

Interactions of massless fermionic fields in three dimensions

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All independent interaction vertices involving massless (Fang-Fronsdal) fermions in three dimensions are classified, completing the classification of interactions of massless fields of any spin. Similar to the bosonic case, we get no independent vertices at quartic or higher order in the fields involving fields with spin $s \geq 3/2$ and cubic vertices only for spins satisfying triangle inequalities, apart from the cases involving (matter) fields with spin $s \leq 1$. Different from the bosonic cases, we get only one vertex for each triple of spins with two Majorana fermions and one boson, which is parity even (odd) when the sum of the spins of all fields involved is odd (even). When the two Majorana fermions are identical, their coupling to an odd-spin boson is trivialized. We comment on the nontrivial holographic dictionary relating these vertices to conformal field theory correlators in two dimensions.

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Introduction. In this Letter, we classify and explicitly construct all independent interactions involving massless fermionic (Fang-Fronsdal) higher-spin (HS) fields [1] in three-dimensional Einstein spacetimes, thus completing the classification of all independent interaction vertices involving bosonic Fronsdal [2] and fermionic Fang-Fronsdal [1] massless fields of any spin in three dimensions [3].

HS gravities [8] are promising candidates for quantum gravity, in particular, as holographic duals (see, e.g., [9–11]) of well-understood conformal field theories (CFTs) (see, e.g., [12–15]). The main problem with known HS gravities [16–18] is the lack of conventional action principles for these theories. This motivates the construction of a conventional action for HS gravity, the so-called Fronsdal

program [19]. Even though the conservative approach to the Fronsdal program has shown an obstruction at quartic order in $d \geq 4$ (see, e.g., [44–51]), the situation is different in $d \leq 3$.

In three dimensions, we make a distinction between the HS gravities describing only edge modes (see, e.g., [52–58]), which can, in general, be given a Chern-Simons action formulation under mild assumptions [59], and those with propagating degrees of freedom [17,60], where HS fields interact with matter, but an action principle is not available so far. In the case when propagating degrees of freedom are present, the situation is analogous to higher dimensions (necessarily infinite tower of HS fields, no action principle known [61]), with a few key differences, that make it plausible that a local action formulation can be achieved via the Fronsdal program. Some of these differences are summarized below.

- (i) Minimal coupling to gravity is available in $d = 3$ (as we show here, also for fermionic fields) for massless fields of any spin, thus avoiding the Aragone-Deser problem [63,64], as opposed to higher dimensions.
- (ii) Interactions do not display differences in flat and (anti-)de Sitter [(A)dS] backgrounds (furthermore, they are available in any Einstein background), as

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opposed to higher dimensions [65–67]. This allows us to study HS gravity around flat spacetime [58,68,69], which offers a simplification in metriclike language.

- (iii) The existence of a Chern-Simons action indicates that, at least for the HS (topological) sector, there may exist also full nonlinear (Fang-)Fronsdal Lagrangians for a large class of models, which are yet to be constructed [70,71].
- (iv) In the metriclike (Fang-)Fronsdal formulation [72,73], it is known how to realize the coupling to matter at least in the lowest order, as opposed to the Chern-Simons setup.

All of these properties single out three-dimensional Minkowski spacetime as the simplest playground where the Fronsdal program can be completed, leading to a nonlinear theory of HS gravity with matter, with further lessons for higher-dimensional HS gravities.

Preliminaries. We work with the mostly plus convention for the metric, and we choose a real representation of the γ matrices,

$$\gamma^0 = i\sigma_2 = -\gamma_0, \quad \gamma^1 = \sigma_3 = \gamma_1, \quad \gamma^2 = \sigma_1 = \gamma_2, \quad (1)$$

where σ 's are Pauli matrices. They satisfy the γ matrix algebra

$$\gamma^\mu \gamma^\nu = \eta^{\mu\nu} + \epsilon^{\mu\nu\rho} \gamma_\rho, \quad (2)$$

where $\epsilon^{\mu\nu\rho}$ is the Levi-Civita antisymmetric tensor with $\epsilon_{012} = 1 = -\epsilon^{012}$ [74]. Throughout the Letter, we use real Majorana spinors ψ satisfying

$$\psi = \psi_c = C(\bar{\psi})^T = C(-\psi^\dagger \gamma^0)^T = -C(\gamma^0)^T \psi^* = \psi^*, \quad (3)$$

where the charge conjugation matrix is $C = -\gamma^0$ in this basis. The formulas we use, however, are also valid for Dirac spinors.

We will work in the realm of perturbative field theory, where any interactions can be switched off consistently, by continuously sending corresponding coupling constants to zero. In this case, the independent vertices are those that involve independent coupling constants that can be continuously sent to zero without affecting any lower-order vertex [75]. This way, we can classify all independent $(n+2)$ th-order vertices by the lowest-order Noether identity they satisfy (see, e.g., [20]),

$$\delta^{(n)} S^{(2)} + \delta^{(0)} S^{(n+2)} = 0, \quad (4)$$

where the superscripts indicate the order in fields for both the action and gauge transformation,

$$S = S^{(2)} + S^{(3)} + \dots, \quad (5)$$

$$\delta = \delta^{(0)} + \delta^{(1)} + \dots \quad (6)$$

We will study Eq. (4) for any n , or rather its weaker version,

$$\delta^{(0)} S^{(n+2)} \approx 0, \quad (7)$$

where $\approx$ denotes equivalence on free shell, to classify all independent vertices involving fermionic fields in three spacetime dimensions.

The lowest order of the action $S^{(2)}$ and gauge transformations $\delta^{(0)}$ are given by (a sum of) (Fang-)Fronsdal actions and corresponding gauge symmetries [1,2],

$$S^{(2)} = \sum_i \int d^3x \mathcal{L}^{(2)}(\phi_i), \quad (8)$$

where the sum goes over all the fields labeled by i , with spins s_i taking integer or half-integer values [76].

For bosonic field $\phi_{\mu_1 \dots \mu_s}^{(s)}$ and fermionic field $\Psi_{\mu_1 \dots \mu_\ell}^{(\ell+1/2)}$ with spins s and $\ell + 1/2$, respectively, the (Fang-)Fronsdal free Lagrangian $\mathcal{L}^{(2)}$ takes the following form:

$$\mathcal{L}^{(2)}(\phi^{(s)}) = \frac{1}{2} \phi_{\mu_1 \dots \mu_s}^{(s)} \square \phi^{(s)\mu_1 \dots \mu_s} + \dots, \quad (9)$$

$$\mathcal{L}^{(2)}(\Psi^{(\ell+1/2)}) = \bar{\Psi}_{\mu_1 \dots \mu_\ell}^{(\ell+\frac{1}{2})} \not{\partial} \Psi^{(\ell+\frac{1}{2})\mu_1 \dots \mu_\ell} + \dots, \quad (10)$$

where $\not{\partial} = \gamma^\mu \partial_\mu$ and the omitted terms contain divergences and (γ) traces of the field that are fixed by the condition that the free action is invariant under free gauge transformations [77],

$$\delta^{(0)} \phi_{\mu_1 \dots \mu_s}^{(s)} = s \partial_{(\mu_1} \epsilon_{\mu_2 \dots \mu_s)}, \quad (11)$$

$$\delta^{(0)} \Psi_{\mu_1 \dots \mu_\ell}^{(\ell+\frac{1}{2})} = \ell \partial_{(\mu_1} \xi_{\mu_2 \dots \mu_\ell)}, \quad (12)$$

where we omit the spinor indices of $\Psi^{(\ell+\frac{1}{2})}$ and ξ .

As in the bosonic case [4–6], for classification of the interactions, it is enough to restrict ourselves to the traceless-transverse part of the vertices. Then, it is sufficient to use the free Fierz equations (see details in [5]),

$$\eta^{\mu_1 \mu_2} \phi_{\mu_1 \mu_2 \dots \mu_s} = 0, \quad \partial^{\mu_1} \phi_{\mu_1 \dots \mu_s} = 0, \quad \square \phi_{\mu_1 \dots \mu_s} = 0, \quad (13)$$

and their fermionic counterparts,

$$\gamma^{\mu_1} \Psi_{\mu_1 \dots \mu_\ell} = 0, \quad \partial^{\mu_1} \Psi_{\mu_1 \dots \mu_\ell} = 0, \quad \not{\partial} \Psi_{\mu_1 \dots \mu_\ell} = 0. \quad (14)$$

As a consequence of (14), we have also

$$\eta^{\mu_1 \mu_2} \Psi_{\mu_1 \mu_2 \dots \mu_s} = 0, \quad \square \Psi_{\mu_1 \dots \mu_s} = 0. \quad (15)$$

When now studying the TT parts of the vertices for massless fields, we impose gauge symmetry with parameters that also satisfy the Fierz equations. The full off-shell vertices can be derived from the TT part by relaxing these

conditions, which will not be pursued here. By applying (2) to a fermionic field and using (14), we can deduce [hereafter $\approx$ is equivalence due to (13) and (14)]

$$\Psi^{\mu_1}_{\mu_2 \dots \mu_\ell} \approx \epsilon^{\mu_1 \alpha \beta} \gamma_\alpha \Psi_{\beta \mu_2 \dots \mu_\ell}, \quad (16)$$

which will play a major role throughout this Letter. Taking the conjugate of the latter equation and using that

$$\gamma_\mu^\dagger = \gamma^0 \gamma_\mu \gamma^0, \quad (17)$$

we find a similar relation for the conjugate field $\bar{\Psi}_{\mu_1 \dots \mu_s} = -\Psi_{\mu_1 \dots \mu_s}^\dagger \gamma^0$,

$$\bar{\Psi}^{\mu_1}_{\mu_2 \dots \mu_\ell} = -\epsilon^{\mu_1 \alpha \beta} \bar{\Psi}_{\beta \mu_2 \dots \mu_\ell} \gamma_\alpha. \quad (18)$$

Another relation can be derived by contracting (2) with a derivative acting on a fermionic field and using Fierz equations (14),

$$0 \approx \gamma^\mu \partial \Psi_{\nu_1 \dots \nu_\ell} = (\partial^\mu + \epsilon^{\mu \nu \lambda} \gamma_\lambda \partial_\nu) \Psi_{\nu_1 \dots \nu_\ell}, \quad (19)$$

which further implies

$$\begin{aligned} 0 &\approx \epsilon^{\alpha \mu \nu} \partial_\mu \Psi_{\nu \nu_2 \dots \nu_\ell} + \delta_{\nu \lambda}^{\alpha \beta} \gamma^\lambda \partial^\nu \Psi_{\beta \nu_2 \dots \nu_\ell} \\ &\approx \epsilon^{\alpha \mu \nu} \partial_\mu \Psi_{\nu \nu_2 \dots \nu_\ell}, \end{aligned} \quad (20)$$

which leads to the following key identities:

$$\partial_{[\mu} \Psi_{\nu] \nu_2 \dots \nu_\ell} \approx 0 \approx \partial_{[\mu} \bar{\Psi}_{\nu] \nu_2 \dots \nu_\ell}. \quad (21)$$

As usual (see, e.g., [20]), when working with symmetric tensor fields, it is useful to introduce an auxiliary vector for each field, to contract the symmetric indices as

$$\phi_i(x_i, a_i) := \frac{1}{s!} \phi_{\mu_1 \dots \mu_{s_i}}(x_i) a_i^{\mu_1} \dots a_i^{\mu_{s_i}}, \quad (22)$$

$$\Psi_i(x_i, a_i) := \frac{1}{s!} \Psi_{\mu_1 \dots \mu_{s_i}}(x_i) a_i^{\mu_1} \dots a_i^{\mu_{s_i}}. \quad (23)$$

Then, one establishes an effective way of contracting Lorentz indices by introducing operators $A_i^\mu := \partial_{a_i^\mu}$, which combined with $P_i := \partial_{x_i^\mu}$ construct the building blocks of vertex operators via scalar contractions,

$$\begin{aligned} s_{ij} &:= P_i \cdot P_j|_{1 \leq i \leq j \leq n}, & y_{ij} &:= A_i \cdot P_j|_{1 \leq i, j \leq n}, \\ z_{ij} &:= A_i \cdot A_j|_{1 \leq i \leq j \leq n}, & \Gamma_{(k)j} &:= \gamma_{(k)}^\mu A_{j\mu}, \\ \Omega_{(k)j} &:= \gamma_{(k)}^\mu P_{j\mu}, \end{aligned} \quad (24)$$

where $\gamma_{(k)}$ is a γ matrix contracting spinor indices of the k th fermion pair in the vertex. We are looking for local scalar vertices of $2f$ fermionic and n bosonic fields given as (the

subscript of the Lagrangian term denotes the spins of involved fields)

$$\begin{aligned} \mathcal{L}_{\ell_1+1/2, \dots, \ell_{2f}+1/2, s_1, \dots, s_n} &= \mathcal{V}(s_{lm}, y_{lm}, z_{lm}, \Gamma_{(k)l}, \Omega_{(k)l}) \\ &\times \left(\prod_{k=1}^f \bar{\Psi}_{2k-1}(x_{2k-1}, a_{2k-1}) \Psi_{2k}(x_{2k}, a_{2k}) \right) \\ &\times \left(\prod_{j=1}^n \phi_j(x_{2f+j}, a_{2f+j}) \right) \Bigg|_{\substack{a_i=0(i=1, \dots, 2f+n) \\ x_i=x(i=1, \dots, 2f+n)}}. \end{aligned} \quad (25)$$

The gauge invariance of the vertices will impose severe restrictions on vertex operators $\mathcal{V}$ as we show below. Note that there are also operators involving the tensor $\epsilon^{\mu\nu\rho}$. As we show in the companion paper [78], these operators can be related to the parity-even operators via relations (16) and (18).

There are several relations to be taken into account. First, the total derivative terms are discarded, which implies the following equations [from now on, we assume the vertex operators to be in the vertex term (25)]:

$$\sum_{j=1}^{2f+n} s_{ij} \approx 0, \quad \sum_{j=1}^{2f+n} y_{ij} \approx 0, \quad \sum_{j=1}^{2f+n} \Omega_{(k)j} \approx 0. \quad (26)$$

Next, we impose the Fierz identities (13)–(15) (no summation over repeating indices)

$$s_{ii} \approx 0, \quad y_{ii} \approx 0, \quad z_{ii} \approx 0, \quad (27)$$

$$\Gamma_{(k)2k-1} \approx 0 \approx \Gamma_{(k)2k}, \quad \Omega_{(k)2k-1} \approx 0 \approx \Omega_{(k)2k}. \quad (28)$$

One has also dimension-dependent Schouten identities that impose further relations involving the operators in the vertex (for details see [20]). In three dimensions, these identities play a crucial role in determining the nontrivial interacting deformations of bosonic-free (Fronsdal) theory [4–6]. They should also be taken into account in the interactions of fermions and will be discussed below.

Cubic vertices with fermions. The general construction of the interacting theory order by order in the power of fields starts from the classification of cubic vertices (see a lightning review in [5]). The vertex operator for the cubic case depends on the following variables [a full basis due to (26)–(28)]:

$$y_i \equiv y_{ii+1}, \quad z_i \equiv z_{i+1i-1}, \quad \Gamma \equiv \Gamma_{(1)3}, \quad (29)$$

where $i = 1, 2, 3$ and $i \pm 3 \equiv i$. Note that $s_{ij} \approx 0$ ($\forall i, j$), as in the bosonic case, and there is only one new operator (Γ) compared to the bosonic case (all other Γ and Ω

operators vanish on shell). Furthermore, we omit the index of $\gamma_{(k)}$ (a cubic vertex has at most one fermion pair).

We will need to solve the following equation:

$$\delta^{(0)} S^{(3)} \approx 0, \quad (30)$$

for the case when the cubic interaction contains two fermionic and one bosonic fields. Given that the fermionic fields satisfy also the Fierz equations for the bosonic fields due to (14) and (15), the vertex operator (dimension-dependent) identities for bosonic fields [4–6] can be directly used also for fermions. However, fermionic equations (14) are stronger than the bosonic ones and imply more identities to be taken into account. More general analysis, involving the parity-odd structures, will be provided in [78]. Here we make a quick derivation of the identities we need. Equations (16) and (18) translate into

$$A_2^\mu \approx \epsilon^{\mu\alpha\beta} \gamma_\alpha A_{2\beta}, \quad A_1^\mu \approx -\epsilon^{\mu\alpha\beta} A_{1\beta} \gamma_\alpha, \quad (31)$$

whereas (21) translates into

$$P_{1[\mu} A_{1\nu]} \approx 0 \approx P_{2[\mu} A_{2\nu]}. \quad (32)$$

Together these relations imply

$$y_1 z_1 \approx y_2 z_2 \approx -y_3 z_3, \quad y_i y_j \approx 0 \quad (i \neq j), \quad (33)$$

and

$$\Gamma y_1 z_1 \approx \Gamma y_2 z_2 \approx -\Gamma y_3 z_3, \quad \Gamma y_i y_j \approx 0 \quad (i \neq j). \quad (34)$$

Note that (34) is not a trivial consequence of (33) as the set of relations does not form an ideal [79]. These equations turn out to be the seeds for all further relations, including Schouten identities. The gauge transformation for each field is given by the operator $T_i = a_i^\mu P_{i\mu}$ of the symmetrized derivative of the parameter, which commutes with y, z, Γ as follows:

$$\begin{aligned} [y_i, T_j] &\approx 0, & [z_i, T_i] &= 0, & [\Gamma, T_i] &\approx 0, \\ [z_{i\pm 1}, T_i] &\approx \pm y_{i\mp 1}. \end{aligned} \quad (35)$$

Therefore, a gauge-invariant vertex has to satisfy

$$\left(y_{i-1} \frac{\partial}{\partial z_{i+1}} - y_{i+1} \frac{\partial}{\partial z_{i-1}} \right) \mathcal{V}(y, z) \approx 0, \quad (36)$$

which is the same as in the bosonic case in any dimensions. The difference, however, is that now the new relations (33) and (34) change the space of functions $\mathcal{V}$ where we solve these equations.

Using (33) and (34), it is straightforward to show that vertices containing more than one derivative are trivial when the spins satisfy triangle inequalities

$$2s_i \leq s_1 + s_2 + s_3. \quad (37)$$

The main point to notice here is that given the relation $y_i y_j \approx 0$ for $i \neq j$ (and similarly $\Gamma y_i y_j \approx 0$), the only way to write a nontrivial higher derivative expression is $y_i^k z_1^{n_1} z_2^{n_2} z_3^{n_3}$ (with or without Γ) with $k > 1$ characterizing the number of derivatives. When $n_i \neq 0$, we can use the identities (33) [(34) in the presence of Γ] to replace $y_i z_i$ with $\pm y_j z_j$ ($j \neq i$), thus getting a vertex containing $y_i y_j$ ($j \neq i$) (and, therefore, trivial). This only leaves the possibility $n_i = 0$ which together with $k > 1$ implies that the triangle inequalities (37) are violated.

When the triangle inequalities between spins are not satisfied and all three massless fields have spins bigger than 1, the requirement of gauge invariance is strong enough to kill all candidate cubic vertices. When one requires only one or two gauge symmetries, some candidate vertices may still survive, which is relevant to the study of interactions involving massive fields [80,81], which is out of the scope of this work.

For general spins $s_i > 1$, satisfying triangle inequalities, a general solution is given by a linear combination of the following vertex operators:

$$\mathcal{V}_E(y, z) = \Gamma y_3 z_1^{n_1} z_2^{n_2} z_3^{n_3+1}, \quad (38)$$

$$\mathcal{V}_O(y, z) = y_3 z_1^{n_1} z_2^{n_2} z_3^{n_3+1}, \quad (39)$$

where $\mathcal{V}_E(\mathcal{V}_O)$ is parity even (odd) and n_i are some non-negative integers. The spins of the fields are given as $s_1 = n_2 + n_3 + 3/2$, $s_2 = n_1 + n_3 + 3/2$ (for both vertices), and $s_3 = n_1 + n_2 + 2$ for the vertex (38), while $s_3 = n_1 + n_2 + 1$ for the vertex (39). It is clear, therefore, that the spins of the gauge parameters, $s_i - 1$, satisfy triangle inequalities, which turns into equality only for (38) when $n_3 = 0$. This aspect is analogous to the cubic vertices of bosonic fields.

Note that there is only one vertex for each given triple of spins satisfying triangle inequalities, different from bosonic cases, where we had one parity-even and one parity-odd vertex. When the two fermionic fields are identical, the vertex (39) trivializes: odd-spin interactions require anti-symmetric Chan-Paton factors (while even-spin interactions require symmetric ones).

As with the bosonic vertices, the general structure of cubic vertices is different when fields carrying propagating degrees of freedom are involved (we will call those “matter fields”). Here, these matter fields can take spin values $s = 0, 1/2, 1$. The special vertices are

$$\mathcal{V}_O^{(1/2, 1/2, s)} = y_3^s, \quad \mathcal{V}_E^{(1/2, 1/2, s)} = \Gamma y_3^{s-1}, \quad (40)$$

$$\mathcal{V}_O^{(1/2, \ell+1/2, 0)} = y_2^\ell, \quad \mathcal{V}_E^{(1/2, \ell+1/2, 1)} = \Gamma y_2^\ell, \quad (41)$$

$$\mathcal{V}_O^{(\ell+1/2, 1/2, 0)} = y_1^\ell, \quad \mathcal{V}_E^{(\ell+1/2, 1/2, 1)} = \Gamma y_1^\ell. \quad (42)$$

The last option to consider is when the bosonic field is a Chern-Simons spin-1 field instead of a Maxwell one. This case is different as the bosonic field equations are stronger,

$$P_{3[\mu}A_{3\nu]} \approx 0. \quad (43)$$

This in turn induces additional identities, including

$$\Gamma y_1 \approx 0 \approx \Gamma y_2, \quad y_3 z_3 \approx 0, \quad (44)$$

which change the classification, allowing only for a zero-derivative vertex. In the end, we find one vertex with arbitrary equal-spin fermions,

$$\mathcal{V}_{\text{CS}}^{\ell+1/2, \ell+1/2, 1} = \Gamma z_3^\ell. \quad (45)$$

This vertex can be derived by covariantizing the derivatives of the Fang-Fronsdal action with $\partial_\mu \rightarrow \partial_\mu + A_\mu$ (here A_μ stands for the standard notation of a spin-1 Chern-Simons field, which should not be confused with A_i^μ). As in all other cases with odd-spin interactions, this vertex requires antisymmetric Chan-Paton factors and is therefore trivial for a single Majorana fermion.

As with the couplings to Fronsdal fields, there is only one fermionic vertex for the coupling to the Chern-Simons field, while there are two bosonic vertices [5]: one parity even and one parity odd.

There is an extra parity-odd vertex for coupling of matter fields with spin 1/2,

$$\mathcal{V}_{\text{CS}, O}^{1/2, 1/2, 1} = y_3. \quad (46)$$

This completes the classification of the cubic vertices involving massless fields of any spin.

Higher-order vertices. By making use of (16) and (18), we can achieve a form of the vertex in which every HS fermion appears together with a γ matrix. The analysis then follows very closely the arguments in [6]. One can first use Schouten identities (potentially after multiplication with a nonvanishing combination of Mandelstam variables s_{ij}) to rewrite the vertex such that there are no contractions between field indices (i.e., no variables z_{ij}) and no contraction between a field index and a γ matrix index (i.e., no variables $\Gamma_{(k)j}$). In the presence of an ϵ tensor, one can use, as in [6], Schouten identities to achieve that the ϵ tensor is contracted only to derivative operators, leading to a gauge-invariant combination that can be ignored in the remaining analysis.

Similarly, Schouten identities can be used to express all y_{ij} in terms of y_{ii+1} and y_{ii+2} , and analogously all $\Omega_{(k)j}$ by $\Omega_{(k)2k+1}$ [using that $\Omega_{(k)2k-1} \approx \Omega_{(k)2k} \approx 0$]. The vertex then depends on the gauge-invariant s_{ij} and $\Omega_{(k)2k+1}$, as well as y_{ii+1} and y_{ii+2} . With respect to the latter two variables,

gauge invariance of the vertex then requires that it can depend only on the gauge-invariant combination

$$Y_i := s_{ii+2}y_{ii+1} - s_{ii+1}y_{ii+2}. \quad (47)$$

As shown in [6], for the bosonic fields these new variables satisfy

$$Y_i^2 \approx 0 \quad (i = 2f + 1, \dots, 2f + n), \quad (48)$$

and therefore a bosonic field can have at most one index (and is at most a spin-1 field). For the fermionic fields, we have instead

$$Y_i \approx 0 \quad (i = 1, \dots, 2f), \quad (49)$$

which follows from (21). [The same condition also holds for a spin-1 Chern-Simons field due to (43).] Interactions, therefore, cannot include fermionic fields with a Lorentz index; hence they can only be spin- $\frac{1}{2}$ fields. One concludes that there are no independent HS vertices of quartic or higher order. A more detailed discussion will be provided in [78] and can also be found in [82]. A similar conclusion was made in a different formulation of the problem in the recent work [83].

Discussion. In this Letter, we have classified and explicitly constructed all independent interaction vertices involving massless fermionic fields in three spacetime dimensions. The characteristic feature of fermionic vertices without propagating (matter) fields is that they all have one derivative. Similar to the bosonic vertices, cubic vertices with higher-spin fermions exist only for spins satisfying triangle inequalities. Different from the bosonic case, where for each collection of spins there is one parity-even and one parity-odd vertex, here we find only one vertex for each triple of spins satisfying triangle inequalities, and the parity symmetry of the vertex depends on the spins. Note that the bosonic vertices are either two or three derivative. The fermionic vertices, in a sense, match all the properties of the two-derivative bosonic vertices: they are unique for a given triplet of spins satisfying triangle inequalities, have the same number of derivatives as the free theory, and can be either parity even or parity odd depending on the spins of the fields involved.

A posteriori, our results can also be interpreted as a nontrivial check of the holographic dictionary of the $\text{AdS}_3/\text{CFT}_2$ correspondence. For bosonic fields in three dimensions, a single Fronsdal field corresponds on the boundary to a pair of holomorphic and antiholomorphic currents. The two cubic vertices of different parity that exist for bosonic fields satisfying the triangle inequality correspond to combinations of the corresponding conformal three-point functions for holomorphic and antiholomorphic currents on the boundary. For fermionic fields in three

spacetime dimensions, a Majorana Fang-Fronsdal field is dual to one Majorana-Weyl fermion on the boundary (see, e.g., [84]). Its chirality depends on a discrete choice of how to couple the Fang-Fronsdal fields to the AdS background and boundary conditions. This is connected to two inequivalent representations of the Clifford algebra related by $\gamma^2 \leftrightarrow -\gamma^2$ which, in turn, is related to a chirality change from the point of view of the boundary. Fixing the choice for all Fang-Fronsdal fields in the same way (for example, to obtain right-moving boundary fermions), the analysis of the vertices carries through and the existence of precisely one vertex for fields satisfying the triangle inequality corresponds to the existence of a unique structure of a three-point function for holomorphic currents satisfying the same inequality. Analogously to the discussion in [7], this match extends to higher-point functions. For completeness, we will present AdS extensions of the vertices presented here in [78].

This interpretation can also be compared to the holography of (HS) supergravity on AdS in a Chern-Simons formulation [81,85–91]. Whereas for bosonic fields the gauge group is of the product form $G \times G$ with the same factors, one can consider [92] different supergroup extensions of the two factors leading at the boundary to different sets of left- and right-moving fermionic charges [93].

The general result [59] suggests that any theory of HS fields without propagating degrees of freedom in three dimensions has a framelike description as a Chern-Simons theory. The results of this Letter are consistent with the extension of this statement to theories with fermions. Using the procedure in Sec. IV of [59], all of the vertices found here can be translated to framelike vertices which can be used in full nonlinear theories of the Chern-Simons form. This, in particular, implies that any theory with massless fermionic HS fields without matter can be described as a Chern-Simons gauge theory with a Lie superalgebra. The first example of such hypergravity was given in [95] (see also [96,97]).

Our analysis, on the other hand, treats propagating (matter) fields and HS fields on the same footing and can be the starting point for the construction of a metriclike theory of HS fields coupled to matter.

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- [1] J. Fang and C. Fronsdal, Massless fields with half integral spin, *Phys. Rev. D* **18**, 3630 (1978).
 - [2] C. Fronsdal, Massless fields with integer spin, *Phys. Rev. D* **18**, 3624 (1978).
 - [3] Purely bosonic vertices were constructed earlier in [4–7].
 - [4] K. Mkrtchyan, Cubic interactions of massless bosonic fields in three dimensions, *Phys. Rev. Lett.* **120**, 221601 (2018).
 - [5] P. Kessel and K. Mkrtchyan, Cubic interactions of massless bosonic fields in three dimensions II: Parity-odd and Chern-Simons vertices, *Phys. Rev. D* **97**, 106021 (2018).
 - [6] S. Fredenhagen, O. Krüger, and K. Mkrtchyan, Vertex-constraints in 3D higher spin theories, *Phys. Rev. Lett.* **123**, 131601 (2019).
 - [7] S. Fredenhagen, O. Krüger, and K. Mkrtchyan, Constraints for three-dimensional higher-spin interactions and conformal correlators, *Phys. Rev. D* **100**, 066019 (2019).
 - [8] Here we use the term “HS gravity” for any consistent full nonlinear theory that includes (some modification of) Einstein gravity with at least one massless field of spin $s > 2$ in its spectrum.
 - [9] I. R. Klebanov and A. M. Polyakov, AdS dual of the critical $O(N)$ vector model, *Phys. Lett. B* **550**, 213 (2002).
 - [10] S. Giombi and X. Yin, The higher spin/vector model duality, *J. Phys. A* **46**, 214003 (2013).
 - [11] M. R. Gaberdiel and R. Gopakumar, Minimal model holography, *J. Phys. A* **46**, 214002 (2013).
 - [12] A. B. Zamolodchikov, Infinite additional symmetries in two-dimensional conformal quantum field theory, *Theor. Math. Phys.* **65**, 1205 (1985).
 - [13] A. Bilal and J.-L. Gervais, Systematic construction of conformal theories with higher spin Virasoro symmetries, *Nucl. Phys. B* **318**, 579 (1989).
 - [14] S. L. Lukyanov and V. A. Fateev, Additional symmetries and exactly soluble models in two-dimensional conformal field theory, *Sov. Sci. Rev. Sect. A* **15**, 1 (1990).
 - [15] K. Lang and W. Rühl, The critical $O(N)$ σ model at dimensions $2 < d < 4$: Fusion coefficients and anomalous dimensions, *Nucl. Phys. B* **400**, 597 (1993).
 - [16] M. A. Vasiliev, Consistent equation for interacting gauge fields of all spins in $(3 + 1)$ -dimensions, *Phys. Lett. B* **243**, 378 (1990).
 - [17] S. F. Prokushkin and M. A. Vasiliev, Higher spin gauge interactions for massive matter fields in 3-D AdS space-time, *Nucl. Phys. B* **545**, 385 (1999).
 - [18] M. A. Vasiliev, Nonlinear equations for symmetric massless higher spin fields in $(A)dS_d$, *Phys. Lett. B* **567**, 139 (2003).
 - [19] See, [20–29] for some key references on the Fronsdal program in dimensions $d \geq 4$. For completeness, we refer the reader to related parallel programs in the light-cone approach [30–34] and the Fradkin-Vasiliev (framelike) approach [35–38]. See also [39–43] for recent reviews.

- [20] S. Fredenhagen, O. Krüger, and K. Mkrtchyan, Restrictions for n -point vertices in higher-spin theories, *J. High Energy Phys.* **06** (2020) 118.
- [21] R. Manvelyan, K. Mkrtchyan, and W. Rühl, General trilinear interaction for arbitrary even higher spin gauge fields, *Nucl. Phys.* **B836**, 204 (2010).
- [22] A. Sagnotti and M. Taronna, String lessons for higher-spin interactions, *Nucl. Phys.* **B842**, 299 (2011).
- [23] E. Joung and M. Taronna, Cubic interactions of massless higher spins in (A)dS: Metric-like approach, *Nucl. Phys.* **B861**, 145 (2012).
- [24] M. Henneaux, G. Lucena Gómez, and R. Rahman, Higher-spin fermionic gauge fields and their electromagnetic coupling, *J. High Energy Phys.* **08** (2012) 093.
- [25] M. Henneaux, G. Lucena Gómez, and R. Rahman, Gravitational interactions of higher-spin fermions, *J. High Energy Phys.* **01** (2014) 087.
- [26] E. Conde, E. Joung, and K. Mkrtchyan, Spinor-helicity three-point amplitudes from local cubic interactions, *J. High Energy Phys.* **08** (2016) 040.
- [27] D. Francia, G.L. Monaco, and K. Mkrtchyan, Cubic interactions of Maxwell-like higher spins, *J. High Energy Phys.* **04** (2017) 068.
- [28] C. Sleight and M. Taronna, Feynman rules for higher-spin gauge fields on AdS_{d+1} , *J. High Energy Phys.* **01** (2018) 060.
- [29] E. Joung and M. Taronna, A note on higher-order vertices of higher-spin fields in flat and (A)dS space, *J. High Energy Phys.* **09** (2020) 171.
- [30] A. K. H. Bengtsson, I. Bengtsson, and L. Brink, Cubic interaction terms for arbitrary spin, *Nucl. Phys.* **B227**, 31 (1983).
- [31] A. K. H. Bengtsson, I. Bengtsson, and N. Linden, Interacting higher spin gauge fields on the light front, *Classical Quantum Gravity* **4**, 1333 (1987).
- [32] E. S. Fradkin and R. R. Metsaev, A cubic interaction of totally symmetric massless representations of the Lorentz group in arbitrary dimensions, *Classical Quantum Gravity* **8**, L89 (1991).
- [33] R. R. Metsaev, Cubic interaction vertices of massive and massless higher spin fields, *Nucl. Phys.* **B759**, 147 (2006).
- [34] R. R. Metsaev, Cubic interaction vertices for fermionic and bosonic arbitrary spin fields, *Nucl. Phys.* **B859**, 13 (2012).
- [35] E. S. Fradkin and M. A. Vasiliev, Cubic interaction in extended theories of massless higher spin fields, *Nucl. Phys.* **B291**, 141 (1987).
- [36] M. A. Vasiliev, Cubic vertices for symmetric higher-spin gauge fields in $(A)dS_d$, *Nucl. Phys.* **B862**, 341 (2012).
- [37] N. Boulanger, D. Ponomarev, and E. D. Skvortsov, Non-Abelian cubic vertices for higher-spin fields in anti-de Sitter space, *J. High Energy Phys.* **05** (2013) 008.
- [38] M. V. Khabarov and Y. M. Zinoviev, Massless higher spin cubic vertices in flat four dimensional space, *J. High Energy Phys.* **08** (2020) 112.
- [39] C. Sleight, Interactions in higher-spin gravity: A holographic perspective, *J. Phys. A* **50**, 383001 (2017).
- [40] D. Ponomarev, Basic introduction to higher-spin theories, *Int. J. Theor. Phys.* **62**, 146 (2023).
- [41] A. Bengtsson, *Interactions*, Higher Spin Field Theory (De Gruyter, Berlin, Boston, 2023).
- [42] S. Pekar, Introduction to higher-spin theories, *Proc. Sci. Modave 2022* (2023) 004.
- [43] A. Campoleoni and S. Fredenhagen, Higher-spin gauge theories in three spacetime dimensions, *Lect. Notes Phys.* **1028**, 121 (2024).
- [44] R. R. Metsaev, Poincaré-invariant dynamics of massless higher spins: Fourth-order analysis on mass shell, *Mod. Phys. Lett. A* **06**, 359 (1991).
- [45] R. R. Metsaev, S matrix approach to massless higher spins theory. 2: The case of internal symmetry, *Mod. Phys. Lett. A* **06**, 2411 (1991).
- [46] P. Dempster and M. Tsulaia, On the structure of quartic vertices for massless higher spin fields on Minkowski background, *Nucl. Phys.* **B865**, 353 (2012).
- [47] A. K. H. Bengtsson, Investigations into light-front quartic interactions for massless fields (I): Non-constructibility of higher spin quartic amplitudes, *J. High Energy Phys.* **12** (2016) 134.
- [48] M. Taronna, On the non-local obstruction to interacting higher spins in flat space, *J. High Energy Phys.* **05** (2017) 026.
- [49] R. Roiban and A. A. Tseytlin, On four-point interactions in massless higher spin theory in flat space, *J. High Energy Phys.* **04** (2017) 139.
- [50] C. Sleight and M. Taronna, Higher-spin gauge theories and bulk locality, *Phys. Rev. Lett.* **121**, 171604 (2018).
- [51] O. A. Gelfond, Moderately non-local $\eta\bar{\eta}$ vertices in the AdS_4 higher-spin gauge theory, *Eur. Phys. J. C* **83**, 1154 (2023).
- [52] M. P. Blencowe, A consistent interacting massless higher spin field theory in $D = (2 + 1)$, *Classical Quantum Gravity* **6**, 443 (1989).
- [53] M. A. Vasiliev, Quantization on sphere and high spin superalgebras, *JETP Lett.* **50**, 374 (1989).
- [54] M. A. Vasiliev, Higher spin algebras and quantization on the sphere and hyperboloid, *Int. J. Mod. Phys. A* **06**, 1115 (1991).
- [55] A. Campoleoni, S. Fredenhagen, S. Pfenninger, and S. Theisen, Asymptotic symmetries of three-dimensional gravity coupled to higher-spin fields, *J. High Energy Phys.* **11** (2010) 007.
- [56] M. Henneaux and S.-J. Rey, Nonlinear W_{∞} as asymptotic symmetry of three-dimensional higher spin anti-de Sitter gravity, *J. High Energy Phys.* **12** (2010) 007.
- [57] A. Campoleoni, S. Fredenhagen, and S. Pfenninger, Asymptotic W -symmetries in three-dimensional higher-spin gauge theories, *J. High Energy Phys.* **09** (2011) 113.
- [58] H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller, and J. Rosseel, Spin-3 gravity in three-dimensional flat space, *Phys. Rev. Lett.* **111**, 121603 (2013).
- [59] M. Grigoriev, K. Mkrtchyan, and E. Skvortsov, Matter-free higher spin gravities in 3D: Partially-massless fields and general structure, *Phys. Rev. D* **102**, 066003 (2020).
- [60] A. V. Korybut, A. A. Sevostyanova, M. A. Vasiliev, and V. A. Vereitin, Disentanglement of topological and dynamical fields in 3d higher-spin theory within shifted homotopy approach, *Phys. Lett. B* **838**, 137718 (2023).
- [61] See, however, [62] and references therein.

- [62] R. Bonezzi, N. Boulanger, E. Sezgin, and P. Sundell, An action for matter coupled higher spin gravity in three dimensions, *J. High Energy Phys.* **05** (2016) 003.
- [63] C. Aragone and S. Deser, Consistency problems of hypergravity, *Phys. Lett.* **86B**, 161 (1979).
- [64] M. Porrati, Old and new no go theorems on interacting massless particles in flat space, in *Proceedings of the 17th International Seminar on High Energy Physics (Quarks 2012) Yaroslavl, Russia, 2012* (2012), arXiv:1209.4876.
- [65] E. S. Fradkin and M. A. Vasiliev, On the gravitational interaction of massless higher spin fields, *Phys. Lett. B* **189**, 89 (1987).
- [66] N. Boulanger, S. Leclercq, and P. Sundell, On the uniqueness of minimal coupling in higher-spin gauge theory, *J. High Energy Phys.* **08** (2008) 056.
- [67] E. Joung and M. Taronna, Cubic-interaction-induced deformations of higher-spin symmetries, *J. High Energy Phys.* **03** (2014) 103.
- [68] H. A. Gonzalez, J. Matulich, M. Pino, and R. Troncoso, Asymptotically flat spacetimes in three-dimensional higher spin gravity, *J. High Energy Phys.* **09** (2013) 016.
- [69] M. Ammon, M. Pannier, and M. Riegler, Scalar fields in 3D asymptotically flat higher-spin gravity, *J. Phys. A* **54**, 105401 (2021).
- [70] P. Neckam, Spin-3-gravity in three-dimensional flat space, Master's thesis, University of Vienna, 2023.
- [71] S. Fredenhagen, K. Mkrtchyan, and P. Neckam (to be published).
- [72] A. Campoleoni, S. Fredenhagen, S. Pfenninger, and S. Theisen, Towards metric-like higher-spin gauge theories in three dimensions, *J. Phys. A* **46**, 214017 (2013).
- [73] S. Fredenhagen and P. Kessel, Metric- and frame-like higher-spin gauge theories in three dimensions, *J. Phys. A* **48**, 035402 (2015).
- [74] Note that in terms of the generalized Kronecker delta $\delta_{\beta_1 \dots \beta_n}^{\alpha_1 \dots \alpha_n} = n! \delta_{[\beta_1}^{\alpha_1} \dots \delta_{\beta_n]}^{\alpha_n}$ this means $\epsilon_{\mu\nu\rho} = \delta_{\mu\nu\rho}^{012}$ and $\epsilon^{\mu\nu\rho} = -\delta_{012}^{\mu\nu\rho}$.
- [75] In general, sending some of the coupling constants to zero might not be possible in the full nonlinear theory, where the independent coupling constants may be related for the consistency of the full theory. However, for the classification of all nontrivial deformations of free action, we do not require the existence of a full nonlinear theory that uses the vertices thus found.
- [76] With a small abuse of the terminology, we use the term spin s for the massless Fronsdal fields in three dimensions, described by symmetric tensors of rank s , and correspondingly, spin $s + 1/2$, for Fang-Fronsdal fields, described by symmetric tensor-spinors of rank s .
- [77] We use round brackets for symmetrization with weight one [e.g., $A_{(\mu\nu)} = \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$] and square brackets for antisymmetrization with weight one [e.g., $A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu})$].
- [78] S. Fredenhagen, F. Lausch, and K. Mkrtchyan (to be published).
- [79] This is a complication that arises from the presence of γ matrices due to the extra relations involving them, such as γ tracelessness of fermions. For example, $\bar{\Psi}\gamma^\mu\Psi_\mu \approx 0$, but $\bar{\Psi}\gamma^\mu\gamma^\rho\Psi_\mu \neq 0$. Therefore, not all identities remain valid when one adds γ matrices in the vertex operator. An example illustrating this is $W = \epsilon_{\mu\nu\rho} A_3^\mu P_3^\nu P_1^\rho \approx 0$ [following from (19)], but $\Gamma W \neq 0$.
- [80] R. R. Metsaev, Cubic interactions of arbitrary spin fields in 3d flat space, *J. Phys. A* **53**, 445401 (2020).
- [81] Y. M. Zinoviev, On higher spin cubic interactions in $d = 3$, *J. High Energy Phys.* **11** (2021) 022.
- [82] F. Lausch, Fermionic interaction vertices in higher-spin theory with a focus on quartic interaction terms, Master's thesis, University of Vienna, 2022.
- [83] E. O. Spirin and M. A. Vasiliev, Off-shell fields and conserved currents, *Phys. Lett. B* **852**, 138625 (2024).
- [84] A. Campoleoni, M. Henneaux, S. Hörtnner, and A. Leonard, Higher-spin charges in Hamiltonian form. II. Fermi fields, *J. High Energy Phys.* **02** (2017) 058.
- [85] A. Achúcarro and P. K. Townsend, A Chern-Simons action for three-dimensional anti-de Sitter supergravity theories, *Phys. Lett. B* **180**, 89 (1986).
- [86] A. Achúcarro and P. K. Townsend, Extended supergravities in $d = (2 + 1)$ as Chern-Simons theories, *Phys. Lett. B* **229**, 383 (1989).
- [87] E. Witten, $(2 + 1)$ -dimensional gravity as an exactly soluble system, *Nucl. Phys.* **B311**, 46 (1988).
- [88] M. Bañados, K. Bautier, O. Coussaert, M. Henneaux, and M. Ortiz, Anti-de Sitter/CFT correspondence in three-dimensional supergravity, *Phys. Rev. D* **58**, 085020 (1998).
- [89] M. Henneaux, L. Maoz, and A. Schwimmer, Asymptotic dynamics and asymptotic symmetries of three-dimensional extended AdS supergravity, *Ann. Phys. (N.Y.)* **282**, 31 (2000).
- [90] Y. Hyakutake, Super Virasoro algebra from supergravity, *Phys. Rev. D* **87**, 045028 (2013).
- [91] M. Henneaux, G. Lucena Gómez, J. Park, and S.-J. Rey, Super- $W(\infty)$ asymptotic symmetry of higher-spin AdS₃ supergravity, *J. High Energy Phys.* **06** (2012) 037.
- [92] M. Günaydin, G. Sierra, and P. K. Townsend, The unitary supermultiplets of $d = 3$ anti-de Sitter and $d = 2$ conformal superalgebras, *Nucl. Phys.* **B274**, 429 (1986).
- [93] Note that the holomorphic factorization is specific to massless fields, corresponding to a trivial representation of one of the simple components of the isometry (see, e.g., [94]).
- [94] S. Gwak, E. Joung, K. Mkrtchyan, and S.-J. Rey, Rainbow valley of colored (anti) de Sitter gravity in three dimensions, *J. High Energy Phys.* **04** (2016) 055.
- [95] C. Aragone and S. Deser, Hypersymmetry in $D = 3$ of coupled gravity massless spin 5/2 system, *Classical Quantum Gravity* **1**, L9 (1984).
- [96] Y. M. Zinoviev, Hypergravity in AdS₃, *Phys. Lett. B* **739**, 106 (2014).
- [97] R. Rahman, The uniqueness of hypergravity, *J. High Energy Phys.* **11** (2019) 115.