

τ -lepton pair spin in proton-proton LHC collisions for anomalous dipole moments

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The spin correlations in the pair of τ leptons produced in high-energy proton-proton collisions are studied. The invariant mass of the τ -lepton pair is chosen close to the Z -boson mass; in these conditions, Z -boson exchange gives the dominant contribution to the Drell-Yan mechanism. The interaction of Z bosons with τ leptons, in addition to the Standard Model couplings, can include weak anomalous magnetic and electric dipole moments (or form factors) of the τ lepton. The dependence of elements of the pp spin-correlation matrix on the weak anomalous moments is determined for various regions of the rapidity of the τ -lepton pair. Based on this calculation, we construct a semirealistic observable built on the momenta of pions in the $\tau^\mp \rightarrow \pi^\mp \nu_\tau$ and $\tau^\pm \rightarrow \rho^\pm \nu_\tau \rightarrow \pi^\pm \pi^0 \nu_\tau$ decay channels, and we show that this observable can be sensitive to the weak anomalous dipole moments.

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I. INTRODUCTION

One of the interesting topics of present-day phenomenology relates to measurement or establishing limits on the muon anomalous dipole moments, both magnetic and electric ones. Numerous papers have been devoted to that subject (see [1,2] and references therein), and experimental measurements [3–5] may indicate that the deviation of the measured value of the anomalous magnetic moment from the Standard Model (SM) prediction is on the edge of discovery. Independently, if this is the case, one should prepare alternative or supplementary measurements and the calculations and software tools necessary for their interpretation.

The production and decay of τ -lepton pairs offer interesting research directions, because anomalous magnetic dipole moments due to possibly enhanced contributions from New Physics (NP) models are expected to be proportional to the squared mass of the lepton, and they are therefore enhanced for the τ lepton by a factor of 280

compared to the muon. This makes these studies important and of contemporary interest.

The LHC experiments [6–8], taking data now, offer immediate opportunity for these measurements. The τ -lepton decays and the τ -decay products are observed and provide access to the physics information embedded in the τ -lepton pair spin state. Neither incoming partons in the hard scattering process nor all τ -decay products (due to neutrinos escaping detection) are available for direct measurement. To simplify the task of observable build and of measurement interpretation, tools like TauSpinner [9–11] can be helpful. In light of Ref. [12], the tool can be used to calculate event weights introducing anomalous dipole moments.

However, these papers do not address the actual impact of dipole moments on τ -decay product distributions. In the present paper, we address the topic in the context of pp collisions in the conditions of the LHC, which was not covered previously. Our consideration is focused on the region of τ -pair invariant mass around the Z -boson mass. In these conditions, the Z -boson exchange mechanism of the Drell-Yan production of the τ pair is the dominant one, and the coupling of the Z boson to τ leptons may carry information on weak anomalous magnetic and/or electric dipole moments of the τ lepton.

In Sec. II, we describe the differential cross section in terms of the spin-correlation matrix in high-energy proton-proton collisions. In Sec. III, some results for elements of an effective proton-proton spin-correlation matrix are

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presented in different kinematical regimes. Section IV is devoted to the definition of a prototype observable that is sensitive to weak anomalous dipole moments. It is distinct from the one used in the TauSpinner algorithm because it relies on detectable momenta only; it does not use the momenta of τ leptons. Conclusions are given in Sec. V.

II. DIFFERENTIAL CROSS SECTION

OF $p + p \rightarrow Z + \dots \rightarrow \tau^- + \tau^+ + \dots$

In this section, we present a differential cross section of the reaction $p + p \rightarrow Z + \dots \rightarrow \tau^- + \tau^+ + \dots$ as a function of the τ -lepton rapidities y_- , y_+ and the τ -pair invariant mass (squared) $M^2 = (p_- + p_+)^2 = x_1 x_2 s$, where s is invariant energy squared in the pp collision. Here, x_1 and x_2 are fractions of the proton momenta p_1 and p_2 carried by partons f and $\bar{f}$, respectively.

The differential cross section of the process $p + p \rightarrow \tau^- + \tau^+ + \dots$ is written as (see, e.g., [13], Chap. 17.4)

$$\begin{aligned} & \frac{d\sigma}{dx_1 dx_2 dt} (pp \rightarrow \tau^- \tau^+ + \dots) \\ &= \sum_{f=q,\bar{q}} F_f(x_1) F_{\bar{f}}(x_2) \frac{d\sigma}{dt} (f\bar{f} \rightarrow \tau^- \tau^+), \end{aligned} \quad (1)$$

where $F_{f,\bar{f}}(x)$ are the parton distribution functions (PDFs), which also depend on an energy scale Q (not indicated explicitly).

The total four-momentum of the τ pair in the parton-parton process $f(x_1 p_1) + \bar{f}(x_2 p_2) \rightarrow \tau^-(p_-) + \tau^+(p_+)$ is

$$\begin{aligned} q^\mu &= (q^0, \vec{q}), \quad \vec{q} = (0, 0, q), \\ q^0 &= E_- + E_+ = \frac{\sqrt{s}}{2} (x_1 + x_2), \\ q &= p_{-\parallel} + p_{+\parallel} = \frac{\sqrt{s}}{2} (x_1 - x_2). \end{aligned} \quad (2)$$

Hereafter, the mass of the proton is neglected. The fractions of the proton momenta

$$x_1 = \frac{M}{\sqrt{s}} \exp(Y_Z), \quad x_2 = \frac{M}{\sqrt{s}} \exp(-Y_Z) \quad (3)$$

are expressed through the invariant mass and rapidity of the τ pair

$$Y_Z = \ln\left(\frac{q^0 + q}{M}\right) = \frac{1}{2} \ln\left(\frac{x_1}{x_2}\right). \quad (4)$$

To include angular dependence of the hard process $f\bar{f} \rightarrow \tau^- \tau^+$, we introduce the longitudinal rapidities of τ^- and τ^+ via the relations [13]

$$\begin{aligned} E_- &= p_\perp \cosh(y_-), & E_+ &= p_\perp \cosh(y_+), \\ p_{-\parallel} &= p_\perp \sinh(y_-), & p_{+\parallel} &= p_\perp \sinh(y_+), \\ \vec{p}_{-\perp} &= \vec{n}_\perp p_\perp, & \vec{p}_{+\perp} &= -\vec{n}_\perp p_\perp, \end{aligned} \quad (5)$$

where “parallel” (“perpendicular”) means longitudinal (transverse) to the beam components of the $\tau^\mp$ momenta, $\vec{n}_\perp$ is the unit vector in the transverse direction, and we assume that initial partons do not have transverse momenta.

It is straightforward to relate rapidities of τ^- and τ^+ with the lepton-pair rapidity Y_Z and variable y^* , which has the meaning of the τ^- rapidity in the center-of-mass (c.m.) frame of the $\tau^- \tau^+$ pair. Namely,

$$y_- = Y_Z + y^*, \quad y_+ = Y_Z - y^*. \quad (6)$$

The rapidity of τ^+ in the c.m. frame is $-y^*$ [14]. Note that the c.m. angle θ^* between the parton f and τ^- is determined from

$$\sin \theta^* = \frac{1}{\cosh(y^*)}, \quad \cos \theta^* = \tanh(y^*). \quad (7)$$

The transverse momentum of τ is $p_\perp = \frac{1}{2} M \sin \theta^*$, and the invariant momentum transfer t of Eq. (1) is equal to $t = -\frac{1}{2} M^2 (1 - \cos \theta^*)$.

We will choose the variables Y_Z , y^* , and M^2 as independent ones. Then, the triply differential cross section can be written as

$$\begin{aligned} & \frac{d\sigma}{dY_Z dy^* dM^2} (pp \rightarrow \tau^- \tau^+ + \dots) \\ &= \frac{x_1 x_2}{2 \cosh^2(y^*)} \sum_{f=q,\bar{q}} F_f(x_1) F_{\bar{f}}(x_2) \frac{d\sigma}{dt} (f\bar{f} \rightarrow \tau^- \tau^+), \end{aligned} \quad (8)$$

where the factor $x_1 x_2$ originates from the Jacobian due to the change of variables from (x_1, x_2, t) to (Y_Z, y^*, M^2) . The cross section of the quark-antiquark production of the polarized τ leptons is [12]

$$\frac{d\sigma}{dt} (q\bar{q} \rightarrow \tau^- \tau^+) = \frac{\beta}{16\pi M^4} \sum_{i,j=1}^4 R_{i,j}^{(q\bar{q})}(M, \theta^*) S_i^- S_j^+. \quad (9)$$

Here, $S_i^\mp \equiv (\vec{S}^\mp, 1)$, $\vec{S}^\mp$ are the polarization vectors of $\tau^\mp$ in their corresponding rest frames. The Cartesian components of vectors $\vec{S}^\mp$ are defined in a system with the axes $(\hat{x}', \hat{y}', \hat{z}')$, where $\hat{z}'$ is along the c.m. momentum of τ^- , the plane $\hat{x}'\hat{z}'$ is spanned on the c.m. momenta of τ^- and q , and $\hat{y}' = \hat{z}' \times \hat{x}'$. The normalized spin-correlation matrix is expressed in terms of the coefficients $R_{i,j}^{(q\bar{q})}(M, \theta^*)$ as $r_{i,j}^{(q\bar{q})} \equiv R_{i,j}^{(q\bar{q})}/R_{44}^{(q\bar{q})}$. Besides this, in Eq. (9), $\beta = (1 - 4m_\tau^2/M^2)^{1/2} \approx 1$ is the τ -lepton velocity in the c.m. frame.

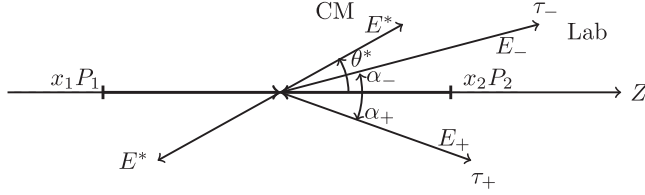


FIG. 1. Configuration of τ^+ and τ^- momenta in the laboratory and c.m. frames.

A. Analysis of kinematics

A typical configuration of τ -lepton momenta in the laboratory frame is shown in Fig. 1. Here, α_+ and α_- are the angles of τ^+ and τ^- with respect to the proton beam axis. They are related to the corresponding rapidities

$$\cos \alpha_{\pm} = \frac{p_{\pm\parallel}}{E_{\pm}} = \tanh(y_{\pm}),$$

$$y_{\pm} = \tanh^{-1}(\cos \alpha_{\pm}) = \frac{1}{2} \ln \left(\frac{1 + \cos \alpha_{\pm}}{1 - \cos \alpha_{\pm}} \right). \quad (10)$$

Note that $0 \leq \alpha_{\pm} \leq \pi/2$ if $y_{\pm} \geq 0$, and $\pi/2 \leq \alpha_{\pm} \leq \pi$ if $y_{\pm} \leq 0$.

Let us analyze the scattering angle θ^* , which determines the $q\bar{q} \rightarrow \tau^-\tau^+$ spin-correlation matrix in the c.m. frame:

- (i) If x_1 corresponds to the quark and x_2 to the antiquark, then θ^* is the angle between the quark and τ^- lepton. In the interval $0 \leq \theta^* \leq \pi$, the sign of $\sin \theta^*$ is positive, while the sign of $\cos \theta^*$ depends on the sign of y^* .
- (ii) If x_1 corresponds to the antiquark and x_2 to the quark, then one needs to change the angle $\theta^* \rightarrow \pi - \theta^*$ in the matrix $R_{i,j}^{(q\bar{q})}(M, \theta^*)$ and perform additional rotation on the latter (see further). The corresponding cross section is denoted below with a tilde.

The cross section taking into account both possibilities is written in the form

$$\frac{d\sigma}{dY_Z dy^* dM^2} (pp \rightarrow \tau^-\tau^+ + \dots)$$

$$= \frac{x_1 x_2}{2 \cosh^2(y^*)} \sum_{q=u,d,s,\dots} \left[F_q(x_1) F_{\bar{q}}(x_2) \frac{d\sigma}{dt} (q\bar{q} \rightarrow \tau^-\tau^+) \right.$$

$$\left. + F_{\bar{q}}(x_1) F_q(x_2) \frac{d\tilde{\sigma}}{dt} (q\bar{q} \rightarrow \tau^-\tau^+) \right]. \quad (11)$$

B. Kinematics for τ spin quantization frames

For the spin-correlation matrix to be useful, we need to define all axes of the corresponding reference frames for τ^- and τ^+ separately.

Further, we need to take into account both τ -lepton production and decay. Because we sum over initial-state

parton contributions without taking into account their spin states, it is straightforward to reuse the description presented a long time ago in [15]. Thus, we can rewrite and extend Eq. (11), which is for stable leptons, to account for the decays $\tau^- \rightarrow X^- \nu_{\tau}$ and $\tau^+ \rightarrow X^+ \bar{\nu}_{\tau}$:

$$\frac{d\sigma}{dY_Z dy^* dM^2 d\Omega_- d\Omega_+} (pp \rightarrow X^- X^+ \nu_{\tau} \bar{\nu}_{\tau} + \dots)$$

$$= \frac{\beta x_1 x_2}{32\pi M^4 \cosh^2(y^*)} \left(\frac{1}{\Gamma_{\tau}} \frac{d\Gamma_-}{d\Omega_-} \right) \left(\frac{1}{\Gamma_{\tau}} \frac{d\Gamma_+}{d\Omega_+} \right)$$

$$\times \sum_{q=u,d,s,\dots} \left[F_q(x_1) F_{\bar{q}}(x_2) \sum_{i,j=1}^4 R_{i,j}^{(q\bar{q})}(M, \theta^*) h_i^- h_j^+ \right.$$

$$\left. + F_{\bar{q}}(x_1) F_q(x_2) \sum_{i,j=1}^4 \tilde{R}_{i,j}^{(q\bar{q})}(M, \pi - \theta^*) h_i^- h_j^+ \right], \quad (12)$$

where $\frac{d\Gamma_{\pm}}{d\Omega_{\pm}}$ is the differential partial width of the decay $\tau^{\pm} \rightarrow X^{\pm} \nu_{\tau}$ for nonpolarized $\tau^{\pm}$, $d\Omega_{\pm}$ denotes the Lorentz invariant phase-space element for the $X^{\pm} \nu_{\tau}$ state, and Γ_{τ} is the total decay width of τ . In Eq. (12), $h_i^{\pm}$ are the spin polarimetric vectors of $\tau^{\pm}$ decays which depend on the decay channels and decay-product momenta. These decay-channel- and decay-kinematics-dependent vectors $h_i^{\pm}$ take the place of $S_i^{\pm}$ in Eq. (9).

The first sum in the brackets of Eq. (12) corresponds to the contribution where the laboratory $\hat{z}$ axis corresponds to the direction of the incoming quark q . For the second part, the laboratory $\hat{z}$ axis is along the direction of the antiquark $\bar{q}$. Because we are interested in the spin correlations, and not in the unpolarized cross section, the second hard-scattering frame needs to be properly oriented. This results in

$$\tilde{R}_{i,j}^{(q\bar{q})}(M, \pi - \theta^*) \equiv \hat{O}_{z'}(\pi) R_{i,j}^{(q\bar{q})}(M, \pi - \theta^*)$$

$$= \varepsilon_i \varepsilon_j R_{i,j}^{(q\bar{q})}(M, \pi - \theta^*), \quad (13)$$

where $\hat{O}_{z'}(\pi)$ is the rotation around the $\hat{z}'$ axis by the angle π . Further, $\varepsilon_i = -1$ for $i = 1, 2$, and $\varepsilon_i = +1$ for $i = 3, 4$.

To calculate the event weight, we sum the spin-correlation coefficients in the $q\bar{q} \rightarrow \tau^-\tau^+$ process over the parton flavors

$$wt_{\text{prod}} wt_{\text{spin}} = \sum_{i,j=1}^4 R_{i,j}^{(pp)} h_i^- h_j^+$$

$$= R_{44}^{(pp)} \left(1 + \sum_{i=1}^3 r_{i,4}^{(pp)} h_i^- + \sum_{j=1}^3 r_{4,j}^{(pp)} h_j^+ \right.$$

$$\left. + \sum_{i,j=1}^3 r_{i,j}^{(pp)} h_i^- h_j^+ \right), \quad (14)$$

where

$$R_{i,j}^{(pp)} \equiv x_1 x_2 \sum_{q=u,d,s,\dots} [F_q(x_1) F_{\bar{q}}(x_2) R_{i,j}^{(q\bar{q})}(M, \theta^*) + F_{\bar{q}}(x_1) F_q(x_2) \tilde{R}_{i,j}^{(q\bar{q})}(M, \pi - \theta^*)] \quad (15)$$

for $i, j = 1, 2, 3, 4$, and we define elements of the effective proton-proton spin-correlation matrix $r_{i,j}^{(pp)}$ and τ^- (τ^+) polarization $r_{i,4}^{(pp)}$ ($r_{4,j}^{(pp)}$) in Eq. (14) as

$$r_{i,j}^{(pp)} = \frac{R_{i,j}^{(pp)}}{R_{44}^{(pp)}}, \quad r_{i,4}^{(pp)} = \frac{R_{i,4}^{(pp)}}{R_{44}^{(pp)}}, \quad r_{4,j}^{(pp)} = \frac{R_{4,j}^{(pp)}}{R_{44}^{(pp)}}, \quad (16)$$

with $i, j = 1, 2, 3$. These elements depend on the variables Y_Z , y^* , and M^2 . We further omit the superscript (pp) for brevity.

The definition of the anomalous magnetic $X(M^2)$ and electric $Y(M^2)$ dipole form factors and their relation to the corresponding weak dipole moments at $M = M_Z$ is given in [12]. Note that in the calculation, only the contributions linear in $X(M^2)$ and $Y(M^2)$ are included, as one can expect their values to be very small. Indeed, the experiment ALEPH [16] determined the following constraints:

$$\begin{aligned} |\text{Re}(\mu_\tau)|_{\text{exp}} &< 1.14 \times 10^{-3}, & |\text{Im}(\mu_\tau)|_{\text{exp}} &< 2.65 \times 10^{-3}, \\ |\text{Re}(d_\tau)|_{\text{exp}} &< 0.91 \times 10^{-3}, & |\text{Im}(d_\tau)|_{\text{exp}} &< 2.01 \times 10^{-3}, \end{aligned} \quad (17)$$

while the SM prediction [17] for the weak anomalous magnetic moment is [18]

$$\mu_{\tau, \text{SM}} = -(2.10 + i0.61) \times 10^{-6}. \quad (18)$$

The detailed definitions of axes of the τ reference frames are not necessary for the formula [Eq. (14)] phenomenology to be discussed. We will return to this point later—in fact, we will rely on the frames built from secondary decay momenta.

III. NUMERICAL RESULTS ON SPIN DENSITY MATRIX

In this section, before we turn attention to results including τ decays, we discuss the results on the spin-correlation matrix averaged over PDFs in Eq. (15). The PDFs “MSTW 2008” from Ref. [19] are used in the calculation.

Let us start a discussion of the numerical results for the elements of the matrix in Eq. (16). There are two interesting kinematic regimes. The first corresponds to the small rapidity of the τ -lepton pair, and the second corresponds to the situation where the τ pair is strongly boosted. In the

TABLE I. Rapidity y^* ; rapidities of τ^- and τ^+ leptons y_- and y_+ ; parton fractions x_1, x_2 ; angles α_- and α_+ of τ^- and τ^+ in the laboratory frame; c.m. angle θ^* ; transverse momentum $p_\perp$ of τ leptons; and their energies E_- , E_+ in the laboratory frame.

Variable	$Y_Z = 0$	$Y_Z = 2$	$Y_Z = 4$
y^*	0.54931	0.54931	0.54931
y_-	0.54931	2.54931	4.54931
y_+	-0.54931	1.45069	3.45069
x_1	0.00671	0.04954	0.36608
x_2	0.00671	0.00091	0.00012
α_- (deg)	60	8.93556	1.21171
α_+ (deg)	120	26.3848	3.63404
θ^* (deg)	60	60	60
$p_\perp$ (GeV)	39.4854	39.4854	39.4854
E_- (GeV)	45.5938	254.214	1867.21
E_+ (GeV)	45.5938	88.8516	622.961

latter case, the quark is more probable to come from the first of the colliding protons. For example, from Table I and also from the behavior of PDFs given in Fig. 2, it is clear that at large rapidity $Y_Z = 4$, the parton, carrying fraction x_1 of the proton momentum, is usually the valence quark, while the other parton is the sea antiquark.

In the figures below, we choose the proton-proton energy $\sqrt{s} = 13.6$ TeV and the $\tau^- \tau^+$ -pair invariant mass $M = M_Z$, and we fix the total τ -pair rapidity Y_Z . To completely determine the kinematics, one also needs to fix the c.m. rapidity y^* . The latter is chosen to correspond to the c.m. angle $\theta^* = 60^\circ$. Values of kinematic variables for the three values $Y_Z = 0, 2, 4$ are shown in Table I.

In Figs. 3–5, we present the dependence of the non-diagonal elements r_{12} , r_{13} , r_{23} , r_{14} , and r_{24} on values of the real and imaginary parts of the weak anomalous magnetic and electric dipole moments X and Y , respectively. The diagonal elements of the spin-correlation matrix, r_{11} , r_{22} , and r_{33} , are less sensitive to X and Y and are not shown. In these figures, only Z-boson exchange is included for simplicity; however, in the Monte Carlo calculation in Sec. IV, γ exchange is included as well.

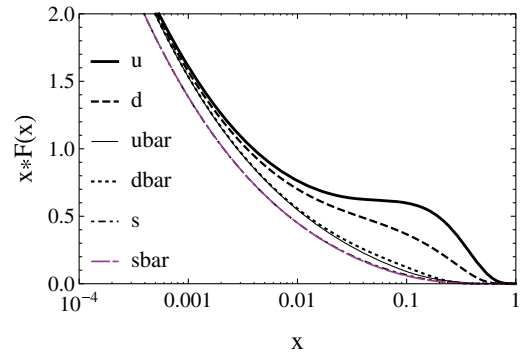


FIG. 2. Parton distribution functions MSTW 2008 at the energy scale $Q = M_Z$.

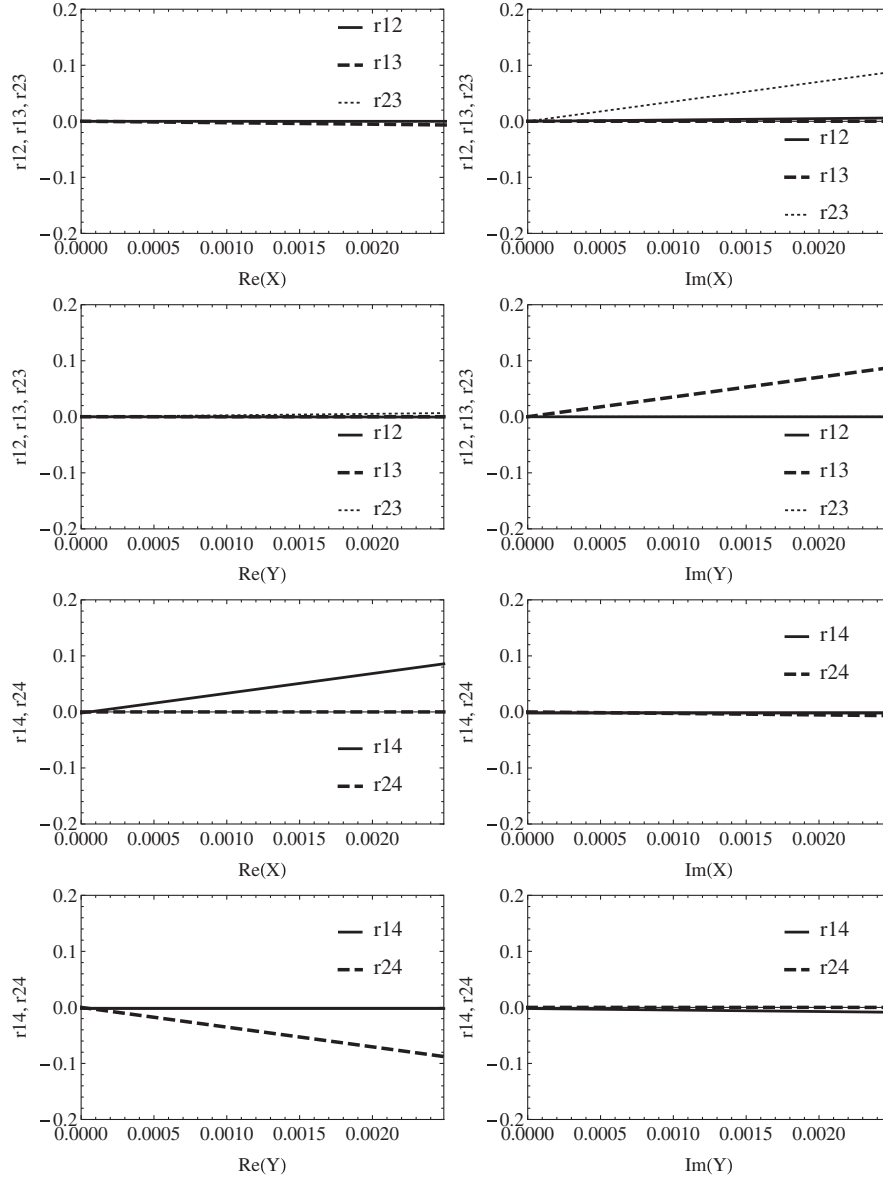


FIG. 3. Dependence of $r_{i,j}$ on the real and imaginary parts of the weak magnetic and electric dipole moments. The proton-proton energy is 13.6 TeV, the τ -pair invariant mass is equal to M_Z , the rapidity is $Y_Z = 0$, and the c.m. angle is 60° . The contribution to the cross section of the quark with the momentum fraction x_1 and the antiquark with fraction x_2 is 55.4%, while the contribution of the antiquark with fraction x_1 and the quark with fraction x_2 is 44.6%. Elements $r_{i,j}$ which are not displayed in the figure are consistent with zero.

The element r_{34} is equal to the longitudinal polarization of the τ^- lepton, and its value depends on the rapidity Y_Z due to the different composition of the parton flavors [an average in Eq. (15)]. In particular, in the SM,

$$\begin{aligned} r_{34} &= -0.150 & \text{for } Y_Z = 0, \\ r_{34} &= -0.386 & \text{for } Y_Z = 2, \\ r_{34} &= -0.725 & \text{for } Y_Z = 4, \end{aligned} \quad (19)$$

and r_{34} depends weakly on the anomalous dipole moments. For this reason, it is not shown in Figs. 3–5.

One can see that some elements of $r_{i,j}$ and $r_{i,4}$ are sensitive to dipole moments. In particular, it can be seen from Fig. 3 that r_{14} and r_{24} are sensitive to $\text{Re}(X)$ and $\text{Re}(Y)$, while r_{23} and r_{13} are sensitive to $\text{Im}(X)$ and $\text{Im}(Y)$. This may be convenient for separating magnetic and electric moment contributions, as well as their real and imaginary parts.

The pattern at larger rapidities in Figs. 4 and 5 becomes more complex, although the sensitivity of $r_{i,j}$ to dipole moments is enhanced.

One can see that regimes of small and large (but still available for measurements at about $Y_Z = 2$) rapidity differ,

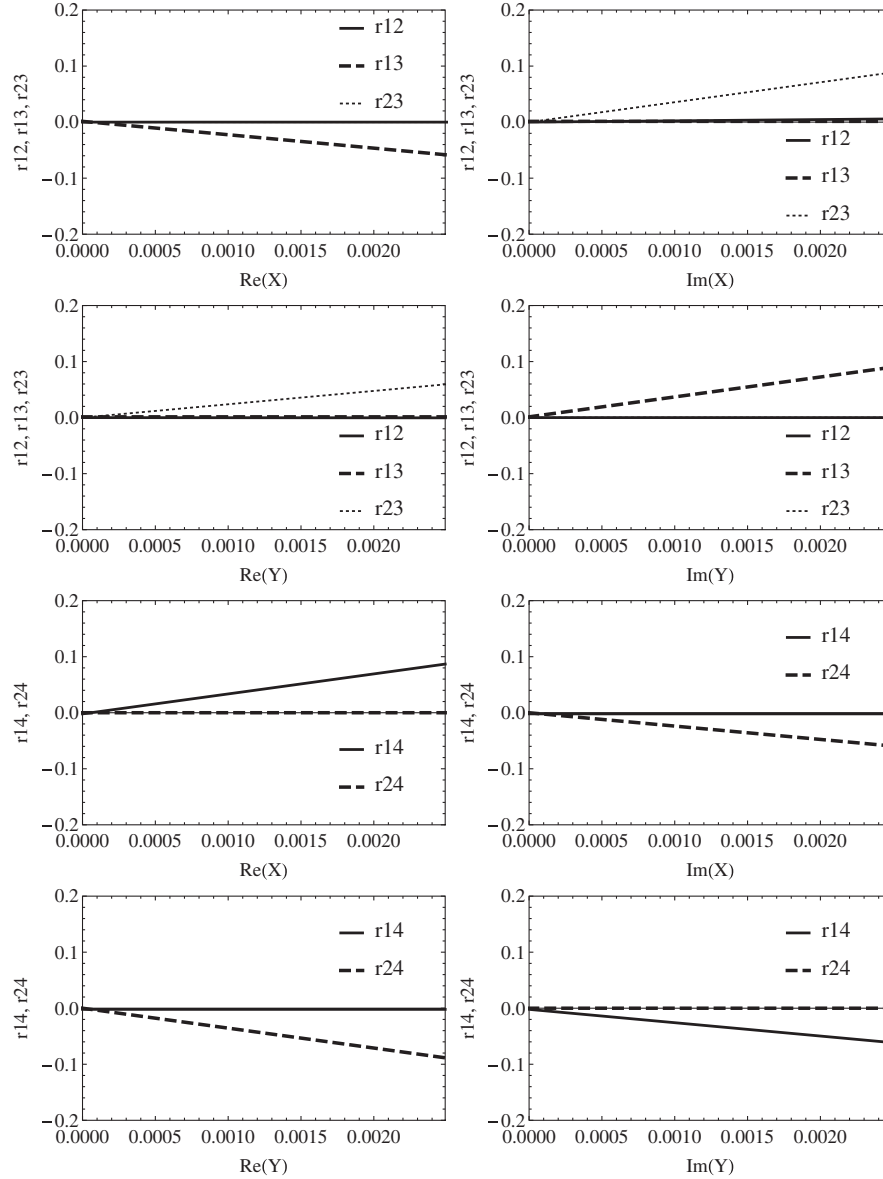


FIG. 4. The same as in Fig. 3, but for rapidity $Y_Z = 2$. The contribution to the cross section of the quark with the momentum fraction x_1 and the antiquark with fraction x_2 is 72.5%, while the contribution of the antiquark with fraction x_1 and the quark with fraction x_2 is 27.5%.

but all regions preserve useful dependence on dipole moments, and thus can be useful for the development of observables. In the following section, we will concentrate on r_{14} and r_{24} , which are sensitive to $\text{Re}(X)$ and $\text{Re}(Y)$. The elements r_{13} and r_{23} require selection on both the $\tau^\pm$ and $\tau^\mp$ configurations. This is more complicated, and we leave their phenomenology to further studies.

IV. NUMERICAL RESULTS ON SENSITIVE OBSERVABLE

In the previous section, we have investigated elements of $r_{i,j}$ and $r_{i,4}$ in Eq. (16). Now, we can use that experience to search for one-dimensional distribution sensitive to

anomalous moments, which can serve as the prototype for an observable, useful for the tests as well. Because of neutrinos escaping detection, and because the most sensitive to dipole moments turn out to be elements r_{14} , r_{24} , r_{13} , and r_{23} , we use $\tau^\mp \rightarrow \pi^\mp \nu_\tau$ decay for its sensitivity to the longitudinal spin, and $\tau^\pm \rightarrow \rho^\pm \nu_\tau \rightarrow \pi^\pm \pi^0 \nu_\tau$ decay for its sensitivity to the transverse spin. In the pp collisions, the reconstruction of τ momenta is difficult, and we can only partly rely on the experience of Ref. [12]. The choice of the first τ -decay channel $\tau^\mp \rightarrow \pi^\mp \nu_\tau$ will not be of large importance later. In principle, any other decay channel can be used as well, but then the discussion of the observables definition would be complicated, because the relation between visible decay products of $\tau^\mp$ and its

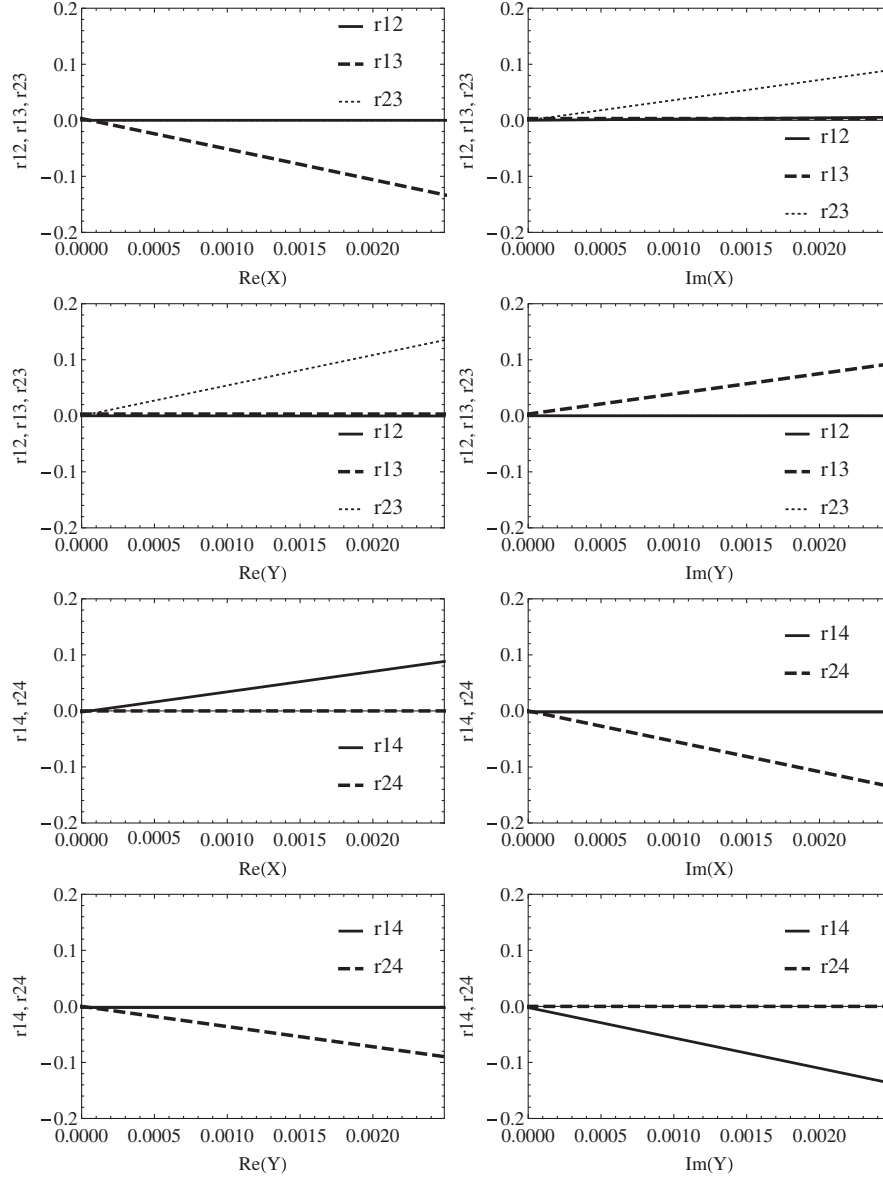


FIG. 5. The same as in Fig. 3, but for rapidity $Y_Z = 4$. The contribution to the cross section of the quark with the momentum fraction x_1 and the antiquark with fraction x_2 is 97.8%, while the contribution of the antiquark with fraction x_1 and the quark with fraction x_2 is 2.2%.

neutrino would differ, and this would call for detailed study of the detector response.

Our test observable was defined from the four-momenta of the visible decay product system. This consists of the acoplanarity angle ϕ between oriented half-planes built on $\vec{p}_{\text{beam}1} \times (\vec{p}_{\pi^\pm} + \vec{p}_{\pi^0})$ and $\vec{p}_{\text{beam}1} \times \vec{p}_{\pi^\mp}$. Let us provide details of the ϕ angle definition. The momentum $\vec{p}_{\text{beam}1}$ denotes the direction of the first beam [20].

As a sensitivity observable, the acoplanarity angle ϕ calculated in the rest frame of visible τ -lepton decay products is used. This angle is defined as follows: first, we define normal versors $\vec{n}_1$ to the plane spanned on the beam direction and $\pi^\mp$ direction of $\tau^\mp \rightarrow \pi^\mp \nu_\tau$ decay, and $\vec{n}_2$ for the plane spanned on the $\pi^\pm$ and π^0 directions for

$\tau^\pm \rightarrow \rho^\pm \nu_\tau$. The acoplanarity angle is then calculated as the arccos of the $\vec{n}_1 \cdot \vec{n}_2$ scalar product, and thus covers the $(0, \pi)$ range only. To extend the definition of acoplanarity to one of two oriented half-planes and to the $(0, 2\pi)$ range, the scalar product of $\pi^\mp$ from $\tau^\mp$ and $\vec{n}_2$ is used; if it is positive, we set the acoplanarity angle equal to $2\pi - \phi$.

Such an observable will still have negligible sensitivity to anomalous couplings. We need to split the sample into two parts according to the sign of the scalar product of the vector $\vec{n}_3$ (normal to the plane spanned on the beam versor and $\vec{n}_1$ versor) and $\vec{n}_2$. We denote them as Region 1 and Region 2.

Figure 6 shows the ratio of the acoplanarity angle distribution for three settings of anomalous dipole moments

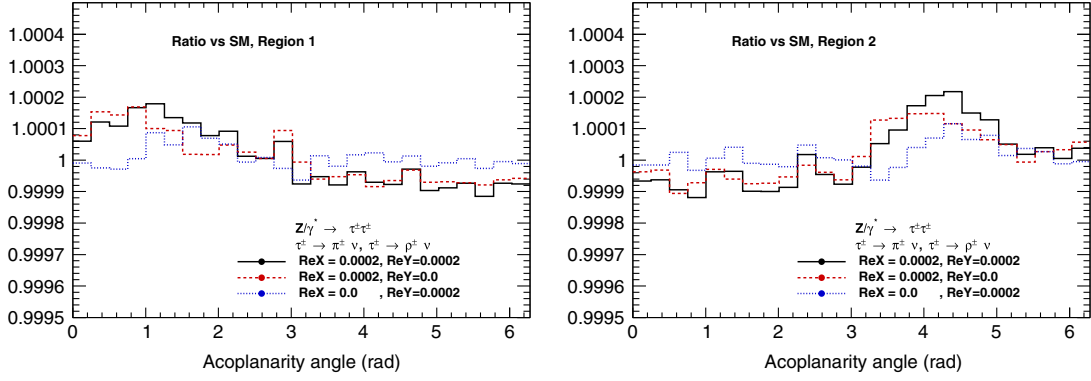


FIG. 6. Ratio of acoplanarity angle ϕ distribution as defined in the text, with (i) anomalous magnetic, (ii) electric, and (iii) both magnetic and electric dipole moments included simultaneously, to the distribution in which dipole moments were absent. Selection cut of Region 1 (left plot) and of Region 2 (right plot).

to the SM predictions and for Region 1 on the left-hand side, and Region 2 on the right-hand side.

We have chosen for the plots $\text{Re}(X) = 2 \times 10^{-4}$, $\text{Re}(Y) = 2 \times 10^{-4}$, and imaginary parts of the dipole moments are set to zero; see Fig. 6. Sensitivity to dipole moments through the τ spin correlations could be demonstrated in the semirealistic condition of an observable build from the visible decay products of τ 's.

The sample used in the analysis has been generated using the PYTHIA 8.2 Monte Carlo generator [21], with pp collision at 13.6 TeV c.m. energy and MSTW2008nnlo68cl.LHgrid parametrization of PDFs. The sample of 4×10^6 events of the hard process $q\bar{q} \rightarrow Z \rightarrow \tau^-\tau^+$ was generated with the invariant mass of a τ -lepton pair restricted to the $M_Z \pm 0.5$ GeV range. Tau leptons were decayed with the Tauola library [22] without spin correlations included, with one tau decaying $\tau^\mp \rightarrow \pi^\mp \nu_\tau$ and another tau decaying $\tau^\pm \rightarrow \rho^\pm \nu_\tau \rightarrow \pi^\pm \pi^0 \nu_\tau$. Spin correlations were included with reweighting provided by the TauSpinner package [10], to which formulas of [12] were implemented [23].

Spin weights were calculated for different models: the SM with $\text{Re}(X) = \text{Re}(Y) = 0$, and a few versions of anomalous moments included cases with (a) $\text{Re}(X) = \text{Re}(Y) = 2 \times 10^{-4}$, (b) $\text{Re}(X) = 2 \times 10^{-4}$, $\text{Re}(Y) = 0$, and (c) $\text{Re}(Y) = 2 \times 10^{-4}$, $\text{Re}(X) = 0$. These dipole couplings are smaller by about a factor of 10^{-3} with respect to the SM vector and axial-vector couplings; therefore, their linear implementation into the differential cross section is reasonable. On the other hand, these dipole couplings are larger than the SM value [Eq. (18)] of the anomalous magnetic moment by a factor of 10^2 , so that the chosen values are sufficient to reflect the effects of NP.

Figure 6 shows the ratio of acoplanarity angle distribution with anomalous moments to that without anomalous moments. Distributions in Region 1 and Region 2 are shown separately. One can observe the effect on the shape of the distribution on the level of 10^{-4} in Region 1 and

Region 2 for the acoplanarity angles $(0, \pi)$ and $(\pi, 2\pi)$, respectively.

As the dipole moments are introduced at the linear level through interference with the SM amplitudes, their effect shown in Fig. 6 can be obtained for other values of $X(M_Z^2)$ and $Y(M_Z^2)$ by a simple rescaling. Note also, that because of weighted event samples, the statistical error affects the NP effect alone, and not the difference of the independently obtained results for distinct anomalous coupling values. Therefore, the obtained numerical results are meaningful even though a sample of only $N = 4 \times 10^6$ events was used. That is why the statistical error on the NP effect is substantially smaller than $1/\sqrt{N} = 0.5 \times 10^{-3}$ expected for the compared distribution features.

In the future, we can use $\pi^\mp$ energy, or an equivalent variable, to explore anomalous moment dependence due to elements r_{13} and r_{23} , which are sensitive to imaginary parts of the weak dipole moments $X(M_Z^2)$ and $Y(M_Z^2)$.

No detector acceptance cuts were taken into account. We believe that this needs to be delegated to the experimental work within collaborations when all details of the detector granularity, backgrounds, and selection cuts can be critically evaluated. Then, other τ -decay channels can probably be used as well, and solutions based on machine learning techniques may become of help.

V. SUMMARY

We have investigated the possibility of using the spin effects of the Drell-Yan τ -pair production and decay in the LHC conditions. For that purpose, we constructed observables sensitive to the presence/absence of anomalous dipole moments of the τ -lepton interaction. To simplify the task, we have concentrated on couplings to the Z boson. Interaction with virtual photons is potentially more involved, as no constraint of the Z peak can be used. The couplings to virtual photons can be measured in the Belle experiments, for relatively low τ -pair virtuality, though. For higher virtuality, a more complex evaluation

of experimental cuts than presented here would be needed. We leave it to the forthcoming work, preferably by the experiments themselves, as discussion of experimental cuts will be of much greater importance.

We expect that our observable, demonstrating the sensitivity of spin-dependent observables to anomalous dipole moments, is primarily for the program tests. In final applications, machine learning (ML) techniques will be far more suitable, but even in those cases, idealized optimal-like variables may be helpful to prepare the primary set of variables used as input for ML algorithm

training. Such variables may help us to study experimental ambiguities, too. The demonstrated extension of the TauSpinner algorithm may be helpful in such projects.

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