

Gravitational form factors of pion from top-down holographic QCD

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The gravitational form factors (GFFs) of pions are calculated from a top-down holographic quantum chromodynamics (QCD) approach with momentum transfer dependence for the first time. It is important because the GFFs of hadrons have information on the internal stress distribution that may provide insight into the mechanism of how QCD forms hadrons. The forward limit value of this GFFs, i.e., the D-term, was also obtained. Furthermore, in this approach, we observe the so-called glueball dominance, in which pions have gravitational interactions via infinite glueball spectra.

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Introduction. How forces act inside hadrons to confine quarks and gluons forming hadrons has not been solved. Addressing this question is inseparable from developing an insight into the nonperturbative properties of quantum chromodynamics (QCD), such as confinement and spontaneous chiral symmetry breaking, which is a crucial problem in modern physics.

Recently, the stress inside hadrons has attracted attention (see a recent review [1,2]) due to the fact that experimentally it has been measured. The stress distribution inside nucleons, extracted from the matrix elements of the nucleon energy-momentum tensor (EMT), is obtained from deeply virtual Compton scattering [3–5]. The form factors that characterize the matrix elements are called gravitational form factors (GFFs), which are calculated theoretically from their coupling with gravity.

The forces inside nucleons were analyzed experimentally in Ref. [6] for the quark part in 2018, and in Ref. [7] for the gluon part in 2022. Experimental results show that around the center of the nucleon, the force pushes the quarks outward, while around the surface it reverses into a confining force, indicating an extremely anisotropic behavior. As a result, the pressure near the center of the nucleon may reach the highest value currently observed in nature. The pion GFFs have also been analyzed experimentally in Refs. [8–10]. The GFFs of nucleons [11–19] and pions [13,17,20–24] have also been

measured by lattice QCD analysis, a first-principles numerical simulation of QCD.

In this letter, we are the first to calculate the momentum transfer dependence of the pion GFFs from a top-down holographic QCD approach. We use the Sakai-Sugimoto model, a top-down holographic QCD model based on the D4-D8 brane construction [25,26]. With this approach, the spectroscopy [25,27–43], decay widths and electromagnetic form factors [26,44–68] of mesons, baryons, and glueballs have been investigated with strong coupling and large N_c limit. Recently, using the same approach, nucleon GFFs have been formulated and D-terms have been estimated [69]. The GFFs of pions [70] and other hadrons [71–77] have also been calculated using bottom-up holographic QCD, which has no direct connection with QCD through string theory.

It is interesting that stress distributions and GFFs of pions [78–96], which are the focus of this letter, show anomalous behavior. The GFFs are very sensitive to interactions in the system. The forward limit of the GFFs giving the stress distribution is called the D-term, which takes the value -1 for spin-zero particles in the free Klein-Gordon theory. On the other hand, even an infinitesimally small interaction causes a significant deviation from -1 [84]. However, the D-term of the Nambu-Goldstone (NG) bosons in the strongly coupled system becomes -1 like the free particles, in the soft pion limit because of the restriction by chiral symmetry [78,84,97].

This fact may provide some insight into the role of chiral symmetry breaking in hadron formation since the D-term carries information about the forces inside hadrons. These insights are expected to be validated in more detail via analysis of hadron GFFs from future hadron experiments [98–102].

Formulation. In this section, we provide the method to derive the GFFs of pions in the top-down holographic

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QCD [25,26]. The method for calculating the GFFs of nucleons with spin 1/2 has been formulated in Ref. [69] and will be applied in the following.

The matrix elements of EMT of a spin-0 particle are expressed as

$$\begin{aligned} & \langle \pi^a(p_2) | \hat{T}^{\mu\nu}(x) | \pi^b(p_1) \rangle \\ &= \delta^{ab} \left[2P^\mu P^\nu A(t) + \frac{1}{2} (\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2) D(t) \right] e^{ix \cdot \Delta}, \end{aligned} \quad (1)$$

where a, b are the indices for isospin and the kinematic variables are defined as $P^\mu = (p_1^\mu + p_2^\mu)/2$, $\Delta^\mu = p_2^\mu - p_1^\mu$ and $t = \Delta^2$ with Lorentz indices μ, ν [1,103,104]. In this paper, we employ the mostly plus metric $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ and the normalization condition as $\langle \vec{p}_2 | \vec{p}_1 \rangle = 2E(2\pi)^3 \delta^{(3)}(\vec{p}_1 - \vec{p}_2)$, $E = \sqrt{\vec{p}_1^2 + m^2}$.

The method for calculating the matrix elements of the EMT by holographic QCD is as follows. Within the framework of holographic QCD, perturbations to the metric caused by matter fields in the bulk propagate to the boundary, allowing us to extract the expectation values of the EMT in the boundary theory from these perturbations. The top-down holographic QCD used in this study, the Sakai-Sugimoto model, consists of N_f D8 branes embedded as a probe in black 4-brane background [105], a holographic dual of N_c D4 branes. This background is a 10-dimensional curved spacetime of type IIA supergravity, derived from 11-dimensional supergravity related to M-theory on doubly Wick rotated AdS_7 black hole $\times S^4$,

$$\begin{aligned} ds^2 &= \frac{r^2}{L^2} [f(r) d\tau^2 - dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_{11}^2] \\ &\quad + \frac{L^2}{r^2} \frac{dr^2}{f(r)} + \frac{L^2}{4} d\Omega_4^2, \\ f(r) &= 1 - \frac{R^6}{r^6}, \quad R = \frac{L^2 M_{KK}}{3}, \quad L^3 = 8\pi g_s N_c l_s^3, \\ R_{11} &= \frac{\lambda}{2\pi N_c M_{KK}}, \quad \lambda N_c = \frac{L^6 M_{KK}}{32\pi g_s l_s^5} = 216\pi^3 \kappa, \end{aligned} \quad (2)$$

where M_{KK} is the Kaluza-Klein mass in the τ direction, g_s string coupling, l_s string length, λ 't Hooft coupling, and R_{11} the radius of the circle compactified in the x_{11} direction.

The expectation values of EMT on the boundary, $\langle T_{\mu\nu} \rangle$, can be extracted from the boundary values of the metric fluctuations in the bulk in the background spacetime (2)

propagated to the boundary as follows,

$$\int_{x_{11}, \tau} \delta g_{\mu\nu} \sim \frac{2\kappa_7^2 L^5}{6} \frac{\langle T_{\mu\nu} \rangle}{r^4} + \dots, \quad (3)$$

where κ_7^2 is related to the 11-dimensional gravitational constant as $\kappa_{11}^2 = \text{Vol}(S^4) \kappa_7^2 = 8\pi^2/3 (L/2)^4 \kappa_7^2$. Note that, to obtain the above, we need to apply the holographic renormalization in the M-theory, developed by Ref. [106]. The reason for considering the 11-dimensional theory is that it is technically convenient to perform holographic renormalization. In the nonconformal 10-dimensional theory, holographic renormalization was developed in Ref. [107].

The metric fluctuations δg_{MN} of the bulk obey the following linearized Einstein equations:

$$\begin{aligned} \bar{\mathcal{H}}_{MN} &= \mathcal{H}_{MN} - \frac{g_{MN}}{2} \mathcal{H}_P^P = -2\kappa_7^2 \mathcal{T}_{MN}, \\ \mathcal{H}_{MN} &\equiv \nabla^2 \delta g_{MN} + \nabla_M \nabla_N \delta g_P^P \\ &\quad - \nabla^P (\nabla_M \delta g_{NP} + \nabla_N \delta g_{MP}) - \frac{12}{L^2} \delta g_{MN}, \end{aligned} \quad (4)$$

where M, N and P are the Lorenz indices of the 7-dimensional space-time after integration over S^4 , as in $M, N, P = 0, 1, 2, 3, \tau, r, 11$, and the source $\mathcal{T}_{MN}$ is the EMT of the matter fields which here considered only the pion field. We solve Eq. (4) under the axial gauge, $\delta g_{Mr} = 0$.

In order to examine the EMT $\mathcal{T}_{MN}$ in the bulk, we embed the D8 branes into the 11-dimensional space, considering the action as

$$\begin{aligned} S_{D8} &= -\frac{C}{2\pi R_{11} g_s} \int d\tau dx_{11} d^4 x dr d\Omega_4 (\delta(\tau) + \delta(\tau - \pi/M_{KK})) \\ &\quad \times \frac{\sqrt{-G_{(11)}}}{\sqrt{G_{\tau\tau}}} \frac{1}{4} G^{MN} G^{PQ} \text{tr}[F_{MP} F_{NQ}] + \dots, \end{aligned} \quad (5)$$

with $C = (64\pi^6 l_s^5)^{-1}$ and the metric tensor G_{MN} in Eq. (2). We define the EMT as

$$\mathcal{T}_{MN} = -\frac{2}{\sqrt{-G_{(11)}}} \frac{\delta S_{D8}}{\delta G_{(11)}^{MN}} \text{Vol}(S^4), \quad (6)$$

which leads to the following expression:

$$\begin{aligned}
\mathcal{T}_{MN}(\vec{\Delta}, r) &\equiv 2\pi R_{11} \int d\tau' \mathcal{T}_{MN}(\tau', \vec{k}, r), \\
\mathcal{T}_{\mu\nu}(\vec{\Delta}, r) &= \frac{L^5}{r^5} \frac{\pi^2 C r L^2}{3g_s \sqrt{f(r)}} \text{tr} \left[F_{\mu\rho} F_{\nu}{}^{\rho} + (1+z^2)^{4/3} M_{KK}^2 F_{\mu z} F_{\nu z} - \frac{\eta_{\mu\nu}}{4} (F_{\rho\sigma} F^{\rho\sigma} + 2(1+z^2)^{4/3} M_{KK}^2 F_{\rho z} F^{\rho}{}_z) \right], \\
\mathcal{T}_{11,11}(\vec{\Delta}, r) &= \frac{L^5}{r^5} \frac{\pi^2 C r L^2}{3g_s \sqrt{f(r)}} \text{tr} \left[-\frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} - \frac{(1+z^2)^{4/3} M_{KK}^2}{2} F_{\rho z} F^{\rho z} \right], \\
\mathcal{T}_{\mu z}(\vec{\Delta}, r) &= \frac{L^5}{r^5} \frac{\pi^2 C r L^2}{3g_s \sqrt{f(r)}} \text{tr} [F_{\mu\rho} F_z{}^{\rho}], \quad \mathcal{T}_{zz}(\vec{\Delta}, r) = \frac{L^5}{r^5} \frac{\pi^2 C r L^2}{3g_s \sqrt{f(r)}} \text{tr} \left[\frac{1}{2} F_{z\rho} F_z{}^{\rho} - \frac{1}{4(1+z^2)^{4/3} M_{KK}^2} F_{\rho\sigma} F^{\rho\sigma} \right], \quad (7)
\end{aligned}$$

where the radial coordinate $r (R \leq r < \infty)$ is related to $z (-\infty < z < \infty)$ as $z = \pm \sqrt{r^6/R^6 - 1}$. In this study, we consider the case where the pion field is the only matter field as the source of the gravitational waves and decompose $A_M(x^\nu, z)$ as follows

$$\begin{aligned}
A_\mu(x^\nu, z) &= 0, \quad A_z(x^\nu, z) = \pi(x^\mu) \phi_0(z), \\
\phi_0(z) &= c_0 (1+z^2)^{-1}, \quad (8)
\end{aligned}$$

where $c_0^{-2} = 2M_{KK}^2 \kappa \int dz (1+z^2) \phi_0(z)^2$ is the normalization constant of the eigenfunction $\phi_0(z)$ [25]. Since the pion field is an external line, it can be replaced by a plane wave as $\pi(x^\nu) \sim e^{ip_\nu x^\nu}$ with incoming momentum p_ν . More precisely, following the method discussed in Ref. [70], we calculate the three-point function $\langle J_A^\alpha T^{\mu\nu} J_A^\beta \rangle$ of the axial vector current J_A^α and the EMT $T^{\mu\nu}$ by the Gubser-Klebanov-Polyakov and Witten (GKP-Witten) relation [108,109] and extract only the pion states using the completeness relation of spectra of the axial vector meson.

Since we are interested in the EMT of QCD at the boundary such as to satisfy $\Delta^\mu \langle T_{\mu\nu} \rangle = 0$, we consider the metric fluctuations that are transverse $\Delta^\mu \delta g_{\mu\nu} \sim 0$ near the boundary, which can be parametrized as $\delta g_{ij} \sim \delta g_{ij}^{\text{TT}} + (\delta_{ij} - \Delta_i \Delta_j / \Delta^2) / 5 \delta g_\alpha^\alpha$ with the transverse-traceless (TT) part $\delta g_{ij}^{\text{TT}}$, $i, j = 1, 2, 3$ and $\alpha = \mu, \tau, 11$. Under the axial gauge, the Einstein equation requires the six-dimensional traceless condition $\delta g_\alpha^\alpha \sim 0$ in the asymptotic AdS region ($r \rightarrow \infty$) labeled by (x^μ, τ, x^{11}) . Therefore, we consider TT modes in the metric fluctuation, which is nothing but glueball spectra. Among the 14 TT modes named in Ref. [28] (see also Ref. [27]), only T_4 and S_4 contribute to the EMT. Actually, substituting the pion field (8) into Eq. (15), we confirm that $\delta g_{\mu\nu}^{\text{other}}$ falls faster than T_4 and S_4 mode at $r \rightarrow \infty$. By decomposing the metric fluctuations into each mode, $\delta g_{\mu\nu} = \delta g_{\mu\nu}^{\text{T}(2)} + \delta g_{\mu\nu}^{\text{T}(0)} + \delta g_{\mu\nu}^{\text{S}(0)} + \delta g_{\mu\nu}^{\text{other}}$, the Einstein equations reduce to

$$\frac{r^2}{L^2} \left[\frac{1}{L^2 r^5} \partial_r ((r^7 - rR^6) \partial_r) - \frac{L^2 \Delta^2}{r^2} \right] \frac{L^2}{r^2} \delta g_{\mu\nu}^{\text{T}} = -2\kappa_7^2 \mathcal{T}_{\mu\nu}^{\text{T}}, \quad (9)$$

$$\begin{aligned}
&\frac{r^2}{L^2} \left[\frac{1}{L^2 r^5} \partial_r \left((r^7 - rR^6) \left(\partial_r + \frac{144r^5 R^6}{(5r^5 - 2R^6)(r^6 + 2R^6)} \right) \right) \right. \\
&\quad \left. - \frac{L^2 \Delta^2}{r^2} \right] \frac{L^2}{r^2} \delta g_{\mu\nu}^{\text{S}(0)} \Big|_{r \rightarrow \infty} = -2\kappa_7^2 \mathcal{T}_{\mu\nu}^{\text{S}(0)}, \quad (10)
\end{aligned}$$

where the source of each mode is decomposed as $\mathcal{T}_{\mu\nu} = \mathcal{T}_{\mu\nu}^{\text{T}} + \mathcal{T}_{\mu\nu}^{\text{S}(0)} + \mathcal{T}_{\mu\nu}^{\text{other}}$ ($\mathcal{T}_{\mu\nu}^{\text{T}} \equiv \mathcal{T}_{\mu\nu}^{\text{T}(2)} + \mathcal{T}_{\mu\nu}^{\text{T}(0)}$), and the two T_4 -modes are combined as $\delta g_{\mu\nu}^{\text{T}} = \delta g_{\mu\nu}^{\text{T}(2)} + \delta g_{\mu\nu}^{\text{T}(0)}$, since they obey the same equation.

Following the same manner as in Ref. [69], each part of EMT is written as

$$2\kappa_7^2 \mathcal{T}_{\mu\nu}^{\text{T}} = 2\kappa_7^2 \mathcal{T}_{\mu\nu} - 2\kappa_7^2 \mathcal{T}_{\mu\nu}^{\text{S}} - 2\kappa_7^2 \mathcal{T}_{\mu\nu}^{\text{other}}, \quad (11)$$

$$\begin{aligned}
2\kappa_7^2 \mathcal{T}_{\mu\nu}^{\text{S}} &= \frac{r^6 + 2R^6}{4(r^6 - R^6)} \left(\eta_{\mu\nu} - \frac{\Delta_\mu \Delta_\nu}{\Delta^2} \right) \\
&\times \left[\frac{1}{L^4 r^3} \partial_r (r(r^6 - R^6) \partial_r (a + b) \right. \\
&\quad \left. - 3R^6(a + b)) - \Delta^2 b \right], \quad (12)
\end{aligned}$$

$$\begin{aligned}
2\kappa_7^2 \mathcal{T}_{\mu\nu}^{\text{other}} &= \frac{1}{L^4 r^3} \partial_r \left[\left(\eta_{\mu\nu} - \frac{\Delta_\mu \Delta_\nu}{\Delta^2} \right) r(r^6 - R^6) \partial_r a \right. \\
&\quad \left. - 3R^6 \eta_{\mu\nu} b \right], \quad (13)
\end{aligned}$$

$$\begin{aligned}
\partial_r a &= -\frac{3R^6}{(5r^6 - 2R^6)} b + \frac{9r^{11}}{(5r^6 - 2R^6)R^6} 2\kappa_7^2 \mathcal{T}_{zz}, \\
b &= -i \frac{\Delta^\mu r^6 \sqrt{r^6 - R^6}}{\Delta^2} 2\kappa_7^2 \mathcal{T}_{\mu z}, \quad (14)
\end{aligned}$$

with the ansatz,

$$\begin{aligned}
\delta g_{\mu\nu}^{\text{other}}(\vec{\Delta}, r) &= \frac{r^2}{L^2} \frac{\Delta_\mu \Delta_\nu}{\Delta^2} a(r, \Delta), \\
\delta g_{11,11}^{\text{other}}(\vec{\Delta}, r) &= \frac{r^2}{L^2} b(r, \Delta), \\
\delta g_{\tau\tau}^{\text{other}}(\vec{\Delta}, r) &= \frac{r^2 f(r)}{L^2} (-b(r, \Delta)). \quad (15)
\end{aligned}$$

We determine the perturbation to the metric caused by the pion field as the bulk EMT source using Green's function method as follows,

$$\begin{aligned}
\delta g_{\mu\nu}^{\text{T/S}}(\vec{\Delta}, r) &\sim \frac{r^2}{L^2} \int dr' \sqrt{-G_{(7)}(r')} G^{\text{T/S}}(\vec{\Delta}, r, r') \\
&\times \frac{L^2}{r^2} 2\kappa_7^2 \mathcal{T}_{\mu\nu}^{\text{T/S}}(\vec{\Delta}, r'), \quad (16)
\end{aligned}$$

where, for S_4 mode, we can write as this form in the $r \rightarrow \infty$ limit, and $\sim$ implies this point. The Green's function $G^{\text{T/S}}$ is written as

$$G^{\text{T/S}}(\vec{\Delta}, r, r') = \sum_{n=1}^{\infty} \frac{\Psi_n^{\text{T/S}}(r) \Psi_n^{\text{T/S}}(r')}{(m_n^{\text{T/S}})^2 + \vec{\Delta}^2}, \quad (17)$$

by expanding in the complete system $\{\Psi_n^{\text{T/S}}(r)\}$ which obeys the eigenvalue equation,

$$\frac{1}{r^3 L^4} \partial_r ((r^7 - rR^6) \partial_r \Psi_n^{\text{T}}) = -(m_n^{\text{T}})^2 \Psi_n^{\text{T}}, \quad (18)$$

$$\begin{aligned}
\frac{1}{r^3 L^4} \partial_r \left((r^7 - rR^6) \left(\partial_r + \frac{144r^5 R^6}{(5r^6 - 2R^6)(r^6 + 2R^6)} \right) \Psi_n^{\text{S}} \right) \\
= -(m_n^{\text{S}})^2 \Psi_n^{\text{S}}, \quad (19)
\end{aligned}$$

where $m_n^{\text{T/S}}$ is n th glueball mass of T/S mode and $\Psi_n^{\text{T/S}}(r)$ satisfy the completeness conditions,

$$\begin{aligned}
\frac{r^3}{L^3} \sum_{n=1}^{\infty} \Psi_n^{\text{T/S}}(r) \Psi_n^{\text{T/S}}(r') &= \delta(r - r'), \\
\int_R^{\infty} dr \frac{r^3}{L^3} \Psi_n^{\text{T/S}}(r) \Psi_m^{\text{T/S}}(r) &= \delta_{mn}. \quad (20)
\end{aligned}$$

Note that the main components of the EMT in the bulk propagated by each of the T(2), T(0), and S(0) states are $\mathcal{T}_{\mu\nu}$, $\mathcal{T}_{11,11}$, and $\mathcal{T}_{\tau\tau}$ respectively. Even though the total $\mathcal{T}_{\tau\tau}$ is zero from Eq. (7), the $\mathcal{T}_{\tau\tau}^{\text{S}}$ satisfying $\mathcal{T}_{\tau\tau}^{\text{S}} + \mathcal{T}_{\tau\tau}^{\text{other}} = 0$ is the source in the bulk propagated by the S_4 mode.

Results. Substituting the pion field into the EMT of the bulk, we read off the matrix elements of the boundary EMT from Eq. (3) with the dictionary Eq. (16) as follows,

$$A(t) = \frac{6}{L^7} \int_R^{\infty} dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\text{T}} \Psi_n^{\text{T}}(r')}{(m_n^{\text{T}})^2 + \vec{\Delta}^2} \frac{r'^3}{L^3} P_{\text{t}}^{\mu\nu} \mathcal{T}_{\mu\nu}(\vec{\Delta}, r'), \quad (21)$$

$$\begin{aligned}
D(t) &= -\frac{6}{L^7} \int_R^{\infty} dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\text{T}} \Psi_n^{\text{T}}(r')}{(m_n^{\text{T}})^2 + \vec{\Delta}^2} \frac{r'^3}{L^3} \\
&\times P_{\text{s}}^{\mu\nu} (\mathcal{T}_{\mu\nu} + \mathcal{T}_{\mu\nu}^{\text{S}} + \mathcal{T}_{\mu\nu}^{\text{other}}) \\
&+ \frac{6}{L^7} \int_R^{\infty} dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\text{S}} \Psi_n^{\text{S}}(r')}{(m_n^{\text{S}})^2 + \vec{\Delta}^2} \frac{r'^3}{L^3} P_{\text{s}}^{\mu\nu} \mathcal{T}_{\mu\nu}^{\text{S}}, \quad (22)
\end{aligned}$$

$$\alpha^{\text{T/S}} = \frac{(m_n^{\text{T/S}})^2 L^4}{6} \int_R^{\infty} dr r^3 \Psi_n^{\text{T/S}}(r), \quad (23)$$

where $P_{\text{t}}^{\mu\nu}$ and $P_{\text{s}}^{\mu\nu}$ are defined as $P_{\text{t}}^{\mu\nu} = (3P^\mu P^\nu / P^2 - \eta^{\mu\nu}) / P^2$ and $P_{\text{s}}^{\mu\nu} = (P^\mu P^\nu / P^2 - \eta^{\mu\nu}) / \Delta^2$.

To calculate infinite sums, we expand as

$$\begin{aligned}
\frac{6}{L^7} \int_R^{\infty} dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\text{T/S}} \Psi_n^{\text{T/S}}(r')}{(m_n^{\text{T/S}})^2 + \vec{\Delta}^2} &= \sum_{k=0}^{\infty} F_k^{\text{T/S}}(r') (-\vec{\Delta}^2)^k, \\
F_k^{\text{T/S}}(r') &= \int_R^{\infty} dr \frac{r^3}{L^3} \sum_{n=0}^{\infty} \frac{\Psi_n^{\text{T/S}}(r) \Psi_n^{\text{T/S}}(r')}{(m_n)^{2k}}. \quad (24)
\end{aligned}$$

From the eigenvalue equations (19), $F_k^{\text{T/S}}(r)$ satisfies the following equations:

$$\frac{1}{r^3 L^4} \partial_r ((r^7 - rR^6) \partial_r F_k^{\text{T}}) = -F_{k-1}^{\text{T}}, \quad (25)$$

$$\begin{aligned}
\frac{1}{r^3 L^4} \partial_r \left((r^7 - rR^6) \left(\partial_r + \frac{144r^5 R^6}{(5r^6 - 2R^6)(r^6 + 2R^6)} \right) F_k^{\text{S}} \right) \\
= -F_{k-1}^{\text{S}}, \quad (26)
\end{aligned}$$

where we can prove $F_0^{\text{T/S}} = 1$ by using Eq. (20). We determine $F_k^{\text{T/S}}(r)$ by solving it as the recurrence formula with boundary conditions on $\partial_r F_k^{\text{T/S}}(r)|_{r \rightarrow R} = 1$, $F_k^{\text{T/S}}(r)|_{r \rightarrow \infty} = 0$.

We show the results of GFFs that truncate the expansion at $k = 10$ using Eq. (24) in Fig. 1 (also including results that truncate at $k = 8, 9$), where we set $\kappa = 0.00745$ and $M_{KK} = 949$ MeV which are determined by ρ meson mass 776 MeV and the pion decay constant $f_\pi = 92.4$ MeV [26]. In the current calculation, $D(t)$ begins to deviate from around $t \sim 0.5$ GeV² while $A(t)$ converges well in the range $0 < t < 0.8$ GeV² on the expansion up to $k = 10$ with momentum transfer. The current approach begins to deviate from QCD in the region where t is close to the M_{KK} , so further efforts are unnecessary.

In the top-down holographic QCD approach, the amplitude of $D(t)$ dumps more rapidly than that of $A(t)$, which is qualitatively consistent with the results of lattice QCD [24]. These differences are due to the contribution of S_4 modes to

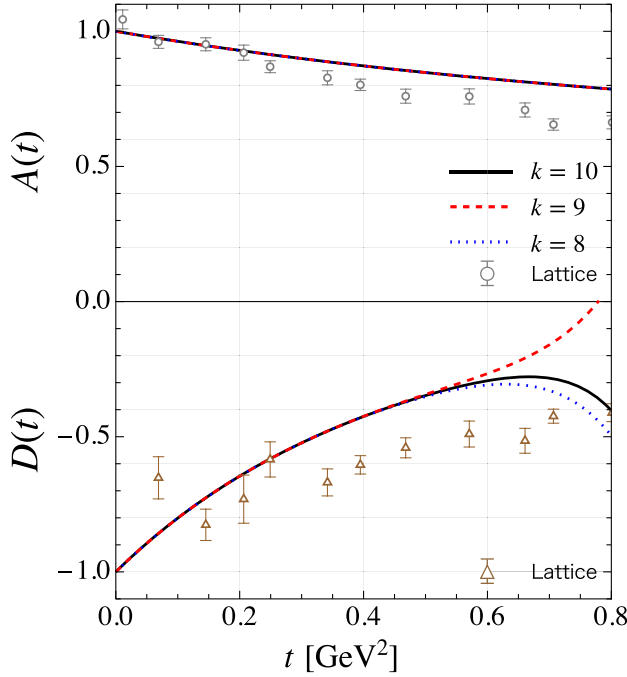


FIG. 1. The pion GFFs $A(t)$ and $D(t)$ with the lattice data [24].

$D(t)$, which is not observed in the bottom-up holographic QCD framework [70]. This is because the usual bottom-up model, as used in Ref. [70], does not have S_4 mode with $\mathcal{T}_{\tau\tau}$ as source. Also in this approach, as in previous studies [69], we observe the so-called glueball dominance, in which pions have gravitational interactions via infinite glueball spectra.

As you can see in Fig. 1, our result shows $D(0) = -1$. Note that this is a result of our calculations with loop effects in the large N_c limit, unlike the fact $A(0) = 1$, given by the general constraints. This is consistent with the low-energy theorem obtained in the soft pion limit [78,84,97]. The result

is as expected since in our present work we consider the Sakai-Sugimoto model, which is the holographic dual of QCD in the chiral limit. Several methods have been proposed on how to introduce quark masses into this model [110–116], thus analysis beyond the chiral limit is a future work.

Summary and outlook. We calculated the GFFs of pions from the top-down holographic QCD. We solve the Einstein equations in bulk theory with pion fields as matter fields and derived the GFFs using the dictionary (3). Our results are qualitatively compatible with those from lattice QCD [24] and successfully reproduce the feature where the behaviors of $A(t)$ and $D(t)$ are different, which is distinct from bottom-up holographic QCD [70]. Additionally, we obtained the D-term as $D(0) = -1$ in the chiral limit.

Finally, this study opens up several avenues for future research. First, it is possible to generalize to other hadrons, such as nucleons and vector mesons with nonzero spin. Also, it is very interesting to investigate beyond the chiral limit, because the connection between the Gell-Mann–Oakes–Renner relation and the stability of the stress distribution has been discussed in Ref. [82]. It is also important to investigate the behavior of the GFFs and the stress distribution inside hadrons under finite temperatures, densities, and magnetic fields, which change the nature of the interaction in the system.

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