

Pion mass dependence in $D\pi$ scattering and the $D_0^*(2300)$ resonance from lattice QCD

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Lattice QCD results for isospin $I = \frac{1}{2}$ $D\pi$ scattering are presented. Utilizing a series of $N_f = 2 + 1$ Wilson-Clover ensembles with pion masses of $m_\pi \approx 133, 208, 305$, and 317 MeV, various two-particle operators are constructed, and the corresponding finite-volume spectra are determined. The S - and P -wave scattering phase shifts are extracted using the Lüscher approach. A clear trend for the motion of the $D_0^*(2300)$ pole is identified. With the physical pion mass configurations also included, this calculation constitutes the first lattice calculation in which the pion mass dependence of the $D_0^*(2300)$ pole is investigated and the scattering lengths are extrapolated/interpolated to the physical pion mass in $D\pi$ scattering.

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I. INTRODUCTION

For over two decades, both experimental and theoretical efforts have been devoted to the study of scalar resonance $D_0^*(2300)$, which has been discovered and verified by many experiments [1–5]. Still, various puzzles remain. The scalar resonance $D_0^*(2300)$ lies so close to its strange partner $D_{s0}^*(2317)$, which is attributed within the quark model to the strong coupling to the $D\pi$ threshold [6]. There are also suggestions of a two-pole structure in this particular channel from unitarized chiral perturbation theory [7,8]. However, this assertion needs to be tested at different quark masses, which is impossible in real-world experiments. It is, therefore, crucial to study $D\pi$ scattering systematically within a nonperturbative framework such as lattice QCD, in which pion mass dependence could be analyzed. This is what we would like to address in this Letter.

The study of the $D\pi$ two-meson system also provides a pivotal input to the more complicated three-body $DD\pi$ system. An example that has drawn much attention recently is the doubly charmed tetraquark candidate, $T_{cc}(3875) \rightarrow DD^* \rightarrow DD\pi$ discovered by the LHCb Collaboration [9,10]. There have been several lattice calculations on DD^* scattering recently, claiming the state that is related to T_{cc} could be identified [11–14]. However, the actual lowest T_{cc} decay channel is a three-body channel, and the large pion mass used in these lattice studies might obscure such complexity by lifting the $DD\pi$ channel beyond the DD^* threshold. The more complete approach thus involves the three-body scattering problem on the lattice which relies on the $D\pi$ two-body scattering amplitudes as an input [15,16].

To search for hadronic resonances on the lattice, the scattering amplitudes of the decay products are obtained by applying Lüscher's formulation [17] to the finite-volume energy levels. The scattering amplitudes are then analytically continued to the complex energy plane. The poles found in these scattering amplitudes correspond to various hadrons found in the experiments. For a review, see [18]. There are, however, indirect extraction of the $I = \frac{1}{2}$ $D\pi$ scattering parameters. In Ref. [19], the scattering length is extracted from the lattice-determined form factor by a combined fit of the scattering length and the Omnès subtraction parameters. In Ref. [20] the scattering length of $I = \frac{1}{2}$ $D\pi$ at physical m_π was obtained within chiral perturbation theory with low-energy constants determined

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from other channels. There are also studies from phenomenological models, e.g., Refs. [21–29].

Direct lattice QCD calculations on $I = \frac{1}{2}$ $D\pi$ scattering were conducted on an $N_f = 2$, $m_\pi \approx 266$ MeV lattice [30] in which a resonance pole was found. Afterward, the Hadron Spectrum Collaboration performed a $D\pi$, $D\eta$, $D_s\bar{K}$ coupled-channel analysis on an $N_f = 2 + 1$ anisotropic lattice ensemble at $m_\pi \approx 391$ MeV and found D_0^* to be a bound state [31]. They also performed a single-channel analysis at $m_\pi \approx 239$ MeV and found a resonance [32]. A calculation at the $SU(3)$ point is also performed [33]. However, focusing on the physical strange sector, the pole positions determined by the lattice studies are all lower than the Particle Data Group (PDG) average value [34]. As m_π decreases toward its physical value, although the scattering lengths appear to move toward the values determined from the experimental results, the pole position derived from these lattice results does not show a monotonic movement. Therefore, it is crucial to examine the movement of the D_0^* pole at different m_π values, particularly its physical value. Additionally, the experimental measurements are presented as the parameters of the relativistic Breit-Wigner (RBW), which could be rather different from the pole positions for broad resonances such as $D_0^*(2300)$. A more consistent comparison is to translate not only the lattice results but also those from the experiments to the pole positions.

In this Letter, we report lattice results on $D\pi$ scattering using a set of $N_f = 2 + 1$ Wilson-clover lattice ensembles at $m_\pi \approx 133, 208, 305$, and 317 MeV to examine the movement of the D_0^* pole in the complex plane. The scattering lengths are interpolated/extrapolated toward the physical pion mass point. The pole positions and the scattering lengths are then compared to the experimental measurements. The behavior of the D_0^* pole exhibits an interesting trend. At heavy m_π , the D_0^* initially appears as a bound state, becoming lighter as m_π decreases and transforming into a virtual state. With further reduction in m_π , the D_0^* transitions into a resonance, showing a complicated movement.

II. FINITE VOLUME SPECTRA

The gauge configurations used in this work were generated by the CLQCD Collaboration with $N_f = 2 + 1$ flavors of dynamical quarks using the tadpole-improved tree-level Symanzik gauge action and the tadpole-improved tree-level Clover fermions [35]. The action of the valence charm quark is the same as the light and strange quark action used in the configurations. Furthermore, in order to investigate the discretization error on the coarsest lattice C48P14 with $m_\pi \approx 133$ MeV, we repeat the analysis using the Fermilab action [36] for the charm quark, which controls discretization errors of $\mathcal{O}(am_c)^n$. The tuning of the parameters in the Fermilab action follows the method applied in Ref. [37]. There are plenty of studies based on

TABLE I. Gauge configurations used in this work. The errors are purely statistical.

Configuration	Volume	a/fm	m_π/MeV	N_{confs}
C48P14	$48^3 \times 96$	0.10530(18)	133.1(1.6)	259
F32P21	$32^3 \times 64$	0.07746(18)	206.8(2.1)	459
F48P21	$48^3 \times 96$	0.07746(18)	208.12(70)	222
F32P30	$32^3 \times 96$	0.07746(18)	305.81(71)	567
F48P30	$48^3 \times 96$	0.07746(18)	304.98(50)	201
H48P32	$48^3 \times 144$	0.05187(26)	317.00(68)	274

these ensembles, see, e.g., [38–48]. The simulations were performed on six ensembles with four different values of pion mass $m_\pi \approx 133, 208, 305$, and 317 MeV. The details of the ensembles are listed in Table I. Among these ensembles, F32P21/F48P21 and F32P30/F48P30 are two couples that share the same pion mass and lattice spacing but different volumes to obtain more kinematic points in the finite-volume spectra, rendering a more stable and precise determination of the scattering parameters.

To obtain the full finite-volume spectra hence the scattering information near the threshold, many operators in irreducible representations (irreps) of the cubic space group are needed. We consider reference frames with a center of mass (c.m.) three-momenta $\frac{L}{2\pi}|\vec{P}| \leq 2$.

The finite-volume spectra can be obtained from the matrix of the correlation functions of the interpolating operators O_i , $C_{ij}(t) = \sum_{t'} \langle O_i(t+t') O_j^\dagger(t') \rangle_T$, where t' is summed over all time slices to enhance the signal. The distillation quark smearing method [49] is used to compute the quark propagators. The details about this method and the explicit forms of operators used in this work can be found in Appendix A, where the strategies proposed in Refs. [50–52] are applied. The construction has been performed with the *Mathematica* tool `Option` [53]. The spectra in each irrep is obtained by solving a generalized eigenvalue problem (GEVP) [54]. The extracted spectra at $m_\pi \approx 305$ MeV are shown in Fig. 1. Non-negligible thermal pollution is found in the correlation functions. And the effect is addressed by generalizing the “weighting and shifting” method in Ref. [55]. We refer the detailed derivation, the solution to Appendix B, and the spectra at other pion masses to Appendix C. There are two volumes at this pion mass. The black lines, red dashed, and green dashed lines represent the $D\pi$, $D^*\pi$, and the $D\pi\pi$ non-interacting levels, respectively. The black points are the energy levels obtained by fitting the eigenvalues of the correlation matrices. The orange bands are predictions from Lüscher’s equation [Eq. (1)] which will be explained in the following.

For two-body scattering near the threshold, lower partial waves dominate. For irreps containing P wave, a level much lower than the $D\pi$ threshold is found, corresponding to a vector D^* bound state. For $[000]T_1^-$, the lowest $D\pi$

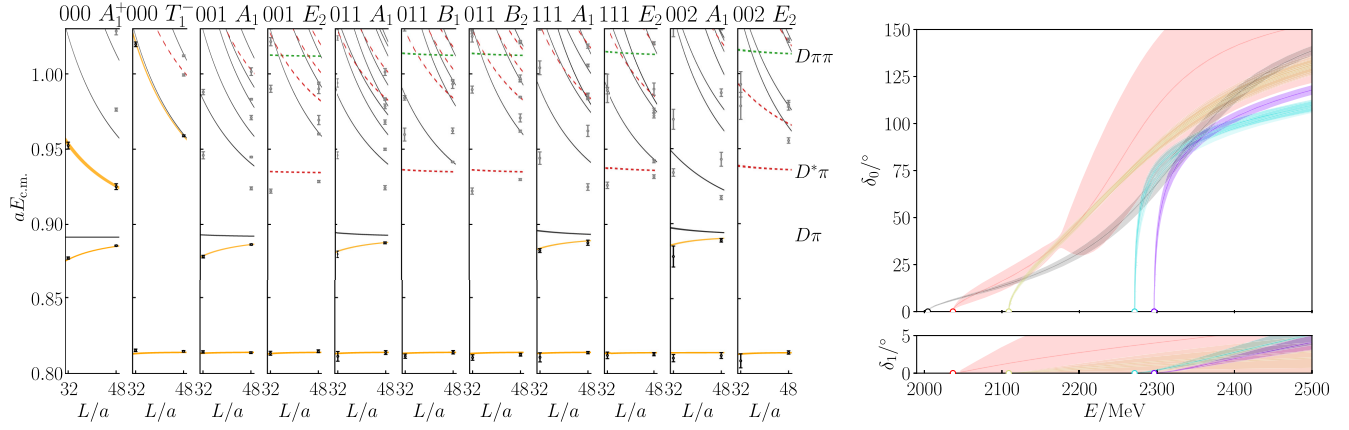


FIG. 1. Left: the finite-volume spectra at $m_\pi \approx 305$ MeV for irreps with leading partial waves being S or P wave. The black and gray points are the energy levels. The black lines, red dashed, and green dashed lines represent the noninteracting energies of the $D\pi$, $D^*\pi$, and the $D\pi\pi$, respectively. Only black points are used in the scattering analysis. The orange bands are the solutions of the Lüscher's equations. Right: the S - and P -wave scattering phase shift δ_0 (upper) and δ_1 (lower). The red, yellow, cyan, and purple bands are results from $m_\pi \approx 133, 208, 305$, and 317 MeV. The gray bands are reconstructed from the PDG value. The bands with the same color represent the results of different parametrizations at each pion mass while the width of each corresponds to the statistical error.

partial wave is the P wave. The first excited states are not far from the noninteracting levels, indicating a weak interaction in the P wave. For irreps containing S wave as the leading partial wave, the $D\pi$ scattering energy levels are substantially lower than the corresponding noninteracting ones, indicating a strong attractive interaction.

III. SCATTERING ANALYSIS

The finite volume spectra are related to the infinite-volume scattering phase shifts via Lüscher's quantization condition, which can be written in the general form as [56,57]

$$\det[M(E, \vec{P}; L) - \cot \delta(E)] = 0, \quad (1)$$

where $M(E, \vec{P}; L)$ contains the information of the finite volume spectra and δ is the phase shift. The scattering t matrix is related to δ by $t_l = \frac{\sqrt{s}}{2} \frac{1}{k \cot \delta_l - ik}$ and can then be deduced from the lattice.

We cut off the infinite-dimensional determinant in Eq. (1) to P wave. To extract both the S - and P -wave phase shifts, a correlated global fit is performed for energy levels below the lowest inelastic thresholds of the irreps with leading partial waves up to P wave, shown in the left panel of Fig. 1. The number of energy levels is $17 \times 2 = 34$ for $m_\pi \approx 305$ and 208 MeV, but is 17 for 133 and 317 MeV, since there are two volumes for $m_\pi \approx 305$ and 208 MeV.

To estimate the model dependencies, we apply many different parametrizations. The main results are reported by using the effective range expansion (ERE), $k^{2l+1} \cot \delta_l = \frac{1}{a_l} + \frac{1}{2} r_l k^2 + P_2 k^4 + \mathcal{O}(k^6)$, where for $l=0$, a_0 is the scattering length and r_0 is the effective range. The scattering momentum k is related to the scattering energy by

$E(\vec{k}) = \sqrt{m_D^2 + Z_D \vec{k}^2} + \sqrt{m_\pi^2 + Z_\pi \vec{k}^2}$, where Z_X is the square of the speed of light on the lattice. Owing to lattice artifacts, Z_X deviates from 1. The values of Z_π , Z_D , and Z_{D^*} are given in Appendix D. For the ensembles except C48P14, we performed the scattering analysis using both the lattice values of Z_X and the continuum value 1. The difference is estimated as a systematic error. The fit results using the ERE are tabulated in Table II. The ensemble at $m_\pi \approx 133$ MeV with the Fermilab charm action is denoted as 133*. Although this procedure may not provide a precise measure of discretization error, the results remain unaffected, as the error estimated from this method is significantly smaller than the statistical error. Consequently, the total error is still dominated by statistical uncertainties, leaving the final results largely unchanged. For the coarsest ensemble, this difference is further highlighted by using two distinct charm actions, demonstrating that the variation lies well within the current statistical error. Other parametrizations and the details about the fitting can be found in Appendix E. In general, little model dependence is found.

TABLE II. The fitting result for the S and P wave of $I = \frac{1}{2}$ $D\pi$ scattering using the ERE parametrization. 133* denotes calculations performed on the same ensemble as 133 but with a Fermilab charm action.

m_π/MeV	a_0/fm	r_0/fm	a_1/fm^3	r_1/fm^{-1}
317	3.36(82)	-0.734(60)	0.0288(70)	32(11)
305	3.9(1.2)	-0.488(90)	0.033(15)	28(13)
208	0.663(60)	-1.15(15)	0.020(21)	127(237)
133	0.33(16)	-2.4(3.8)	0.17(32)	45(58)
133*	0.26(14)	-4.0(5.5)	0.28(18)	63(110)

The energy spectra for any L can then be reconstructed from the parametrized Lüscher's formula, which are plotted as the orange bands in Fig. 1. The solutions from the Lüscher's formula describe the data well.

The phase shifts for all the ensembles used in this work are plotted in the right panel of Fig. 1. There are four bands for $m_\pi \approx 305, 317$ MeV and two for $m_\pi \approx 208, 133$ MeV. The bands with the same color represent the results from various parametrizations at each pion mass with the width of the bands representing the statistical error. The differences among the bands are estimated as the systematic error due to different parametrizations. The gray curves are reconstructed from the PDG-averaged RBW parameters. The S -wave phase shift of all m_π 's rises from 0 to 180 degrees, indicating a nontrivial structure. The P -wave phase shift δ_1 is small compared to the S wave in the elastic zone.

IV. POLE POSITIONS

Observable structures normally appear as poles in the scattering amplitudes in the complex $s = E_{\text{c.m.}}^2$ plane. If there is an isolated pole near the real axis beyond the threshold, the scattering phase shifts, and the cross section would signal a sharp change close to the pole position. The t matrix near a pole behaves as $t \sim \frac{c^2}{s-s_0}$ with s_0 being the pole position. If there is a resonance in the channel, the mass m and the width Γ will be related to the pole position by $\sqrt{s_0} = m - \frac{i}{2}\Gamma$. To compare the results from different lattice groups, we focus on the mass of the structure subtracted by spin-averaged mass of D and D^* , as was done in Ref. [30], $\sqrt{s_0} = \sqrt{s_0} - \frac{1}{4}(m_D + 3m_{D^*})$, which we refer to as the spin-average-subtracted pole position.

For the S wave of the $D\pi$ scattering, we found a virtual state for $m_\pi \approx 305$ and 317 MeV, and a resonance pole for $m_\pi \approx 133$ and 208 MeV, from all parametrizations. These poles are identified to the scalar D_0^* found in the experiment. The spin-average-subtracted pole positions on the Riemann sheet are tabulated in Table III and shown as the covariance ellipses and the red error bars in Fig. 2. The poles from different parametrizations at the same m_π are represented by the overlapping ellipses. The small

TABLE III. The pole positions for S wave of $I = \frac{1}{2}$ $D\pi$ scattering. The errors account for the statistical error, the variance across multiple parametrizations, and the estimate of the lattice discretization effects. 133* denotes calculations performed on the same ensemble as 133 but with a Fermilab charm action.

m_π/MeV	$\text{Re}\sqrt{s} - \frac{1}{4}(D + 3D^*)/\text{MeV}$	$\text{Im}\sqrt{s}/\text{MeV}$
317	211.6(4.9)	0
305	216.1(4.8)	0.00(11)
208	260(16)	162(27)
133	285(93)	88(134)
133*	239(57)	53(86)

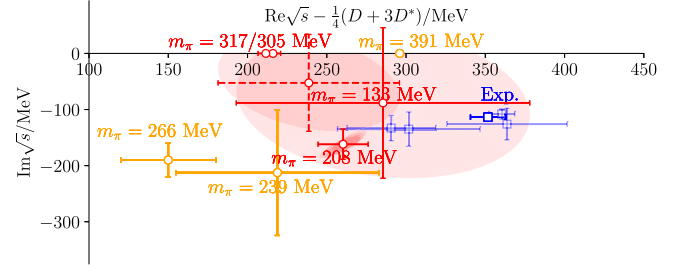


FIG. 2. The poles for S wave of $I = \frac{1}{2}$ $D\pi$ scattering. The poles from this work are the covariance ellipses and the red points that wrap over them. The dashed red error bar indicates the result at $m_\pi \approx 133$ MeV obtained using the Fermilab action. The blue points are the experimental measurements [1,3–5]. The solid blue point is the PDG average [34]. The orange points are the results from other lattice calculations [30–32].

deviations among different ellipses indicate that the parametrization dependence of the pole positions is small compared to the statistical error. The red error bars at $m_\pi \approx 208$ MeV cover the range of the pole positions from all parametrizations, and the discretization error is added in quadrature. The dashed red error bar indicates the result at $m_\pi \approx 133$ MeV obtained using the Fermilab action. The results at $m_\pi \approx 317$ MeV and $m_\pi \approx 305$ MeV are close to each other. Since the two results share similar pion masses but rather different lattice spacings, this indicates that the lattice discretization effect should be small for this lattice spacing. For the coarsest lattice C48P14 with $m_\pi \approx 133$ MeV, we have presented results with the charm quark being either conventional Clover action or the Fermilab formulation to give an estimate of the lattice artifact. Although we cannot entirely rule out the possibility that the effects of differing pion masses and lattice spacings may cancel each other out, we still expect the discretization effect to be minimal due to the similarity in pion masses, at least for the current statistical error. In the figure, we have also presented the spin-average-subtracted pole positions from other lattice calculations which were done at $m_\pi \approx 266, 239$, and 391 MeV. [30–32]. The plot also presents the pole positions of the experimental results that describe D_0^* by RBW. These are the blue points. The solid blue data are from the PDG average value [34], and the transparent ones are the experimental measurements [1,3–5] it uses.

Combining the results from the previous lattice calculations [30–32], an interesting running behavior of the D_0^* pole as a function of m_π emerges: the pole starts at $m_\pi \approx 391$ MeV on the real axis as a bound state and becomes lighter as m_π decreases. At some point in $317 \text{ MeV} \lesssim m_\pi \lesssim 391 \text{ MeV}$, the pole penetrates the Riemann sheet and passes to the unphysical sheet where it becomes a virtual state. As m_π further decreases, the pole evolves to the complex plane at $266 \text{ MeV} \lesssim m_\pi \lesssim 305 \text{ MeV}$ and becomes a resonance, and the pole stops moving leftward

but turns rightward and begins to move on a complicated trajectory. The pole at the physical pion mass is consistent with the experiments. However, owing to the large uncertainty, no conclusive statement should be made. It is also interesting to note that the trajectory of the D_0^* poles is qualitatively similar to the predictions from the chiral perturbation theory [23,58]. See Refs. [59,60] for a general discussion.

V. CHIRAL INTERPOLATIONS OF a_0^{-1}

In this section, we interpolate the S -wave scattering length to the physical pion mass $m_\pi^{\text{phys}} = 135$ MeV. The systematic error of the scattering length from the model dependence is included by wrapping the spread of results of $\lim_{k \rightarrow 0} k \cot \delta_0$ from all parametrizations. The discretization error is also included. We use a polynomial form to describe the chiral behavior,

$$a_0^{-1}(m_\pi) = c_0 + c_1 m_\pi^2 + c_2 m_\pi^4. \quad (2)$$

Similar extrapolations were done in Ref. [13]. The fit uses determinations of a_0^{-1} from simulations with the Clover charm action to maintain consistency in the action; hence, the 133* results are not included in the extrapolation.

The fit is shown in Fig. 3, where the results from other lattice results in Refs. [30–32] are also shown. The blue band is the reconstructed curve using the fitted parameters. The blue point is the extrapolated value at the physical pion mass. The extrapolation proceeds by first sampling a_0^{-1} evenly from its distribution for each parametrization at each m_π , followed by performing extrapolations for each sample. This approach incorporates the distribution of all parametrizations across all m_π values into the fit.

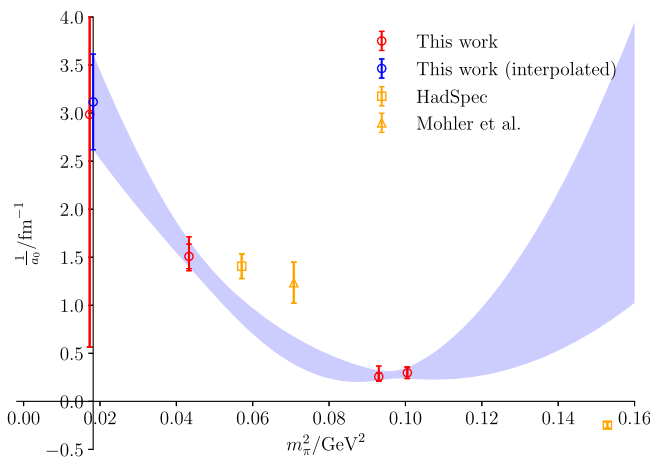


FIG. 3. The chiral interpolation of the inverse of the $I = \frac{1}{2}$ $D\pi$ scattering length. The fit takes data from this work, shown as the red data. The orange points are from previous lattice works [30–32]. The blue band is the reconstructed fitting model. The blue point is the prediction of $\frac{1}{a_0}$ at the physical pion mass.

The fitting results are $c_0 = 4.66(95) \text{ fm}^{-1}$, $c_1 = -94(29) \text{ MeV}^{-2} \cdot \text{fm}^{-1}$, $c_2 = 502(203) \text{ MeV}^{-4} \cdot \text{fm}^{-1}$ with $\chi^2/\text{d.o.f.} = 1.02$. The inverse scattering length at the physical point is then

$$a_0^{-1}(m_\pi^{\text{phys}}) = 3.12(50) \text{ fm}^{-1}. \quad (3)$$

Adding to Eq. (2) an a^2 term yields consistent results, indicating that the data points reasonably incorporate the estimation of the discretization error. Owing to the large statistical error at the physical pion mass, the chiral extrapolation/interpolation is essentially constrained by data points at 208 MeV and 305/317 MeV. However, two observations most likely support the reliability of the extrapolation/interpolation. First, data points from previous lattice calculations align closely with the chiral curves obtained in this study. Second, the value of $a_0^{-1}(m_\pi^{\text{phys}})$ is consistent with the direct calculation at the physical ensemble. These factors suggest that the results are dependable despite the aforementioned limitations.

We infer the experimental measurement of the scattering length by analytically continuing the RBW amplitudes and taking the threshold limit. The result from [30] is also presented in a similar way. The scattering lengths in this work together with previous lattice results [30–32], the deduced PDG average [34], the experimental measurements [1,3–5], and the phenomenological models [20,23–25] are shown in Fig. 4. Limited by only one or two values in previous lattice calculations, the dependence on the pion mass is hardly recognized, which leads to poor knowledge of the discrepancy with experiments. Given the red points in this work, a clearly visible m_π dependency is

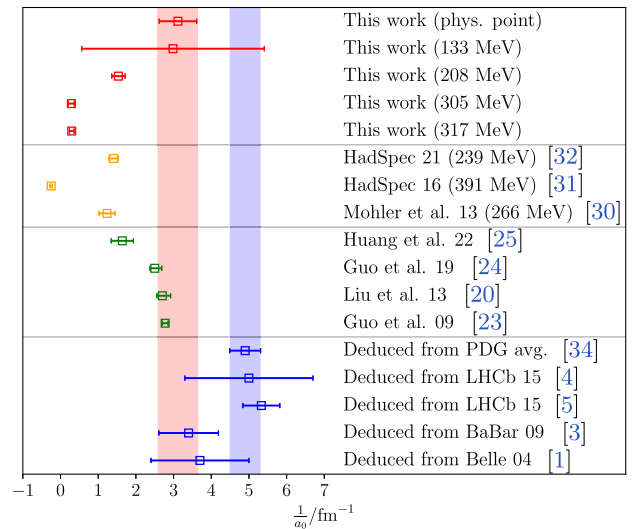


FIG. 4. The comparison of $\frac{1}{a_0}$ from this work (red), the previous lattice results [30–32] (orange), the experiments [1,3–5,34] (blue), and the phenomenological models [20,23–25] (green). The experimental values are obtained by analytically continuing the RBW amplitudes from the measured parameters.

identified, and the discrepancy with the PDG value is also greatly alleviated. In fact, our result is consistent with the *BABAR* and *Belle* experiments within the range of error. The result is also consistent with those in the phenomenological models in Refs. [20,23,24].

As the paper was under review, we noticed a recent measurement from the ALICE Collaboration [61] using momentum correlation functions. Their result for the scattering length, $a_0 = 0.02(3)(1)$ fm, is consistent with zero and notably 1 order of magnitude smaller than our calculation.

VI. CONCLUSIONS

Using a series of $N_f = 2 + 1$ Wilson-Clover configurations, the S - and P -wave $I = \frac{1}{2}$ $D\pi$ scattering phases are studied at $m_\pi \approx 133, 208, 305$, and 317 MeV within Lüscher's formalism. Various single-meson and two-meson interpolating operators are constructed, which leads to stable excited states energy spectra from variational analysis of the correlation matrices. A strong attractive interaction in irreps that contain S wave is observed while that of P wave is small. The scattering phase shifts are extracted by solving the Lüscher's equation. The D_0^* found on our configurations are virtual states at $m_\pi \approx 305, 317$ MeV and are resonance states at $m_\pi \approx 133, 208$ MeV. Together with the results from previous works, we arrive at the following picture for the movement of the D_0^* pole when m_π varies: D_0^* pole is a bound state at $m_\pi \gtrsim 391$ MeV; it then evolves into a virtual state somewhere in $317 \text{ MeV} \lesssim m_\pi \lesssim 391$ MeV and finally becomes a resonance at $266 \text{ MeV} \lesssim m_\pi \lesssim 305$ MeV. The pole moves to the left at first, but then turns around and moves to the right and shows a convoluted behavior. This trajectory of the D_0^* pole as a function of pion mass provides an essential clue for phenomenological models.

The scattering lengths are interpolated to the physical pion mass, which is then compared to the deduced experimental measurements. There is still tension between our result and the PDG-averaged result, especially the recent measurement from the ALICE Collaboration [61]. Future calculations performed at finer lattice spacings and more pion masses would be valuable to further clarify this. Furthermore, the results presented in this Letter on the $D\pi$ scattering amplitude near the threshold could be utilized for future $DD\pi$ three-body studies.

Finally, we found only one pole near the threshold region. The missing of the two-pole structure [7,8] is likely due to the ignorance of the coupled-channel effects. To solve the puzzle of the mass ordering of D_0^* and D_{s0}^* , a similar investigation in DK scattering is underway.

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APPENDIX A: TABLE OF OPERATORS

Since the physical process happens on a cubic spacetime, the rotational symmetries are broken into the cubic group, and its subgroups which are further reduced by the Lorentz boost. The finite-volume spectra are also subduced into the irreducible representations (irreps) of the little groups. We consider reference frames with a c.m. three-momenta $\vec{P} = [0, 0, 0], [0, 0, 1], [0, 1, 1], [1, 1, 1], [0, 0, 2]$ in the unit of $\frac{2\pi}{L}$.

For the construction of the single-hadron operators, the helicity strategy proposed by the Hadron Spectrum Collaboration [50,51] is applied. For the construction of the two-hadron operators, we propose a generalized version of the projection formula put forward by Prelovsek *et al.* [52],

$$O_{\Gamma, \mu}^{[k]}(\vec{P}) = \sum_{R \in G} T_{\mu, \mu}^{\Gamma}(R) R O_1(\vec{k}) O_2(\vec{P} - \vec{k}) R^\dagger, \quad (\text{A1})$$

where $\vec{k}$ designates the relative momentum between the two single hadrons. $T_{\mu, \mu}^{\Gamma}$ is the diagonal matrix elements μ, μ of the irrep Γ . $O_{1,2}$ are single-hadron operators of the form $\bar{\psi} \Gamma \psi$ with $\bar{\psi}$ and ψ being light or charm quarks, and the gamma matrices transform as scalar and vector.

We consider reference frames with a c.m. three-momenta $\vec{P} = [0, 0, 0], [0, 0, 1], [0, 1, 1], [1, 1, 1], [0, 0, 2]$ in the unit of $\frac{2\pi}{L}$, which corresponds to the cubic group O_h and the little groups $C_{4v}, C_{2v}, C_{3v}, C_{4v}$, respectively. We consider all

TABLE IV. Operators for $\vec{P} = [000]$ used to interpolate the $D\pi$ system. Each irrep contains both one-meson type and two-meson type operators. The symbols $\eta_{\mu_i}^{\alpha_i}$ are defined in the context. Each operator consists of a linear combination of one- or two-hadron quark bilinears. η denotes the coefficients of certain operators. μ indicates the Cartesian gamma matrices. α denotes the momentum vector of the single hadron or the relative momentum vector of the two hadrons.

Irrep	type	Operator
A_1^+	One	$(+1)_0^0$
	Two	$(+1)_5^0$ $(+1)_5^z, (+1)_5^y, (+1)_5^x, (+1)_5^{-z}, (+1)_5^{-y}, (+1)_5^{-x}$ $(+1)_5^{yz}, (+1)_5^{xz}, (+1)_5^{xy}, (+1)_5^{-y,z}, (+1)_5^{-x,z}, (+1)_5^{-x,y}, (+1)_5^{y,-z}, (+1)_5^{x,-z}, (+1)_5^{x,-y}, (+1)_5^{-y,-z}, (+1)_5^{-x,-z}, (+1)_5^{-x,-y}$
T_1^-	One	$(+1)_z^0$
	Two	$(+1)_5^z, (-1)_5^{-z}$ $(+1)_5^{xz}, (+1)_5^{-x,z}, (+1)_5^{yz}, (+1)_5^{-y,z}, (-1)_5^{x,-z}, (-1)_5^{-x,-z}, (-1)_5^{y,-z}, (-1)_5^{-y,-z}$ $(+1)_x^y, (-1)_x^{-y}, (+1)_y^{-x}, (-1)_y^x$
T_1^+	One	$(+1)_{5z}^0$
	Two	$(+1)_z^0$ $(+1)_z^z, (+1)_z^y, (+1)_z^x, (+1)_z^{-z}, (+1)_z^{-y}, (+1)_z^{-x}$ $(-2)_z^z, (+1)_z^y, (+1)_z^x, (-2)_z^{-z}, (+1)_z^{-y}, (+1)_z^{-x}$

irreps with leading partial waves up to P wave for $D\pi$ scattering. Note that since $m_D \neq m_\pi$, the S wave and P wave can mix in moving frames where parity is no longer a good quantum number. For the $D\pi$ system, up to P wave, the irreps A_1 of C_{4v}, C_{2v}, C_{3v} contain both the S wave and P wave; irreps T_1^- of O_h, E_2 of C_{4v}, C_{3v} and B_1, B_2 of C_{2v} house only the P wave; A_1^+ of O_h houses only the S wave.

It is useful to do a linear transformation of the operator set if possible. We perform the transformation in the irreps in the c.m. frame to let the operators transform in a certain partial wave, as constructed by the partial-wave method in Ref. [52]. In this way, the operators mainly couple to the low partial waves we are concerned about.

The operators need to be further projected to isospin $I = \frac{1}{2}$,

$$O_{D^{(*)}\pi}^{I=\frac{1}{2}} = 2D^0\pi^+ - \sqrt{2}D^{(*)}\pi^0, \quad (\text{A2})$$

The single-hadron operators that appeared with different electric charges had the same flavor structure and differed only in isospin.

Each operator is then a linear combination of quark bilinear or the product of two bilinears. The operators are represented as

$$\begin{cases} O_{\text{one}}(\vec{P}) = \sum_i \eta_i D_{\mu_i}(\vec{P}), \\ O_{\text{two}}(\vec{P}) = \sum_i \eta_i D_{\mu_i}(\vec{p}_i) \pi(\vec{P} - \vec{p}_i), \end{cases} \quad (\text{A3})$$

and are uniquely identified by the parameters η_i, μ_i , and $\vec{p}_i$ for one- and two-meson operators, respectively. $\mu_i \in \{0, 5, x, y, z, 5x, 5y, 5z\}$ is the Cartesian gamma matrices

and corresponds to $\gamma_0, \gamma_5, \gamma_x, \gamma_y, \gamma_z, \gamma_5\gamma_x, \gamma_5\gamma_y, \gamma_5\gamma_z$. $\vec{p}_i$ is the relative momenta. For convenience, the parameters are denoted by

$$\eta_{\mu_i}^{\alpha_i}, \quad (\text{A4})$$

where α_i one-to-one corresponds to the momentum vector. One direction notation in α_i means one unit of momentum in that direction. For example, $\alpha = yz \rightarrow \vec{p} = [011]$, $\alpha = -2x \rightarrow \vec{p} = [-200]$, etc. Operators for isospin $I = \frac{1}{2}$ for $\vec{P} = [000]$ are shown in Table IV; $\vec{P} = [001]$ are shown in Table V; $\vec{P} = [011]$ are shown in Table VI; $\vec{P} = [111]$ are shown in Table VII; and $\vec{P} = [002]$ are shown in Table VIII.

TABLE V. Operators for $\vec{P} = [001]$ used to interpolate the $D\pi$ system. Notations as in Table IV.

Irrep	Type	Operator
A_1	One	$(+1)_0^z$
	Two	$(+1)_z^z$ $(+1)_5^0$ $(+1)_5^z$ $(+1)_5^{-x}, (+1)_5^x, (+1)_5^{-y}, (+1)_5^y$ $(+1)_5^{xz}, (+1)_5^{-x,z}, (+1)_5^{yz}, (+1)_5^{-y,z}$ $(+1)_5^{-z}$ $(+1)_5^{2z}$
E_2	One	$(+1)_y^z$ $(+1)_{5x}^z$
	Two	$(+1)_5^y, (-1)_5^{-y}$ $(+1)_5^{yz}, (-1)_5^{-y,z}$ $(+1)_x^0$ $(+1)_x^z$

TABLE VI. Operators for $\vec{P} = [011]$ used to interpolate the $D\pi$ system. Notations as in Table IV.

Irrep	Type	Operator
A_1	One	$(+1)_0^{yz}$
		$(+1)_y^{yz}, (+1)_z^{yz}$
	Two	$(+1)_5^0$
		$(+1)_5^{yz}$
		$(+1)_5^y, (+1)_5^z$
		$(+1)_5^{xy}, (+1)_5^{xz}, (+1)_5^{-x,y}, (+1)_5^{-x,z}$ $(+1)_5^{xyz}, (+1)_5^{-x,yz}$ $(+1)_x^y, (-1)_x^z$
B_1	One	$(+1)_y^{yz}, (-1)_z^{yz}$
		$(+1)_{5x}^{yz}$
	Two	$(+1)_5^y, (-1)_5^z$
		$(+1)_5^{xy}, (-1)_5^{xz}, (+1)_5^{-x,y}, (-1)_5^{-x,z}$ $(+1)_x^y, (+1)_x^z$
B_2	One	$(+1)_x^{yz}$
		$(+1)_{5y}^{yz}, (-1)_{5z}^{yz}$
	Two	$(+1)_5^{xy}, (+1)_5^{xz}, (-1)_5^{-x,y}, (-1)_5^{-x,z}$
		$(+1)_5^x, (-1)_5^{-x}$
		$(+1)_5^{xyz}, (-1)_5^{-x,yz}$
		$(+1)_y^y, (-1)_z^z$ $(+1)_z^y, (-1)_y^z$ $(+1)_y^0, (-1)_z^0$ $(+1)_y^{yz}, (-1)_z^{yz}$

We have also tested that adding to the operator set the covariant derivatives makes the resulting energy levels plateau a little earlier, but the signals become significantly worse. As a result, the operators with covariant derivatives are not applied in this work.

TABLE VII. Operators for $\vec{P} = [111]$ used to interpolate the $D\pi$ system. Notations as in Table IV.

Irrep	Type	Operator
A_1	One	$(+1)_0^{xyz}$
		$(+1)_x^{xyz}, (+1)_y^{xyz}, (+1)_z^{xyz}$
	Two	$(+1)_5^0$
		$(+1)_5^{xyz}$
		$(+1)_5^x, (+1)_5^y, (+1)_5^z$
		$(+1)_5^{yz}, (+1)_5^{xz}, (+1)_5^{xy}$
E_2	One	$(+1)_y^x, (-1)_x^z, (+1)_z^y, (-1)_y^x, (+1)_x^z, (-1)_y^z$
		$(+1)_x^{xy}, (-1)_y^{xy}, (+1)_y^{yz}, (-1)_z^{yz}, (+1)_z^{xz}, (-1)_x^{xz}$
	Two	$(+1)_x^{xyz}, (-1)_y^{xyz}$
		$(+1)_{5x}^{xyz}, (+1)_{5y}^{xyz}, (-2)_{5z}^{xyz}$
		$(+1)_5^x, (-1)_5^y$
		$(+1)_5^{yz}, (-1)_5^{xz}$
		$(+1)_x^0, (+1)_y^0, (-2)_z^0$
		$(+1)_x^{xyz}, (+1)_y^{xyz}, (-2)_z^{xyz}$
		$(+1)_x^x, (+1)_y^y, (-2)_z^z$
		$(+1)_x^{yz}, (+1)_y^{xz}, (-2)_z^{xy}$

TABLE VIII. Operators for $\vec{P} = [002]$ used to interpolate the $D\pi$ system. Notations as in Table IV.

Irrep	Type	Operator
A_1	One	$(+1)_0^{2z}$
		$(+1)_z^{2z}$
	Two	$(+1)_5^0$
		$(+1)_5^{2z}$
		$(+1)_5^z$
		$(+1)_5^{xz}, (+1)_5^{-x,z}, (+1)_5^{yz}, (+1)_5^{-y,z}$
E_2	One	$(+1)_y^{2z}$
		$(+1)_{5x}^{2z}$
	Two	$(+1)_5^{yz}, (-1)_5^{-y,z}$
		$(+1)_x^0$
		$(+1)_x^z$
		$(+1)_x^{yz}, (+1)_x^{-y,z}$

We have developed a *Mathematica* package `OpTion` (Operator construcTion) that can construct general N -hadron operators. By setting the type of hadrons, the desired quantum numbers, the number of derivatives, and the maximum relative momentum, the operator set can be obtained by one line of straightforward commands. The formalism of the operator construction will be described in detail by an independent publication [53].

APPENDIX B: CORRELATOR ANALYSIS AND THERMAL POLLUTIONS

The distillation method [49] is used to reduce the cost of generating propagators, which are calculated only in the subspace spanned by the N_v lowest eigenvectors of the covariant Laplacian operator. The quark fields are smeared by the distillation operator such that the high-lying modes are suppressed. And $C_{ij}(t)$'s are then estimated using relevant perambulators. The N_v applied in this work is 100, 200 for ensembles with volume 32^3 and 48^3 , respectively. It has been checked that the spectra are insensitive to N_v and the error would slightly increase with smaller values of N_v .

To extract the finite-volume spectra, we solve the generalized eigenvalue problem (GEVP) [54] of the correlation matrices for each irrep,

$$C(t)v_n(t, t_0) = \lambda_n(t, t_0)C(t_0)v_n(t, t_0), \quad (\text{B1})$$

where the reference time $t_0 = 20$ for H48P32 and $t_0 = 10$ for other configurations. Changing t_0 hardly affects the resulting spectra. The energy levels of the n th excited states are extracted by fitting the eigenvalues $\lambda_n(t, t_0)$:

$$\lambda_n(t, t_0) = (1 - A_n)e^{-E_n(t-t_0)} + A_ne^{-E'_n(t-t_0)}. \quad (\text{B2})$$

It is found that the effective mass of some eigenvalues does not plateau at large time, but keeps rising or dropping.

We found such behavior is due to the non-negligible thermal pollution which can be explained by considering the decomposition of the correlation matrix. We attribute the effect to the elements whose source and sink are both $D\pi$ -like two-body operators with the same momentum structure:

$$\begin{aligned}
 C_{[D\pi]_i, [D\pi]_j}(t) &= \langle D_{\vec{k}}^-(t) \pi_{\vec{P}-\vec{k}}^-(t) D_{\vec{k}}^+(0) \pi_{\vec{P}-\vec{k}}^+(0) \rangle_T \\
 &\supset \frac{1}{Z_T} \left[e^{-E_\pi(\vec{P}-\vec{k})T} \langle \pi_{\vec{P}-\vec{k}}^- | D_{\vec{k}}^-(t) \pi_{\vec{P}-\vec{k}}^-(t) | D_{\vec{k}}^+ \rangle \langle D_{\vec{k}}^+ | D_{\vec{k}}^+(0) \pi_{\vec{P}-\vec{k}}^+(0) | \pi_{\vec{P}-\vec{k}}^- \rangle \right. \\
 &\quad \left. + e^{-E_D(\vec{k})T} \langle D_{\vec{k}}^- | D_{\vec{k}}^-(t) \pi_{\vec{P}-\vec{k}}^-(t) | \pi_{\vec{P}-\vec{k}}^+ \rangle \langle \pi_{\vec{P}-\vec{k}}^+ | D_{\vec{k}}^+(0) \pi_{\vec{P}-\vec{k}}^+(0) | D_{\vec{k}}^- \rangle \right] \\
 &= \frac{1}{Z_T} |z_{\vec{k}}^D|^2 |z_{\vec{P}-\vec{k}}^\pi|^2 \left[e^{-E_\pi(\vec{P}-\vec{k})T - (E_D(\vec{k}) - E_\pi(\vec{P}-\vec{k}))t} + e^{-E_D(\vec{k})T - (E_\pi(\vec{P}-\vec{k}) - E_D(\vec{k}))t} \right] \\
 &\approx \frac{1}{Z_T} |z_{\vec{k}}^D|^2 |z_{\vec{P}-\vec{k}}^\pi|^2 e^{-E_\pi(\vec{P}-\vec{k})T - (E_D(\vec{k}) - E_\pi(\vec{P}-\vec{k}))t}, \tag{B3}
 \end{aligned}$$

where $z_{\vec{k}}^X \equiv \langle X_{\vec{k}}^+ | X_{\vec{k}}^+ | \Omega \rangle$. In the last step, the second thermal state is discarded since it is subleading, compared to the first term.

We generalize the “weighting and shifting” method as described in Ref. [55] to the $D\pi$ system to eliminate the leading thermal pollution. The leading thermal pollution can be removed by constructing the weighted-shifted correlation matrices,

$$\begin{aligned}
 \tilde{C}_{ij}(t) &= e^{(E_D(\vec{k}) - E_\pi(\vec{P}-\vec{k}))t} C_{ij}(t) \\
 &\quad - e^{(E_D(\vec{k}) - E_\pi(\vec{P}-\vec{k}))(t+1)} C_{ij}(t+1). \tag{B4}
 \end{aligned}$$

Most elements have no such thermal state, but the procedure does not affect their behavior under t . The spectra

should be shifted up by $(E_D(\vec{k}) - E_\pi(\vec{P}-\vec{k}))$ to compensate for the weighting procedure. The weighted-shifted correlation matrices $\tilde{C}_{ij}(t)$ can then be sent through the usual GEVP process, yielding the right energy levels. By doing correlated fits of these eigenvalues, towers of energy spectra in different irreps can be obtained.

APPENDIX C: FINITE-VOLUME SPECTRA

The lattice spectra and the predictions from Lüscher’s analysis for $I = \frac{1}{2}$ $D\pi$ scattering at $m_\pi = 317$, 208, and 133 MeV are shown in Figs. 5–7, respectively. The fact that the first few noninteracting levels of $m_\pi = 133$ MeV increase as L increases is a reflection of the discretization artifact since the configuration is coarser than the others.

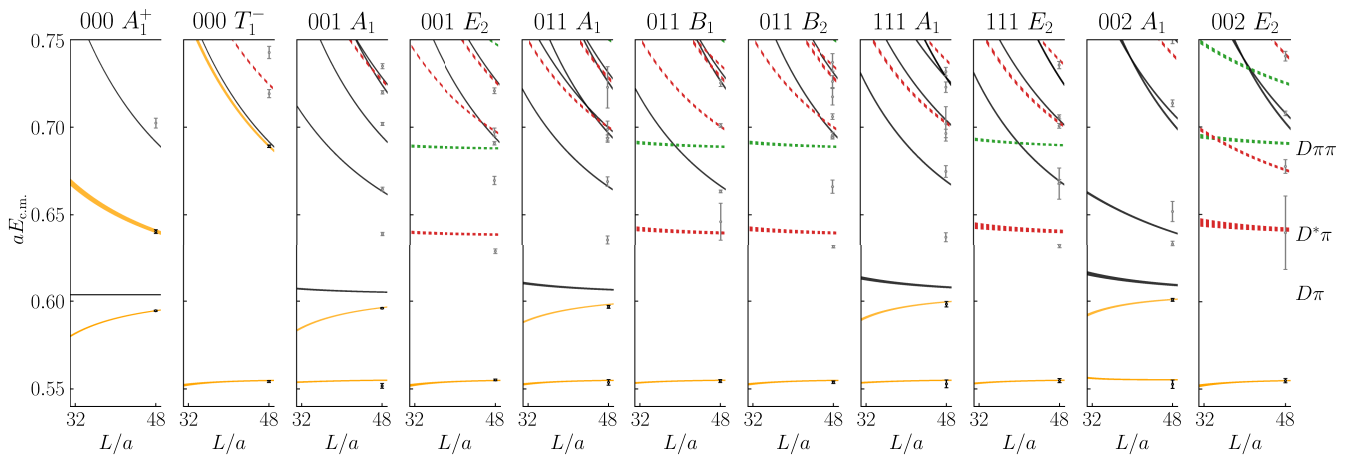
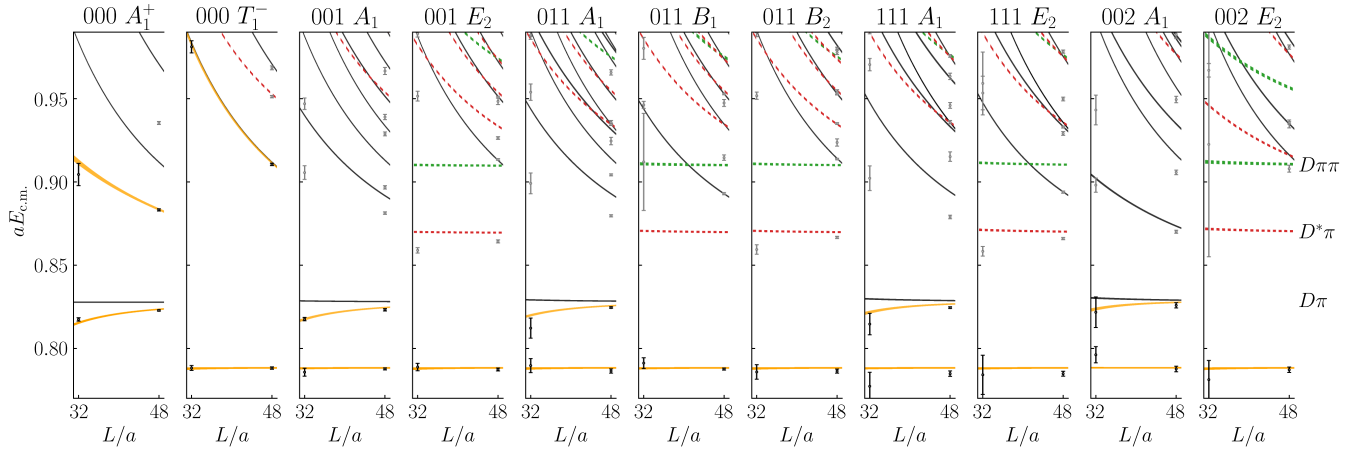
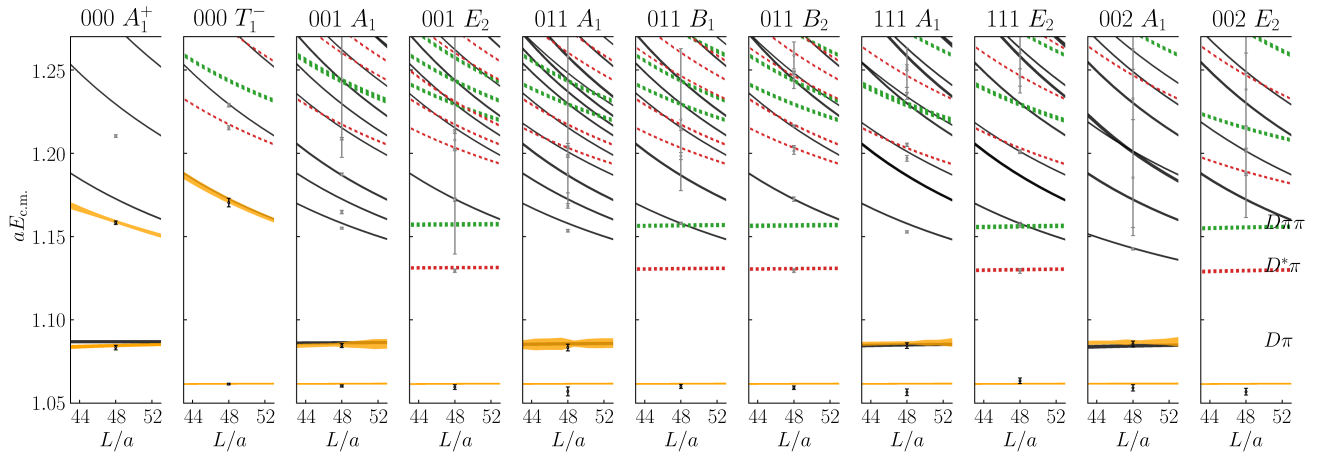


FIG. 5. The finite-volume spectra at $m_\pi \approx 317$ MeV for irreps with leading partial waves being S or P wave. The black and gray points are the energy levels. The black lines, red dashed, and green dashed lines represent the noninteracting energies of the $D\pi$, $D^*\pi$, and the $D\pi\pi$, respectively. Only black points are used in the scattering analysis. The orange bands are the solutions of the Lüscher’s equations.

FIG. 6. $I = \frac{1}{2} D\pi$ finite-volume spectra at $m_\pi \approx 208$ MeV. Descriptions as in Fig. 5.FIG. 7. $I = \frac{1}{2} D\pi$ finite-volume spectra at $m_\pi \approx 133$ MeV. Descriptions as in Fig. 5.

APPENDIX D: DISCRETIZATION ERRORS

The dispersion relations are examined and well described via $E_X = \sqrt{m_X^2 + Z_X \vec{k}^2}$, where $\vec{k}$ is the scattering momentum. We find Z_π of all configurations is close to 1. However, Z_D and Z_{D^*} are statistically smaller than 1, indicating a non-negligible discretization effect. The extracted value of the meson masses, Z_D and Z_π are tabulated in Table IX.

The coefficients Z_X show more deviation from 1 at coarser configurations and less at finer configurations, which reflects the degree of the discretization artifact. The discretization error is addressed by comparing the difference between the corrected dispersion relation and the continuum dispersion relation, in other words, the difference of the observables by the Z_X measured on the lattice and 1 in

TABLE IX. The meson mass and coefficients in the dispersion relation of the configurations.

Configurations	a/fm	m_π/MeV	m_D/MeV	m_{D^*}/MeV	Z_π	Z_D	Z_{D^*}
C48P14	0.10530(18)	133.1(1.6)	1903.73(34)	1988.15(72)	1.0103(33)	0.861(14)	0.865(11)
F32P21	0.07746(18)	206.8(2.1)	1901.3(1.1)	2007.2(2.2)	1.0409(84)	0.9283(48)	0.905(16)
F48P21	0.07746(18)	208.12(70)	1900.71(48)	2006.3(1.2)	1.0157(49)	0.9302(31)	0.9360(45)
F32P30	0.07746(18)	305.81(71)	1965.7(1.0)	2076.6(1.5)	0.9977(56)	0.9083(74)	0.9027(93)
F48P30	0.07746(18)	304.98(50)	1966.20(57)	2074.1(1.1)	1.0054(24)	0.9262(41)	0.9243(70)
H48P32	0.05187(26)	317.00(68)	1979.41(85)	2108.9(2.0)	1.0115(41)	0.9551(66)	0.935(15)

$$E(\vec{k}) = \sqrt{m_D^2 + Z_D \vec{k}^2} + \sqrt{m_\pi^2 + Z_\pi \vec{k}^2}. \quad (\text{D1})$$

The errors are found to be small and are added in quadrature with the parametrization errors.

APPENDIX E: FITTING DETAILS AND PARAMETRIZATION VARIATIONS

The determinant in

$$\det[M(E, \vec{P}; L) - \cot \delta(E)] = 0 \quad (\text{E1})$$

contains an infinite number of partial waves. Since we mainly focus on the energy region near the threshold where the l wave is suppressed by k^{2l} , the high partial waves can

be safely discarded, and the equation then becomes finite dimensional. The partial wave cutoff is taken to be P wave in this work.

After the cutoff, Lüscher's equation is still an under-constrained problem. One way to proceed is to parametrize the scattering phase shifts in terms of models and extract the model parameters from the lattice spectra.

With appropriate parametrization of the scattering phase shifts, the lattice spectra below the inelastic threshold in any irrep can be solved from Lüscher's formula given the specific parameters. The explicit form with an angular momentum cutoff and specific c.m. momentum can be found in Ref. [57]. What we need to solve is the opposite: given the lattice spectra in many irreps, the parameters are determined by minimizing the χ^2 , as in Ref. [55],

$$\chi^2(\{\vec{a}\}) = \sum_L \sum_{\substack{\vec{P}\Gamma n \\ \vec{P}'\Gamma' n'}} [E_{\text{c.m.}}(L; \vec{P}\Gamma n) - E_{\text{c.m.}}^{\text{det}}(L; \vec{P}\Gamma n; \{\vec{a}\})] \text{Cov}^{-1}(L; \vec{P}\Gamma n; \vec{P}'\Gamma' n') [E_{\text{c.m.}}(L; \vec{P}'\Gamma' n') - E_{\text{c.m.}}^{\text{det}}(L; \vec{P}'\Gamma' n'; \{\vec{a}\})], \quad (\text{E2})$$

where $\vec{a}$ represents the list of the parameters that need to be determined. $\text{Cov}(L; \vec{P}\Gamma n; \vec{P}'\Gamma' n')$ is the covariance matrix between the energy levels. Note that the covariance between a different irrep of the same configuration is not zero, but the covariance between different configurations is zero. $E_{\text{c.m.}} = \sqrt{E^2 - |\vec{P}|^2}$ is the c.m. energy fitted from the GEVP eigenvalues, while $E_{\text{c.m.}}^{\text{det}}$ is the solution of Lüscher's equation given the parameters.

When accounting for the discretization errors, Eq. (D1) is used. This corrected dispersion relation moves the noninteracting levels from the singularities of the conventional zeta function. In order not to miss any energy levels, the energy range is divided into small ranges with a step of 0.0005 to search for the solutions of Lüscher's equation.

Besides the ERE used in the main context, we also used the K matrix parametrization, which provides many flexible forms for each partial wave. For elastic scattering, the t matrix is related to the K matrix by

$$t_l^{-1} = \frac{1}{(2k)^{2l}} K_l^{-1} + I_l, \quad (\text{E3})$$

where the Chew-Mandelstam phase space I_l is taken to be $-i\rho$. The K matrix can be parametrized to be

$$K_l(s) = \frac{g_l^2}{m_l^2 - s} + \sum_n \gamma_l^{(n)} s^n \quad (\text{E4})$$

or

$$K_l(s) = \frac{1}{\sum_{n=0} c_l^{(n)} s^n}. \quad (\text{E5})$$

The number of parameters can be varied to give many different parametrizations. The K matrix with pole parametrizations is used to fit the attractive $I = \frac{1}{2}$ channel. We do not apply the RBW parameterization here since the RBW applies only to resonances that are narrow and there are no relevant thresholds or other nearby resonances. The parametrizations that yield pole positions too deep that touch the left-hand cut are eliminated from this work.

By testing various types of parametrizing the phase shifts, the model dependence is measured. As seen from the right panel of Fig. 1 in the main text, there is little systematic error from the parametrization. The parametrizations we have used in this work for $I = \frac{1}{2}$ $D\pi$ are tabulated in Table X. The parametrizations that generate poles in the left-hand region or have large unreasonable $\chi^2/\text{d.o.f.}$ are not presented and are used.

As discussed in the main text, a virtual state or a resonance state D_0^* is found for the S wave, while for the P wave, a bound state is found for all pion masses and corresponds to the vector D^* . No simple movement of the pole position under m_π is observed. Note that although D^* should be a resonance in nature, in our physical ensemble, the hyperfine splitting $m_{D^*} - m_D$ becomes smaller than m_π due to the lattice artifacts, making D^* still a bound state. This effect suggests that the results of the ensemble lack precision and are not well determined. However, owing to

TABLE X. Parametrizations used for $I = \frac{1}{2} D\pi$ scattering analysis.

Configurations	S wave parametrizations	P wave parametrizations	$\chi^2/\text{d.o.f.}$
F32P30 / F48P30	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2$	$k^3 \cot \delta_1 = \frac{1}{a_1} + \frac{1}{2} r_1 k^2$	1.10
	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + P_{2,0} k^4$	$k^3 \cot \delta_1 = \frac{1}{a_1} + \frac{1}{2} r_1 k^2$	1.14
	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + P_{2,0} k^4$	$K_1 = \frac{g_1^2}{m_1^2 - s}$	1.55
	$K_0 = \frac{g_0^2}{m_0^2 - s}$	$k^3 \cot \delta_1 = \frac{1}{a_1} + \frac{1}{2} r_1 k^2$	1.10
F32P21 / F48P21	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2$	$k^3 \cot \delta_1 = \frac{1}{a_1} + \frac{1}{2} r_1 k^2$	1.94
	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + P_{2,0} k^4$	$k^3 \cot \delta_1 = \frac{1}{a_1} + \frac{1}{2} r_1 k^2$	1.76
	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2$	$K_1 = \frac{g_1^2}{m_1^2 - s}$	2.01
	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + P_{2,0} k^4$	$K_1 = \frac{g_1^2}{m_1^2 - s}$	1.82
H48P32	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2$	$k^3 \cot \delta_1 = \frac{1}{a_1} + \frac{1}{2} r_1 k^2$	1.17
	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + P_{2,0} k^4$	$k^3 \cot \delta_1 = \frac{1}{a_1} + \frac{1}{2} r_1 k^2$	1.27
C48P14	$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2$	$k^3 \cot \delta_1 = \frac{1}{a_1} + \frac{1}{2} r_1 k^2$	1.39

the larger uncertainty in the ensemble, it imposes fewer constraints on the extrapolations, so the final results are not significantly impacted. Moreover, despite the notable qualitative difference in the P wave, this effect should not raise concerns about the determination of the S wave. Although the D^* is close to the threshold and more

susceptible to discretization effects, the mass of the D_0^* is much higher and is less affected by these issues.

We have also attempted to fit the spectra separately for the S and P wave. We use $[000]A_1^+$ to fit the S wave and $[000]T_1^-$ to fit the P wave. The results are consistent with that of the combined fit described above.

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