

Semileptonic decay of the triply heavy Ω_{ccb} to the observed Ξ_{cc}^{++} state

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We investigate the weak semileptonic decay of the $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell$, where a triply heavy baryon with spin 1/2 decays into the observed doubly heavy baryon with spin 1/2, using quantum chromodynamics (QCD) sum rule method in all lepton channels. We compute the six relevant vector and axial vector form factors entering the low energy matrix elements in full theory. The invariant form factors are building blocks, using the fit faction of which in terms of q^2 in whole physical region, we calculate the exclusive widths in three lepton channels. Our predictions may help the present and future experiments in the course of their search for doubly and triply heavy baryons.

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I. INTRODUCTION

Research on heavy baryons has gained significant momentum in the last decade due to notable experimental discoveries in the spectroscopy of heavy hadrons. These advancements have driven considerable progress in the field. Among the various types of heavy baryons, those containing one heavy quark (b , c) have been extensively investigated and observed in numerous experimental facilities. In 2017, the LHCb collaboration reported the discovery of the doubly charmed baryon, Ξ_{cc}^{++} , well described by the quark model in its lowest state of doubly heavy baryons, in the final state $\Lambda_c^+ K^- \pi^+ \pi^+$ [1]. Subsequently, they measured the lifetime of Ξ_{cc}^{++} [2] and identified another decay mode, $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ [3]. In 2022, LHCb announced the observation of a new channel $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ from an experimental perspective [4]. Triply heavy baryons, consisting of three heavy quarks (b , c), represent the last missing category of standard hadrons. Following the discovery of the Ξ_{cc}^{++} , the detection of triply heavy baryons now seems increasingly likely. While theoretical studies of triply heavy baryons have largely focused on their mass spectra [5–49], relatively less attention has been given to their decay properties [50–56]. Decay processes discussed in the literature predominantly involve transitions from spin

3/2 to final states with spin 1/2 [50–55], or from spin 1/2 to 3/2 [56].

The present study focuses on the investigation of the semileptonic decay of the triply heavy baryon Ω_{ccb}^+ to the doubly heavy baryon Ξ_{cc}^{++} , where both the initial and final particles have spin 1/2. To find the relevant form factors and estimate the decay width for the process $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell$ in all lepton channels, we employ the quantum chromodynamics (QCD) sum rule method, introduced by Shifman *et al.* [57,58], a well-established nonperturbative approach recognized for its effectiveness in predicting hadronic properties [59–63]. This methodology evaluates the decay from two distinct perspectives: the first is the physical side, where results are expressed in terms of the hadronic parameters like mass and residue of both the initial and final baryons as well as the transition form factors; the second is the QCD side, where results are derived in terms of the fundamental degrees of freedom like quark masses, quark-gluon coupling constant and condensates associated with the quark-quark, gluon-gluon, and quark-gluon interactions through the QCD vacuum. The form factors are subsequently computed by correlating the outcomes of these two approaches, utilizing principles of quark-hadron duality assumption, double Borel transformation as well as continuum subtraction. Ultimately, the obtained form factors are used to determine the relevant semileptonic decay widths at different lepton channels.

The structure of the rest of this article is as follows: Section II outlines the formalism of QCD sum rules used to calculate an appropriate correlation function in both the physical and QCD sectors. We derive the desired sum rules for the form factors based on the standard prescription of

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the method. Section III is dedicated to the numerical analysis of the form factors. In Sec. IV, we present numerical results for the semileptonic decay widths across all lepton channels. Section V includes our concluding remarks and some details of the calculations are presented in the Appendixes.

II. FORMALISM

In the semileptonic decay $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell$ decay, the charm quarks act as the spectators and the transition is conducted by the weak current $\mathcal{J}_\mu^{tr} = \bar{u}\gamma_\mu(1-\gamma_5)b$ through the process $b \rightarrow u \ell \bar{\nu}_\ell$ at quark level (see Fig. 1). The effective Hamiltonian describing this process is given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell, \quad (1)$$

where G_F represents the Fermi coupling constant and V_{ub} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element. By sandwiching the effective Hamiltonian between the initial triply and the final doubly heavy baryon states, the corresponding amplitude is obtained

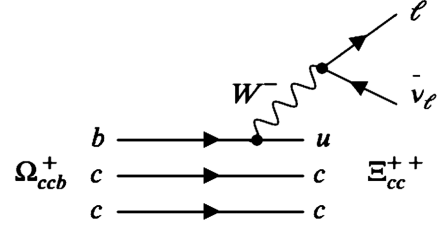


FIG. 1. The $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell$ semileptonic decay.

$$M = \langle \Xi_{cc}^{++} | \mathcal{H}_{\text{eff}} | \Omega_{ccb}^+ \rangle = \frac{G_F}{\sqrt{2}} V_{ub} \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell \langle \Xi_{cc}^{++} | \bar{u} \gamma_\mu (1 - \gamma_5) b | \Omega_{ccb}^+ \rangle. \quad (2)$$

As is seen, the transition current consists of two components: the vector ($V^\mu = \bar{u}\gamma_\mu b$) and the axial vector ($A^\mu = \bar{u}\gamma_\mu\gamma_5 b$), each of which contains information about three transition form factors. Therefore, the following low energy matrix elements are defined in terms of six form factors by taking into account the Lorentz invariance and parity considerations

$$\begin{aligned} \langle \Xi_{cc}^{++}(p', s') | V^\mu | \Omega_{ccb}^+(p, s) \rangle &= \bar{u}_{\Xi_{cc}^{++}}(p', s') \left[F_1(q^2) \gamma^\mu + F_2(q^2) \frac{p^\mu}{m_{\Omega_{ccb}^+}} + F_3(q^2) \frac{p'^\mu}{m_{\Xi_{cc}^{++}}} \right] u_{\Omega_{ccb}^+}(p, s), \\ \langle \Xi_{cc}^{++}(p', s') | A^\mu | \Omega_{ccb}^+(p, s) \rangle &= \bar{u}_{\Xi_{cc}^{++}}(p', s') \left[G_1(q^2) \gamma^\mu + G_2(q^2) \frac{p^\mu}{m_{\Omega_{ccb}^+}} + G_3(q^2) \frac{p'^\mu}{m_{\Xi_{cc}^{++}}} \right] \gamma_5 u_{\Omega_{ccb}^+}(p, s), \end{aligned} \quad (3)$$

where F_i and G_i ($i = 1, 2, 3$) represent the form factors associated with the vector and axial components within the full framework, respectively. The four-momenta of the initial and final baryons are denoted as p and p' , respectively, while $q = p - p'$ represents the four-momentum transferred to the lepton pair. The symbols $u_{\Omega_{ccb}^+}(p, s)$ and $u_{\Xi_{cc}^{++}}(p', s')$ correspond to the Dirac spinors of the initial and final states. As mentioned, the form factors are fundamental parameters that can be extracted utilizing the QCD sum rule method. The first step in this process involves selecting an appropriate correlation function. To derive the form factors, we express the three-point correlation function as follows:

$$\begin{aligned} \Pi_\mu(p, p', q) &= i^2 \int d^4x e^{-ip \cdot x} \int d^4y e^{ip' \cdot y} \langle 0 | \\ &\times \mathcal{T} \{ \mathcal{J}_{cc}^{tr, V(A)}(y) \mathcal{J}_\mu^{tr, V(A)}(0) \bar{\mathcal{J}}_{ccb}^{\Omega_{ccb}^+}(x) \} | 0 \rangle, \end{aligned} \quad (4)$$

where the symbol $\mathcal{T}$ denotes the time-ordered product, while $\mathcal{J}_\mu^{tr, V(A)}(0)$ represents the transition current of

semileptonic decay. $\mathcal{J}_{ccb}^{\Omega_{ccb}^+}(x)$ and $\mathcal{J}_{cc}^{\Xi_{cc}^{++}}(y)$ signify the interpolating currents of the initial and final particles, respectively. In this case, the initial particle is a triply heavy baryon with spin 1/2, and the final particle is a symmetric doubly heavy spin 1/2 baryon. The general expressions of the interpolating currents for these specified particles are given as follows:

$$\begin{aligned} \mathcal{J}_{QQ'Q''}^{\Omega_{ccb}^+}(x) &= 2\epsilon^{abc} \{ (Q^{aT}(x) C Q'^b(x)) \gamma_5 Q^c(x) \\ &+ \beta (Q^{aT}(x) C \gamma_5 Q'^b(x)) Q^c(x) \}, \end{aligned} \quad (5)$$

and

$$\begin{aligned} \mathcal{J}_{QQ'Q''}^{\Xi_{cc}^{++}}(x) &= \frac{2}{\sqrt{2}} \epsilon^{abc} \{ (Q^{aT}(x) C q^b(x)) \gamma_5 Q^c(x) \\ &+ \beta (Q^{aT}(x) C \gamma_5 q^b(x)) Q^c(x) \}. \end{aligned} \quad (6)$$

Here the superscripts a , b , and c denote color indices, C represents the charge conjugation operator, and β is an

arbitrary auxiliary parameter. The variable q correspond to light quark, u , while $Q^{(l)}$ denotes the heavy quarks $c(b)$. Within the framework of QCD sum rules, the correlation function must be computed from both the hadronic and the QCD sides. Then the form factors can be achieved by relating the results derived from these two regimes, applying dispersion integrals and quark-hadron duality assumption and utilizing the Borel transformation technique and continuum subtraction to suppress contributions of higher excited states.

A. Hadronic side

In this subsection, the correlation function is evaluated within the timelike region of the light cone. To achieve this, it is imperative to incorporate two relevant complete sets of states that possess the same quantum numbers as the initial, Ω_{ccb}^+ , and final, Ξ_{cc}^{++} , states at the appropriate stage. Subsequently, by performing the integrals over four dimensional coordinates x and y , the form of the correlation function in the hadronic sector is derived as follows:

$$\Pi_{\mu}^{\text{Had}}(p, p', q) = \frac{\langle 0 | \mathcal{J}_{cc}^{\Xi_{cc}^{++}}(0) | \Xi_{cc}^{++}(p') \rangle \langle \Xi_{cc}^{++}(p') | \mathcal{J}_{\mu}^{tr, V(A)}(0) | \Omega_{ccb}^+(p) \rangle \langle \Omega_{ccb}^+(p) | \bar{\mathcal{J}}_{ccb}^{\Omega_{ccb}^+}(0) | 0 \rangle}{(p'^2 - m_{\Xi_{cc}^{++}}^2)(p^2 - m_{\Omega_{ccb}^+}^2)} + \dots, \quad (7)$$

where the first term is the ground state contribution and dots refer to the contributions from higher resonances and continuum modes. The hadronic matrix elements referenced in the correlation function are defined by the subsequent equations

$$\begin{aligned} \langle 0 | \mathcal{J}_{cc}^{\Xi_{cc}^{++}}(0) | \Xi_{cc}^{++}(p') \rangle &= \lambda_{\Xi_{cc}^{++}} u_{\Xi_{cc}^{++}}(p', s'), \\ \langle \Omega_{ccb}^+(p) | \bar{\mathcal{J}}_{ccb}^{\Omega_{ccb}^+}(0) | 0 \rangle &= \lambda_{\Omega_{ccb}^+}^{\dagger} \bar{u}_{\Omega_{ccb}^+}(p, s), \end{aligned} \quad (8)$$

the symbols $\lambda_{\Omega_{ccb}^+}$ and $\lambda_{\Xi_{cc}^{++}}$ correspond to the residue of the initial and the final states, respectively. $u_{\Omega_{ccb}^+}(p, s)$ and

$u_{\Xi_{cc}^{++}}(p', s')$ satisfy the following identities:

$$\begin{aligned} \sum_{s'} u_{\Xi_{cc}^{++}}(p', s') \bar{u}_{\Xi_{cc}^{++}}(p', s') &= \not{p}' + m_{\Xi_{cc}^{++}}, \\ \sum_s u_{\Omega_{ccb}^+}(p, s) \bar{u}_{\Omega_{ccb}^+}(p, s) &= \not{p} + m_{\Omega_{ccb}^+}. \end{aligned} \quad (9)$$

By substituting the lately straightforward mathematical relations into Eq. (7), the general form of the correlation function in the hadronic sector can be expressed as follows:

$$\begin{aligned} \Pi_{\mu}^{\text{Had}}(p, p', q) &= \frac{\lambda_{\Xi_{cc}^{++}} \lambda_{\Omega_{ccb}^+}^{\dagger} (\not{p}' + m_{\Xi_{cc}^{++}})(\not{p} + m_{\Omega_{ccb}^+})}{(p'^2 - m_{\Xi_{cc}^{++}}^2)(p^2 - m_{\Omega_{ccb}^+}^2)} \left(\left[F_1(q^2) \gamma^{\mu} + F_2(q^2) \frac{p^{\mu}}{m_{\Omega_{ccb}^+}} + F_3(q^2) \frac{p'^{\mu}}{m_{\Xi_{cc}^{++}}} \right] \right. \\ &\quad \left. - \left[G_1(q^2) \gamma^{\mu} \gamma_5 + G_2(q^2) \frac{p^{\mu} \gamma_5}{m_{\Omega_{ccb}^+}} + G_3(q^2) \frac{p'^{\mu} \gamma_5}{m_{\Xi_{cc}^{++}}} \right] \right) + \dots \end{aligned} \quad (10)$$

To suppress the contributions from excited and continuum states, we employ the double Borel transformation

$$\hat{\mathbf{B}} \frac{1}{(p^2 - s)^m} \frac{1}{(p'^2 - s')^n} \longrightarrow (-1)^{m+n} \frac{1}{\Gamma[m] \Gamma[n]} \frac{1}{(M^2)^{m-1}} \frac{1}{(M'^2)^{n-1}} e^{-s/M^2} e^{-s'/M'^2}, \quad (11)$$

in this transformation, M^2 and M'^2 represent Borel parameters, which are determined by establishing working windows in the numerical analysis section. In the Borel scheme, we have

$$\begin{aligned} \hat{\mathbf{B}} \Pi_{\mu}^{\text{Had}}(p, p', q) &= \lambda_{\Xi_{cc}^{++}} \lambda_{\Omega_{ccb}^+}^{\dagger} (\not{p}' + m_{\Xi_{cc}^{++}})(\not{p} + m_{\Omega_{ccb}^+}) e^{-\frac{m_{\Omega_{ccb}^+}^2}{M^2}} e^{-\frac{m_{\Xi_{cc}^{++}}^2}{M'^2}} \\ &\quad \times \left(\left[F_1(q^2) \gamma^{\mu} + F_2(q^2) \frac{p^{\mu}}{m_{\Omega_{ccb}^+}} + F_3(q^2) \frac{p'^{\mu}}{m_{\Xi_{cc}^{++}}} \right] - \left[G_1(q^2) \gamma^{\mu} \gamma_5 + G_2(q^2) \frac{p^{\mu} \gamma_5}{m_{\Omega_{ccb}^+}} + G_3(q^2) \frac{p'^{\mu} \gamma_5}{m_{\Xi_{cc}^{++}}} \right] \right) + \dots \end{aligned} \quad (12)$$

B. QCD side

As previously indicated, within the QCD sum rule formalism, it is crucial to compute the correlation function in the QCD sector in the deep Euclidean spacelike region through the application of operator product expansion (OPE). To accomplish this objective, one must first

substitute the interpolating current of the initial and final particle given in Eqs. (5) and (6) within Eq. (4). Following this substitution, and applying Wick's theorem to compute all possible contractions of the corresponding quark fields, a clear expression emerges that includes the heavy and light quark propagators for the correlation function

$$\begin{aligned} \Pi_\mu(p, p') = & 2\sqrt{2}\epsilon^{abc}\epsilon^{a'b'c'} \int d^4x e^{-ip \cdot x} \int d^4y e^{ip' \cdot y} \{ \gamma_5 S_c^{cc'}(y-x) \gamma_5 \text{Tr}[S_c'^{aa'}(y-x) S_u^{bi}(y) \gamma_\mu (1-\gamma_5) S_b^{ib'}(-x)] \\ & - \gamma_5 S_c^{ca'}(y-x) S_b^{ib'}(-x) (1-\gamma_5) \gamma_\mu S_u^{bi}(y) S_c^{ac'}(y-x) \gamma_5 \\ & + \beta (\gamma_5 S_c^{cc'}(y-x) \text{Tr}[S_c'^{aa'}(y-x) S_u^{bi}(y) \gamma_\mu (1-\gamma_5) S_b^{ib'}(-x) \gamma_5] \\ & - \gamma_5 S_c^{ca'}(y-x) \gamma_5 S_b^{ib'}(-x) (1-\gamma_5) \gamma_\mu S_u^{bi}(y) S_c^{ac'}(y-x) \\ & + S_c^{ca'}(y-x) \gamma_5 \text{Tr}[S_c'^{aa'}(y-x) \gamma_5 S_u^{bi}(y) \gamma_\mu (1-\gamma_5) S_b^{ib'}(-x) \\ & - S_c^{ca'}(y-x) S_b^{ib'}(-x) (1-\gamma_5) \gamma_\mu S_u^{bi}(y) \gamma_5 S_c^{ac'}(y-x) \gamma_5] \\ & + \beta^2 (S_c^{cc'} \text{Tr}[S_c'^{aa'}(y-x) \gamma_5 S_u^{bi}(y) \gamma_\mu (1-\gamma_5) S_b^{ib'}(-x) \gamma_5] \\ & - S_c^{ca'}(y-x) \gamma_5 S_b^{ib'}(-x) (1-\gamma_5) \gamma_\mu S_u^{bi}(y) \gamma_5 S_c^{ac'}(y-x)) \}. \end{aligned} \quad (13)$$

Here, $S_{c(b)}$ denotes the heavy quark propagator, S_u represents the light quark propagator, and $S' = CS^T C$. To conduct calculations in coordinate space, one should utilize the following explicit formulas for the heavy and light propagators, respectively. We take:

$$\begin{aligned} S_q^{ab}(x) = & i\delta_{ab} \frac{\not{x}}{2\pi^2 x^4} - \delta_{ab} \frac{m_q}{4\pi^2 x^2} - \delta_{ab} \frac{\langle \bar{q}q \rangle}{12} + i\delta_{ab} \frac{\not{x} m_q \langle \bar{q}q \rangle}{48} - \delta_{ab} \frac{x^2}{192} \langle \bar{q}g\sigma Gq \rangle + i\delta_{ab} \frac{x^2 \not{x} m_q}{1152} \langle \bar{q}g\sigma Gq \rangle \\ & - i \frac{g G_{ab}^{\alpha\beta}}{32\pi^2 x^2} [\not{x} \sigma_{\alpha\beta} + \sigma_{\alpha\beta} \not{x}] - i\delta_{ab} \frac{x^2 \not{x} g^2 \langle \bar{q}q \rangle^2}{7776} - \delta_{ab} \frac{x^4 \langle \bar{q}q \rangle \langle g^2 G^2 \rangle}{27648} + \dots, \end{aligned} \quad (14)$$

and

$$\begin{aligned} S_Q^{ab}(x) = & i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left\{ \frac{\delta_{ab}(\not{k} + m_Q)}{k^2 - m_Q^2} - \frac{g_s G_{ab}^{\alpha\beta} \sigma_{\alpha\beta}(\not{k} + m_Q) + (\not{k} + m_Q) \sigma_{\alpha\beta}}{4(k^2 - m_Q^2)^2} \right. \\ & \left. + \frac{g_s^2 G^2}{12} \delta_{ab} m_Q \frac{k^2 + m_Q \not{k}}{(k^2 - m_Q^2)^4} + \frac{g_s^3 G^3}{48} \delta_{ab} \frac{(\not{k} + m_Q)}{(k^2 - m_Q^2)^6} [k(k^2 - 3m_Q^2) + 2m_Q(2k^2 - m_Q^2)](\not{k} + m_Q) + \dots \right\}, \end{aligned} \quad (15)$$

where $m_{Q(q)}$ denote the mass of the heavy (light) quark and k represents the four-momentum. $G_{ab}^{\mu\nu} = G_A^{\mu\nu} t_{ab}^A$, $t^A = \lambda^A/2$, $G^2 = G_A^{\mu\nu} G_{\mu\nu}^A$, $G_{\mu\nu}$ denotes the gluon field strength tensor, λ^A illustrate the Gell-Mann matrices which $A = 1, 2, \dots, 8$; $\langle \bar{q}q \rangle$, $\langle G^2 \rangle$ and $\langle \bar{q}g\sigma Gq \rangle$ represent quark-quark, gluon-gluon and quark-gluon condensates, respectively. At this stage, following the applying the quark propagators in coordinate space to the correlation function, we proceed to compute the integrals within the QCD side utilizing a series of mathematical techniques. These techniques include Fourier transforms, Feynman parametrization, and various identities, as detailed. An illustration of this process is provided in

$$\int d^4k_1 \int d^4k_2 \int d^4k_3 \int d^4x e^{i(k_1+k_2+k_3-p) \cdot x} \int d^4y e^{i(-k_1-k_2+p') \cdot y} \frac{f(k_1, k_2, k_3, y)}{(k_1^2 - m_c^2)^{n_1} (k_2^2 - m_c^2)^{n_2} (k_3^2 - m_b^2)^{n_3} y^{2n_4}}. \quad (16)$$

In the initial step, we employ the following identity [64]:

$$\frac{1}{y^{2n}} = \int \frac{d^D t}{(2\pi)^D} e^{-it \cdot y} i(-1)^{n+1} 2^{D-2n} \pi^{D/2} \frac{\Gamma(D/2 - n)}{\Gamma(n)} \left(-\frac{1}{t^2} \right)^{D/2-n}, \quad (17)$$

inserting of $y_\mu \rightarrow -i\frac{\partial}{\partial p'_\mu}$ into the aforementioned relation yields

$$\int d^D t \int d^4 k_1 \int d^4 k_2 \int d^4 k_3 \int d^4 x e^{i(k_1+k_2+k_3-p)\cdot x} \int d^4 y e^{i(-k_1-k_2+p'-t)\cdot y} \times \frac{f(k_1, k_2, k_3)}{(k_1^2 - m_c^2)^{n_1} (k_2^2 - m_c^2)^{n_2} (k_3^2 - m_b^2)^{n_3}} \left(-\frac{1}{t^2}\right)^{D/2-n_4}. \quad (18)$$

In the subsequent step, we apply Fourier integrals with respect to the four variables x and y

$$\int d^4 x e^{i(k_1+k_2+k_3-p)\cdot x} \int d^4 y e^{i(-k_1-k_2+p'-t)\cdot y} = (2\pi)^4 \delta^4(k_1+k_2+k_3-p) (2\pi)^4 \delta^4(-k_1-k_2+p'-t). \quad (19)$$

Making use of the two Dirac delta functions derived in Eq. (A3), the four-integrals over k_2 and k_3 can be easily performed. In the third step, the remaining four-integrals over k_1 and t are evaluated using the Feynman parametrization

$$\frac{1}{A_1^{n_1} A_2^{n_2} A_3^{n_3} A_4^{n_4}} = \frac{\Gamma(n_1+n_2+n_3+n_4)}{\Gamma(n_1)\Gamma(n_2)\Gamma(n_3)\Gamma(n_4)} \int_0^1 \int_0^{1-r} \int_0^{1-r-z} dv dz dr \times \frac{r^{n_1-1} z^{n_2-1} v^{n_3-1} (1-r-z-v)^{n_4-1}}{[rA_1 + zA_2 + vA_3 + (1-r-z-v)A_4]^{n_1+n_2+n_3+n_4}}. \quad (20)$$

In the fourth step, we use the following identity to address the remaining integrals [64]:

$$\int d^D t \frac{(t^2)^m}{(t^2 + \Delta)^n} = \frac{i\pi^2 (-1)^{m-n} \Gamma(m+2) \Gamma(n-m-2)}{\Gamma(2) \Gamma(n) [-\Delta]^{n-m-2}}. \quad (21)$$

In the final step, we apply the following identity to extract the imaginary parts [64]

$$\Gamma\left[\frac{D}{2} - n\right] \left(-\frac{1}{\Delta}\right)^{D/2-n} = \frac{(-1)^{n-1}}{(n-2)!} (-\Delta)^{n-2} \ln[-\Delta]. \quad (22)$$

It is important to note that for dimensions devoid of an imaginary component, their contributions are calculated

directly using the standard procedures of the method. Ultimately, the correlation function within the QCD framework is expressed as follows, representing a function of twenty-four distinct Lorentz structures

$$\Pi_\mu^{\text{QCD}}(p, p', q) = \sum_i \Pi_i^{\text{QCD}}(p^2, p'^2, q^2) s_i, \quad (23)$$

where i runs from 1 to 24, and S_i denotes the various Lorentz structures presented in Table I. The $\Pi_i^{\text{QCD}}(p^2, p'^2, q^2)$ functions computed on the QCD side are Lorentz invariants, expressed in terms of double dispersion integrals as

$$\Pi_i^{\text{QCD}}(p^2, p'^2, q^2) = \int_{(2m_c+m_b)^2}^\infty ds \int_{(2m_c)^2}^\infty ds' \frac{\rho_i^{\text{QCD}}(s, s', q^2)}{(s-p^2)(s'-p'^2)} + \Gamma_i(p^2, p'^2, q^2), \quad (24)$$

where ρ_i is referred as the spectral density, $\rho_i^{\text{OPE}}(s, s', q^2) = \frac{1}{\pi} \text{Im} \Pi_i^{\text{OPE}}(p^2, p'^2, q^2)$. An alternative procedure for obtaining the two-variable spectral density, $\rho_i(s, s')$, for a simple term as an example in the QCD side, and subtraction of higher states and continuum contributions is provided in the Appendix A (for some details about the dispersion relation for the two-variable complex function see also Refs. [65,66].) ρ_i^{OPE} encompasses the following components: (i) the perturbative part, (ii) the three-mass dimension or the condensation of two quarks, $\langle \bar{q}q \rangle$, (iii) the four-mass dimension or the

condensation of two gluons, $\langle G^2 \rangle$. Γ_i represents the contribution of the five-mass dimension, which is calculated directly. The ρ_i and Γ_i for the structure $\gamma_\mu \gamma_5$, as an example, are presented in the Appendix B. We present the results for this structure related to the form factor G_1 in the Appendix B, since this structure revives nonzero expressions for all the perturbative and nonperturbative contributions in momentum space. By adopting the quark-hadron duality assumption, continuum thresholds, denoted as s_0 and s'_0 , emerge in both the initial and final states. Ultimately, by applying double Borel transformation to

TABLE I. Various Lorentz structures appeared in the calculations.

i	S_i	i	S_i	i	S_i
1	$\not{p}'\gamma_\mu\not{p}$	9	$\not{p}'\gamma_\mu$	17	$p_\mu\not{p}\gamma_5$
2	$p_\mu\not{p}'\not{p}$	10	$p_\mu\not{p}'\gamma_5$	18	$p'_\mu\not{p}\gamma_5$
3	$p'_\mu\not{p}'\not{p}$	11	$p'_\mu\not{p}'$	19	$p'_\mu\not{p}$
4	$p'_\mu\not{p}'\gamma_5$	12	$p_\mu\not{p}'$	20	$p_\mu\not{p}$
5	$p'_\mu\not{p}'\not{p}\gamma_5$	13	$\gamma_\mu\not{p}\gamma_5$	21	p'_μ
6	$\not{p}'\gamma_\mu\gamma_5$	14	γ_μ	22	$p'_\mu\gamma_5$
7	$\not{p}'\gamma_\mu\not{p}\gamma_5$	15	$\gamma_\mu\not{p}$	23	p_μ
8	$p_\mu\not{p}'\not{p}\gamma_5$	16	$\gamma_\mu\gamma_5$	24	$p_\mu\gamma_5$

suppress the contributions coming from the excited and continuous states, the form of the correlation function utilized in the QCD side is obtained as follows:

$$\begin{aligned} \Pi_i^{\text{QCD}}(M^2, M'^2, s_0, s'_0, q^2) \\ = \int_{(2m_c+m_b)^2}^{s_0} ds \int_{(2m_c)^2}^{s'_0} ds' e^{-s/M^2} e^{-s'/M'^2} \rho_i^{\text{QCD}}(s, s', q^2) \\ + \tilde{\Gamma}_i(M^2, M'^2, q^2), \end{aligned} \quad (25)$$

where $\tilde{\Gamma}_i(M^2, M'^2, q^2)$ is the double Borel transformed form of $\Gamma_i(p^2, p'^2, q^2)$. Note that, the contribution of five-mass dimensional nonperturbative operator vanishes after applying the double Borel transformation for the structure $\gamma_\mu\gamma_5$. To ultimately derive the six form factors, various corresponding coefficients of structures from both the hadronic and QCD sides are matched.

III. NUMERICAL ANALYSIS

In this section, we present the numerical results for the form factors defining the semileptonic decay $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell$. A comprehensive set of input parameters utilized in the calculations is depicted in Table II. In addition to the parameters listed in Table II, five auxiliary parameters are introduced, which were referenced throughout the calculations: the Borel parameters M^2 and M'^2 , the continuum thresholds s_0 and s'_0 , as well as the arbitrary mixing parameter β . The physical quantities must exhibit minimal dependence on variations in these parameters. Within the framework of QCD sum rule method, to ascertain the operational window for the aforementioned auxiliary parameters, two conditions must be satisfied: the pole contribution (PC) dominance and convergence of the OPE series. These conditions can be imposed as follows:

$$PC = \frac{\Pi^{\text{QCD}}(M^2, M'^2, s_0, s'_0)}{\Pi^{\text{QCD}}(M^2, M'^2, \infty, \infty)} \geq 0.5, \quad (26)$$

and

TABLE II. Input parameters used in calculations.

Parameters	Values
m_b	$(4.18^{+0.03}_{-0.02}) \text{ GeV}$ [67]
m_c	$(1.27 \pm 0.02) \text{ GeV}$ [67]
m_e	0.51 MeV [67]
m_μ	105 MeV [67]
m_τ	1.776 GeV [67]
$m_{\Omega_{ccb}^+}$	$(8.15^{0.27}_{0.23}) \text{ GeV}$ [49]
$m_{\Xi_{cc}^{++}}$	$(3621.6 \pm 0.4) \text{ MeV}$ [67]
G_F	$1.17 \times 10^{-5} \text{ GeV}^{-2}$ [67]
V_{ub}	$(3.82 \pm 0.20) \times 10^{-3}$ [67]
m_0^2	$(0.8 \pm 0.2) \text{ GeV}^2$ [68–70]
$\langle \bar{u}u \rangle$	$-(0.24 \pm 0.01)^3 \text{ GeV}^3$ [68,69]
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	$(0.012 \pm 0.004) \text{ GeV}^4$ [68–70]
$\lambda_{\Omega_{ccb}^+}$	$0.44^{0.23}_{0.25} \text{ GeV}^3$ [49]
$\lambda_{\Xi_{cc}^{++}}$	$0.16 \pm 0.04 \text{ GeV}^3$ [71]

$$R(M^2, M'^2) = \frac{\Pi^{\text{QCD}}(\text{dim}4+\text{dim}5)(M^2, M'^2, s_0, s'_0)}{\Pi^{\text{QCD}}(M^2, M'^2, s_0, s'_0)} \leq 0.05. \quad (27)$$

The PC criterion, as articulated in Eq. (26), requires that the PC exceeds the contributions coming from the higher and continuum states, and help us determine the upper limits for the Borel parameters. The convergence of the OPE series, as expressed in Eq. (27), establishes the lower limits for these parameters. Consequently, based on these criteria, the Borel windows are implemented as follows:

$$\begin{aligned} 9 \text{ GeV}^2 \leq M^2 \leq 14 \text{ GeV}^2, \\ \text{and} \\ 4 \text{ GeV}^2 \leq M'^2 \leq 6 \text{ GeV}^2. \end{aligned} \quad (28)$$

The parameters s_0 and s'_0 are auxiliary variables whose values are not entirely optional; their values depend on the energy of the excited states in both the initial and final channels. Although there is a lack of experimental data regarding the excited states of triply and doubly heavy baryons, our analysis affirm the optimal stability of the sum rules within the following intervals:

$$\begin{aligned} (m_{\Omega_{ccb}^+} + 0.1)^2 \text{ GeV}^2 \leq s_0 \leq (m_{\Omega_{ccb}^+} + 0.5)^2 \text{ GeV}^2, \\ \text{and} \\ (m_{\Xi_{cc}^{++}} + 0.1)^2 \text{ GeV}^2 \leq s'_0 \leq (m_{\Xi_{cc}^{++}} + 0.5)^2 \text{ GeV}^2. \end{aligned} \quad (29)$$

In addition to the Borel parameters and continuum thresholds, the other auxiliary parameter to be set is β entered Eqs. (5) and (6). While β can be chosen to be different for the two interpolating currents in initial and final channels, our analyses show that they belong to working regions that have a major overlap. The optimization of the physical quantities with respect to these parameters lead us to take

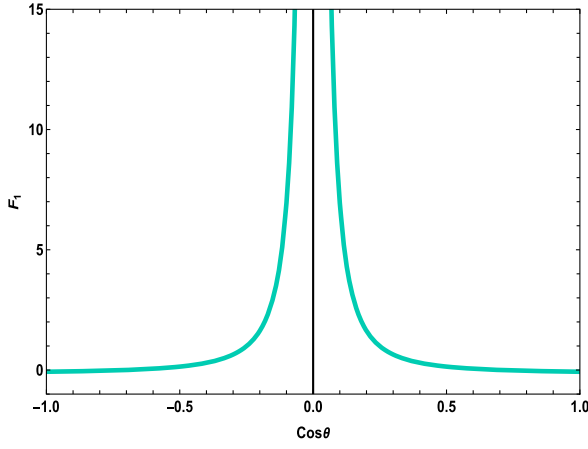


FIG. 2. Variations of F_1 form factor corresponding to the structure $\not{p}'\gamma_\mu$ with respect to $\cos\theta$ at the central values of s_0 , s'_0 , M^2 and M'^2 and at $q^2 = 0$.

this parameter the same for two interpolating currents. The range of the auxiliary mathematical parameter β can be evaluated as follows: since this parameter spans the entire interval from $-\infty$ to $+\infty$, we introduce a new parameter defined as $\beta = \tan\theta$ to incorporate all possible values for β such that $-1.0 \leq \cos\theta \leq 1.0$. Figure 2 exemplifies the variation of F_1 as a function of $\cos\theta$, allowing for the identification of the operational region for $\cos\theta$. Based on numerical analysis and insights from Fig. 2, we conclude

that the optimal regions conducive to the stability of form factors with respect to variations in $\cos\theta$ are delineated as follows:

$$-0.7 \leq \cos\theta \leq -0.5 \quad \text{and} \quad 0.5 \leq \cos\theta \leq 0.7. \quad (30)$$

To demonstrate the stability of the six form factors with respect to the auxiliary parameters, Figs. 3–6 are provided. In these figures, F_1 , F_2 , F_3 , G_1 , G_2 , and G_3 correspond to the structures denoted as $\not{p}'\gamma_\mu$, $p_\mu\not{p}'$, $p'_\mu\not{p}$, $\not{p}'\gamma_\mu\not{p}\gamma_5$, $p_\mu\not{p}'\gamma_5$, and $p_\mu\gamma_5$, respectively. In Figs. 3 and 4 the variations of form factors are depicted as functions of M^2 and M'^2 respectively, at three fixed values of s_0 along with other parameters at their average values. In Figs. 5 and 6 the variations of form factors are depicted as functions of M^2 and M'^2 respectively, at three fixed values of s'_0 while other parameters are kept at their average values. It is important to note that all graphs are generated at the average value of $\cos\theta$ with $q^2 = 0$. We see that the form factors demonstrate good stability with respect to the auxiliary parameters in their working intervals.

Following the identification of the operational windows for all the auxiliary parameters, we proceed to examine the behavior of the form factors as functions of q^2 . Our analysis indicates that the fitting of the form factors is effectively achieved using the following expression:

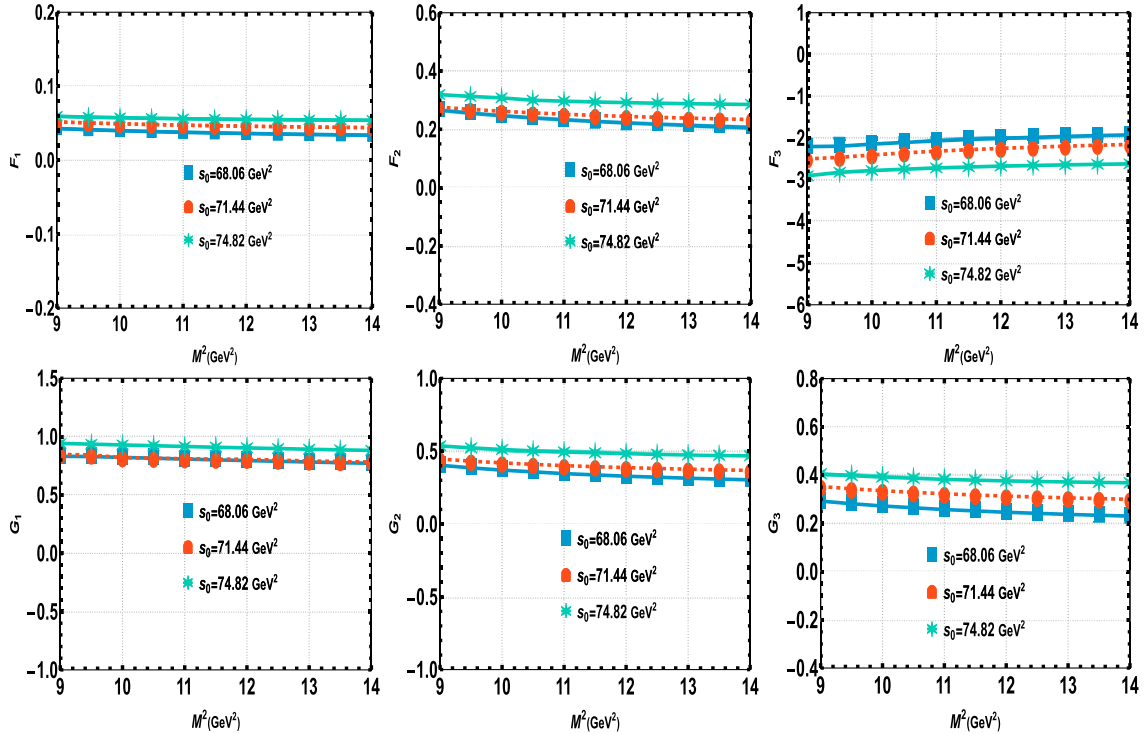


FIG. 3. Form factors as functions of the Borel parameter M^2 at various values of the parameter s_0 , $q^2 = 0$ and average values of other auxiliary parameters ($s'_0 = 15.40 \text{ GeV}^2$, $M'^2 = 5.00 \text{ GeV}^2$, and $\cos\theta = 0.60$).

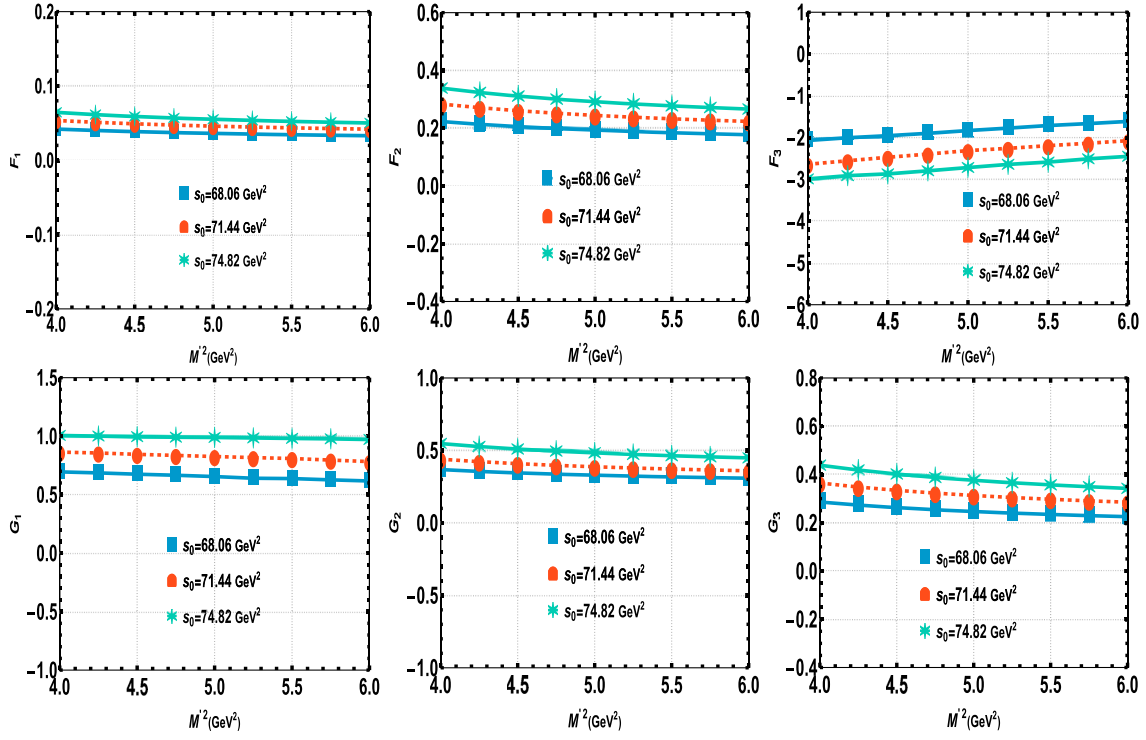


FIG. 4. Form factors as functions of the Borel parameter M'^2 at various values of the parameter s_0 , $q^2 = 0$ and average values of other auxiliary parameters ($s'_0 = 15.40 \text{ GeV}^2$, $M^2 = 11.50 \text{ GeV}^2$, and $\cos \theta = 0.60$).

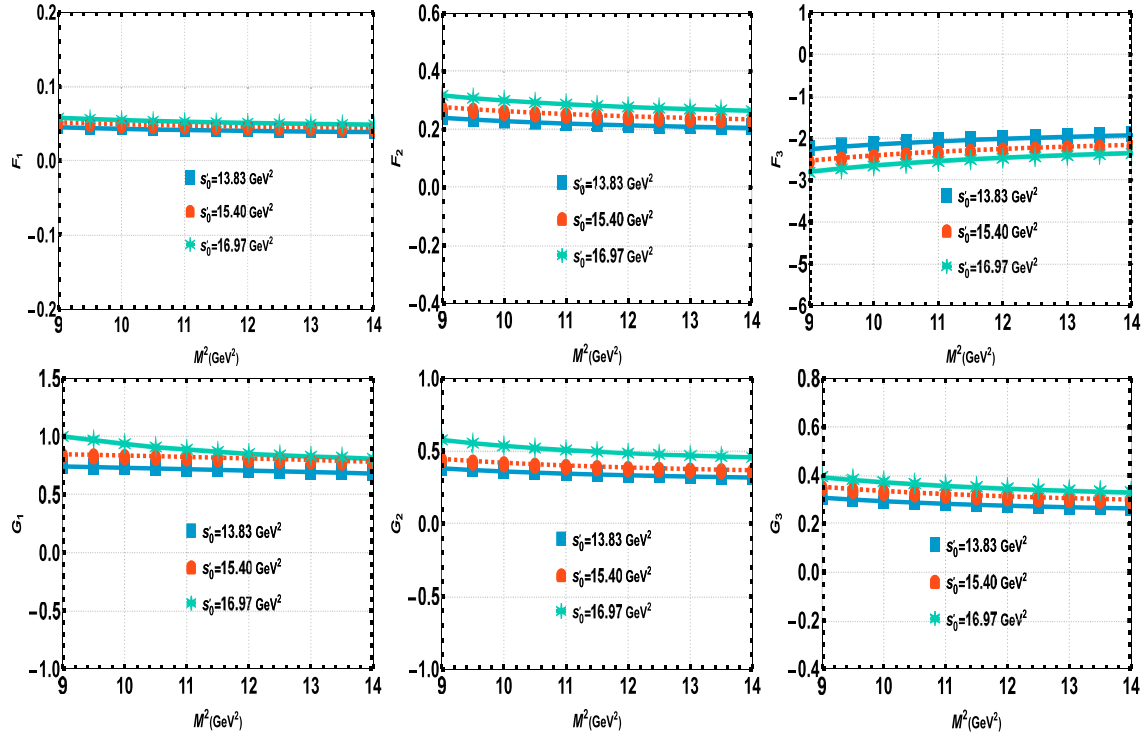


FIG. 5. Form factors as functions of the Borel parameter M^2 at various values of the parameter s'_0 , $q^2 = 0$ and average values of other auxiliary parameters ($s_0 = 71.44 \text{ GeV}^2$, $M'^2 = 5.00 \text{ GeV}^2$, and $\cos \theta = 0.60$).

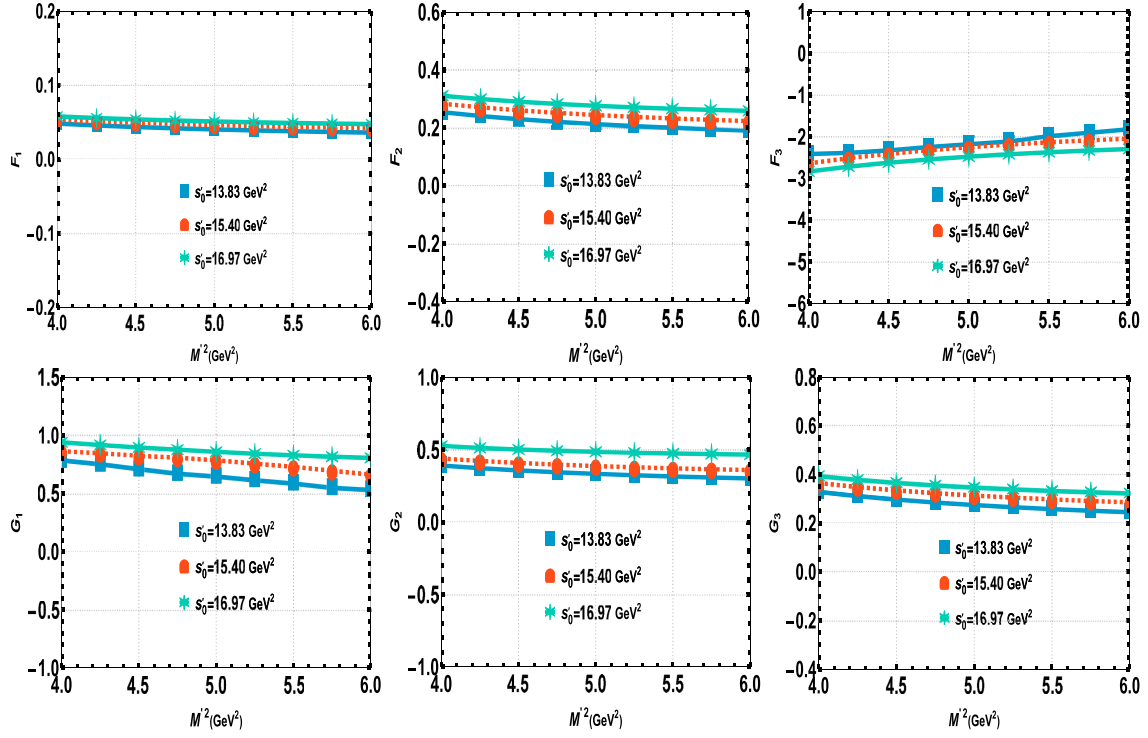


FIG. 6. Form factors as functions of the Borel parameter M'^2 at various values of the parameter s'_0 , $q^2 = 0$ and average values of other auxiliary parameters ($s'_0 = 71.44$ GeV 2 , $M^2 = 11.50$ GeV 2 , and $\cos \theta = 0.60$).

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{\left(1 - a_1 \frac{q^2}{m_{\Omega_{ccb}^+}^2} + a_2 \left(\frac{q^2}{m_{\Omega_{ccb}^+}^2}\right)^2 + a_3 \left(\frac{q^2}{m_{\Omega_{ccb}^+}^2}\right)^3 + a_4 \left(\frac{q^2}{m_{\Omega_{ccb}^+}^2}\right)^4\right)}. \quad (31)$$

In this context, the parameters, $\mathcal{F}(0)$, a_1 , a_2 , a_3 , and a_4 serve as the fitting parameters corresponding to the structures delineated in Table III. It is important to note that these quantities are computed in the average values of the auxiliary parameters. It is noteworthy that the uncertainty associated with the form factors presented in Table III arises from both of the auxiliary parameters and the input ones. It should be noted that the structures in Table III are not unique to find the form factors as is clear from the presented results in both the physical and QCD

sides. In the framework of sum rule method, the form factors are practically obtained to be structure-dependent. Generally, structures with more momenta lead to more stability with respect to the variations of the auxiliary parameters. In this study, the selected structures are based on the operational domains of the auxiliary parameters, wherein the uncertainty of the results is relatively minimal and requirements of the method, discussed above, are better satisfied. The variations of the form factors F_1 , F_2 , F_3 , G_1 , G_2 , and G_3 as functions of q^2 within the permissible

TABLE III. Parameters of the fit functions for different form factors corresponding to $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} l \bar{\nu}_\ell$ transition.

	$F_1(q^2): \not{p}' \gamma_\mu$	$F_2(q^2): p_\mu \not{p}'$	$F_3(q^2): p'_\mu \not{p}$	$G_1(q^2): \not{p}' \gamma_\mu \not{p} \gamma_5$	$G_2(q^2): p_\mu \not{p}' \gamma_5$	$G_3(q^2): p_\mu \gamma_5$
$\mathcal{F}(q^2 = 0)$	0.06 ± 0.01	$0.26^{+0.07}_{-0.05}$	$-2.52^{+0.74}_{-0.75}$	0.84 ± 0.25	0.45 ± 0.13	0.34 ± 0.10
a_1	2.52	2.92	1.41	1.80	3.12	2.14
a_2	-0.25	-0.82	-15.54	-17.87	-10.14	-5.94
a_3	3.43	11.13	78.45	92.95	67.61	29.93
a_4	0.99	-11.32	-124.53	-145.86	-106.61	-39.92

range $m_l^2 \leq q^2 \leq (m_{\Omega_{ccb}^+} - m_{\Xi_{cc}^{++}})^2$ for the structures $\not{p}'\gamma_\mu$, $p_\mu\not{p}'$, $p'_\mu\not{p}$, $\not{p}'\gamma_\mu\not{p}\gamma_5$, $p_\mu\not{p}'\gamma_5$, and $p_\mu\gamma_5$, respectively, are illustrated in Fig. 7. Figure 8 depicts these variations, incorporating uncertainties in the calculation of form

factors. In the subsequent section, we will employ the fitting functions of the form factors as primary input parameters to evaluate the decay widths at different lepton channels.

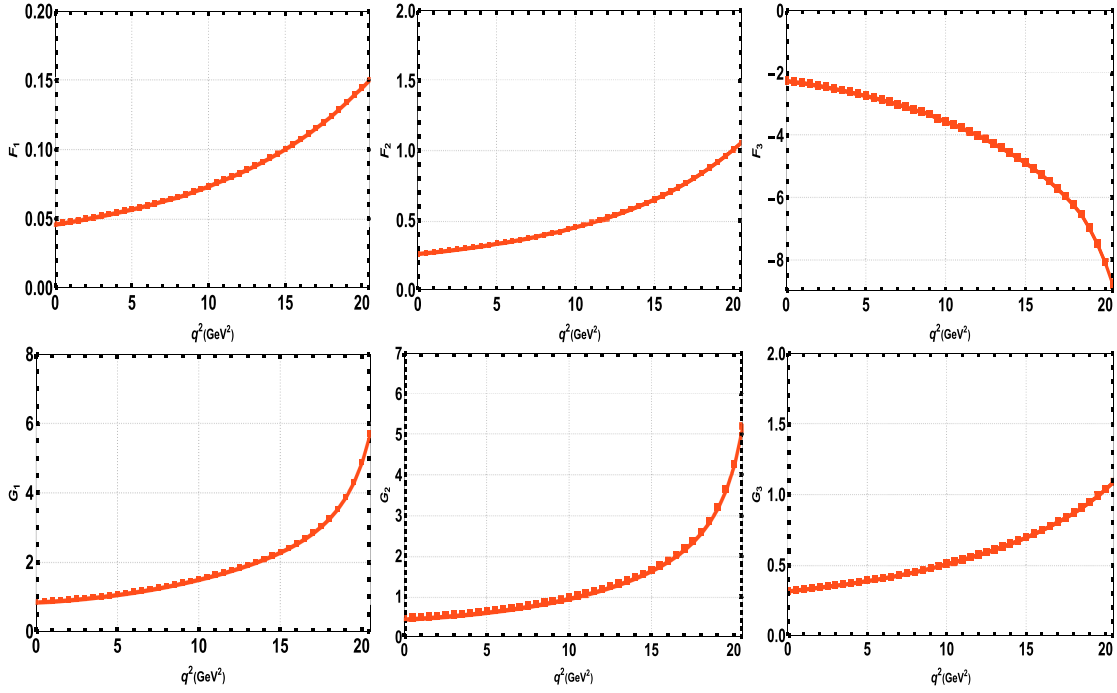


FIG. 7. The form factors F_1 , F_2 , F_3 , G_1 , G_2 , and G_3 , corresponding to the structures in Table III, as functions of q^2 at average values of auxiliary parameters.

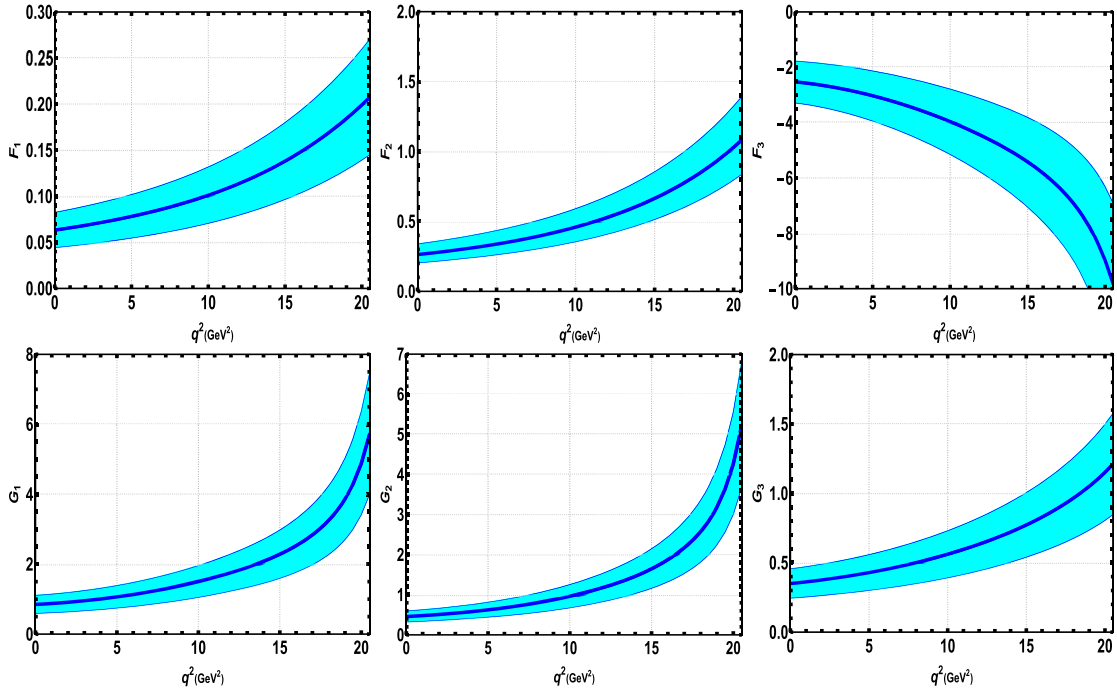


FIG. 8. The form factors corresponding to the structures in Table III with their errors at the average values of auxiliary parameters.

IV. DECAY WIDTH

In this section, the decay widths of semileptonic mode $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell$ across all lepton channels are computed utilizing the fitted form factors derived in the preceding section. The differential decay width is given by:

$$\frac{d\Gamma(\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell)}{dq^2} = \frac{G_F^2}{(2\pi)^3} |V_{ub}|^2 \frac{\lambda^{1/2}(q^2 - m_\ell^2)^2}{48m_{\Omega_{ccb}^+}^3 q^2} \mathcal{H}_{\text{tot}}(q^2), \quad (32)$$

where $\lambda \equiv \lambda(m_{\Omega_{ccb}^+}^2, m_{\Xi_{cc}^{++}}^2, q^2) = m_{\Omega_{ccb}^+}^4 + m_{\Xi_{cc}^{++}}^4 + q^4 - 2(m_{\Omega_{ccb}^+}^2 m_{\Xi_{cc}^{++}}^2 + m_{\Omega_{ccb}^+}^2 q^2 + m_{\Xi_{cc}^{++}}^2 q^2)$, and m_ℓ represents the lepton mass (e, μ, τ). $\mathcal{H}_{\text{tot}}(q^2)$ denotes total helicity, which is defined as

$$\mathcal{H}_{\text{tot}}(q^2) = [\mathcal{H}_U(q^2) + \mathcal{H}_L(q^2)] \left(1 + \frac{m_\ell^2}{2q^2}\right) + \frac{3m_\ell^2}{2q^2} \mathcal{H}_S(q^2). \quad (33)$$

The relevant parity-conserving helicity structures are expressed in terms of the total helicity amplitudes as follows:

$$\begin{aligned} \mathcal{H}_U(q^2) &= |H_{+1/2,+1}|^2 + |H_{-1/2,-1}|^2, \\ \mathcal{H}_L(q^2) &= |H_{+1/2,0}|^2 + |H_{-1/2,0}|^2, \\ \mathcal{H}_S(q^2) &= |H_{+1/2,t}|^2 + |H_{-1/2,t}|^2, \end{aligned} \quad (34)$$

In this set of expressions, different helicity amplitude are expressed in terms of the transition F_i and G_i form factors

$$\begin{aligned} H_{+1/2,0}^{V,A} &= \frac{1}{\sqrt{q^2}} \sqrt{2m_{\Omega_{ccb}^+} m_{\Xi_{cc}^{++}} (\sigma \mp 1)} [(m_{\Omega_{ccb}^+} \pm m_{\Xi_{cc}^{++}}) \mathcal{F}_1^{V,A}(\sigma) \pm m_{\Xi_{cc}^{++}} (\sigma \pm 1) \mathcal{F}_2^{V,A}(\sigma) \pm m_{\Omega_{ccb}^+} (\sigma \pm 1) \mathcal{F}_3^{V,A}(\sigma)], \\ H_{+1/2,1}^{V,A} &= -2\sqrt{m_{\Omega_{ccb}^+} m_{\Xi_{cc}^{++}} (\sigma \mp 1)} \mathcal{F}_1^{V,A}(\sigma), \\ H_{+1/2,t}^{V,A} &= \frac{1}{\sqrt{q^2}} \sqrt{2m_{\Omega_{ccb}^+} m_{\Xi_{cc}^{++}} (\sigma \pm 1)} [(m_{\Omega_{ccb}^+} \mp m_{\Xi_{cc}^{++}}) \mathcal{F}_1^{V,A}(\sigma) \pm (m_{\Omega_{ccb}^+} - m_{\Xi_{cc}^{++}}) \mathcal{F}_2^{V,A}(\sigma) \pm (m_{\Omega_{ccb}^+} \sigma - m_{\Xi_{cc}^{++}}) \mathcal{F}_3^{V,A}(\sigma)], \end{aligned} \quad (35)$$

where

$$\sigma = \frac{m_{\Omega_{ccb}^+}^2 + m_{\Xi_{cc}^{++}}^2 - q^2}{2m_{\Omega_{ccb}^+} m_{\Xi_{cc}^{++}}}. \quad (36)$$

In this context, $\mathcal{F}_i^V \equiv F_i$ and $\mathcal{F}_i^A \equiv G_i$ ($i = 1, 2, 3$) illustrate vector and axial form factors. The upper (lower) sign is associated with the vector (axial) contribution. $H_{h',h_W}^{V,A}$ are utilized to represent the helicity amplitudes for weak decays including the vector (V) and the axial vector (A) currents, and h' and h_W indices stand for the helicities of the final baryon and the virtual W-boson, respectively. The amplitudes for negative helicity values can be derived using the relation

$$H_{-h',-h_W}^{V,A} = \pm H_{h',h_W}^{V,A}. \quad (37)$$

The total helicity amplitude for the V-A current is thus expressed by

$$H_{h',h_W} = H_{h',h_W}^V - H_{h',h_W}^A. \quad (38)$$

The calculated decay widths for all lepton channels, along with their associated errors, are provided in Table IV. Due to the lack of experimental data on the lifetimes of

triple heavy baryons, it is currently not possible to compute the branching fractions. However, the decay width itself serves as an important indicator for guiding experimental searches and theoretical analysis. The decay width is expected to show variations based on the choice of lepton channels, primarily due to the mass differences among the leptons involved. Specifically, as the lepton mass increases (moving from e or μ to τ), the decay width tends to decrease. This trend is consistent with the fact that decays involving heavier leptons are kinematically suppressed. These findings are crucial in providing a benchmark for upcoming experiments, as they enable experimentalists to estimate decay probabilities and search for corresponding signals. Assessing the ratio of decay width in τ and e/μ channels is advantageous due to its reduced uncertainty. We find:

$$R = \frac{\Gamma[\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \tau \bar{\nu}_\tau]}{\Gamma[\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} e(\mu) \bar{\nu}_{e(\mu)}]} = 0.60_{-0.02}^{+0.07}. \quad (39)$$

These results, combined with the mass and residue calculated in our prior work [49], are valuable for refining or finalizing the properties of Ω_{ccb}^+ through future experiments. Comparing our results as Standard Model (SM) theory prediction with future experimental data, could

TABLE IV. Decay widths (in GeV) for the semileptonic $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell$ transition at different channels.

$\Gamma[\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} e \bar{\nu}_e] \times 10^{14}$	$\Gamma[\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \mu \bar{\nu}_\mu] \times 10^{14}$	$\Gamma[\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \tau \bar{\nu}_\tau] \times 10^{15}$
$1.23^{+0.28}_{-0.56}$	$1.22^{+0.28}_{-0.55}$	$7.32^{+1.48}_{-3.16}$

confirm the SM or open new avenue for discovering possible new physics in heavy baryon decays.

V. CONCLUSION

So far, there have been no experimental observations of the triply heavy baryons as the last generation of the standard heavy baryons predicted by the quark model. Few theoretical analysis concerning the weak decay of this group of baryons have been conducted, often focusing on the decay processes involving triply heavy baryons with spin 3/2 transitioning to other triply heavy baryons, or those with spin 1/2 decaying into triply heavy baryons with spin 3/2. This study represents a pioneering investigation into the decay of triply heavy particles with spin 1/2 serving as both initial and final ones in the all lepton channels. In this work, we employed the standard QCD sum rule method to compute the relevant correlation function, which here is the three-point correlation function. This computation was conducted first on the physical side in terms of the vector and axial vector form factors. Subsequently, we derived the three-point correlation function on the QCD side through a series of manual algebraic relations in terms of fundamental QCD parameters like the quark masses, condensation of quarks, gluons, and their combinations, the strong coupling constant and $\dots$. In the final step, the form factors were calculated by assuming quark-hadron duality and applying Borel transformations and matching the same Lorentz structures from both sides. Upon completion of the form factor calculations, we were positioned to determine the decay width of the semileptonic transformation $\Omega_{ccb}^+ \rightarrow \Xi_{cc}^{++} \ell \bar{\nu}_\ell$ across all lepton channels. The order of exclusive widths indicate that these channels are accessible in near future.

The results presented in this study and those on the spectroscopic properties of the triply heavy baryons in Ref. [49] can help experimental groups at different colliders in their search for these interesting heavy particles. New developments in the experimental side tempt us to believe that we may not be far from observing such states at various hadron colliders.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: DOUBLE SPECTRAL DENSITY

In this appendix, we present an alternative procedure for a simple term in QCD side, as an example, to derive a two-variable dependent spectral density. For this aim, we start with

$$\begin{aligned} \Pi &= \int d^4 k_1 \int d^4 k_2 \int d^4 k_3 \int d^4 x e^{i(k_1+k_2+k_3-p) \cdot x} \\ &\times \int d^4 y e^{i(-k_1-k_2+p') \cdot y} \\ &\times \frac{1}{(k_1^2 - m_c^2)^2 (k_2^2 - m_c^2) (k_3^2 - m_b^2) y^2}. \end{aligned} \quad (\text{A1})$$

Using the identity presented in Eq. (17), we get

$$\begin{aligned} \Pi &= C \int d^4 t \int d^4 k_1 \int d^4 k_2 \int d^4 k_3 \int d^4 x e^{i(k_1+k_2+k_3-p) \cdot x} \\ &\times \int d^4 y e^{i(-k_1-k_2+p'-t) \cdot y} \\ &\times \frac{1}{(k_1^2 - m_c^2)^2 (k_2^2 - m_c^2) (k_3^2 - m_b^2)} \left(-\frac{1}{t^2} \right), \end{aligned} \quad (\text{A2})$$

where $C = \frac{i}{4\pi^2}$ is a constant. Now, we use

$$\begin{aligned} &\int d^4 x e^{i(k_1+k_2+k_3-p) \cdot x} \int d^4 y e^{i(-k_1-k_2+p'-t) \cdot y} \\ &= (2\pi)^4 \delta^4(k_1+k_2+k_3-p) (2\pi)^4 \delta^4(-k_1-k_2+p'-t), \end{aligned} \quad (\text{A3})$$

where the resultant Dirac deltas are used to perform the four-integrals over k_3 and k_2 . We employ the Feynman parametrization as the body text, which leads to

$$\begin{aligned} \Pi &= C_1 \int d^4 t \int d^4 k_1 \int_0^1 \int_0^{1-r} \int_0^{1-r-z} dr dz dv \\ &\times \frac{r}{[r(k_1^2 - m_c^2) + z(-m_c^2 + (-k_1 + p' - t)^2) + v(-m_b^2 + (p - p' + t)^2) + (1 - r - z - v)t^2]^5}, \end{aligned} \quad (\text{A4})$$

where $C_1 = i(2\pi)^6$. In the next step, we use the identity presented in Eq. (21) to perform the remaining integrals, which leads to

$$\Pi = C_1 \int_0^1 \int_0^{1-r} \int_0^{1-r-z} dr dz dv \frac{r}{\Delta}, \quad (\text{A5})$$

where

$$\Delta = (-1+r)[p'^2 r z A_1 - (m_c^2(r+z)A_2 + v[p^2 r z A_2^2 + m_b^2 A_2 - q^2(r+z)A_1])] \quad (\text{A6})$$

and

$$A_1 = \frac{(-1+r+v+z)}{(r^2+r(-1+z)+(-1+z)z)^2},$$

$$A_2 = \frac{1}{(r^2+r(-1+z)+(-1+z)z)}. \quad (\text{A7})$$

Applying double Borel transformation with respect to the p^2 and p'^2 using the

$$\mathcal{B}(M'^2) \frac{1}{\alpha' - p'^2} = e^{\frac{-\alpha'}{M'^2}},$$

$$\mathcal{B}(M^2) e^{-\beta p^2} = \delta(1/M^2 - \beta), \quad (\text{A8})$$

we get

$$\mathcal{B}(M^2) \mathcal{B}(M'^2) \Pi = C_1 \int_0^1 \int_0^{1-r} \int_0^{1-r-z} dr dz dv B_1 e^{\frac{\alpha}{M'^2}} \times \delta\left(\frac{1}{M^2} - \frac{\beta}{M'^2}\right), \quad (\text{A9})$$

where $\alpha = m_c^2 B_2 + m_b^2 B_3 - q^2 B_4$ and $\beta = \frac{v}{(-1+r+v+z)}$. The variables B_1 , B_2 , B_3 , and B_4 are defined by

$$B_1 = \frac{1}{(-1+r)zA_1},$$

$$B_2 = \frac{(r+z)(r^2+r(-1+z)+(-1+z)z)}{rz(-1+r+v+z)},$$

$$B_3 = \frac{(r^2+r(-1+z)+(-1+z)z)v}{rz(-1+r+v+z)},$$

$$B_4 = \frac{(r+z)v(-1+r+v+z)}{rz(-1+r+v+z)}. \quad (\text{A10})$$

Performing integral over v , one obtains

$$\mathcal{B}(M^2) \mathcal{B}(M'^2) \Pi = C_1 \int_0^1 \int_0^{1-r} dr dz \times D_4 \frac{M^2}{M^2 + M'^2} e^{\frac{q^2 D_1}{M'^2 + M^2}} e^{\frac{-m_b^2 D_2}{M'^2}} e^{\frac{-m_c^2 D_3}{M'^2}}, \quad (\text{A11})$$

where $M_1^2 = \frac{M^2 M'^2}{M'^2 + M^2}$ and we have applied the following shorthand notations:

$$D_1 = \frac{(-1+r+z)(r+z)}{rz},$$

$$D_2 = \frac{-(r^2+r(-1+z)+(-1+z)z)}{rz},$$

$$D_3 = \frac{(r+z)(r^2+r(-1+z)+(-1+z)z)}{rz(-1+r+z)},$$

$$D_4 = \frac{(-1+r+z)}{(r^2+r(-1+z)+(-1+z)z)^2(-1+r)z}. \quad (\text{A12})$$

In the last step, we discuss how contribution of the continuum and higher states are subtracted (see also [72–74]). To this end, we consider a generic term of the form

$$A = \int_0^1 \int_0^{1-r} dr dz \frac{M^2}{M^2 + M'^2} e^{\frac{-m_b^2 D_2}{M'^2}} e^{\frac{-m_c^2 D_3}{M'^2}}. \quad (\text{A13})$$

at $q^2 = 0$. We introduce new variables by defining $\sigma_1 = \frac{1}{M^2}$ and $\sigma_2 = \frac{1}{M'^2}$. We have

$$A = \left(\frac{\sigma_2}{\sigma_1 + \sigma_2}\right) \int_0^1 \int_0^{1-r} dr dz e^{-(m_b^2 D_2 + m_c^2 D_3) \sigma_1} e^{-m_c^2 D_3 \sigma_2}$$

$$= \sigma_2 \int_0^1 \int_0^{1-r} dr dz e^{-(m_b^2 D_2 + m_c^2 D_3) \sigma_1} e^{-m_c^2 D_3 \sigma_2}$$

$$\times \int_0^\infty d\xi e^{-\xi(\sigma_1 + \sigma_2)}$$

$$= \sigma_2 \int_0^1 \int_0^{1-r} dr dz \int_0^\infty d\xi e^{-(m_b^2 D_2 + m_c^2 D_3 + \xi) \sigma_1} e^{-(m_c^2 D_3 + \xi) \sigma_2}$$

$$= - \int_0^1 \int_0^{1-r} dr dz \times \int_0^\infty d\xi \left(\left(\frac{d}{d\xi} \right) e^{-(m_c^2 D_3 + \xi) \sigma_2} \right) e^{-(m_b^2 D_2 + m_c^2 D_3 + \xi) \sigma_1}. \quad (\text{A14})$$

We obtain the double spectral density by performing another double Borel transformation with respect to $\sigma_1 \rightarrow \frac{1}{s}$ and $\sigma_2 \rightarrow \frac{1}{s'}$

$$\rho(s, s') = - \int_0^1 \int_0^{1-r} dr dz \int_0^\infty d\xi \left(\frac{d}{d\xi} \delta[s' - (\xi + m_c^2 D_3)] \right) \times \delta[s - (\xi + m_b^2 D_2 + m_c^2 D_3)]. \quad (\text{A15})$$

Performing the ξ integral, finally the following expression for the double spectral density is found

$$\rho(s, s') = - \int_0^1 \int_0^{1-r} dr dz \left(\frac{d}{ds} \delta(s' - s + m_b^2 D_2) \right) \times \theta(s - (m_b^2 D_2 + m_c^2 D_3)), \quad (\text{A16})$$

where θ is the Heaviside Theta function. Now, we use this spectral density to write the subtracted correlation function corresponding to the considered term in Borel scheme as

$$\Pi^{\text{sub}} = \int_{s_L}^{s_0} ds \int_{s'_L}^{s'_0} ds' \rho(s, s') e^{-s/M^2} e^{-s'/M'^2}. \quad (\text{A17})$$

where $s_L = (m_b + 2m_c)^2$ and $s'_L = (2m_c)^2$. From the same manner, we can generalized the calculations to the case $q^2 \neq 0$ and write

$$\Pi'^{\text{sub}} = \int_{s_L}^{s_0} ds \int_{s'_L}^{s'_0} ds' \rho(s, s', q^2) e^{-s/M^2} e^{-s'/M'^2}, \quad (\text{A18})$$

which is the double Borel performed form of the two-variable spectral representation of the related correlation function.

APPENDIX B: OPE RESULTS FOR ONE OF THE INVOLVED STRUCTURES

In this appendix, we depict the explicit forms of spectral densities $\rho^i(s, s', q^2)$ and functions $\Gamma^i(p^2, p'^2, q^2)$ corresponding to the structure $\gamma_\mu \gamma_5$

$$\begin{aligned} \rho_{\gamma_\mu \gamma_5}^{\text{pert}}(s, s', q^2) = & \frac{1}{\sqrt{2512}\pi^4} \int_0^1 du \int_0^{1-u} dv \int_0^{1-u-v} dz \frac{3D(s, s', q^2) \Theta[D(s, s', q^2)]}{K_2 W_1 W_2 W_3^5} \\ & \times \{ [D(s, s', q^2) W_2 (2u W_3 (W_3 + 15v) - v(W_3 (2 + 11K_2 W_3) + 9W_3 v + 11v^2)) \\ & + 2K_1 W_3 ((W_3 + v)(-3q^2 W_1 + s'(8u W_3 + 23uv + 8W_4 v) \\ & + 3s(u W_3 + v(W_3 + v))) - (3q^2 - 3s - 8s')(W_3 u + (4u + W_4)v)z)] \\ & - 8\beta [3D(s, s', q^2) W_2 (u W_3 (W_3 - 2v) + v(W_3 (-1 + 3K_2 W_3) + 4W_3 v + 3v^2)) \\ & + 4K_1 ((W_3 + v)[s(W_1 - u W_1) + q^2 W_1 W_3 + s'v(u - u^2 + 2K_2 W_3^2 + 2(u + W_4)v)] \\ & + [(q^2 - s)u W_3^2 + W_3(-K_2 s' + (-2 + K_2)s'u + s(1 + K_2 - (2 + K_2)u) \\ & + q^2(-1 + 2u + K_2 W_3))v + (2q^2 - 2s + s')W_3 v^2 + (q^2 - s + s')v^3]z)] \\ & + \beta^2 [D(s, s', q^2) W_2 (2u W_3 (W_3 + 15v) - v(W_3 (2 + 11K_2 W_3) + 9W_3 v + 11v^2)) \\ & + 2K_1 W_3 ((W_3 + v)[-3q^2(u(u + W_4) + W_4 v) + s'(8u W_3 + 23uv + 8W_4 v) + 3s(u W_3 + v(W_3 + v))] \\ & - (3q^2 - 3s - 8s')[W_3 u + (4u + W_4)v]z)] \}, \end{aligned} \quad (\text{B1})$$

$$\rho_{\gamma_\mu \gamma_5}^3(s, s', q^2) = \int_0^1 du \int_0^{1-u} dv \frac{(-1 + \beta)^2 m_b \langle \bar{u} u \rangle}{8\sqrt{2}\pi^2 (W_5)^3} \Theta[L(s, s', q^2)], \quad (\text{B2})$$

$$\begin{aligned} \rho_{\gamma_\mu \gamma_5}^4(s, s', q^2) = & \int_0^1 du \int_0^{1-u} dv \int_0^{1-u-v} dz \frac{\Theta[D(s, s', q^2)]}{12288\sqrt{2}K_2^3 W_1 W_3^7 \pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \\ & \times \{ [3(-85u^2 W_3^3 + u W_3^2 (129 - 333u + 44K_2 W_3)v - W_3 (44 + 8K_2 W_3 + u(-486 + 585u + 16K_2 W_3))v^2 \\ & + W_3 (80 - 373u + 8K_2 W_3)v^3 + 4(7 - 13u)v^4 + 8v^5) + (24u(-25 + 21u)W_3^2 + 3W_3 (200 - 110K_2 W_3 \\ & + u(-648 + 580u + 95K_2 W_3))v + W_3 (-1158 + 765u + 242K_2 W_3)v^2 + 5(-160 + 151u)v^3 + 242v^4)z \\ & + 6(2u W_3 (50W_3 + 101v) + 5v(W_3 (-20 + 11K_2 W_3) + 31W_3 v + 11v^2))z^2] \\ & + \beta [96(-u^5 + u^4(3 + (-9 + K_2)v) + u^3(-3 + (21 + K_2(-3 + v) - 22v)v) + 2W_4 v^2(-1 + K_2 + v^2) \\ & + u W_4 v(-3 + K_2 - 4K_2 v + v(18 + v)) + u^2[1 + v(-15 + (41 - 15v)v + K_2(3 + 2(-2 + v)v)])) \\ & + (-W_3^2 u(11 + 18u) + (3K_2 W_3^2(-20 + 13u) + u(-30 + (84 - 43u)u) + 11W_4)v \\ & + (58K_2 W_3^2 + (27 - 85u)u - 47W_4)v^2 + 2(42u + 29W_4)v^3)z \} \end{aligned}$$

$$\begin{aligned}
& + (11W_3^2u + (60K_2W_3^2 - u(1 + 10u) - 11W_4)v + (71u + 60W_4)v^2)z^2] \\
& + \beta^2[-282W_3^3u^2 + 6(57 + 17K_2W_3 - 127u)W_3^2uv + 2[u(-468 + 933u - 495u^2 + K_2(331 + u(-410 + 163u))) \\
& + 84K_2u^4 + 30u^4]v^2 + 8((20 + 21K_2(-2 + u) - 26u)u + 21W_4^2)v^3 + 434uv^4 \\
& + [9W_3^2u(-18 + 5u) + (-K_2W_3^2(-38 + 79u) + u(499 + u(-755 + 418u)) + 162W_4)v \\
& + 2(27K_2W_3^2 + (319 - 303u)u + 97W_4)v^2 + (-181u + 54W_4)v^3]z \\
& + 2(81W_3^2u - (19K_2W_3^2 + (305 - 224u)u + 81W_4)v + (62u - 19W_4)v^2)z^2]\}, \tag{B3}
\end{aligned}$$

and

$$\begin{aligned}
\Gamma_{\gamma_\mu\gamma_5}^5(p^2, p'^2, q^2) &= \frac{\langle \bar{u}u \rangle m_o^2}{32\sqrt{2}\pi^2 W_5^7 L_1^3(p^2, p'^2, q^2)} \\
&\times \left\{ (-1 + \beta)^2 [(-m_b uv(L_1(p^2, p'^2, q^2)(L_1(p^2, p'^2, q^2) - 2m_c^2)W_5^2 + 2m_c^2 uv p'^2))] \right. \\
&- \left[(1 + \beta) uv p'^2 (2(-1 + \beta)L_1(p^2, p'^2, q^2)m_c W_5^4 + uW_5((-1 + \beta)m_c(p^2 + p'^2 - q^2)u \right. \\
&- 2(1 + \beta)L_1(p^2, p'^2, q^2)m_b W_5)v + (1 + \beta)m_b u^2 v^2 p'^2) \frac{2}{W_5^2} \Big] \\
&+ [(-1 + \beta)(1 + \beta)m_c(L_1(p^2, p'^2, q^2)W_5(3(p^2 + p'^2 - q^2)u^2 v \\
&+ L_1(p^2, p'^2, q^2)W_5^2(u + 2v)) + 2u^2 v^2 p'^4)] \Big\}, \tag{B4}
\end{aligned}$$

where we have defined

$$\begin{aligned}
D(s, s', q^2) &= \left(\frac{W_3(m_c^2 W_1 W_5 - s' u W_2 v + (m_b^2 W_1 - q^2 W_2 W_5 + s u v)z)}{W_1^2} \right), \\
L(s, s', q^2) &= \left(-m_c^2 + \frac{(m_b^2 - q^2)W_5 W_6 + s' u v}{W_5^2} \right), \\
L_1(p^2, p'^2, q^2) &= \left(-m_c^2 + \frac{(m_b^2 - q^2)W_5 W_6 + p'^2 u v}{W_5^2} \right), \\
K_1 &= \frac{uv W_2 W_3}{W_1^2}, \\
K_2 &= \frac{-W_1}{W_3^2}. \tag{B5}
\end{aligned}$$

Here, we have applied the following shorthand symbolizations, as well:

$$\begin{aligned}
W_1 &= (u^2 + u(-1 + v) + (-1 + v)v), \\
W_2 &= (-1 + u + v + z), \\
W_3 &= (-1 + u), \\
W_4 &= (-1 + v), \\
W_5 &= (u + v), \\
W_6 &= (-1 + u + v). \tag{B6}
\end{aligned}$$

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