

W-boson exotic decay into three charged leptons at the LHCYifan Fei^{1,*}, Peiran Li^{2,†}, Zhen Liu^{2,‡}, Kun-Feng Lyu^{2,§} and Maxim Pospelov^{2,3,||}¹*Department of Physics and Center for Field Theory and Particle Physics,
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We investigate the W boson's exotic decay channel, $W \rightarrow \ell\ell\ell\nu$, at the LHC. Although the four-body final states suppress the decay branching ratio, the large production of W bosons makes detecting and precisely measuring this decay probability entirely feasible. Our simulation study indicates that this tiny branching ratio can be measured with subpercent precision at the HL-LHC. This decay channel can also constrain Standard Model extensions. Using the $L_\mu - L_\tau$ model as a benchmark, we find that the current bound on the gauge coupling for Z' mass in the range of [4, 75] GeV can significantly improve.

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Introduction. Precision measurements of the Standard Model (SM) are a powerful method to test its internal consistency and explore new physics, i.e., possible beyond SM (BSM) extensions. W bosons, predicted as the minimal UV completion of the four-Fermi weak interactions, were discovered over 40 years ago. One might think that many decay channels of W bosons have been studied by now. However, only a limited number of decay modes have been measured thus far. The Large Hadron Collider (LHC) has been extremely successful as a Higgs machine, a top quark machine, and a direct BSM search machine. Future plans aim to extend the data sample to an unprecedented integrated luminosity of 3 ab^{-1} [1]. We argue that it will also produce and record the highest number of W bosons [2–5] in accelerators, $N_W \sim O(10^{11})$, and hence can reveal new physics through the W boson's rare decays. Specifically, we show that the LHC has collected enough data to provide a few percent level precision measurement on a rare W boson decay [6–9] within the SM through the radiative process. Additionally, this channel can probe various interesting new physics, e.g., HNL searches [10–12] with competitive sensitivity.

Our paper should be considered complementary to (and independent from) the recent study by ATLAS of the exotic Z' searches via the $W \rightarrow 3\ell + \nu$ mode [13].

(On-shell) decay width from theory. We first compute the theoretical prediction for the on-shell decay branching ratio. In the SM, multiple Feynman diagrams contribute to the 1-to-4 decay process $W^+ \rightarrow \mu^+ \nu_\mu \ell^+ \ell^-$, where ℓ denotes electron or muon. The dominant contribution is from the two diagrams shown in Fig. 1. The W^+ boson decays into an antimuon and a muon neutrino, followed by a virtual photon converting to a pair of electron and positron. The off-shell photon can originate from either the muon or the W boson. The invariant mass of the electron pair determines the off-shellness of the intermediate photon. If the electron pair is too soft or nearly collinear with the decaying W , then the photon propagator would approach the pole, requiring further resummation. Practically, to measure the final three charged leptons, we need to apply kinematic cuts on the electron pair to make them visible and separable. Hence, we impose a threshold on the electron pair invariant mass and estimate the decay branching ratio. The partial decay width for the process $W^+(p) \rightarrow \mu^+(p_1) \nu_\mu(p_2) e^+(k_1) e^-(k_2)$ is given by integrating over the phase space of the four final particles.

We use the package FeynCalc [14,15] to perform the numerical integration over the phase space region. As a cross-check, the package MadGraph5_aMC@NLO [16] is used to generate the decay sample files, and the invariant mass cuts are imposed to obtain the corresponding particle decay branching ratio. The only cut is on the invariant mass of the electron pair. We set its invariant mass $M_{\ell\ell}$ larger than $M_{\ell\ell}^{\text{thresh}}$ and then scan this threshold parameter. In Fig. 2, we

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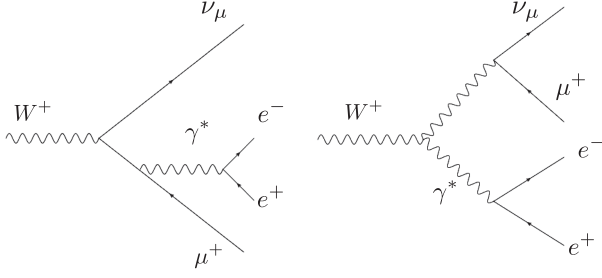


FIG. 1. The dominant Feynman diagrams for the decay process $W^+ \rightarrow \mu^+ \nu_\mu e^+ e^-$.

plot the decay branching ratio as a function of $M_{\ell\ell}^{\text{thresh}}$. The invariant mass starts from 1 GeV, at which the branching ratio is as high as $O(10^{-5})$. As the cut value increases, the amplitude becomes more suppressed due to the photon propagator.

Note that in this calculation the initial W boson is exactly on-shell. However, due to the large decay width of the W boson, the real production of the decay process cannot be factorized exactly. In main context, we define the fiducial decay branching ratio and compare it with the on-shell decay width we have computed.

Observable fiducial decay width. We first focus on the measurable exotic decay branching ratio of $W^+ \rightarrow \mu^+ \nu_\mu \ell^+ \ell^-$, where ℓ denotes electron or muon. The expected signal final states are the three charged leptons from the W boson decay. Under the zero-width approximation, the signal cross section can be factorized into the product of the W boson production rate and the decay branching ratio, namely $\sigma(pp \rightarrow W) \times \text{Br}(W \rightarrow \ell\ell\ell\nu_\ell)$. The branching ratio computation is discussed in Supplemental Material [17]. However, the produced $3\ell\nu_\ell$ invariant mass is not highly peaked at m_W but rather has a broader extension with long tails. Furthermore, additional diagrams contribute to the channel $pp \rightarrow \ell\ell\ell\nu_\ell$.

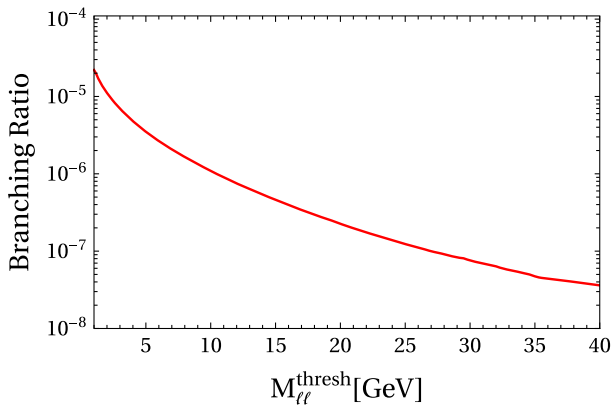


FIG. 2. The branching ratio of the decay channel $W^+ \rightarrow \ell^+ \ell^- \ell^+ \nu_\ell$ as a function of the charged lepton pair invariant mass $M_{\ell\ell}^{\text{thresh}}$ threshold value.

To address these subtleties, a clear definition should be made to separate the signal from the background. Because a slightly off-shell W boson can turn into an on-shell W boson with a virtual photon, the signal must be defined in a narrow window, unlike the inclusive W/Z boson measurements [18,19]. Any event with the four-lepton invariant mass within the on-shell region (OR), $m_W - 2\Gamma_W < \sqrt{\hat{s}} < m_W + 2\Gamma_W$, is counted as the signal. Events outside this range are considered background. At the parton level, this accounts for most of the W bosons near the pole, making the rate defined this way unambiguous. It is a typical subtlety in attempts to extract decay branching fractions of unstable particles, where one must compare and convert the convoluted rate around the pole region to an on-shell quantity. One can choose to enlarge the $\pm 2\Gamma_W$ region when defining the signal region at the parton level, and the result will be robust against such choices as long as signal and background are treated consistently during the simulation process. We aim to extract the branching ratio dominantly from the W decay and avoid contamination from other t -channel diagrams. The fiducial branching ratio can be extracted by

$$\text{Br}^{\text{fid}} = \frac{\sigma_{\text{signal}}^{\text{fid}}}{\sigma_W} \times f_W, \quad (1)$$

where $\sigma_{\text{signal}}^{\text{fid}}$ is the signal cross section after required kinematic selections at the reconstruction level, σ_W is the inclusive W boson cross section, and f_W is the theoretical ratio of the fiducial cross section of $pp \rightarrow W \rightarrow \ell\ell\ell\nu$ to our fiducial signal cross section:

$$f_W = \frac{\sigma(pp \rightarrow W) \times \text{Br}(W \rightarrow \ell\ell\ell\nu_\ell)}{\sigma_{\text{signal}}^{\text{fid}}}. \quad (2)$$

For consistency, we generate the on-shell decay sample files in the rest frame of the W boson and then boost them to the lab frame, imposing the same cuts as the fiducial signal. f_W is found to be larger than 1 due to the negative interference between the s -channel and t -channel diagrams. Such negative interference will cause a smaller signal cross section, resulting in a slightly larger f_W . It will also decrease if the OR is widened. Nevertheless, we aim to estimate the precision $\frac{\delta \text{Br}}{\text{Br}}$, which is independent of f_W . This is another interesting example of nonfactorizable SM on-shell interference [20]; it is related to the radiation zeros [21–24] and warrants further theory explorations.

W-boson exotic decay precision measurement. After defining the observable fiducial branching fraction, we study the projected sensitivity. We choose two benchmark collider configurations. The primary benchmark is based on the future 14 TeV HL-LHC [25] with an integrated luminosity of 3 ab^{-1} . The second benchmark implements

a conservative analysis of the 13 TeV run-2 stage following CMS/ATLAS configurations with an integrated luminosity of 150 fb^{-1} . Before enumerating all possible signals and backgrounds, we highlight that the signal process exhibits a distinct characteristic of three leptons in the final state with a low invariant mass:

$$M_{\ell\ell\ell} < 80 \text{ GeV}. \quad (3)$$

This unique characteristic allows us to discriminate this rare decay from the majority of multilepton background.

Next, we classify the signal and background channels. The parton-level events are generated via the package MadGraph5_aMC@NLO. They are subsequently passed to the interfaced PYTHIA8 [26] and DELPHES3 [27,28] to simulate the reconstruction-level events under the two benchmark detector configurations. Various kinematic selections and deep neural network (DNN) analyses are implemented to extract the final precision of the branching ratio measurement.

There have been dedicated measurements of W boson production at the LHC [19,29–35]. Theoretical predictions of the total cross section involve NNLO QCD corrections [36–38] plus complete NLO electroweak corrections [39–41]. The dominant systematic uncertainty stems from the proton PDFs. We generate the parton-level events at the tree level and then perform jet matching to resolve the infrared divergence. The leading order channel for the on-shell W boson production is the 2-to-1 process: $u + \bar{d} \rightarrow W^+$ with its conjugate process. Additionally, the associated multiple jets production is generated to account for the QCD correction. Overall, we run the following processes in MadGraph5_aMC@NLO at the parton level:

$$pp \rightarrow \ell\ell\ell\nu_\ell + (j), \quad M_{\ell\ell\ell\nu} \in \text{OR},$$

where ℓ denotes e and μ , and j refers to the jet including gluons and light quarks. The τ lepton also contributes to this channel; however, the charged lepton from τ decay would be too soft to pass the trigger, and hence, its contribution is negligible. One can study a τ -specific exotic decay using various hadronic τ taggers [42–45]. Therefore, the main contribution is from the e, μ final states. The parton-level generator cut, mainly for the purpose of improving signal generation efficiency, for the HL-LHC (run-2) benchmark is

$$p_T(j) > 20 \text{ GeV}, \quad p_T(\ell) > 3 \text{ GeV}, \quad \eta(\ell) < 5.0(2.5), \\ \eta(j) < 5.0, \quad \Delta R(\ell\ell) > 0.2, \quad M_{\ell\ell} > 1 \text{ GeV},$$

where $M_{\ell\ell}$ is the invariant mass of the dilepton. The signal events then pass through the interfaced PYTHIA for parton shower and jet matching. We match up to one jet and choose the merging scale to be 15 GeV. After jet matching, the cross section is 0.61 pb.

The primary backgrounds can be classified as

- (1) $pp \rightarrow \ell\ell\ell\nu_\ell + (j), M_{\ell\ell\ell\nu} \notin \text{OR}$.
- (2) $pp \rightarrow \ell\ell\ell\ell + (j)$.
- (3) $pp \rightarrow t\bar{t} + (j)$.

The first class of background, $pp \rightarrow \ell\ell\ell\nu_\ell + (j), M_{\ell\ell\ell\nu} \notin \text{OR}$, involves three charged leptons plus one missing neutrino final state with the phase space outside the OR regime. The four-lepton invariant mass can be either below $m_W - 2\Gamma_W$ or above $m_W + 2\Gamma_W$. In the low invariant mass region, the four-lepton final states can come from various off-shell states or low-lying hadron decays and are difficult to veto. The events with the lepton invariant masses greater than $m_W + 2\Gamma_W$ are dominated by the on-shell W and Z boson production with leptonic decay, typically having $M_{\ell\ell\ell} > m_Z$.

For the second class, $pp \rightarrow \ell\ell\ell\ell + (j)$, we consider the four charged leptons final states with one lepton lost at the reconstruction level. One of the primary contributions is the single Z boson exotic decay, similar to what we are focusing on [46,47]. Both the single Z boson production rate and its exotic decay branching ratio are smaller than the W boson by a factor of a few. Other principal channels include the production of a double Z boson or a single Z boson with a virtual photon, followed by decay into two pairs of leptons.

The $t\bar{t}$ production is also included as part of the main backgrounds. At $\sqrt{s} = 13 \text{ TeV}$, the cross section of top quark pair production is around 800 pb. A top quark can decay into a bottom quark plus W boson followed by leptonic decay final states. The b jets can be further misidentified as an isolated lepton. As a result, we may observe two real leptons plus one faked “lepton” at the reconstruction level. Although the mistag rate is roughly 10^{-3} [48], it cannot be ignored due to the huge production rate.

All backgrounds are implemented with the same parton-level preselection as the signal events. We also include jet matching and choose the same merging scale as the signal. After merging, the cross section for each background process is listed in the second column of Table I (run-2 benchmark can be found in Supplemental Material [17]).

After passing through PYTHIA8 for parton shower and jet merging, the truth-level sample is sent to DELPHES3 for fast parametric simulation to include various detector effects. We carry out two comparative analyses: one is cut based, and the other is DNN based. A reconstruction-level initial selection according to both the current trigger system from ATLAS and CMS [49,50] and future HL-LHC detector performance [51,52] is shown in this section.

The three charged leptons from the W boson tend to share half of the total invariant mass due to the favored-low invariant mass of two same-flavor charged leptons from the fermion propagator suppression. Hence, keeping the soft charged leptons is key to sizable signal statistics. In the following, we show two analyses based on future and

TABLE I. Cutflow table of signal and backgrounds under 14 TeV HL-LHC benchmark. The p_T , η , and ΔR cuts are absorbed into $n(\ell) = 3$. The machine learning analysis is separated from the cut-based analysis. There is no successive relation between the last two columns.

Cross section [pb]	Parton level	$n(\ell) = 3$	$n(j) \leq 2, M_{\ell\ell} > 4 \text{ GeV}$	$M_{\ell\ell\ell} < 80 \text{ GeV}$	Cut-based result $M_{\ell\ell\ell} < 60 \text{ GeV}$	DNN-based result DNN selection
Signal	0.61(100%)	0.036(5.9%)	0.021(3.5%)	0.021(3.5%)	0.021(3.4%)	0.017(2.7%)
$pp \rightarrow \ell\ell\ell\nu, M_{\ell\ell\ell\nu} \notin \text{OR}$	0.95(100%)	0.22(23%)	0.2(21%)	0.013(1.4%)	8×10^{-3} (0.87%)	3.3×10^{-3} (0.3%)
$pp \rightarrow \ell\ell\ell\ell$	0.34(100%)	0.068(20%)	0.061(18%)	0.017(5%)	7.2×10^{-3} (2.1%)	3.2×10^{-3} (0.95%)
$pp \rightarrow t\bar{t} + (j)$	688(100%)	0.19(0.027%)	0.11(0.016%)	$0.023(3 \times 10^{-5})$	$0.01(1 \times 10^{-5})$	$2.1 \times 10^{-3}(3 \times 10^{-6})$

current multilepton triggers and the projected sensitivity for HL-LHC and LHC run-2, respectively.

HL-LHC benchmark. The multilepton requirement primarily determines the signal efficiency. According to the future HL-LHC multilepton trigger system [51,52], the low threshold of transverse momentum is 5 GeV. The rapidity coverage will be also improved. Thus, the HL-LHC benchmark is treated under an initial selection of

$$\begin{aligned} \eta(\mu) < 2.8, \quad \eta(e, j) < 4, \quad \Delta R(\ell\ell, j\ell) > 0.2, \\ n(\ell) = 3, \quad p_T(\ell) > 5 \text{ GeV}, \\ n(j) \leq 2 \text{ with } p_T(j) > 20 \text{ GeV}, \quad M_{\ell\ell} > 4 \text{ GeV}, \end{aligned}$$

where $n(j)$ and $n(\ell)$ stand for the number of jets and charged leptons. The additional jet veto is to reject the $t\bar{t}$ background because hadronic processes generally produce more jets. The invariant mass of the same-flavor, opposite-charged lepton pair is required to be sizable to avoid the low mass meson resonance. Future detailed analysis could extend the search to this low invariant mass regime.

The cutflow table is shown in Table I. Requiring $n(\ell) = 3$ excludes a large portion of the signal events since there is a higher probability of two soft same-flavor charged leptons appearing. After the initial selection and requiring $M_{\ell\ell\ell} < 80 \text{ GeV}$, we proceed with a cut-based analysis and DNN-based machine learning analysis to separate signal and background. The DNN-based analysis

details can be found in the Supplemental Material [17]. The projected sensitivities of these analyses, based on statistical uncertainty, are

$$\text{Cut-based: } \delta\text{Br}^{\text{fid}}/\text{Br}^{\text{fid}} = 0.71\%, \quad (4)$$

$$\text{DNN-based: } \delta\text{Br}^{\text{fid}}/\text{Br}^{\text{fid}} = 0.62\%. \quad (5)$$

As mentioned previously, the invariant mass of the three charged leptons is a strong feature to reject most of the background processes, as seen in Fig. 3. Cross-checked with the machine learning results, a simple cut-based analysis is also implemented by only requiring $M_{\ell\ell\ell} < 60 \text{ GeV}$, leading to a cut-based analysis precision projection at HL-LHC of 0.71%. The three-lepton invariant mass $M_{\ell\ell\ell}$ is peaked at the lower mass regime, as the neutrino tends to carry away half of the energy in the center-of-mass frame. The $\ell\ell\ell\nu_\ell$ background is suppressed in the low mass regime. Starting from around 100 GeV, the differential distribution rate sharply increases. The on-shell W boson plus γ^*/Z starts to play a role. For the 4ℓ final states channel, there is a peak at around m_Z . This implies that the three ℓ are primarily from the single Z boson exotic decay and one soft lepton is invisible. The γ^*/Z may not contribute significantly, as the virtual photon tends to be forward. Interestingly, the 4ℓ final state decayed from Z boson is significantly vetoed by the invariant mass selection. For Z boson exotic decay, it will likely miss a soft lepton, which only carries a tiny fraction of the total energy. Consequently, most of the Z boson background will still have an invariant mass larger than 80 GeV. While for W boson exotic decay, the neutrino would take away half of the energy in the center-of-mass frame, hence suppressing the three charged leptons' invariant mass. This is the key distinction between W and Z boson exotic decay.

Run-2 benchmark. The analysis of 13 TeV run-2 benchmark is similar but with more conservative selections, which is discussed in Supplemental Material [17]. We end with a precision of

$$\text{Cut-based: } \delta\text{Br}^{\text{fid}}/\text{Br}^{\text{fid}} = 6.5\%, \quad (6)$$

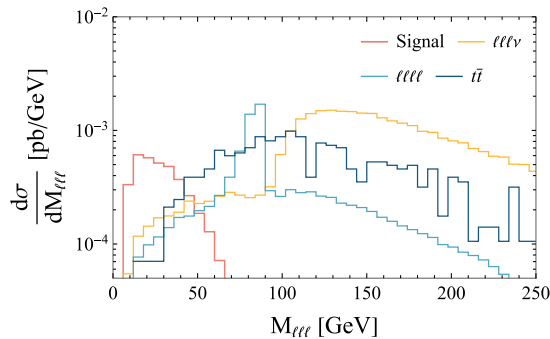


FIG. 3. Reconstruction level distribution of $M_{\ell\ell\ell}$ after initial selection cuts under HL-LHC benchmark.

$$\text{DNN-based: } \delta \text{Br}^{\text{fid}} / \text{Br}^{\text{fid}} = 4.4\% \quad (7)$$

based on statistical uncertainty.

The precision is comparable with the statistical uncertainty of the $Z \rightarrow 4\ell$ branching ratio, which is around 3% [46,47]. In general, the single W boson production rate at the LHC is higher than the Z boson by a factor of 3. Furthermore, the branching ratio of W boson four-lepton decays should also be larger than that of the Z boson, which can be realized from their dilepton decay. However, we conservatively use $\Delta R(\ell\ell) > 0.2$, which significantly reduces signal efficiency. A lower threshold, similar to that in Refs. [46,47], could further improve the measurement.

We comment on the systematic uncertainties here. One of the major systematics is the W boson production rate from PDFs and higher-order corrections. In the HL-LHC benchmark analysis, the low statistical uncertainty of 0.62% motivates us to reduce the systematic uncertainty of $O(1\%)$ as much as possible. For instance, the proton PDF uncertainty is around 3%. However, the measured parameter can be chosen as $\frac{\text{Br}(W \rightarrow \ell\ell\nu_\ell)}{\text{Br}(W \rightarrow \ell\nu_\ell)}$, which is largely independent of proton PDF choices. The pileup also affects our channel since this is a relatively low p_T channel. However, this study mainly depends on the three charged leptons reconstruction with clear separation $\Delta R(\ell\ell) > 0.2$. The LHC has lepton angular resolution of 10^{-3} rad level [51,52]. Therefore, the reconstruction on the main vertex for our channel can be reasonably differed from pileup event. With future HL-LHC upgrade, such as HGTD [53] in ATLAS and MTD [54] in CMS, the pileup effects can be further mitigated.

Sensitivities in Z' models. In this section, we aim to exploit the W boson exotic decay channels and probe directly the BSM models. We first take the $L_\mu - L_\tau$ model and then also discuss the results for Z' with anomalous couplings.

The minimal $L_\mu - L_\tau$ model [55–61] posits that both muon and tau lepton numbers are oppositely charged. It can be spontaneously broken, leading to a massive Z' boson. In the IR, this scenario can be parametrized in terms of the gauge coupling $g_{Z'}$ and the mass $m_{Z'}$. The Z' boson can be radiated from both of the lepton legs from the two-body leptonic decays of the W boson. The relevant Lagrangian is given by

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} m_{Z'}^2 Z_\mu'^2 + g_{Z'} Z'_\nu (\bar{\mu} \gamma^\nu \mu - \bar{\tau} \gamma^\nu \tau) \\ & + g_{Z'} Z'_\nu (\bar{\nu}_\mu \gamma^\nu P_L \nu_\mu - \bar{\nu}_\tau \gamma^\nu P_L \nu_\tau). \end{aligned}$$

The signal event is chosen to be the trimuon final state:

$$pp \rightarrow W^{(*)} + (j) \rightarrow Z' \mu \nu_\mu + (j) \rightarrow \mu \mu \mu \nu_\mu + (j). \quad (8)$$

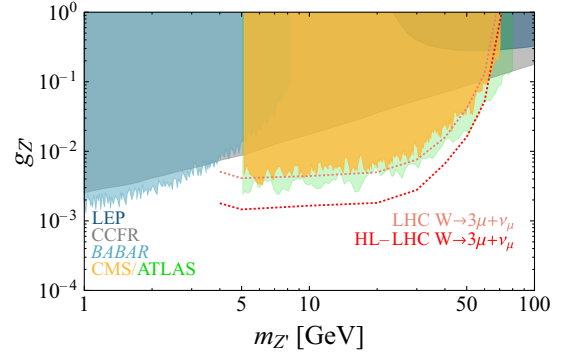


FIG. 4. The 95% C.L. projected sensitivity of $g_{Z'}$ as a function of mass $m_{Z'}$. Shaded regions are excluded by neutrino trident in CCFR [61,63], LEP [64], BABAR [65], CMS $Z \rightarrow 4\mu$ analysis [62], and the recent ATLAS run-2 $W \rightarrow 3\mu + \nu$ analysis [13]. The dashed lines correspond to run-2 (light-red) and HL-LHC (red) benchmarks.

Similar to before, τ -flavored events have very soft leptons with low tagging efficiency and hence require a separate study to optimize a search. We hence focus on the trimuon final state here. As shown in Table I, after requiring $M_{\ell\ell\ell} < 80$ GeV, the dominant background will be the SM process of W boson decay, which was the subject of the previous section. Now, it has become the leading background. For detailed analysis, please see the Supplemental Material [17].

The 95% C.L. projected sensitivities from this W exotic decay channel on coupling strength $g_{Z'}$ versus the Z' mass are shown in Fig. 4 with light-red and red dashed lines for our run-2 analysis and HL-LHC, respectively. We also show the current exclusions from other existing searches. $Z \rightarrow 4\mu$ [62] is an established channel to look for light leptonic Z' . It enjoys a low SM background and is hence helpful. Our run-2 benchmark is generally better than the $Z \rightarrow 4\mu$ channel due to a higher production rate and a higher leptonic decay branching fraction of the W boson. For the Z' mass below 20 GeV, the bound for the coupling $g_{Z'}$ is almost flat since the background is mainly from SM W boson exotic decay and the internal fermion propagator suppression is the same. Above 20 GeV, the SM background reduces, but the signal rate drops quickly due to the phase space suppression, and hence, the sensitivity drops. Close to the final stage of this study, ATLAS put out a Z' search using this exotic decay channel of the W boson [13], which is shown as the green shaded region in Fig. 4. One can see that apart from the fluctuations from the experimental data, the projected sensitivity from our study (light-red dashed line) is highly compatible with the ATLAS search results, which indicates the robustness of our proposed analysis and hence our projections for the SM W exotic decay measurement in the previous section. ATLAS could already carry out a few percent-level precision on this SM W decay, and it is also compatible with the study's background simulation. Even with a slight

subtlety of on-shell rate vs W^* contributions discussed in our paper, the collaboration may want to extend their study to expand the list of the rare W decay branching ratios.

Conclusion and outlook. We explore the W boson four-lepton decay branching ratio at LHC run-2 and HL-LHC and propose to conduct new searches in this channel at the LHC. This new decay channel will not only add new information about the W boson decays but it can also be used to search for interesting SM interference effects and directly for new physics such as Z' models. This channel is statistically competitive with the $Z \rightarrow 4\ell$ channels.

Due to the sizeable decay width of the W boson, the four-lepton final states also receive contributions from other diagrams. We define the fiducial branching ratio for the invariant mass within the $[m_W - 2\Gamma_W, m_W + 2\Gamma_W]$ range. The measurement precision from statistics can reach around 4.4% for run-2 and 0.62% for HL-LHC. In addition, we exploit this decay channel to put constraints on Z' models. The projected sensitivity for the

gauge coupling $g_{Z'}$ at HL-LHC can be improved by a factor of a few. Many new directions to further this study should be carried out in future works. For example, the flavor universality can be tested by examining the different flavor combinations for the four-lepton final states. One can also further explore the Z boson propagator contribution in the exotic decay channel.

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Data availability. The data associated with the figures in this paper can be accessed via Github [66].

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