quad \left. + 9 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + 3(\text{tr}(Y_E Y_E^+))^2 + 9 \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) \right] + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \quad (\text{B6})
\end{aligned}$$

$$\begin{aligned}
\gamma_\mu(\alpha, Y) &= \frac{1}{2} (\gamma_{H_u}(\alpha, Y) + \gamma_{H_d}(\alpha, Y)) \\
&= -\frac{3\alpha_1}{20\pi} - \frac{3\alpha_2}{4\pi} + \frac{1}{16\pi^2} \text{tr}(3Y_U Y_U^+ + 3Y_D Y_D^+ + Y_E Y_E^+) + \frac{1}{2\pi^2} \left[\frac{9\alpha_1^2}{400} + \frac{9\alpha_2^2}{16} + \frac{9}{40} \alpha_1 \alpha_2 \right. \\
&\quad \left. - \frac{9\alpha_2^2}{2} \left(\ln a_{\varphi,2} + 1 + \frac{A}{2} \right) + \frac{99\alpha_1^2}{100} \left(\ln a_1 + 1 + \frac{A}{2} \right) + \frac{21\alpha_2^2}{4} \left(\ln a_2 + 1 + \frac{A}{2} \right) \right] + \frac{1}{16\pi^3} \\
&\quad \times \text{tr}(Y_E Y_E^+) \left[\frac{3\alpha_1}{10} + (B-A) \left(\frac{9\alpha_1}{20} + \frac{3\alpha_2}{4} \right) \right] + \frac{1}{16\pi^3} \text{tr}(Y_U Y_U^+) \left[\frac{\alpha_1}{5} + 4\alpha_3 + (B-A) \right. \\
&\quad \left. \times \left(\frac{13\alpha_1}{20} + \frac{9\alpha_2}{4} + 4\alpha_3 \right) \right] + \frac{1}{16\pi^3} \text{tr}(Y_D Y_D^+) \left[-\frac{\alpha_1}{10} + 4\alpha_3 + (B-A) \left(\frac{7\alpha_1}{20} + \frac{9\alpha_2}{4} + 4\alpha_3 \right) \right] \\
&\quad - \frac{1}{(16\pi^2)^2} \left[3 \text{tr}((Y_E Y_E^+)^2) + 6 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + 9 \text{tr}((Y_U Y_U^+)^2) + 9 \text{tr}((Y_D Y_D^+)^2) \right] \\
&\quad + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6). \quad (\text{B7})
\end{aligned}$$

APPENDIX C: EXPRESSIONS FOR RGFS IN THE $\overline{\text{DR}}$ SCHEME

To verify Eqs. (24) and (25) in the $\overline{\text{DR}}$ scheme, we present the expressions for the three-loop MSSM β functions calculated in [48] and rewritten in the notation adopted in this paper,

$$\begin{aligned}
\frac{\beta_1(\alpha, Y)}{\alpha_1^2} &= -\frac{1}{2\pi} \cdot \frac{3}{5} \left\{ -11 - \frac{199\alpha_1}{60\pi} - \frac{9\alpha_2}{4\pi} - \frac{22\alpha_3}{3\pi} + \frac{1}{8\pi^2} \text{tr} \left(\frac{13}{3} Y_U Y_U^+ + \frac{7}{3} Y_D Y_D^+ + 3Y_E Y_E^+ \right) \right. \\
&\quad + \frac{1}{2\pi^2} \left(\frac{32117\alpha_1^2}{1800} + \frac{27\alpha_2^2}{8} - \frac{121\alpha_3^2}{18} + \frac{23\alpha_1\alpha_2}{40} + \frac{137\alpha_1\alpha_3}{45} + \alpha_2\alpha_3 \right) + \frac{1}{8\pi^3} \text{tr}(Y_U Y_U^+) \left(\frac{169\alpha_1}{360} \right. \\
&\quad \left. + \frac{29\alpha_2}{8} + \frac{44\alpha_3}{9} \right) + \frac{1}{8\pi^3} \text{tr}(Y_D Y_D^+) \left(\frac{49\alpha_1}{360} + \frac{11\alpha_2}{8} + \frac{32\alpha_3}{9} \right) + \frac{1}{8\pi^3} \text{tr}(Y_E Y_E^+) \left(\frac{27\alpha_1}{40} + \frac{21\alpha_2}{8} \right) \\
&\quad - \frac{1}{(8\pi^2)^2} \left[7 \text{tr}((Y_U Y_U^+)^2) + \frac{9}{2} \text{tr}((Y_D Y_D^+)^2) + \frac{9}{2} \text{tr}((Y_E Y_E^+)^2) + \frac{29}{6} \text{tr}(Y_D Y_D^+ Y_U Y_U^+) \right. \\
&\quad \left. + \frac{15}{2} (\text{tr}(Y_U Y_U^+))^2 + 3(\text{tr}(Y_D Y_D^+))^2 + 2(\text{tr}(Y_E Y_E^+))^2 + 7 \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) \right] \Big\} \\
&\quad + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \quad (\text{C1})
\end{aligned}$$

$$\begin{aligned}
\frac{\beta_2(\alpha, Y)}{\alpha_2^2} = & -\frac{1}{2\pi} \left\{ -1 - \frac{9\alpha_1}{20\pi} - \frac{25\alpha_2}{4\pi} - \frac{6\alpha_3}{\pi} + \frac{1}{8\pi^2} \text{tr}(3Y_U Y_U^+ + 3Y_D Y_D^+ + Y_E Y_E^+) + \frac{1}{2\pi^2} \right. \\
& \times \left(\frac{457\alpha_1^2}{200} - \frac{35\alpha_2^2}{8} - \frac{11\alpha_3^2}{2} - \frac{9\alpha_1\alpha_2}{40} + \frac{\alpha_1\alpha_3}{5} - 3\alpha_2\alpha_3 \right) + \frac{1}{8\pi^3} \text{tr}(Y_U Y_U^+) \left(\frac{29\alpha_1}{40} + \frac{33\alpha_2}{8} \right. \\
& + 4\alpha_3 \left. \right) + \frac{1}{8\pi^3} \text{tr}(Y_D Y_D^+) \left(\frac{11\alpha_1}{40} + \frac{33\alpha_2}{8} + 4\alpha_3 \right) + \frac{1}{8\pi^3} \text{tr}(Y_E Y_E^+) \left(\frac{21\alpha_1}{40} + \frac{11\alpha_2}{8} \right) - \frac{1}{(8\pi^2)^2} \\
& \times \left[6 \text{tr}((Y_U Y_U^+)^2) + 6 \text{tr}((Y_D Y_D^+)^2) + 2 \text{tr}((Y_E Y_E^+)^2) + \frac{9}{2} (\text{tr}(Y_U Y_U^+))^2 + \frac{9}{2} (\text{tr}(Y_D Y_D^+))^2 \right. \\
& + \frac{1}{2} (\text{tr}(Y_E Y_E^+))^2 + 3 \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) + 3 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) \left. \right\} + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \quad (C2)
\end{aligned}$$

$$\begin{aligned}
\frac{\beta_3(\alpha, Y)}{\alpha_3^2} = & -\frac{1}{2\pi} \left\{ 3 - \frac{11\alpha_1}{20\pi} - \frac{9\alpha_2}{4\pi} - \frac{7\alpha_3}{2\pi} + \frac{1}{8\pi^2} \text{tr}(2Y_U Y_U^+ + 2Y_D Y_D^+) + \frac{1}{2\pi^2} \left(\frac{851\alpha_1^2}{300} \right. \right. \\
& + \frac{27\alpha_2^2}{8} - \frac{347\alpha_3^2}{24} + \frac{3\alpha_1\alpha_2}{40} - \frac{11\alpha_1\alpha_3}{60} - \frac{3\alpha_2\alpha_3}{4} \left. \right) + \frac{1}{8\pi^3} \text{tr}(Y_U Y_U^+) \left(\frac{11\alpha_1}{30} + \frac{3\alpha_2}{2} + \frac{13\alpha_3}{3} \right) \\
& + \frac{1}{8\pi^3} \text{tr}(Y_D Y_D^+) \left(\frac{4\alpha_1}{15} + \frac{3\alpha_2}{2} + \frac{13\alpha_3}{3} \right) - \frac{1}{(8\pi^2)^2} \left[3 \text{tr}((Y_U Y_U^+)^2) + 3 \text{tr}((Y_D Y_D^+)^2) \right. \\
& + \frac{9}{2} (\text{tr}(Y_U Y_U^+))^2 + \frac{9}{2} (\text{tr}(Y_D Y_D^+))^2 + \frac{3}{2} \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) + 2 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) \left. \right] \left. \right\} \\
& + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6). \quad (C3)
\end{aligned}$$

The anomalous dimensions entering Eqs. (24) and (25) can also be calculated with the help of Eqs. (13) and (14) because in the $\overline{\text{DR}}$ scheme all renormalization constants contain only ε poles. The two-loop anomalous dimensions for all MSSM chiral matter superfields were first calculated in [55]. They are also given in [48]. In the notation adopted in this paper, the corresponding results can be found in [51]. Using them, after some algebraic transformations we obtain the anomalous dimensions needed for checking Eqs. (24) and (25) for the $\overline{\text{DR}}$ renormalization prescription,

$$\begin{aligned}
\gamma_{\det Y_U}(\alpha, Y) = & \frac{1}{2} (3\gamma_{H_u}(\alpha, Y) + \text{tr} \gamma_Q(\alpha, Y) + \text{tr} \gamma_U(\alpha, Y)) \\
= & -\frac{13\alpha_1}{20\pi} - \frac{9\alpha_2}{4\pi} - \frac{4\alpha_3}{\pi} + \frac{1}{16\pi^2} \text{tr}(12Y_U Y_U^+ + Y_D Y_D^+) + \frac{1}{2\pi^2} \left[\frac{2743\alpha_1^2}{1200} + \frac{45\alpha_2^2}{16} - \frac{2\alpha_3^2}{3} \right. \\
& + \frac{3\alpha_1\alpha_2}{8} + \frac{17\alpha_1\alpha_3}{15} + 3\alpha_2\alpha_3 \left. \right] + \frac{1}{16\pi^3} \text{tr}(Y_U Y_U^+) \left[\frac{7\alpha_1}{10} + \frac{3\alpha_2}{2} + 12\alpha_3 \right] + \frac{1}{16\pi^3} \text{tr}(Y_D Y_D^+) \\
& \times \frac{\alpha_1}{10} - \frac{1}{(16\pi^2)^2} \left[31 \text{tr}((Y_U Y_U^+)^2) + 2 \text{tr}((Y_D Y_D^+)^2) + 11 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + 9 (\text{tr}(Y_U Y_U^+))^2 \right. \\
& + 3 (\text{tr}(Y_D Y_D^+))^2 + \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) \left. \right] + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \quad (C4)
\end{aligned}$$

$$\begin{aligned}
\gamma_{\det Y_D}(\alpha, Y) = & \frac{1}{2} (3\gamma_{H_d}(\alpha, Y) + \text{tr} \gamma_Q(\alpha, Y) + \text{tr} \gamma_D(\alpha, Y)) \\
= & -\frac{7\alpha_1}{20\pi} - \frac{9\alpha_2}{4\pi} - \frac{4\alpha_3}{\pi} + \frac{1}{16\pi^2} \text{tr}(Y_U Y_U^+ + 12Y_D Y_D^+ + 3Y_E Y_E^+) + \frac{1}{2\pi^2} \left[\frac{287\alpha_1^2}{240} + \frac{45\alpha_2^2}{16} \right. \\
& - \frac{2\alpha_3^2}{3} + \frac{3\alpha_1\alpha_2}{8} + \frac{\alpha_1\alpha_3}{3} + 3\alpha_2\alpha_3 \left. \right] + \frac{1}{16\pi^3} \text{tr}(Y_U Y_U^+) \cdot \frac{\alpha_1}{5} + \frac{1}{16\pi^3} \text{tr}(Y_D Y_D^+) \left[-\frac{\alpha_1}{10} + \frac{3\alpha_2}{2} \right. \\
& + 12\alpha_3 \left. \right] + \frac{1}{16\pi^3} \text{tr}(Y_E Y_E^+) \cdot \frac{9\alpha_1}{10} - \frac{1}{(16\pi^2)^2} \left[2 \text{tr}((Y_U Y_U^+)^2) + 31 \text{tr}((Y_D Y_D^+)^2) \right. \\
& + 9 \text{tr}((Y_E Y_E^+)^2) + 11 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + 3 (\text{tr}(Y_U Y_U^+))^2 + 9 (\text{tr}(Y_D Y_D^+))^2 + 3 \text{tr}(Y_E Y_E^+) \\
& \times \text{tr}(Y_D Y_D^+) \left. \right] + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6); \quad (C5)
\end{aligned}$$

$$\begin{aligned}
\gamma_{\det Y_E}(\alpha, Y) &= \frac{1}{2} (3\gamma_{H_d}(\alpha, Y) + \text{tr} \gamma_L(\alpha, Y) + \text{tr} \gamma_E(\alpha, Y)) \\
&= -\frac{27\alpha_1}{20\pi} - \frac{9\alpha_2}{4\pi} + \frac{1}{16\pi^2} \text{tr}(9Y_D Y_D^+ + 6Y_E Y_E^+) + \frac{1}{2\pi^2} \left[\frac{81\alpha_1^2}{16} + \frac{45\alpha_2^2}{16} + \frac{27\alpha_1\alpha_2}{40} \right] + \frac{1}{16\pi^3} \\
&\quad \times \text{tr}(Y_D Y_D^+) \left[-\frac{3\alpha_1}{10} + 12\alpha_3 \right] + \frac{1}{16\pi^3} \text{tr}(Y_E Y_E^+) \left[\frac{9\alpha_1}{10} + \frac{3\alpha_2}{2} \right] - \frac{1}{(16\pi^2)^2} \left[27 \text{tr}((Y_D Y_D^+)^2) \right. \\
&\quad \left. + 13 \text{tr}((Y_E Y_E^+)^2) + 9 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + 3(\text{tr}(Y_E Y_E^+))^2 + 9 \text{tr}(Y_E Y_E^+) \text{tr}(Y_D Y_D^+) \right] \\
&\quad + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6);
\end{aligned} \tag{C6}$$

$$\begin{aligned}
\gamma_\mu(\alpha, Y) &= \frac{1}{2} (\gamma_{H_u}(\alpha, Y) + \gamma_{H_d}(\alpha, Y)) \\
&= -\frac{3\alpha_1}{20\pi} - \frac{3\alpha_2}{4\pi} + \frac{1}{16\pi^2} \text{tr}(3Y_U Y_U^+ + 3Y_D Y_D^+ + Y_E Y_E^+) + \frac{1}{2\pi^2} \left(\frac{207\alpha_1^2}{400} + \frac{15\alpha_2^2}{16} + \frac{9\alpha_1\alpha_2}{40} \right) \\
&\quad + \frac{1}{16\pi^3} \left[\text{tr}(Y_E Y_E^+) \frac{3\alpha_1}{10} + \text{tr}(Y_U Y_U^+) \left(\frac{\alpha_1}{5} + 4\alpha_3 \right) + \text{tr}(Y_D Y_D^+) \left(-\frac{\alpha_1}{10} + 4\alpha_3 \right) \right] \\
&\quad - \frac{1}{(16\pi^2)^2} \left[3 \text{tr}((Y_E Y_E^+)^2) + 6 \text{tr}(Y_D Y_D^+ Y_U Y_U^+) + 9 \text{tr}((Y_U Y_U^+)^2) + 9 \text{tr}((Y_D Y_D^+)^2) \right] \\
&\quad + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6).
\end{aligned} \tag{C7}$$

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- [1] J. R. Ellis, S. Kelley, and D. V. Nanopoulos, *Phys. Lett. B* **260**, 131 (1991).
- [2] U. Amaldi, W. de Boer, and H. Furstenau, *Phys. Lett. B* **260**, 447 (1991).
- [3] P. Langacker and M. X. Luo, *Phys. Rev. D* **44**, 817 (1991).
- [4] P. C. West, *Introduction to Supersymmetry and Supergravity* (World Scientific, Singapore, 1990), p. 425.
- [5] I. L. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity: Or a Walk Through Superspace* (IOP, Bristol, United Kingdom, 1998), p. 656.
- [6] R. N. Mohapatra, *Unification and Supersymmetry. The Frontiers of Quark—Lepton Physics: The Frontiers of Quark-Lepton Physics* (Springer, New York, 2002), p. 441.
- [7] M. S. Chanowitz, J. R. Ellis, and M. K. Gaillard, *Nucl. Phys. B* **128**, 506 (1977).
- [8] A. J. Buras, J. R. Ellis, M. K. Gaillard, and D. V. Nanopoulos, *Nucl. Phys. B* **135**, 66 (1978).
- [9] H. Georgi and D. V. Nanopoulos, *Nucl. Phys. B* **159**, 16 (1979).
- [10] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [11] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys. B* **229**, 381 (1983).
- [12] D. R. T. Jones, *Phys. Lett.* **123B**, 45 (1983).
- [13] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Phys. Lett.* **166B**, 329 (1986); *Yad. Fiz.* **43**, 459 (1986) [*Sov. J. Nucl. Phys.* **43**, 294 (1986)].
- [14] M. A. Shifman and A. I. Vainshtein, *Nucl. Phys. B* **277**, 456 (1986); *Zh. Eksp. Teor. Fiz.* **91**, 723 (1986) [*Sov. Phys. JETP* **64**, 428 (1986)].
- [15] M. A. Shifman, *Int. J. Mod. Phys. A* **11**, 5761 (1996).
- [16] D. Ghilencea, M. Lanzagorta, and G. G. Ross, *Nucl. Phys. B* **511**, 3 (1998).
- [17] G. Amelino-Camelia, D. Ghilencea, and G. G. Ross, *Nucl. Phys. B* **528**, 35 (1998).
- [18] L. Mihaila, *Adv. High Energy Phys.* **2013**, 607807 (2013).
- [19] S. Heinemeyer, M. Mondragón, N. Tracas, and G. Zoupanos, *Phys. Rep.* **814**, 1 (2019).
- [20] A. L. Kataev and K. V. Stepanyantz, *arXiv:2410.12070*.
- [21] S. L. Adler, *Phys. Rev. D* **10**, 3714 (1974).
- [22] M. Shifman and K. Stepanyantz, *Phys. Rev. Lett.* **114**, 051601 (2015).
- [23] M. Shifman and K. V. Stepanyantz, *Phys. Rev. D* **91**, 105008 (2015).
- [24] A. A. Slavnov, *Nucl. Phys. B* **331**, 301 (1971).
- [25] A. A. Slavnov, *Teor. Mat. Fiz.* **13**, 174 (1972) [*Theor. Math. Phys.* **13**, 1064 (1972)].
- [26] A. A. Slavnov, *Teor. Mat. Fiz.* **33**, 210 (1977) [*Theor. Math. Phys.* **33**, 977 (1977)].
- [27] V. K. Krivoshchekov, *Teor. Mat. Fiz.* **36**, 291 (1978) [*Theor. Math. Phys.* **36**, 745 (1978)].
- [28] P. C. West, *Nucl. Phys. B* **268**, 113 (1986).

- [29] A. L. Kataev and K. V. Stepanyantz, *Nucl. Phys.* **B875**, 459 (2013).
- [30] K. Stepanyantz, *Eur. Phys. J. C* **80**, 911 (2020).
- [31] A. E. Kazantsev, V. Y. Shakhmanov, and K. V. Stepanyantz, *J. High Energy Phys.* 04 (2018) 130.
- [32] M. D. Kuzmichev, N. P. Meshcheriakov, S. V. Novgorodtsev, I. E. Shirokov, and K. V. Stepanyantz, *Eur. Phys. J. C* **79**, 809 (2019).
- [33] S. S. Aleshin *et al.*, *Nucl. Phys.* **B956**, 115020 (2020).
- [34] I. Shirokov and V. Shirokova, *Eur. Phys. J. C* **84**, 249 (2024).
- [35] V. Y. Shakhmanov and K. V. Stepanyantz, *Phys. Lett. B* **776**, 417 (2018).
- [36] W. Siegel, *Phys. Lett.* **84B**, 193 (1979).
- [37] W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, *Phys. Rev. D* **18**, 3998 (1978).
- [38] I. Jack, D. R. T. Jones, and C. G. North, *Phys. Lett. B* **386**, 138 (1996).
- [39] I. Jack, D. R. T. Jones, and C. G. North, *Nucl. Phys.* **B486**, 479 (1997).
- [40] I. Jack, D. R. T. Jones, and A. Pickering, *Phys. Lett. B* **435**, 61 (1998).
- [41] R. V. Harlander, D. R. T. Jones, P. Kant, L. Mihaila, and M. Steinhauser, *J. High Energy Phys.* 12 (2006) 024.
- [42] J. Hisano and M. A. Shifman, *Phys. Rev. D* **56**, 5475 (1997).
- [43] I. Jack and D. R. T. Jones, *Phys. Lett. B* **415**, 383 (1997).
- [44] L. V. Avdeev, D. I. Kazakov, and I. N. Kondrashuk, *Nucl. Phys.* **B510**, 289 (1998).
- [45] L. Girardello and M. T. Grisaru, *Nucl. Phys.* **B194**, 65 (1982).
- [46] D. Korneev, D. Plotnikov, K. Stepanyantz, and N. Tereshina, *J. High Energy Phys.* 10 (2021) 046.
- [47] M. T. Grisaru, W. Siegel, and M. Rocek, *Nucl. Phys.* **B159**, 429 (1979).
- [48] I. Jack, D. R. T. Jones, and A. F. Kord, *Ann. Phys. (Amsterdam)* **316**, 213 (2005).
- [49] M. Maniatis, *Int. J. Mod. Phys. A* **25**, 3505 (2010).
- [50] U. Ellwanger, C. Hugonie, and A. M. Teixeira, *Phys. Rep.* **496**, 1 (2010).
- [51] O. Haneychuk, V. Shirokova, and K. Stepanyantz, *J. High Energy Phys.* 09 (2022) 189.
- [52] A. Kazantsev and K. Stepanyantz, *J. High Energy Phys.* 06 (2020) 108.
- [53] S. S. Aleshin, A. E. Kazantsev, M. B. Skoptsov, and K. V. Stepanyantz, *J. High Energy Phys.* 05 (2016) 014.
- [54] A. E. Kazantsev, M. B. Skoptsov, and K. V. Stepanyantz, *Mod. Phys. Lett. A* **32**, 1750194 (2017).
- [55] J. E. Bjorkman and D. R. T. Jones, *Nucl. Phys.* **B259**, 533 (1985).