

***CP* asymmetries corresponding to the imaginary parts of the interference terms in cascade decays of heavy hadrons**

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A mechanism of generating *CP* violation through the imaginary part of the interference of two amplitudes is proposed. This mechanism has shown clear evidence in decays such as $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$. The proposed mechanism is helpful in searching for *CP* violation in bottom and charmed baryon decay processes.

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CP violation (CPV) plays an important role in the Standard Model of particle physics and cosmology. It has been observed in the decays of *K*, *B*, and *D* mesons [1–6]. On the contrary, CPV has not been observed in any baryon decay process, which deserves further theoretical and experimental study.

CPV induced by time-reversal-odd (T-odd) correlations plays a special role in searching for CPV in heavy hadron decays [7,8], among which those induced by the triple-product asymmetries (TPAs), i.e., the distribution asymmetry corresponding to the signature of the triple products of the momenta and/or spins of the particle involved, are the most extensively studied ones [8–18]. TPA induced *CP* asymmetries (CPAs) have been investigated experimentally in decay channels such as: $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$, $D^+ \rightarrow K^+ K^- \pi^+ \pi^0$, $D_{(s)}^+ \rightarrow K^+ K^- \pi^+ \pi^0$, $D_{(s)}^+ \rightarrow K^+ \pi^- \pi^+ \pi^0$, $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$, $\Lambda_b^0 \rightarrow p K^- \pi^+ \pi^-$, $\Lambda_b^0 \rightarrow p K^- K^+ K^-$, and $\Xi_b^0 \rightarrow p K^- K^- \pi^+$ [19–29]. Although a nonzero TPA was observed in the decay channel $\Lambda_b^0 \rightarrow p \pi^- \pi^+ \pi^-$ [28]: $a_P^{\hat{T}\text{-odd}} = (-4.0 \pm 0.7 \pm 0.2)\%$, TPA induces *CP* asymmetry (TP-CPA) has never been observed yet.

It happens that TPAs usually get contributions from the imaginary part of some interfering terms of two amplitudes. It can be shown that TP-CPA is usually proportional to the cosine of the relative strong phases:

$$\Im(ab^*) - \Im(\bar{a}\bar{b}^*) \sim \sin \phi \cos \delta, \quad (1)$$

where *a* and *b* are the two amplitudes, ϕ is the weak phase between them, while δ is the strong phase difference, $\bar{a}$ and $\bar{b}$ are respectively the *CP* conjugates of *a* and *b*. In this short paper, we propose another mechanism which can generate CPAs that are also proportional to the imaginary part of some interfering terms in multibody decays of heavy hadrons, just as TP-CPA does. In this mechanism, the interference occurs between the amplitudes corresponding to two intermediated resonances. Unlike the situation of TP-CPA which has never been observed experimentally, we will show that the CPV induced by the currently proposed mechanism has in fact already left clear evidence in three-body decays of bottom mesons such as $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$.

Let us take a three-body decay $\mathbb{H} \rightarrow abc$ to illustrate, where $\mathbb{H}$ is the mother hadron, and *a*, *b*, and *c* are the three hadrons in the final state. We will consider cascade decays of the form $\mathbb{H} \rightarrow rc$, $r \rightarrow ab$, where *r* represents an intermediate resonance. The decay amplitudes can be expressed in the form of

$$\mathcal{M} = \sum_r \frac{\mathcal{A}_r}{s_r}, \quad (2)$$

where $\mathcal{A}_r$ is the decay amplitude for the cascade process $\mathbb{H} \rightarrow rc$, $r \rightarrow ab$, s_r is the reciprocal of the Breit–Wigner factor, which approximately takes the form $s_r = s - m_r^2 + im_r \Gamma_r$, with *s* the invariant mass of the *ab* system, the summation over *r* indicates that there can be more than one intermediate resonances. The decay amplitude squared can then be expressed as

$$|\overline{\mathcal{M}}|^2 = \sum_{r,r'} \frac{\mathcal{A}_r \mathcal{A}_{r'}^*}{s_r s_{r'}^*}, \quad (3)$$

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where the overline above the decay amplitude squared indicates that there may be spin or helicity summation over the initial and the final states. Note that the possible spin or helicity summations have been omitted on the right-hand side of the above equation. One can see that there can be interferences between different intermediate resonances, provided that the decay amplitude is dominated by at least two intermediate resonances in some regions of the phase space. This happens when the masses of the intermediate resonances are close to each other, and their decay widths are not small. We consider the simplest situation when two intermediate resonances dominate in some regions of the phase space. In the interfering region of the phase space, the decay amplitude squared can then be expressed as

$$|\overline{\mathcal{M}}|^2 \approx \frac{|\mathcal{A}_1|^2}{|s_1|^2} + \frac{|\mathcal{A}_2|^2}{|s_2|^2} + 2\Re\left(\frac{\mathcal{A}_1\mathcal{A}_2^*}{s_1s_2^*}\right). \quad (4)$$

For the convenience of the subsequent analysis, we can further express the interfering term as

$$\Re\left(\frac{\mathcal{A}_1\mathcal{A}_2^*}{s_1s_2^*}\right) = \frac{\Re(\mathcal{A}_1\mathcal{A}_2^*)\Re(s_1s_2^*) + \Im(\mathcal{A}_1\mathcal{A}_2^*)\Im(s_1s_2^*)}{|s_1s_2|^2}. \quad (5)$$

Note that we have divided the two-resonance-interfering term into two different terms, which are respectively proportional to $\Re(s_1s_2^*) = m_1\Gamma_1m_2\Gamma_2 + (s - m_1^2)(s - m_2^2)$ and $\Im(s_1s_2^*) = (s - m_2^2)m_1\Gamma_1 - (s - m_1^2)m_2\Gamma_2$. The second term on the right-hand side of Eq. (5) is especially interesting since it is proportional to the imaginary part of the interfering term $\mathcal{A}_1\mathcal{A}_2^*$. By generally expressing $\mathcal{A}_1$ and $\mathcal{A}_2$ in terms of amplitudes with different weak phases, $\mathcal{A}_1 = \sum_k \mathcal{A}_1^k$ and $\mathcal{A}_2 = \sum_{k'} \mathcal{A}_2^{k'}$, it can be seen that CPA induced by the imaginary part of the interfering term $\mathcal{A}_1\mathcal{A}_2^*$ will be a collection of terms which are proportional to the cosine of some unitary phases:

$$\begin{aligned} A_{\Im(\mathcal{A}_1\mathcal{A}_2^*)} &\equiv \Im(\mathcal{A}_1\mathcal{A}_2^*) - \Im(\overline{\mathcal{A}_1}\overline{\mathcal{A}_2^*}) \\ &\propto \sum_{kk'} |\mathcal{A}_1^k \mathcal{A}_2^{k'}| \sin \phi_{kk'} \cos \delta_{kk'}, \end{aligned} \quad (6)$$

where $\overline{\mathcal{A}_1}$ and $\overline{\mathcal{A}_2}$ are the corresponding amplitudes for the CP -conjugate process, $\phi_{kk'}$ and $\delta_{kk'}$ are respectively the weak and unitary phase differences between $\mathcal{A}_1^k$ and $\mathcal{A}_2^{k'}$. In comparison, CPA induced by the real part of the interfering term $\mathcal{A}_1\mathcal{A}_2^*$ takes the form

$$\begin{aligned} A_{\Re(\mathcal{A}_1\mathcal{A}_2^*)} &\equiv \Re(\mathcal{A}_1\mathcal{A}_2^*) - \Re(\overline{\mathcal{A}_1}\overline{\mathcal{A}_2^*}) \\ &\propto \sum_{kk'} |\mathcal{A}_1^k \mathcal{A}_2^{k'}| \sin \phi_{kk'} \sin \delta_{kk'}. \end{aligned} \quad (7)$$

It is more common that the decay width of one of the resonances is considerably larger than that of the other one. For the current situation, suppose that $\Gamma_1 > \Gamma_2$. We also expect that the masses of the two resonances are close to

each other, so that we have $\Gamma_1 > m_1 - m_2$. Besides, since we are focusing on the interfering effect, it will be enough for us to constrain s in the interfering region, so that we have $\Gamma_1 > |s - m_1^2|/m_1$ and $\Gamma_1 > |s - m_2^2|/m_2$. With the above consideration, one can see that $\Re(s_1s_2^*)$ is dominated by the first term, so that one has $\Re(s_1s_2^*) \approx m_1\Gamma_1m_2\Gamma_2$. Meanwhile, we can rearrange $\Im(s_1s_2^*)$ into the form

$$\Im(s_1s_2^*) = m_1\Gamma_1 \left(1 - \frac{m_2\Gamma_2}{m_1\Gamma_1}\right) (s - m_2^2), \quad (8)$$

where $m_2' = m_2 \sqrt{\left(1 - \frac{m_1\Gamma_2}{m_2\Gamma_1}\right) / \left(1 - \frac{m_2\Gamma_2}{m_1\Gamma_1}\right)}$. Now the difference between the behavior of $\Re(s_1s_2^*)$ and $\Im(s_1s_2^*)$ is obvious: while the latter tends to change sign as s passes through m_2^2 , the former does not.

The aforementioned behavior difference allows us to isolate the contributions of the term $\Im(\mathcal{A}_1\mathcal{A}_2^*)$ to CPV. For a CPV observable which is defined on the phase space region $m_2^2 - \Delta_- < s < m_2^2 + \Delta_+$, with Δ_- and Δ_+ about the same order as Γ_2^2 and/or $(m_1 - m_2)^2$,

$$A_{CP} \equiv \frac{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} (|\overline{\mathcal{M}}|^2 - |\mathcal{M}|^2) ds}{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} (|\overline{\mathcal{M}}|^2 + |\mathcal{M}|^2) ds}, \quad (9)$$

where $\bar{\mathcal{M}}$ is the decay amplitude for the CP -conjugate process; the contribution of the term $\Im(\mathcal{A}_1\mathcal{A}_2^*)$ to A_{CP} is pretty much negligible, because that there is strong cancellation for the term $\Im(s_1s_2^*)$ when integrating over $m_2^2 - \Delta_- < s < m_2^2$ and $m_2^2 < s < m_2^2 + \Delta_+$. To avoid this cancellation, one can introduce a sign factor in the definition of the above CPV observables:

$$A_{CP}^{\Im} \equiv \frac{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} (|\overline{\mathcal{M}}|^2 - |\mathcal{M}|^2) \text{sgn}(s - m_2^2) ds}{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} (|\overline{\mathcal{M}}|^2 + |\mathcal{M}|^2) ds}, \quad (10)$$

where the sign function sgn is defined as

$$\text{sgn}(x) = \begin{cases} +1, & x > 0; \\ -1, & x < 0. \end{cases} \quad (11)$$

It can be shown that the newly introduced CPV observable $A_{CP}^{\Im}$ gets its contribution mainly from the term $\Im(\mathcal{A}_1\mathcal{A}_2^*)$. To see this, one needs to reexpress Eq. (10) as

$$\begin{aligned} A_{CP}^{\Im} &\propto A_{\Im(\mathcal{A}_1\mathcal{A}_2^*)} I_1^{(\Delta_-, \Delta_+)} + A_{\Re(\mathcal{A}_1\mathcal{A}_2^*)} I_2^{(\Delta_-, \Delta_+)} \\ &\quad + A_{|\mathcal{A}_1|^2} I_3^{(\Delta_-, \Delta_+)} + A_{|\mathcal{A}_2|^2} I_4^{(\Delta_-, \Delta_+)}, \end{aligned} \quad (12)$$

where $A_{|\mathcal{A}_1|^2} \equiv |\mathcal{A}_1|^2 - |\overline{\mathcal{A}_1}|^2$ and $A_{|\mathcal{A}_2|^2} \equiv |\mathcal{A}_2|^2 - |\overline{\mathcal{A}_2}|^2$ are respectively measures of CPV in the two-body decays $\mathbb{H} \rightarrow 1c$ and $\mathbb{H} \rightarrow 2c$, and the four integrals are respectively defined as

$$I_1^{(\Delta_-, \Delta_+)} \equiv m_1 \Gamma_1 \left(1 - \frac{m_2 \Gamma_2}{m_1 \Gamma_1} \right) \int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \frac{(s - m_2^2) \text{sgn}(s - m_2^2)}{|s_1 s_2|^2} ds, \quad (13)$$

$$I_2^{(\Delta_-, \Delta_+)} \equiv \int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \frac{[m_1 \Gamma_1 m_2 \Gamma_2 + (s - m_1^2)(s - m_2^2)] \text{sgn}(s - m_2^2)}{|s_1 s_2|^2} ds, \quad (14)$$

$$I_3^{(\Delta_-, \Delta_+)} \equiv \int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \frac{\text{sgn}(s - m_2^2)}{|s_1|^2} ds, \quad (15)$$

$$I_4^{(\Delta_-, \Delta_+)} \equiv \int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \frac{\text{sgn}(s - m_2^2)}{|s_2|^2} ds. \quad (16)$$

As can be seen from the above equations, the presence of the factor $\text{sgn}(s - m_2^2)$ makes sure that there are cancellations for the integrals between $m_2^2 - \Delta_- < s < m_2^2$ and $m_2^2 < s < m_2^2 + \Delta_+$ parts in $I_2^{(\Delta_-, \Delta_+)}$, $I_3^{(\Delta_-, \Delta_+)}$, and $I_4^{(\Delta_-, \Delta_+)}$. In contrast, it is the same factor, together with $(s - m_2^2)$, that guarantees that there are no such kind of cancellations in $I_1^{(\Delta_-, \Delta_+)}$. Consequently, the contribution of the first term on the right-hand side of Eq. (12) containing $\Im(\mathcal{A}_1 \mathcal{A}_2^*)$ is amplified, while the last three terms are attenuated.

Clearly, Eq. (10) cannot isolate the contribution of the term $\Im(\mathcal{A}_1 \mathcal{A}_2^*)$ in an ideal way, in the sense that the introduction of the sign factor cannot entirely eliminate the contributions of other terms to $A_{CP}^{\Im}$. Despite that, it does not affect the fact that $A_{CP}^{\Im}$ gets its main contribution from the term $\Im(\mathcal{A}_1 \mathcal{A}_2^*)$. It is possible to further suppress the contribution of other terms in Eq. (12). For example, one can arrange the values of Δ_- and Δ_+ to fulfill the condition that $I_2^{(\Delta_-, \Delta_+)} = 0$, $I_3^{(\Delta_-, \Delta_+)} = 0$, or $I_4^{(\Delta_-, \Delta_+)} = 0$. However, it does not seem necessary to do so, because $A_{CP}^{\Im}$ is worth being measured especially when one cannot find CPV elsewhere, which means that $A_{\Re(\mathcal{A}_1 \mathcal{A}_2^*)}$, $A_{|\mathcal{A}_1|^2}$, and $A_{|\mathcal{A}_2|^2}$ are likely to be small. Consequently, it is unnecessary to arrange the values of Δ_- and Δ_+ . One can even just simply choose $\Delta_- = \Delta_+$.

This seems to be a good place to stress that although we adopt the Breit–Wigner formula for the propagator factors, the basic idea does not depend on this choice. Explicitly, the sign changing characteristics in Eq. (8) is independent of the

choice of the propagator factor formula. However, the value of the zero point m_2^2 does depend on the choice; making the value of m_2^2 may be different in different choices of propagator factor formulas. To reduce the uncertainties caused by the values of the zero point m_2^2 , an alternative choice for Eq. (10) is to remove a small section in the integral

$$A_{CP}^{\Im, \text{alt}} \equiv \frac{(\int_{m_2^2 - \Delta_-}^{m_2^2} - \int_{m_2^2}^{m_2^2 + \Delta_+})(|\overline{\mathcal{M}}|^2 - |\mathcal{M}|^2) ds}{(\int_{m_2^2 - \Delta_-}^{m_2^2} + \int_{m_2^2}^{m_2^2 + \Delta_+})(|\overline{\mathcal{M}}|^2 + |\mathcal{M}|^2) ds}, \quad (17)$$

where m'_{2-} and m'_{2+} are chosen to fulfill the condition $m'_{2-} < m_2^2 < m'_{2+}$.

It often happens that the quantum number of the two resonances are different, which means that the kinematical behavior of the interfering terms is different from the noninterfering terms. For example, the interference of a scalar resonance and a vector resonance leads to forward-backward asymmetry (FBA) in the decay-angular distributions, which can further induce corresponding CPAs. In the phase space region $m_2^2 - \Delta_- < s < m_2^2 + \Delta_+$, the FBA induced CPA¹ can be defined as [30–32]

$$A_{CP}^{FB} \equiv \frac{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \int_{-1}^{+1} (|\overline{\mathcal{M}}|^2 - |\mathcal{M}|^2) \text{sgn}(c_\theta) dc_\theta ds}{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \int_{-1}^{+1} (|\overline{\mathcal{M}}|^2 + |\mathcal{M}|^2) dc_\theta ds}, \quad (18)$$

where $c_\theta \equiv \cos \theta$, with θ being a helicity angle which is defined as the angle between momenta of a and c in the center-of-mass frame of the ab system. It can be easily shown that this A_{CP}^{FB} gets the contribution mainly from $\Re(\mathcal{A}_1 \mathcal{A}_2^*)$. Just as the above, by introducing the sign factor $\text{sgn}(s - m_2^2)$, one can obtain a CPV observable which gets its contribution mainly from the term $\Im(\mathcal{A}_1 \mathcal{A}_2^*)$

$$A_{CP}^{FB, \Im} \equiv \frac{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \int_{-1}^{+1} (|\overline{\mathcal{M}}|^2 - |\mathcal{M}|^2) \text{sgn}(c_\theta) \text{sgn}(s - m_2^2) dc_\theta ds}{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \int_{-1}^{+1} (|\overline{\mathcal{M}}|^2 + |\mathcal{M}|^2) dc_\theta ds}. \quad (19)$$

In general, it is also possible that the interfering terms enter in the angular distributions of the final particles. For example, for the partial-wave CPA which can be defined as [33,34]

¹The FB-CPA defined in Eq. (18) is actually called direct-CPA-subtracted FB-CPA in Ref. [30].

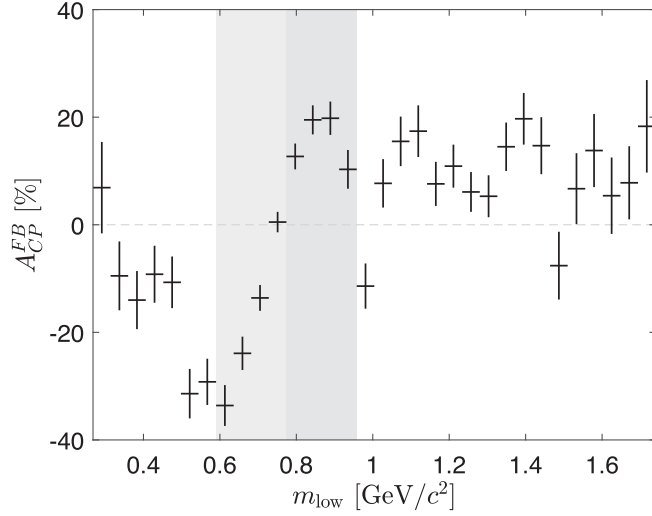


FIG. 1. The FB-CPA of each bin distributed in the low m_{low} region for $B^\pm \rightarrow \pi^+ \pi^- \pi^\pm$. To obtain the FB-CPA of each bin, both the differences between the event yields of the CP conjugate processes of each bin, $N_{B^-} - N_{B^+}$, and the sum between them, $N_{B^-} + N_{B^+}$, are necessary. The former can be directly read off from Fig. 12 of Ref. [36], while the latter can be obtained according to $N_{B^-} + N_{B^+} = \sigma^2$, where σ is the error of $N_{B^-} - N_{B^+}$ and can be read off from the same figure. Note that we have simply assumed that σ contains only the statistical error.

$$A_{CP}^{(l)} \equiv \frac{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \int_{-1}^{+1} (|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2) P_l(c_\theta) dc_\theta ds}{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \int_{-1}^{+1} (|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2) dc_\theta ds}, \quad (20)$$

with P_l the l -Legendre polynomial, it gets the contribution from the real parts of certain interfering terms. The contribution of the imaginary parts of these interfering

terms to CPV are mainly contained in

$$A_{CP}^{(l), \Im} \equiv \frac{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \int_{-1}^{+1} (|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2) P_l(c_\theta) \text{sgn}(s - m_2^2) dc_\theta ds}{\int_{m_2^2 - \Delta_-}^{m_2^2 + \Delta_+} \int_{-1}^{+1} (|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2) dc_\theta ds}. \quad (21)$$

Unlike the TP-CPAs, CPA induced by the imaginary part of the interfering term $\Im(\mathcal{A}_i \mathcal{A}_j^*)$ has in fact been observed experimentally in a sense. Take the decay $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ as an example [35,36]. The experimental results of the LHCb collaboration indicate that there is a strong interfering effect between $\rho(770)^0$ and $f_0(500)$ in the phase space of the intermediate resonance $\rho(770)^0$ region [37]. As is pointed out by the author of Ref. [38], the results of LHCb imply strongly that the imaginary part of the interfering terms $\text{Im}(c_-^{*\rho} c_-^\rho + c_-^\rho c_-^{*\sigma})$ gives large contribution to the regional CPAs [38]. Based on the data extracted from Fig. 12 of Ref. [36], we obtain the FB-CPA of each bin distributed in the invariant mass of the $\pi^+ \pi^-$ pair with lower invariant mass m_{low} , as shown in Fig. 1. As can be inferred from Eq. (18), FB-CPA of each bin is defined as

$$A_{CP,k}^{FB} = \frac{(N_{B^-} - N_{B^+})_{\cos \theta_{\text{hel}} > 0,k} - (N_{B^-} - N_{B^+})_{\cos \theta_{\text{hel}} < 0,k}}{(N_{B^-} + N_{B^+})_{\cos \theta_{\text{hel}} > 0,k} + (N_{B^-} + N_{B^+})_{\cos \theta_{\text{hel}} < 0,k}}, \quad (22)$$

where k is the bin index. From Fig. 1 one can see that A_{CP}^{FB} changes rapidly around the $\rho(770)^0$ resonance. For the eight bins around the $\rho(770)^0$ resonance which are highlighted with gray shaded areas in Fig. 1, the averaged FB-CPA is obtained to be

$$A_{CP}^{FB, \text{ave}} = \frac{\sum_{k=8}^{15} [(N_{B^-} - N_{B^+})_{\cos \theta_{\text{hel}} > 0,k} - (N_{B^-} - N_{B^+})_{\cos \theta_{\text{hel}} < 0,k}]}{\sum_{k=8}^{15} [(N_{B^-} + N_{B^+})_{\cos \theta_{\text{hel}} > 0,k} + (N_{B^-} + N_{B^+})_{\cos \theta_{\text{hel}} < 0,k}]} = (0.8 \pm 1.0)\%. \quad (23)$$

This result shows clearly that while the FB-CPA of each bin around the $\rho(770)^0$ resonance are relatively large, there is a strong cancellation when combining all the eight bins to obtain the averaged one. On the other hand, by introducing a sign factor $\text{sgn}(s - m_2^2)$ for m_2^2 close to the mass of the $\rho(770)^0$ resonance, one can avoid the cancellation²

$$A_{CP}^{FB, \Im} = \frac{(\sum_{k=12}^{15} - \sum_{k=8}^{11}) [(N_{B^-} - N_{B^+})_{\cos \theta_{\text{hel}} > 0,k} - (N_{B^-} - N_{B^+})_{\cos \theta_{\text{hel}} < 0,k}]}{\sum_{k=8}^{15} [(N_{B^-} + N_{B^+})_{\cos \theta_{\text{hel}} > 0,k} + (N_{B^-} + N_{B^+})_{\cos \theta_{\text{hel}} < 0,k}]} = (13.2 \pm 1.0)\%. \quad (24)$$

The analysis of this paper is especially helpful for CPV searching in baryon decay processes. Unlike the bottom

²It will not make much difference if we include more bins. For example, if bin 7 and bin 16 are included, Eqs. (23) and (24) become $A_{CP}^{FB, \text{ave}} = (-1.1 \pm 0.9)\%$ and $A_{CP}^{FB, \Im} = (12.8 \pm 0.9)\%$, respectively.

and charm meson decays, the bottom and charmed baryon decay processes are quite limited by the substantial lower statistics. It is impossible to see the interfering effect as clear as that in the B meson decay processes such as $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$. As a consequence, a measurement of decay-angular distribution CPV observables such as the aforementioned A_{CP}^{FB} and $A_{CP}^{(l)}$ in the interfering region only

reveal the contribution of the term $\Re(\mathcal{A}_1\mathcal{A}_2^*)$. The contribution of the term $\Im(\mathcal{A}_1\mathcal{A}_2^*)$ to CPV will simply be missed. In order to search for the CPV effect caused by the term $\Im(\mathcal{A}_1\mathcal{A}_2^*)$ in bottom and charmed baryon decay processes, which has never been raised up explicitly in the literature, we strongly suggest measuring the observables such as $A_{CP}^{FB,\Im}$ and $A_{CP}^{(I),\Im}$.

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