

Mass spectra of doubly charmed tetraquarks T_{cc}

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Motivated by the first observation, to the best of our knowledge, of a doubly charmed tetraquark candidate $T_{cc}(3875)^+$, we perform a systematic calculation of the masses of some constituent diquarks and the doubly charmed tetraquark states from $1S$ to $2P$ excitations in a nonrelativistic constituent quark potential model. First, the mass of the charmed spin-1 diquark is predicted with ~ 3500 MeV, and the masses of the light “good” (spin-0 and isospin-0) and “bad” (spin-1 and isospin-1) antidiquark are predicted with ~ 670 MeV and ~ 840 MeV, respectively. The mass difference between a light good and a light bad diquark is ~ 170 MeV, which is consistent with previous phenomenological analyses. The interaction between the charmed diquark and the light antidiquark is then modulated by the quark-antiquark potential in the model, and the mass spectra of doubly charmed tetraquarks from $1S$ to $2P$ excitations are calculated with the obtained masses of diquarks/antidiquarks and refitted parameters from $T_{cc}(3875)^+$ and $X(3872)$. The pattern of the mass spectra of doubly charmed tetraquarks is obtained explicitly. The $1S - 1P$ and $1S - 2S$ mass splittings of doubly charmed tetraquarks are about 410–430 MeV and 610–650 MeV, respectively, and this mass splittings pattern is similar to that for ordinary D mesons. The mass splittings of doubly charmed tetraquarks with a good diquark is about 10–25 MeV higher than the corresponding mass splittings of doubly charmed tetraquarks with a bad diquark from $1S$ to $2P$ excitations. When $T_{cc}(3875)^+$ is assumed as a $IJ^P = 01^+$ ground doubly charmed tetraquark with a light good antidiquark, the next P -wave excited doubly charmed tetraquarks are predicted to have masses with an average 420–430 MeV higher. A $IJ^P = 10^+$ ground doubly charmed tetraquark $T_{cc}(3834)$ located around 3834 MeV is also expected.

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I. INTRODUCTION

Over the past two decades, numerous exotic hadrons have been found in experiments, and the “particle zoo” is rapidly expanding. In particular, four quark states have raised physicists’ great concern since the discovery of $X(3872)$. Allowed by QCD theory, there exist several kinds of four quark states: tetraquark state [1–6] in diquark-antidiquark configuration, molecular state [7–15] in meson-meson configuration, hadro-charmonium [16,17], and adjoint charmonium [18], etc. In Ref. [19], seven kinds of explanations of XYZ mesons have been listed, though there is no compelling explanation for the pattern of XYZ mesons. A doubly charmed tetraquark has quark component $cc\bar{u}\bar{d}$. As a possible partner of $X(3872)$, the first, to the best of our knowledge, doubly charmed tetraquark candidate $T_{cc}(3875)^+$ [20,21] was reported by LHCb in a $D^0D^0\pi^+$ invariant mass spectrum in proton-proton

collisions in 2021. $T_{cc}(3875)^+$ is located just below the $D^{*+}D^0$ mass threshold with a very narrow width. With a Breit-Wigner parametrization, its mass relative to the charmed meson pair is measured

$$\delta m_{\text{BW}} \equiv m_{\text{BW}} - (m_{D^{*+}} + m_{D^0}) = -273 \pm 61 \pm 5_{-14}^{+11} \text{ keV}, \quad (1)$$

where the charmed mesons have masses [22]

$$\begin{aligned} m_{D^{*+}} &= 2010.26 \pm 0.05 \text{ MeV}, \\ m_{D^0} &= 1864.84 \pm 0.05 \text{ MeV}. \end{aligned} \quad (2)$$

For the proximity of $T_{cc}(3875)^+$ to the $D^{*+}D^0$ mass threshold, a unitarized scheme incorporating the threshold effect is employed, and the mass parameter is shifted to [21]

$$\delta m_{\text{pole}} = -360 \pm 40_{-0}^{+4} \text{ keV}. \quad (3)$$

Moreover, the spin-parity quantum numbers of the isoscalar $T_{cc}(3875)^+$ are in favor of $J^P = 1^+$ in experiment.

So far, experimental data of the doubly heavy tetraquark are scarce, but there are many theoretical studies of doubly charmed four quark states. In the 1980s, most studies of

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doubly charmed four quark states concentrated on their stability and existence [23–25]. In recent years, as more experimental data of heavy baryons and multiquarks accumulated, the mass spectra, decay widths, and production rates of doubly charmed four quark states have been investigated with various methods [15,26–28], such as constituent quark models [5,19,26,29–44], flux-tube model [45], QCD sum rule [46–50], Bethe-Salpeter equation [51,52], and lattice QCD simulation [53–55]. More references could be found in relevant reviews [13,56–60] and references therein.

There are manifest bifurcations on the nature of the doubly charmed $T_{cc}(3875)^+$. There are generally two kinds of interpretations. As $T_{cc}(3875)^+$ has mass very close to the $D^{*+}D^0$ mass threshold, the interpretation as a doubly charmed hadronic molecular state has been discussed extensively [15,27,28,43,50]. However, the possibility as a doubly charmed tetraquark cannot be ruled out. The properties of $T_{cc}(3875)^+$ as a doubly charmed tetraquark in diquark-antidiquark or a compact tetraquark picture have also been investigated [42,44].

Different interactions and models could be employed to compute the mass spectra of doubly charmed four quark states, and therefore the pattern of the mass spectra can exhibit much information about the interactions and structures inside hadrons. The calculation of mass spectra is an important way to detect the dynamics and structure in tetraquarks. In practice, the interpretation of four quark states candidates also depends heavily on the internal quark interactions and models.

There are four components in a doubly charmed tetraquark, and the internal interactions are complex. The mass spectra of the doubly charmed four quark states have been calculated in different approximations. In a four-body picture, the mass of the ground doubly charmed tetraquark with isospin-0 is calculated with 3916 MeV (1 MeV below the lowest meson-meson threshold) based on a nonrelativistic potential which reproduced very well the masses of mesons and baryons [25]. A similar four-body computation with a modified potential indicates that the ground doubly charmed four quark state has mass 23 MeV below the $D^{*+}D^0$ threshold [40]. In a three-body picture, the mass of the ground doubly charmed four quark state is obtained with 3961 MeV in a chiral diquark model [44], where two light antiquarks make a pointlike antidiquark, and then the antidiquark interacts with other two charmed quarks. In a two-body picture, the mass spectra of the doubly charmed four quark states were also investigated in a relativistic potential model [5], where the diquark mass is calculated with the same potential between the diquark and the antidiquark. The ground doubly charmed tetraquark with $IJ^P = 01^+$ is predicted with 3935 MeV. Systemic computation of the mass spectra of doubly charmed tetraquarks including high excitations is scarce and urgently required.

Though the internal structure of a tetraquark is not clear, many studies indicate that the diquark and antidiquark are good stuff to construct the tetraquark. A diquark was first mentioned by Gell-Mann [61] and prevailed in multiquark hadronic systems as a fundamental antisymmetric composite of two strongly correlated quarks [2–4,56,62–71]. It is also pointed out that a charmed diquark is indispensable in the construction of the doubly charmed baryons and tetraquarks in many investigations [34–37,39,46,51].

In nature, it is possible that a doubly charmed tetraquark consists of a fundamental charmed diquark and light antidiquark where two charmed quarks make a compact heavy diquark and two light quarks make a compact light antidiquark. On the other hand, the diquark-antidiquark picture of the tetraquark is a good approximation for simplicity. It is expected that the calculation of the mass spectra of doubly charmed tetraquarks $[cc][\bar{u}\bar{d}]$ in the diquark-antidiquark picture may discover more information on the dynamics and structure in tetraquark. Another aim is to show the pattern of mass spectra of doubly charmed tetraquarks around 4 GeV and to provide more information for the search of tetraquark in experiments.

For these purposes, the potential proposed by Semay *et al.* [25,72] is employed in our calculation, and a similar potential has already been employed in some studies of doubly charmed tetraquarks [25,36,40,44,73]. In contrast with normal treatment of tetraquarks in terms of diquarks and antidiquarks, masses of the charmed diquark and light antidiquarks are obtained through a quark-quark interaction instead of parameters input. Subsequently, the mass spectra of some doubly charmed tetraquarks are calculated through a similar quark-antiquark interaction.

The work is organized as follows: the construction of the potential model and the wave functions is presented in Sec. II after the Introduction. The numerical calculation and the mass spectra are given in Sec. III. The last section is devoted to a brief summary.

II. CONSTITUENT DIQUARK-ANTIDIQUARK POTENTIAL MODEL

A. Potential model

To describe the mass spectra of mesons more accurately, based on the familiar “Coulomb + linear” potentials, Semay *et al.* proposed a modified potential for quark-antiquark interaction in the following explicit form in a nonrelativistic potential model [25,72]:

$$\begin{aligned}
 V_{q\bar{q}} &= V_0 + (\vec{s}_1 \cdot \vec{s}_2) V_{ss} \\
 &= -\frac{\alpha(1 - e^{-r/r_c})}{r} + \lambda r^p + C \\
 &\quad + (\vec{s}_1 \cdot \vec{s}_2) \frac{8\kappa(1 - e^{-r/r_c}) e^{-r^2/r_0^2}}{3m_1 m_2 \sqrt{\pi} r_0^3}, \quad (4)
 \end{aligned}$$

where r_0 is determined by

$$r_0 = A \left(\frac{2m_1 m_2}{m_1 + m_2} \right)^{-B}, \quad (5)$$

and the details of other parameters can be found in Refs. [25,72]. In baryon systems with a heavy quark, the mass spectra of baryons are also well described by the potential. Of course, the quark-quark interaction potential in baryon or multiquark states is obtained by a $V_{qq} = \frac{1}{2} V_{q\bar{q}}$ prescription.

The parameters r_c and p indicate the deviation of the potential from the Cornell potential at short and long distances, respectively, and they may take different values in mesons and baryons [25,72]. Four kinds of potentials were proposed based on the two parameters:

$$\begin{aligned} AL1 \quad & \text{potential: } p = 1, r_c = 0; \\ AL2 \quad & \text{potential: } p = 1, r_c \neq 0; \\ AP1 \quad & \text{potential: } p = \frac{2}{3}, r_c = 0; \\ AP2 \quad & \text{potential: } p = \frac{2}{3}, r_c \neq 0, \end{aligned} \quad (6)$$

where the letter L or P denotes the linear ($p = 1$) or the $\frac{2}{3}$ -Power ($p = \frac{2}{3}$) confinement, respectively. The parameter $p = \frac{2}{3}$ gives the correct asymptotic (for large angular momentum) Regge trajectories of mesons and baryons [25,74]. The number 1 or 2 indicates two types of the parameter r_c (zero or finite number) where $r_c = 0$ means that α and κ are constant parameters.

The spectra of baryons and mesons from these kinds of potentials are better described than the common Coulomb-plus-linear potential [25,72]. These potentials are employed in our calculation of the masses of diquarks/antiquarks and tetraquarks. Since the diquark or antiquark is assumed as a compact component where there does not exist an angular momentum between the quarks or antiquarks, the AL potentials are employed. And only the tetraquarks from S -wave excitations to P -wave excitations are concerned in our calculation; the AL potentials are also preferred.

Thus, the full Hamiltonian of the doubly charmed tetraquark is constructed as

$$H = m_1 + m_2 + \frac{p^2}{2\mu} + V_{al} + V_{sl}, \quad (7)$$

where m_1 and m_2 are the mass of the charmed diquark and the light antiquark, respectively. $\frac{p^2}{2\mu}$ is the kinetic energy of the tetraquark in the center-of-mass system (c.m.s.), determined by the relative momentum p and reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$. V_{al} is the AL -type potential [72] in Eq. (4).

The last term V_{sl} in Eq. (7) represents the coupling between the spin and the orbital angular momentum. A universal Breit-Fermi approximation is employed as [75]

$$\begin{aligned} V_{sl} = V_{so} + V_{ten} = & \frac{\alpha(1 - e^{-r/r_c})}{r^3} \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \\ & \times \left(\frac{\vec{s}_1}{m_1} + \frac{\vec{s}_2}{m_2} \right) \cdot \vec{L} - \frac{1}{2r} \frac{\partial V_{\text{conf}}}{\partial r} \left(\frac{\vec{s}_1}{m_1^2} + \frac{\vec{s}_2}{m_2^2} \right) \cdot \vec{L} \\ & + \frac{1}{3m_1 m_2} \left(\frac{1}{r} \frac{\partial V_{\text{Coul}}}{\partial r} - \frac{\partial^2 V_{\text{Coul}}}{\partial r^2} \right) \\ & \times \left(\frac{3\vec{s}_1 \cdot \vec{r} \vec{s}_2 \cdot \vec{r}}{r^2} - \vec{s}_1 \cdot \vec{s}_2 \right), \end{aligned} \quad (8)$$

where V_{Coul} is the modified Coulomb potential in this work, and thus, the explicit form follows:

$$\begin{aligned} V_{sl} = & V_1 \vec{L} \cdot \vec{s}_1 + V_2 \vec{L} \cdot \vec{s}_2 + V_t \left(\frac{3\vec{s}_1 \cdot \vec{r} \vec{s}_2 \cdot \vec{r}}{r^2} - \vec{s}_1 \cdot \vec{s}_2 \right) \\ = & \left[\frac{\alpha(1 - e^{-r/r_c})}{m_1 m_2 r^3} + \frac{1}{m_1^2} \left(\frac{\alpha(1 - e^{-r/r_c})}{2r^3} - \frac{\lambda}{2r} \right) \right] \vec{L} \cdot \vec{s}_1 \\ & + \left[\frac{\alpha(1 - e^{-r/r_c})}{m_1 m_2 r^3} + \frac{1}{m_2^2} \left(\frac{\alpha(1 - e^{-r/r_c})}{2r^3} - \frac{\lambda}{2r} \right) \right] \\ & \times \vec{L} \cdot \vec{s}_2 + \frac{\alpha(1 - e^{-r/r_c})}{m_1 m_2 r^3} \left(\frac{3\vec{s}_1 \cdot \vec{r} \vec{s}_2 \cdot \vec{r}}{r^2} - \vec{s}_1 \cdot \vec{s}_2 \right), \end{aligned} \quad (9)$$

where $\vec{s}_1$ and $\vec{s}_2$ denote the spin angular momentum of the charmed diquark and the light antiquark, respectively, and $\vec{L}$ stands for their orbital angular momentum. $r_c = 0$ and $r_c \neq 0$ corresponding to the $AL1$ and $AL2$ potential, respectively, will be taken use of.

To compute the masses of the charmed diquark and light antiquarks, similar equations as Eq. (7) hold except that the quark-quark potential inside the charmed diquark and light antiquark has the forms as follows:

$$\begin{aligned} V_{qq} = & \frac{1}{2} \left[-\frac{\alpha(1 - e^{-r/r_c})}{r} + \lambda r + C \right. \\ & \left. + (\vec{s}_q \cdot \vec{s}_{\bar{q}}) \frac{8\kappa(1 - e^{-r/r_c}) e^{-r^2/r_0^2}}{3m_q m_{\bar{q}} \sqrt{\pi} r_0^3} \right]. \end{aligned} \quad (10)$$

As the potential is proportional to the product of two Gell-Mann matrices, $\vec{\lambda}_i \cdot \vec{\lambda}_j$, the potential between two quarks is half the potential between a quark and an antiquark when the diquark is of the color antitriplet.

B. Wave function

In the diquark-antidiquark picture, the total wave function of a doubly charmed tetraquark is composed of color, flavor, spin, and spatial parts,

$$\Psi_{\text{JM}} = \chi_f \otimes \chi_c \otimes [\Psi_{lm}(\vec{r}) \otimes \chi_s]_{\text{JM}}. \quad (11)$$

χ_f is the flavor wave function

$$\chi_f = |\{cc\}\{\bar{u}\bar{d}\}\rangle, \quad (12)$$

where the curly bracket outside the charmed diquark represents its symmetric flavor wave function, and there are two possibilities for the light antidiquark with different spin and isospin. In the following, a square bracket outside the $\bar{u}\bar{d}$ antidiquark indicates the antisymmetric flavor structure with a spin and isospin 0, while a curly bracket outside the $\bar{u}\bar{d}$ antidiquark indicates the symmetric flavor structure with a spin and isospin 1. They are the so-called “good” and “bad” antidiquarks in Ref. [66], respectively, which are sometimes denoted with the “scalar” and “vector” antidiquarks directly. In what follows, these two notations will be employed for light antidiquarks everywhere. As a diquark is assumed without spatial structure, the wave function of a diquark consists of the color, spin, and flavor wave function. Thus, four kinds of color-spin wave functions are allowed in the $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$ color configuration:

$$\chi_{3,1} = |\{cc\}_1^{\bar{\mathbf{3}}}\{\bar{u}\bar{d}\}_0^{\mathbf{3}}\rangle_1, \quad (13)$$

$$\chi_{3,0} = |\{cc\}_1^{\bar{\mathbf{3}}}\{\bar{u}\bar{d}\}_1^{\mathbf{3}}\rangle_0, \quad (14)$$

$$\chi_{3,1} = |\{cc\}_1^{\bar{\mathbf{3}}}\{\bar{u}\bar{d}\}_1^{\mathbf{3}}\rangle_1, \quad (15)$$

$$\chi_{3,2} = |\{cc\}_1^{\bar{\mathbf{3}}}\{\bar{u}\bar{d}\}_1^{\mathbf{3}}\rangle_2, \quad (16)$$

where the notations $|\{cc\}_{\text{spin}}^{\text{color}}(\bar{u}\bar{d})_{\text{spin}}^{\text{color}}\rangle_{\text{total spin}}$ were employed. The first wave function corresponds to the tetraquark composed of a good diquark, while the other three are those with a bad diquark.

Similarly, two kinds of color-spin wave functions are allowed in the $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$ color configuration for the tetraquark,

$$\chi_{6,1} = |\{cc\}_0^{\mathbf{6}}\{\bar{u}\bar{d}\}_1^{\bar{\mathbf{6}}}\rangle_1, \quad (17)$$

$$\chi_{6,0} = |\{cc\}_0^{\mathbf{6}}\{\bar{u}\bar{d}\}_0^{\bar{\mathbf{6}}}\rangle_0. \quad (18)$$

However, the color interactions within these kinds of configurations are repulsive [66], and thus, these diquarks/antidiquarks are not likely to make compact correlated clusters. Therefore, the configurations in the $\mathbf{6}_c$ color representation are not discussed in this work.

The spatial wave function $\psi_{lm}(\vec{r})$ is obtained through a Gaussian expansion method (GEM) [76], in which the wave function is expanded in terms of Gaussian functions as the trial wave function in the Schrödinger equation,

$$\Psi_{lm}(\vec{r}) = \sum_{n=1}^{n_{\max}} c_{nl} \phi_{nlm}(\vec{r}), \quad (19)$$

$$\phi_{nlm}(\vec{r}) = N_{nl} r^l e^{-r^2/r_n^2} Y_{lm}(\hat{r}), \quad (20)$$

where N_{nl} is the normalization coefficient of the state with radial and orbital angular momentum quantum numbers n and l . The size parameter r_n is obtained in a geometric progression

$$r_n = r_1 a^{n-1} (n = 1, \dots, n_{\max}), \quad (21)$$

from three parameters $\{n_{\max}, r_1, r_{n_{\max}}\}$ [76].

Through the Rayleigh-Ritz variational principle, the Schrödinger equation is transformed to the following matrix eigenvalue problem:

$$\sum_{n=1}^{n_{\max}} \sum_{n'=1}^{n_{\max}} (H_{nn'} - E_{nl} N_{nn'}) c_{nl} = 0, \quad (22)$$

where $H_{nn'}$ and $N_{nn'}$ are the matrix element and the overlap integral

$$H_{nn'} = \langle \phi_{n'lm}(\vec{r}) | H | \phi_{nlm}(\vec{r}) \rangle, \quad (23)$$

$$N_{nn'} = \langle \phi_{n'lm}(\vec{r}) | \phi_{nlm}(\vec{r}) \rangle. \quad (24)$$

The expansion coefficients c_{nl} in Eq. (19) and the eigenenergy E are then obtained. To obtain convergent results, we fix r_0 with 0.1 fm as a start, which affects little on the final results, and we modify the values of r_n and $n_{\max}$ as was done in Refs. [43,77]. In our geometric progression, the variational parameters $\{r_1, r_{n_{\max}}, n_{\max}\}$ are finally fixed as $\{0.1 \text{ fm}, 5 \text{ fm}, 25\}$.

C. Matrix elements

After the diquark masses have been computed, the mass spectra of the tetraquark states are obtained from Eq. (7). To obtain the contributions of spin-dependent terms, we first expand the wave function in the LS coupling scheme. That is to say, the spin angular momentum of the diquark and the antidiquark couples to $\vec{s}_1 + \vec{s}_2 = \vec{S}$, and then couples with the orbital angular momentum $\vec{L}$ to get the total angular momentum $\vec{S} + \vec{L} = \vec{J}$ of the tetraquark state that is denoted as $|^{2S+1}L_J\rangle$. The mass matrix of the spin-dependent interaction is expressed as

$$H_{\text{SD}} = \begin{pmatrix} H_{11} & H_{12} & \cdots \\ H_{21} & H_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}. \quad (25)$$

Similar to the heavy-light meson, there are mixings between $[cc][\bar{u}\bar{d}]$ states with different total spins but the

same total angular momentum; $|^{2S+1}L_J\rangle \leftrightarrow |^{2S'+1}L_J\rangle$ resulted from the spin-orbit interaction [78].

For the S -wave and P -wave excited doubly charmed tetraquarks without radial excitation, according to the wave functions, there are one S -wave and three P -wave excited tetraquarks made from a charmed diquark with spin-1 and a light good antiquark. There are three S -wave and seven P -wave excited tetraquarks made from a charmed diquark with spin-1 and a light bad antiquark.

For the seven P -wave excited tetraquarks with a light bad antiquark, the mass matrix elements are expressed in the $|^{2S+1}L_J\rangle$ basis as follows:

- (i) For the states 3P_0 and 5P_3 with $S = 1$ and $S = 2$, no mixing is expected, and the contributions of the spin-dependent terms are

$$J = 0: H_{SD} = -V_{ss} - V_1 - V_2 - 2V_t, \quad (26)$$

$$J = 3: H_{SD} = V_{ss} + V_1 + V_2 - \frac{2}{5}V_t. \quad (27)$$

- (ii) For $J = 1$ states, there is mixing among 1P_1 , 3P_1 , and 5P_1 . The mass matrix elements are

$$H_{11} = -2V_{ss}, \quad (28)$$

$$H_{12} = H_{21}^* = \frac{2}{\sqrt{3}}V_1 - \frac{2}{\sqrt{3}}V_2, \quad (29)$$

$$H_{13} = H_{31}^* = \frac{2}{\sqrt{5}}V_t, \quad (30)$$

$$H_{22} = -V_{ss} - \frac{1}{2}V_1 - \frac{1}{2}V_2 + V_t, \quad (31)$$

$$H_{23} = H_{32}^* = \frac{\sqrt{15}}{6}V_1 - \frac{\sqrt{15}}{6}V_2, \quad (32)$$

$$H_{33} = V_{ss} - \frac{3}{2}V_1 - \frac{3}{2}V_2 - \frac{7}{5}V_t. \quad (33)$$

- (iii) For $J = 2$ states, there is mixing between 3P_2 and 5P_2 . The mass matrix elements are

$$H_{11} = -V_{ss} + \frac{1}{2}V_1 + \frac{1}{2}V_2 - \frac{1}{5}V_t, \quad (34)$$

$$H_{12} = H_{21}^* = \frac{\sqrt{3}}{2}V_1 - \frac{\sqrt{3}}{2}V_2, \quad (35)$$

$$H_{22} = V_{ss} - \frac{1}{2}V_1 - \frac{1}{2}V_2 + \frac{7}{5}V_t. \quad (36)$$

The V_{ss} , V_1 , V_2 , V_t are the spin-spin, spin-orbit, and tensor interaction potentials displayed in Sec. II A.

III. NUMERICAL RESULTS

A. Model parameters

The mass of constituent quarks and the parameters in the AL quark-quark potentials for the calculation of charmed diquark and light antiquarks are taken as those in Refs. [25,72] where the mass spectra of mesons and baryons are simultaneously reproduced well. In Table I, the parameters adopted in our computation are presented.

With these parameters, the masses of relevant diquarks and antiquarks are computed with the AL potentials, and the results are presented in Table II. The light good antiquark has a mass ~ 170 MeV lower than the light bad antiquark. This result is consistent with the phenomenological analysis in Ref. [66], where the diquark mass differences between good and bad diquarks is ~ 200 MeV. In particular, our obtained diquark mass difference is spectator quark independent while the diquark mass difference extracted from the phenomenological analysis is spectator quark dependent.

The mass of the vector charmed diquark is almost the same as the predicted 3.51 GeV in QCD sum rule [49], while the masses of the light antiquarks are larger than those fitted by phenomenological analysis [4] or calculated with the QCD sum rule [79]. However, the constituent mass difference of the light good antiquark and the light antiquark, $m_{\bar{u}\bar{d}} - m_{\bar{u},\bar{d}} \sim 350$ MeV, is only 40 MeV larger than the spectator quark dependent ~ 310 MeV extracted from phenomenological analysis [4,66].

Smaller masses of the ground doubly and hidden charmed tetraquarks are obtained when we proceed with the calculation in terms of the parameters in Table I. The

TABLE I. Parameters in two AL potentials.

Parameters	$AL1$	$AL2$
$m_{u,d}$ (GeV)	0.315	0.320
m_c (GeV)	1.836	1.851
α	0.5069	0.5871
κ	1.8609	1.8475
λ (GeV ²)	0.1653	0.1673
C (GeV)	-0.8321	-0.8182
B	0.2204	0.2132
$A(\text{GeV}^{B-1})$	1.6553	1.6560
$r_c(\text{GeV}^{-1})$	0	0.1844

TABLE II. Mass (in MeV) of the charmed diquark and light antiquarks in AL potentials.

	Spin	$AL1$	$AL2$
m_{cc}	$s = 1$	3499.5	3518.6
$m_{\bar{u}\bar{d}}$	$s = 0$	666.2	673.9
	$s = 1$	834.1	841.8

obtained masses of the ground doubly and hidden charmed tetraquarks are about 130–160 MeV smaller than the masses of $T_{cc}(3875)$ and $X(3872)$. The predicted smaller masses may result from the inner structure of tetraquarks. As known, for the cluster structure of diquarks/antidiquarks and a relative larger separation between a diquark and an antidiquark in comparison to the distance between a quark and an antiquark in ordinary mesons, the interaction intensity and accordingly the parameters in the AL quark-antiquark potential between the charmed diquark and the light antidiquark may not be the same as those in ordinary mesons. In Ref. [42], two schemes of parameters are adopted to study the $(qq)(\bar{Q}\bar{Q})$ tetraquarks for the difference of spatial configurations in the conventional hadrons and tetraquarks. In this work, the parameters α and λ are refixed through the two observed tetraquark candidates $T_{cc}(3875)^+$ and $X(3872)$.

There are many explanations of $X(3872)$ and $T_{cc}(3875)$, but we are far from understanding their structure and dynamics. In Ref. [42], a lowest $IJ^P = 01^+ T_{cc}$ with $(qq)(\bar{c}\bar{c})$ configurations is predicted with mass 3869 MeV in their scheme II, and in Ref. [44], a lowest $J^P = 1^+ T_{cc}$ with $(cc)(\bar{q}\bar{q})$ configurations with the flavor 3 antidiquarks is predicted with mass ~ 3961 MeV. In contrast to mostly advocated molecule interpretation, we think it is rational to assume $T_{cc}(3875)^+$ as a ground $IJ^P = 01^+$ doubly charmed tetraquark.

Recently, a measurement of the radiative decay ratio $\frac{\Gamma_{\chi_{c1}(3872) \rightarrow \Psi(2S)\gamma}}{\Gamma_{\chi_{c1}(3872) \rightarrow J/\Psi\gamma}}$ does not support the popular molecule interpretation of $X(3872)$ [80,81]. In particular, $X(3872)$ is assumed to have a $c\bar{c}$ charmonium component in some analyses. Obviously, the mixing would make the behaviors of $X(3872)$ complex and would make the mass of the pure hidden charmed tetraquark deviate from the mass of $X(3872)$. In this paper, we will not focus on the mixing before some detailed mixing information has been discovered by experiments. Instead, the mixing effect will be taken into account through some uncertainties of the masses of $X(3872)$ and $T_{cc}(3875)$. $X(3872)$ is assumed as a ground $J^{PC} = 1^{++}$ hidden charmed tetraquark made from a cq diquark and a $\bar{c}\bar{q}$ antidiquark in our study [82].

In fact, the explanation of exotic states depends heavily on the quark dynamics and employed model. We hope this prescription could make a good description of the doubly charmed tetraquarks. The spin-spin interaction does not contribute to the mass of these two ground states, and the values of κ , C , B , A , and r_c follow the original sets in Refs. [25,72] with uniformity.

There are two types of AL potentials in the calculation of the masses of diquark/antidiquark and tetraquark. Once the charmed diquark and light antidiquark are computed with the same $r_c = 0$ or $r_c \neq 0$ simultaneously, four sets of parameters for α and λ are fixed. Hereafter for simplicity, they are denoted as Set I to Set IV. The obtained parameters

TABLE III. Parameters of Set I to Set IV.

Set	I	II	III	IV
α	0.3910	0.4205	0.4290	0.4600
$\lambda(\text{GeV}^2)$	0.1941	0.1916	0.1957	0.1932

of Set I to Set IV resulted from potentials $AL1$ and $AL1$ -type, $AL2$ and $AL1$ -type, $AL1$ and $AL2$ -type, $AL2$ and $AL2$ -type, respectively, in the diquark and tetraquark presented in Table III. The fitted α (modified Coulomb term coefficient) in tetraquarks is smaller than that in ordinary mesons, while the fitted λ (modified confinement term coefficient) is larger than that in ordinary mesons.

The isospin breaking and mixing effects play an important role in molecule states, where bosons are exchanged between color-singlet hadrons to bind hadronic molecules both in the constituent quark potential model and in the chiral Lagrangian theory [83–87]. The long-distance isospin dependence of the interactions in molecular models follows primarily from the isospin content of exchanged bosons. The isospin breaking and mixing effects may play an essential role in diquark-antidiquark states. In Refs. [83,88,89], an isospin-dependent interaction was introduced in the diquark-antidiquark picture for the hidden-charm tetraquarks, which brings in less than 10 MeV to the mass spectra. This isospin-dependent interaction provides a dynamical explanation of the isospin breaking or mixing effects. In Ref. [85], a mixing scheme of the molecule and diquark-antidiquark is proposed, and the effects of the D^*D isospin are well analyzed.

B. Mass spectra of doubly charmed tetraquarks

With those parameters and the computed masses of diquarks and antidiquarks, the mass spectra of doubly charmed tetraquarks from $1S$ to $2P$ excitations are computed. The results of the doubly charmed tetraquarks composed of light good and bad antidiquarks are presented in Tables IV and V, respectively.

In these tables, the first column gives all the allowed tetraquark states from $1S$ - to $2P$ -wave excitations with the spectroscopic notations in the LS coupling scheme. The spin-parity quantum numbers J^P are listed in the second column with the parity $P = (-1)^L$. The third to the sixth columns present the obtained mass spectra with the refitted parameters Sets I–IV, respectively. For a comparison, the mass spectra in the chiral diquark model [44] are presented in the last columns of Tables IV and V.

As shown in Tables IV and V, when $T_{cc}(3875)^+$ is assumed as the $J^P = 1^+$ ground doubly charmed tetraquark candidate with a light good antidiquark, there exists another $J^P = 0^+$ ground doubly charmed tetraquark with a light bad antidiquark. This ground doubly charmed tetraquark with isospin-1 [denoted with “ $T_{cc}(3834)$ ”] has mass

TABLE IV. Mass spectra (in MeV) of the doubly charmed tetraquark in the $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$ color configuration with a light good antiquark.

$n^{2S+1}L_J$	J^P	Set I	Set II	Set III	Set IV	Reference [44]
1^3S_1	1^+	3874.83	3874.78	3875.42	3874.84	3961
1^3P_0	0^-	4274.51	4276.50	4278.91	4280.18	4253
1^3P_1	1^-	4284.51	4287.53	4290.24	4292.61	4253
1^3P_2	2^-	4304.50	4309.58	4312.90	4317.47	4253
2^3S_1	1^+	4511.38	4511.17	4522.19	4521.77	4363
2^3P_0	0^-	4792.06	4790.46	4802.52	4800.42	
2^3P_1	1^-	4802.07	4801.38	4813.60	4812.45	
2^3P_2	2^-	4822.07	4823.23	4835.74	4836.52	

TABLE V. Mass spectra (in MeV) of the doubly charmed tetraquark in the $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$ color configuration with a light bad antiquark.

$n^{2S+1}L_J$	J^P	Set I	Set II	Set III	Set IV	Reference [44]
1^1S_0	0^+	3834.11	3831.98	3837.14	3834.90	4132
1^3S_1	1^+	3907.65	3906.38	3909.15	3907.59	4151
1^5S_2	2^+	4054.74	4055.18	4053.15	4052.98	4185
1^1P_1	1^-	4335.02	4342.18	4341.32	4347.61	4423
1^3P_0	0^-	4350.83	4348.80	4349.46	4346.39	4430
1^3P_1	1^-	4416.72	4414.12	4417.13	4413.93	4430
1^3P_2	2^-	4359.25	4367.61	4367.18	4375.05	4430
1^5P_1	1^-	4379.91	4379.52	4380.69	4379.33	4442
1^5P_2	2^-	4432.25	4434.85	4437.39	4439.34	4442
1^5P_3	3^-	4399.57	4409.28	4409.58	4418.93	4442
2^1S_0	0^+	4526.52	4528.09	4539.88	4541.39	4546
2^3S_1	1^+	4558.48	4559.71	4570.33	4571.46	4560
2^5S_2	2^+	4622.39	4622.95	4631.24	4631.61	4585
2^1P_1	1^-	4836.59	4839.76	4848.30	4850.10	
2^3P_0	0^-	4835.60	4831.55	4841.65	4836.95	
2^3P_1	1^-	4889.44	4885.39	4897.07	4892.82	
2^3P_2	2^-	4859.44	4864.39	4872.76	4877.33	
2^5P_1	1^-	4863.13	4860.77	4870.73	4868.33	
2^5P_2	2^-	4911.47	4911.38	4922.15	4921.61	
2^5P_3	3^-	4896.31	4902.31	4910.84	4916.35	

3834 MeV in the $AL1$ ($r_c = 0$) potential and 3837 MeV in the $AL2$ ($r_c \neq 0$) potential, where the $r_c = 0$ and $r_c \neq 0$ bring very small differences to the predicted mass. Though the light good antiquark has a mass ~ 170 MeV lower than the light bad antiquark, the ground $J^P = 1^+$ doubly charmed tetraquark $T_{cc}(3875)^+$ with a light good antiquark has a mass ~ 40 MeV higher than the $J^P = 0^+$ ground doubly charmed tetraquark “ $T_{cc}(3834)$ ” with a light bad antiquark. The existence of such an “exotic” ground tetraquark is an explicit indication of the diquark-antiquark structure and dynamics in the doubly charmed tetraquarks.

TABLE VI. Mass splittings (in MeV) of radially and orbitally excited $[cc][\bar{u}\bar{d}]$ with a light good antiquark, where the parameters are chosen as Sets I–IV.

	Set I	Set II	Set III	Set IV	Reference [44]	D meson [90]
$1P - 1S$	419.67	423.77	426.15	430.20	292	456.00
$2S - 1S$	636.55	636.39	646.77	646.93	402	634.27
$2P - 1P$	517.56	513.75	523.10	519.45	...	...
$2P - 2S$	300.69	301.14	302.48	302.72	...	...

TABLE VII. Mass splittings (in MeV) of radially and orbitally $[cc][\bar{u}\bar{d}]$ with a light bad antiquark, where the parameters are chosen as Sets I–IV.

	Set I	Set II	Set III	Set IV	Reference [44]	D meson [90]
$1P - 1S$	407.70	413.05	413.55	418.73	292	456.00
$2S - 1S$	609.24	610.55	619.64	621.25	402	634.27
$2P - 1P$	490.06	487.07	495.69	492.83	...	...
$2P - 2S$	288.52	289.57	289.60	290.31	...	...

To see the pattern of the mass spectra of doubly charmed tetraquarks, the mass splittings between different excitations are also computed and listed in Tables VI and VII, respectively. As a comparison, the mass splittings in the chiral diquark model and the mass splittings between D mesons [90] corresponding to the same excitations are presented in the next two columns. The mass of each multiplet is taken as the spin-weighted average mass

$$\overline{M}_{nL} = \frac{\sum_J (2J+1) M_{nLJ}}{\sum_J (2J+1)}. \quad (37)$$

For the mass splittings, the data of established D mesons in PDG [90] are listed in Table VIII.

As shown in Tables VI and VII, the $1S - 1P$ and $1S - 2S$ mass splittings of the doubly charmed tetraquarks are about 410–430 MeV and 610–650 MeV, respectively. The corresponding mass splittings of the doubly charmed tetraquarks with a light good antiquark is about 10–25 MeV higher than those of the doubly charmed tetraquarks with a light bad antiquark. These mass splittings are similar to those for normal D mesons. The $2S - 2P$ mass splittings are about 120 MeV smaller than the $1S - 1P$ mass splittings, and the $1P - 2P$ mass splittings are also about 120 MeV smaller than the $1S - 2S$ mass splittings. In our calculation, many-body interactions and residual interaction [91] have not been taken into account.

To take into account the mixing and isospin breaking effects on the spectrum of tetraquarks, we vary the mass of $X(3872)$ with 50 MeV around its experimental data, and

TABLE VIII. Mass spectrum (in MeV) of charmed meson from PDG [90].

State	$n^{2S+1}L_J$	J^P	Mass [90]
$D^\pm$	1^1S_0	0^-	1869.66 ± 0.05
D^0	1^1S_0	0^-	1864.84 ± 0.05
$D^*(2007)^0$	1^3S_1	1^-	2006.85 ± 0.05
$D^*(2010)^\pm$	1^3S_1	1^-	2010.26 ± 0.05
$D_1(2430)^0$	1^1P_1	1^+	2412 ± 9
$D_0^*(2300)$	1^3P_0	0^+	2343 ± 10
$D_1((2420)^0)$	3^1P_1	1^+	2422.1 ± 0.6
$D_2^*(2460)$	1^3P_2	2^+	2461.1 ± 0.8
$D_0(2550)^0$	2^1S_0	0^-	2549 ± 19
$D_1^*(2600)^0$	2^3S_1	1^-	2627 ± 10

our fitting procedure indicates that the parameters α and λ decrease with the increase of the mass. The α varies about 0.2, and the λ varies about 0.06. The variation of the parameters leads to some variations of the mass spectra: the average mass of $1S$ -wave states with a light bad diquark varies about 15 MeV, and that of $1P$ -wave states varies about 76 MeV. We also vary the mass of $T_{cc}(3875)^+$ with 10 MeV around its experimental data in the fitting procedure, and the parameters vary little and increase with the increase of the mass. The α varies about 0.02, the λ varies about 0.01, which brings the average mass of $1S$ -wave states with a light bad diquark a variation about 8 MeV, and that of $1P$ -wave states a variation about 27 MeV.

IV. SUMMARY

In this work, a doubly charmed tetraquark is assumed consisting of a charmed diquark and a light antiquark, the interactions in the tetraquark are modulated as two kinds of interactions: the first type is the interaction between a quark and another quark or the interaction between an antiquark and another antiquark, and the second type is the interaction between the charmed diquark and the light antiquark. In practical calculations, the modified AL -type quark-quark and quark-antiquark potentials proposed by Semay *et al.* are employed for the interactions. Masses of some constituent diquarks and doubly charmed tetraquarks are calculated in terms of those potentials.

In the calculation of the vector charmed diquark and light antiquark, the same parameters fitted from ordinary mesons and baryons are employed. The mass of the charmed diquark with spin-1 is calculated with 3499.5 and 3518.6 MeV in the $AL1$ and $AL2$ potentials, respectively. The calculated mass of the vector charmed diquark is almost the same as the predicted 3.51 GeV in the QCD sum rule.

For the light antiquarks, there are good and bad antiquarks. The light good antiquark is predicted to have masses 666.2 and 673.9 MeV in the $AL1$ and $AL2$ potentials, respectively. The light bad antiquark is

predicted to have masses 834.1 and 841.8 MeV in the $AL1$ and $AL2$ potentials, respectively. The light good antiquark has a mass ~ 170 MeV lower than the light bad antiquark, which is consistent with previous phenomenological analysis: ~ 200 MeV. Our predicted diquark mass difference is spectator quark independent. The obtained masses of the light antiquarks change small in different AL potentials, and they are much larger than the popular ones obtained in other models.

After detailed construction of the wave functions of the doubly charmed tetraquarks, we have computed their mass spectra from S -wave to P -wave excitations in terms of the fitted parameters from $T_{cc}(3875)^+$ and $X(3872)$. For the doubly charmed tetraquarks with a light good antiquark, there are one S -wave excitation with $J^P = 1^+$ and three P -wave excitations with $J^P = 0^-$, $J^P = 1^-$, and $J^P = 2^-$, respectively. For the doubly charmed tetraquarks with a light bad antiquark, there are three S -wave excitations with $J^P = 0^+$, $J^P = 1^+$, and $J^P = 2^+$, respectively. Furthermore, there are seven P -wave excitations with $J^P = 0^-$ (one), $J^P = 1^-$ (three), $J^P = 2^-$ (two), and $J^P = 3^-$ (one), respectively. These features of the emergence of tetraquarks are explicit indications of the quark structure in tetraquarks.

When $T_{cc}(3875)^+$ is assumed as the $J^P = 1^+$ ground doubly charmed tetraquark candidate with isospin-0 made from a light good antiquark. A $IJ^P = 10^+$ ground doubly charmed tetraquark with a light bad antiquark inside is also expected. This $IJ^P = 10^+$ ground tetraquark has a mass about 3831–3837 MeV in the AL potentials, which exhibits an explicit indication of the diquark-antidiquark structure and dynamics in the doubly charmed tetraquarks. The next P -wave excited doubly charmed tetraquarks corresponding to $T_{cc}(3875)^+$ are predicted to have masses with an average 410–430 MeV higher. These numerical results can serve as a guide for experiments in the search of tetraquarks, and we expect the $IJ^P = 10^+$ ground doubly charmed tetraquark “ $T_{cc}(3834)$ ” to be found in forthcoming experiments.

The mass splittings among doubly charmed tetraquarks in different excitations are similar to those for ordinary D mesons. The $1S - 1P$ and $1S - 2S$ mass splittings of the doubly charmed tetraquarks are about 410–430 MeV and 610–650 MeV, respectively. The corresponding mass splittings of the doubly charmed tetraquarks with a light good antiquark is about 10–25 MeV higher than those of the doubly charmed tetraquarks with a light bad antiquark from $1S$ to $2P$ excitations. The $2S - 2P$ mass splittings are about 120 MeV smaller than the $1S - 1P$ mass splittings, and the $1P - 2P$ mass splittings are also about 120 MeV smaller than the $1S - 2S$ mass splittings. The pattern of mass splittings in tetraquarks exhibits the properties of quark interactions and could serve as a support of the constituent diquark-antidiquark potential model.

The effects of mixing, many-body interactions, and cluster structure, etc., in doubly charmed tetraquarks have not been taken into account explicitly, the isospin-dependent interaction has not been taken into account either. The many-body interactions and cluster effects are possibly small if the quarks and antiquarks in tetraquarks make a pointlike compact cluster in the real world. The parameters in the AL potentials fitted by $T_{cc}(3875)^+$ and $X(3872)$ may bring in an uncertainty to the predicted mass spectra of doubly charmed tetraquarks if these two states are not the assumed ones. However, through a simple uncertainties estimate, those uncertainties of mass spectra

are not large. Of course, if more doubly or hidden charmed tetraquarks are observed and definitely identified, the experimental data can be employed for the fitting procedure to improve our predictions.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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- [1] R. L. Jaffe, Multi-quark hadrons. I. The phenomenology of $Q^2\bar{Q}^{-2}$ mesons, *Phys. Rev. D* **15**, 267 (1977).
 - [2] R. L. Jaffe and F. Wilczek, Diquarks and exotic spectroscopy, *Phys. Rev. Lett.* **91**, 232003 (2003).
 - [3] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, A new look at scalar mesons, *Phys. Rev. Lett.* **93**, 212002 (2004).
 - [4] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Diquark-antidiquarks with hidden or open charm and the nature of $X(3872)$, *Phys. Rev. D* **71**, 014028 (2005).
 - [5] D. Ebert, R. N. Faustov, V. O. Galkin, and W. Lucha, Masses of tetraquarks with two heavy quarks in the relativistic quark model, *Phys. Rev. D* **76**, 114015 (2007).
 - [6] G. Yang, J. Ping, and J. Segovia, Tetra- and penta-quark structures in the constituent quark model, *Symmetry* **12**, 1869 (2020).
 - [7] M. B. Voloshin and L. B. Okun, Hadron molecules and charmonium atom, *JETP Lett.* **23**, 333 (1976).
 - [8] J. Weinstein and N. Isgur, $qq\bar{q}\bar{q}$ system in a potential model, *Phys. Rev. D* **27**, 588 (1983).
 - [9] N. A. Tornqvist, Possible large deuteron-like meson meson states bound by pions, *Phys. Rev. Lett.* **67**, 556 (1991).
 - [10] N. A. Tornqvist, From the deuteron to deusons, an analysis of deuteron-like meson meson bound states, *Z. Phys. C* **61**, 525 (1994).
 - [11] C.-Y. Wong, Molecular states of heavy quark mesons, *Phys. Rev. C* **69**, 055202 (2004).
 - [12] T. Barnes, F. E. Close, and H. J. Lipkin, Implications of a DK molecule at 2.32 GeV, *Phys. Rev. D* **68**, 054006 (2003).
 - [13] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, Exotic hadrons with heavy flavors: X, Y, Z, and related states, *Prog. Theor. Exp. Phys.* **2016**, 062C01 (2016).
 - [14] M.-Z. Liu, T.-W. Wu, M. Pavon Valderrama, J.-J. Xie, and L.-S. Geng, Heavy-quark spin and flavor symmetry partners of the $X(3872)$ revisited: What can we learn from the one boson exchange model?, *Phys. Rev. D* **99**, 094018 (2019).
 - [15] A. Feijoo, W. H. Liang, and E. Oset, $D^0 D^0 \pi^+$ mass distribution in the production of the T_{cc} exotic state, *Phys. Rev. D* **104**, 114015 (2021).
 - [16] S. Dubynskiy and M. B. Voloshin, Hadro-charmonium, *Phys. Lett. B* **666**, 344 (2008).
 - [17] Q. Wang, M. Cleven, F.-K. Guo, C. Hanhart, U.-G. Meißner, X.-G. Wu, and Q. Zhao, $Y(4260)$: Hadronic molecule versus hadro-charmonium interpretation, *Phys. Rev. D* **89**, 034001 (2014).
 - [18] E. Braaten, How the $Z_c(3900)$ reveals the spectra of quarkonium hybrid and tetraquark mesons, *Phys. Rev. Lett.* **111**, 162003 (2013).
 - [19] E. Braaten, C. Langmack, and D. H. Smith, Born-Oppenheimer approximation for the XYZ mesons, *Phys. Rev. D* **90**, 014044 (2014).
 - [20] R. Aaij *et al.* (LHCb Collaboration), Observation of an exotic narrow doubly charmed tetraquark, *Nat. Phys.* **18**, 751 (2022).
 - [21] R. Aaij *et al.* (LHCb Collaboration), Study of the doubly charmed tetraquark T_{cc}^+ , *Nat. Commun.* **13**, 3351 (2022).
 - [22] P. A. Zyla *et al.* (Particle Data Group), Review of particle physics, *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
 - [23] H. J. Lipkin, A model independent approach to multi-quark bound states, *Phys. Lett. B* **172**, 242 (1986).
 - [24] J. Carlson, L. Heller, and J. A. Tjon, Stability of dimesons, *Phys. Rev. D* **37**, 744 (1988).
 - [25] C. Semay and B. Silvestre-Brac, Diquonia and potential models, *Z. Phys. C* **61**, 271 (1994).
 - [26] J. Vijande, F. Fernandez, A. Valcarce, and B. Silvestre-Brac, Tetraquarks in a chiral constituent quark model, *Eur. Phys. J. A* **19**, 383 (2004).
 - [27] M. Albaladejo, T_{cc}^+ coupled channel analysis and predictions, *Phys. Lett. B* **829**, 137052 (2022).
 - [28] B. Wang and L. Meng, Revisiting the DD^* chiral interactions with the local momentum-space regularization up to the third order and the nature of T_{cc}^+ , *Phys. Rev. D* **107**, 094002 (2023).
 - [29] B. A. Gelman and S. Nussinov, Does a narrow tetraquark $cc\bar{u}\bar{d}$ state exist?, *Phys. Lett. B* **551**, 296 (2003).
 - [30] J. Vijande, E. Weissman, A. Valcarce, and N. Barnea, Are there compact heavy four-quark bound states?, *Phys. Rev. D* **76**, 094027 (2007).

- [31] S. H. Lee and S. Yasui, Stable multiquark states with heavy quarks in a diquark model, *Eur. Phys. J. C* **64**, 283 (2009).
- [32] R. T. Kleiv, T. G. Steele, A. Zhang, and I. Blokland, Heavy-light diquark masses from QCD sum rules and constituent diquark models of tetraquarks, *Phys. Rev. D* **87**, 125018 (2013).
- [33] S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, Exotic tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration, *Eur. Phys. J. C* **77**, 709 (2017).
- [34] M. Karliner and J. L. Rosner, Discovery of doubly-charmed Ξ_{cc} baryon implies a stable ($bb\bar{u}\bar{d}$) tetraquark, *Phys. Rev. Lett.* **119**, 202001 (2017).
- [35] E. J. Eichten and C. Quigg, Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$, *Phys. Rev. Lett.* **119**, 202002 (2017).
- [36] J.-M. Richard, A. Valcarce, and J. Vijande, Few-body quark dynamics for doubly heavy baryons and tetraquarks, *Phys. Rev. C* **97**, 035211 (2018).
- [37] E. Braaten, L.-P. He, and A. Mohapatra, Masses of doubly heavy tetraquarks with error bars, *Phys. Rev. D* **103**, 016001 (2021).
- [38] Q.-F. Lü, D.-Y. Chen, and Y.-B. Dong, Masses of doubly heavy tetraquarks $T_{QQ'}$ in a relativized quark model, *Phys. Rev. D* **102**, 034012 (2020).
- [39] J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, and T. Yao, Double-heavy tetraquark states with heavy diquark-antiquark symmetry, *Chin. Phys. C* **45**, 043102 (2021).
- [40] Q. Meng, E. Hiyama, A. Hosaka, M. Oka, P. Gubler, K. U. Can, T. T. Takahashi, and H. S. Zong, Stable double-heavy tetraquarks: Spectrum and structure, *Phys. Lett. B* **814**, 136095 (2021).
- [41] Q. Qin, Y.-F. Shen, and F.-S. Yu, Discovery potentials of double-charm tetraquarks, *Chin. Phys. C* **45**, 103106 (2021).
- [42] X.-Z. Weng, W.-Z. Deng, and S.-L. Zhu, Doubly heavy tetraquarks in an extended chromomagnetic model *, *Chin. Phys. C* **46**, 013102 (2022).
- [43] C. Deng and S.-L. Zhu, T_{cc}^+ and its partners, *Phys. Rev. D* **105**, 054015 (2022).
- [44] Y. Kim, M. Oka, and K. Suzuki, Doubly heavy tetraquarks in a chiral-diquark picture, *Phys. Rev. D* **105**, 074021 (2022).
- [45] C. Deng, H. Chen, and J. Ping, Systematical investigation on the stability of doubly heavy tetraquark states, *Eur. Phys. J. A* **56**, 9 (2020).
- [46] F. S. Navarra, M. Nielsen, and S. H. Lee, QCD sum rules study of $QQ - \bar{u}\bar{d}$ mesons, *Phys. Lett. B* **649**, 166 (2007).
- [47] M.-L. Du, W. Chen, X.-L. Chen, and S.-L. Zhu, Exotic $QQ\bar{q}\bar{q}$, $QQ\bar{q}\bar{s}$ and $QQ\bar{s}\bar{s}$ states, *Phys. Rev. D* **87**, 014003 (2013).
- [48] Z.-G. Wang and Z.-H. Yan, Analysis of the scalar, axial-vector, vector, tensor doubly charmed tetraquark states with QCD sum rules, *Eur. Phys. J. C* **78**, 19 (2018).
- [49] S. Esau, A. Palameta, R. T. Kleiv, D. Harnett, and T. G. Steele, Axial Vector cc and bb diquark masses from QCD Laplace sum-rules, *Phys. Rev. D* **100**, 074025 (2019).
- [50] S. S. Agaev, K. Azizi, and H. Sundu, Hadronic molecule model for the doubly charmed state T_{cc}^+ , *J. High Energy Phys.* **06** (2022) 057.
- [51] G. Q. Feng, X. H. Guo, and B. S. Zou, $QQ'\bar{u}\bar{d}$ bound state in the Bethe-Salpeter equation approach, [arXiv:1309.7813](https://arxiv.org/abs/1309.7813).
- [52] Z.-M. Ding, H.-Y. Jiang, and J. He, Molecular states from $D^{(*)}\bar{D}^{(*)}/B^{(*)}\bar{B}^{(*)}$ and $D^{(*)}D^{(*)}/\bar{B}^{(*)}\bar{B}^{(*)}$ interactions, *Eur. Phys. J. C* **80**, 1179 (2020).
- [53] Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura, and K. Sasaki, Charmed tetraquarks T_{cc} and T_{cs} from dynamical lattice QCD simulations, *Phys. Lett. B* **729**, 85 (2014).
- [54] P. Junnarkar, N. Mathur, and M. Padmanath, Study of doubly heavy tetraquarks in lattice QCD, *Phys. Rev. D* **99**, 034507 (2019).
- [55] M. Padmanath and S. Prelovsek, Signature of a doubly charm tetraquark pole in DD^* scattering on the lattice, *Phys. Rev. Lett.* **129**, 032002 (2022).
- [56] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, Heavy-quark QCD exotica, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
- [57] A. Esposito, A. Pilloni, and A. D. Polosa, Multiquark resonances, *Phys. Rep.* **668**, 1 (2017).
- [58] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Hadronic molecules, *Rev. Mod. Phys.* **90**, 015004 (2018); **94**, 029901(E) (2022).
- [59] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Pentaquark and tetraquark states, *Prog. Part. Nucl. Phys.* **107**, 237 (2019).
- [60] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, An updated review of the new hadron states, *Rep. Prog. Phys.* **86**, 026201 (2023).
- [61] M. Gell-Mann, A schematic model of baryons and mesons, *Phys. Lett.* **8**, 214 (1964).
- [62] M. Ida and R. Kobayashi, Baryon resonances in a quark model, *Prog. Theor. Phys.* **36**, 846 (1966).
- [63] D. B. Lichtenberg and L. J. Tassie, Baryon mass splitting in a boson-fermion model, *Phys. Rev.* **155**, 1601 (1967).
- [64] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, and D. B. Lichtenberg, Diquarks, *Rev. Mod. Phys.* **65**, 1199 (1993).
- [65] M. Karliner and H. J. Lipkin, A diquark—triquark model for the KN pentaquark, *Phys. Lett. B* **575**, 249 (2003).
- [66] R. L. Jaffe, Exotica, *Phys. Rep.* **409**, 1 (2005).
- [67] L. Maiani, A. D. Polosa, and V. Riquer, The new pentaquarks in the diquark model, *Phys. Lett. B* **749**, 289 (2015).
- [68] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, The hidden-charm pentaquark and tetraquark states, *Phys. Rep.* **639**, 1 (2016).
- [69] L. Maiani, A. D. Polosa, and V. Riquer, A theory of X and Z multiquark resonances, *Phys. Lett. B* **778**, 247 (2018).
- [70] S. L. Olsen, T. Skwarnicki, and D. Zieminska, Nonstandard heavy mesons and baryons: Experimental evidence, *Rev. Mod. Phys.* **90**, 015003 (2018).
- [71] M. Y. Barabanov *et al.*, Diquark correlations in hadron physics: Origin, impact and evidence, *Prog. Part. Nucl. Phys.* **116**, 103835 (2021).
- [72] B. Silvestre-Brac, Spectrum and static properties of heavy baryons, *Few-Body Syst.* **20**, 1 (1996).
- [73] Q. Meng, M. Harada, E. Hiyama, A. Hosaka, and M. Oka, Doubly heavy tetraquark resonant states, *Phys. Lett. B* **824**, 136800 (2022).
- [74] M. Fabre De La Ripelle, A confining potential for quarks, *Phys. Lett. B* **205**, 97 (1988).

- [75] A. De Rujula, H. Georgi, and S. L. Glashow, Hadron masses in a gauge theory, *Phys. Rev. D* **12**, 147 (1975).
- [76] E. Hiyama, Y. Kino, and M. Kamimura, Gaussian expansion method for few-body systems, *Prog. Part. Nucl. Phys.* **51**, 223 (2003).
- [77] X. Chen, F.-L. Wang, Y. Tan, and Y. Yang, Double-heavy tetraquarks with strangeness in the chiral quark model*, *Chin. Phys. C* **47**, 023102 (2023).
- [78] J.-B. Liu and M.-Z. Yang, Spectrum of Higher excitations of B and D mesons in the relativistic potential model, *Phys. Rev. D* **91**, 094004 (2015).
- [79] A. Zhang, T. Huang, and T. G. Steele, Diquark and light four-quark states, *Phys. Rev. D* **76**, 036004 (2007).
- [80] B. Grinstein, L. Maiani, and A. D. Polosa, Radiative decays of $X(3872)$ discriminate between the molecular and compact interpretations, *Phys. Rev. D* **109**, 074009 (2024).
- [81] R. Aaij *et al.* (LHCb Collaboration), Probing the nature of the $\chi_{c1}(3872)$ state using radiative decays, *J. High Energy Phys.* **11** (2024) 121.
- [82] Y.-Y. Lin, J.-Y. Wang, and A. Zhang, $X(3872)$ and hidden charmed tetraquarks (to be published).
- [83] Z.-F. Sun, N. Li, and X. Liu, Isospin violation effect and three-body decays of the T_{cc} state, *Phys. Rev. D* **110**, 094025 (2024).
- [84] N. Li and S.-L. Zhu, Isospin breaking, coupled-channel effects and diagnosis of $X(3872)$, *Phys. Rev. D* **86**, 074022 (2012).
- [85] R. F. Lebed and S. R. Martinez, T_{cc} in the diabatic diquark model: Effects of D^*D isospin, *Phys. Rev. D* **110**, 034033 (2024).
- [86] S. Takeuchi, K. Shimizu, and M. Takizawa, On the origin of the narrow peak and the isospin symmetry breaking of the $X(3872)$, *Prog. Theor. Exp. Phys.* **2014**, 123D01 (2014); **2015**, 079203(E) (2015).
- [87] L. R. Dai, L. M. Abreu, A. Feijoo, and E. Oset, The isospin and compositeness of the $T_{cc}(3875)$ state, *Eur. Phys. J. C* **83**, 983 (2023).
- [88] S. J. Brodsky, D. S. Hwang, and R. F. Lebed, Dynamical picture for the formation and decay of the exotic XYZ mesons, *Phys. Rev. Lett.* **113**, 112001 (2014).
- [89] R. F. Lebed, The pentaquark candidates in the dynamical diquark picture, *Phys. Lett. B* **749**, 454 (2015).
- [90] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [91] Y.-Y. Lin, J.-Y. Wang, and A. Zhang, Mass spectrum of fully charmed $[cc][\bar{c}\bar{c}]$ tetraquarks, *Eur. Phys. J. Plus* **139**, 707 (2024).