

New approach to finding invisible states in e^+e^- annihilation and application to BESIII data

Glennys R. Farrar^{1,*}, Qi-Ming Li (李启铭)^{2,3,†} and Chang-Zheng Yuan (苑长征)^{2,3,‡}

¹*Center for Cosmology and Particle Physics, New York University, New York, New York 10003, USA*

²*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*

³*University of Chinese Academy of Sciences, Beijing 100049, China*



(Received 7 November 2023; accepted 14 August 2024; published 24 September 2024)

We compare precision $e^+e^- \rightarrow \mu^+\mu^-$ cross section measurements by BESIII in the $E_{\text{cm}} = 3.8\text{--}4.6$ GeV range, to predictions based on measured R_{had} data. The consistency is poor (p-value < 0.012). Allowing for resonance contributions not seen in R_{had} gives an excellent fit. The 4.6σ state required at 4421 MeV suggests that a substantial portion of $\psi(4415)$ decays are not identified, while the 3.2σ structure at 4211 MeV, if confirmed, points to a new, very narrow resonance. This analysis shows the power of precision $e^+e^- \rightarrow \mu^+\mu^-$ measurements to uncover or probe otherwise difficult to access states.

DOI: 10.1103/PhysRevD.110.L051901

Introduction. Certain types of final states in e^+e^- collisions would not have been detected in existing e^+e^- experiments, due to event acceptance requirements [1]. We call such final states ‘invisible’ for the purposes of this paper. In addition to truly invisible beyond the Standard Model states, hadronic states in which less than $\approx 30\%$ of the energy is in visible hadrons or leptons, or whose energy deposit is too asymmetric, would generally have been rejected [1]. The production of invisible final states in e^+e^- annihilation can be investigated in two ways:

- (i) Measuring the total cross section inclusively, through inclusive initial state radiation (ISR); this approach is limited by the relatively poor resolution of the ISR photon energy and very challenging background.
- (ii) Measuring $\sigma_{e^+e^- \rightarrow \mu^+\mu^-}$ very accurately, since the $\mathcal{O}(\alpha^2)$ correction to the Born contribution depends on the true total e^+e^- cross section through a dispersion relation—an approach pointed out in [2].

Here we develop the second approach. The results are surprising. The combined $\sigma_{e^+e^- \rightarrow \mu^+\mu^-}$ and $\sigma_{e^+e^- \rightarrow \text{hadrons}}$ data implies, at 4.6σ significance, a resonance at 4421 MeV which decays into invisible final states. Whether the undetected final states contain Beyond the Standard Model particles, or a configuration of hadrons that escapes

detection, is not presently known. The mass and width of the resonance is compatible with its being the $\psi(4415)$; in that case, about half its decays are not detected by the R_{had} experiments. The analysis also returns an exceptionally narrow resonance at 4211 MeV whose significance is 3.2σ ; if confirmed this is an entirely new state with exclusively invisible decays.

Our analysis consists of comparing precision data on $e^+e^- \rightarrow \mu^+\mu^-$ from BESIII [3] to predictions using R_{had} data [2]. We find that the measured $e^+e^- \rightarrow \mu^+\mu^-$ cross section shows sharp structure in the 4.2–4.5 GeV range that is in significant conflict with predictions from R_{had} data. We look for possible sources of energy dependent systematic uncertainties in the $e^+e^- \rightarrow \mu^+\mu^-$ measurements, but none appear capable of explaining the data. An excellent fit to the data is obtained by allowing for resonances whose contributions to R_{had} have gone undetected. We report the (small) contribution of these states to g-2 of the muon, the running of the QED coupling, and the intermediate and short distance window functions for comparison to precision lattice QCD calculations.

Data used: The BESIII Collaboration measured the cross section $\sigma_{\mu^+\mu^-}$ for $e^+e^- \rightarrow \mu^+\mu^-$ at center-of-mass (c.m.) energies from 3.8 to 4.6 GeV [3]. The raw cross section data is shown in the left panel of Fig. 1. Dividing the cross section by the Born cross section for easier visibility, the center panel shows $R_{\mu^+\mu^-} \equiv \sigma(e^+e^- \rightarrow \mu^+\mu^-)/\sigma^B(e^+e^- \rightarrow \mu^+\mu^-)$, where $\sigma^B(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$ and α is the QED fine structure constant.

Data samples were collected at 133 c.m. energies with the BESIII detector, operating at the BEPCII collider from 2011 to 2017. The c.m. energy of each dataset is measured

* Contact author: gf25@nyu.edu

† Contact author: liqiming@ihep.ac.cn

‡ Contact author: yuancz@ihep.ac.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI. Funded by SCOAP³.

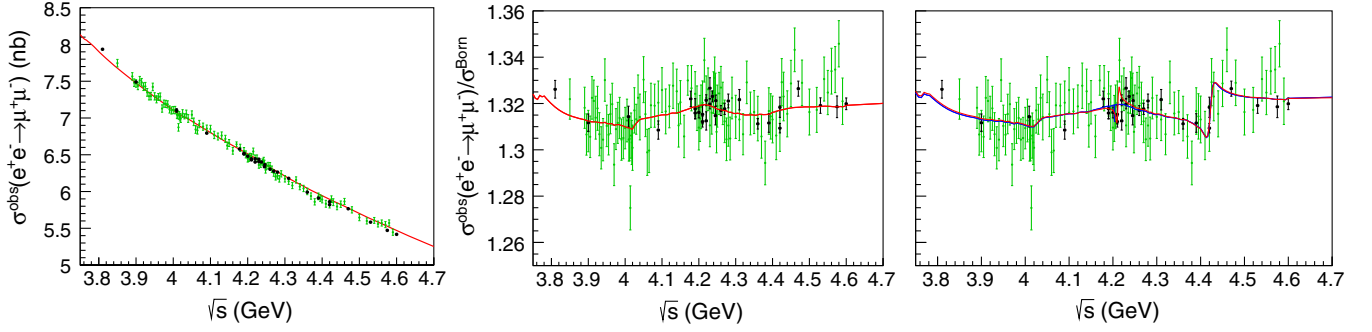


FIG. 1. Left: $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ measured by the BESIII experiment [3]. Center and right: The black (green) points show the corresponding $R_{\mu\mu}$ from high-luminosity (scan) data, with statistical errors. The red curve in the center panel shows the prediction calculated from R_{had} , shifted by the rescaling in Eq. (9) to account for systematics. The right panel shows the fit with one (blue) or two (red, mostly covering blue) constant-width resonances added to the R_{had} data.

using dimuon events, with an uncertainty of ± 0.8 MeV [4]. The statistical precision of the BESIII measurements is about 0.7% for the energy scan datasets and is from 0.16% to 0.35% for the other datasets. The systematic uncertainty of the measurement is estimated by BESIII to be about 2.91% and is common to all the datasets; it originates from several sources including the luminosity measurement (1%), uncertainty in tracking efficiency (1%), detector acceptance, signal extraction, Monte Carlo generator, and so on. We allow for this with a linear rescaling of the $e^+e^- \rightarrow \mu^+\mu^-$ measurement, as discussed in detail below. Our analysis concentrates on a possible shape difference between the observed and predicted $R_{\mu^+\mu^-}$ as a function of energy, which can supply information about final states which have escaped detection in the direct measurement of the hadronic cross section. The precision of our numerical analysis is at the 0.1% level, with the uncertainty mainly introduced by the uncertainties of the R_{had} data used in the calculations.

Theoretical framework. The total cross section $\sigma_{\mu^+\mu^-}$, for the exclusive process $e^+e^- \rightarrow \mu^+\mu^-$, is related to the vacuum polarization (VP), $\Pi(s)$, by

$$\sigma_{\mu^+\mu^-}(s) = \frac{\sigma_{\mu^+\mu^-}^{\text{B}}(s)}{|1 - \Pi(s)|^2}, \quad (1)$$

where s is the c.m. energy squared. The charged lepton contribution to $\Pi(s)$ can be calculated in perturbation theory and we omit it in subsequent formulae for clarity. The contribution to $\Pi(s)$ from virtual states other than leptons, $\Pi_{nl}(s)$, is related to the total cross section $\sigma_{nl}(s)$ to not-leptons final states in the one-photon exchange approximation through a dispersion relation (see, e.g., [2]),

$$\Pi_{nl}(s) = \frac{s}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \frac{\sigma_{nl}(s')}{s - s' + i\epsilon} ds'. \quad (2)$$

Here σ_{nl} is the cross section for e^+e^- to hadronic final states (more precisely, nonleptonic final states) with no

photon emission. Measured cross sections include soft photon emission, but published R_{had} data corrects for this; details for how this can be done iteratively are given in [2]. Using the identity $(x + i\epsilon)^{-1} = P(1/x) - i\pi\delta(x)$, gives

$$\Pi_{nl}(s) = -\frac{s}{4\pi^2\alpha} P \int_{4m_\pi^2}^{\infty} \frac{\sigma_{nl}(s')}{s' - s} ds' - i\frac{s}{4\pi\alpha} \sigma_{nl}(s), \quad (3)$$

where P represents principal value. The integration is performed analytically for narrow resonances J/ψ , $\psi(2S)$, $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$. The high-energy part of the integration assumes that $R_{\text{had}}(s) = R_{\text{had}}(s_1)$ is constant above a certain value s_1 (taken to be above the Υ resonance region), and the integral between threshold and s_1 is carried out numerically after separation of the principle value part. Thus,

$$\begin{aligned} \text{Re}\Pi_{nl}(s) = & \frac{3s}{\alpha} \sum_j \frac{\Gamma_{e^+e^-}^j}{M_j} \frac{s - M_j^2}{(s - M_j^2)^2 + M_j^2\Gamma_j^2} \\ & + \frac{\alpha}{3\pi} R_{\text{had}}(s_1) \ln \left| \frac{s - s_1}{s_1} \right| \\ & - \frac{s}{4\pi^2\alpha} \int_{4m_\pi^2}^{s_1} \frac{\sigma_{\text{nr}}(s') - \sigma_{\text{nr}}(s)}{s' - s} ds' \\ & - \frac{s\sigma_{\text{nr}}(s)}{4\pi^2\alpha} \ln \left| \frac{s_1 - s}{4m_\pi^2 - s} \right|, \end{aligned} \quad (4)$$

where Γ_j , $\Gamma_{e^+e^-}^j$, and M_j denote the total width, partial width to e^+e^- , and mass of the resonance j , respectively. Here, $\sigma_{\text{nr}}(s)$ is the $\sigma_{nl}(s)$ in (3) with the contributions from narrow resonances subtracted.

The measured cross section for $e^+e^- \rightarrow \mu^+\mu^-$ includes unobserved radiated photons, so before comparing to data we integrate the exclusive cross section $\sigma_{\mu^+\mu^-}(s)$ calculated from Eq. (1), with the ISR function as discussed in [2].

We model a possible undetected contribution to σ_{nl} , either unresolved due to gaps in the R_{had} measurements or decaying dominantly into undetected final states, as being

due to one or two resonances, R1 and R2. The generalization to more resonances is straightforward from this case, and allowing for two unseen states proves sufficient for our purposes. For this initial study, we assume R1 and R2 can be described by constant-width Breit-Wigner (BW) functions,

$$A = \frac{\sqrt{12\pi}\Gamma_{ee}\Gamma}{s - m^2 + im\Gamma}, \quad (5)$$

with m , Γ , and Γ_{ee} being the mass, width, and electronic partial width of the resonance. We allow for the possibility that the more massive resonance (R2) can have common, interfering decay channels with R1.

The resonances R1 and R2 may or may not have final states contributing to the detected R_{had} , but their undetected contributions simply add to σ_{nl} without interference since the two types of final states are inherently orthogonal. If there is only one resonance, R1, it can be treated using Eqs. (3) and (4), just like the known vector mesons. This fit entails three new parameters—the mass, width and electronic width of R1: m_1 , Γ_1 , $\Gamma_{ee,1}$. However in computing $|1 - \Pi_{R_{\text{had}}} - \Pi_R|^{-2}$, where Π_R is the contribution from the new sector, one must take care about the proper treatment of possible interference terms within the new sector. When there are two resonances, we have two masses, widths and Γ_{ee} 's, and in addition the branching fraction $\mathcal{B}$ and relative

phase ϕ of R2 decay into R1 final states, which we take to be energy independent constants. If some resonance has both visible and unseen channels, the values of m , Γ and Γ_{ee} should be compatible, when proper account is taken of interference within each sector; for this, a more sophisticated treatment than the constant width BW used here could be warranted.

The contribution to σ_{nl} of the resonances R1 and R2 not already included via the measured R_{had} is then, in this approximation:

$$\begin{aligned} \sigma_{R1+R2} &= |A_{R1} + e^{i\phi}\sqrt{\mathcal{B}}A_{R2}|^2 + (1 - \mathcal{B})|A_{R2}|^2 \\ &\equiv \sigma_{R1} + \sigma_{R2} + \sigma_{R1R2}, \end{aligned} \quad (6)$$

where $A_{R1,R2}$ are the appropriate BW functions, given in Eq. (5) for the constant width treatment, and σ_{R1R2} is merely the shorthand

$$\sigma_{R1R2} \equiv 2\sqrt{\mathcal{B}}\text{Re}[A_{R1}^* A_{R2} e^{i\phi}]. \quad (7)$$

The first two terms in Eq. (6) are just Breit-Wigner cross sections and their contributions to $\Pi_{nl}(s)$ in Eq. (3) are treated in the standard way. The last term in Eq. (6)'s contribution to $\text{Re}\Pi_{nl}$ in (3) must be directly integrated. Taking the BW's for R1 and R2 to be momentum-independent as in Eq. (5) we find

$$\begin{aligned} \text{Re}\Pi_{R1R2}(s) &\equiv -\frac{s}{4\pi^2\alpha}P \int_{4m_\pi^2}^{\infty} \frac{2\sqrt{\mathcal{B}}\text{Re}[A_{R1}^* A_{R2} e^{i\phi}](s')}{(s' - s)} ds' \\ &= K \text{Re} \left[\frac{e^{i\phi}}{(s - a)(s - b)(b - a)} \left(b \ln \frac{a}{s} - a \ln \frac{b}{s} + s \ln \frac{b}{a} \right) \right], \end{aligned} \quad (8)$$

where $a = m_1(m_1 + i\Gamma_1)$, $b = m_2(m_2 - i\Gamma_2)$, $K = 6s/(\pi\alpha)\sqrt{\mathcal{B}\Gamma_{ee,1}\Gamma_1\Gamma_{ee,2}\Gamma_2}$ and $\ln(|z|e^{i\theta}) = \ln|z| + i\theta$. If the resonance contributions are not constant-width Breit-Wigner functions, the integral can be done numerically.

Before closing this discussion, let us comment on how the characteristic dip-bump structure from the virtual contribution of resonances to $R_{\mu^+\mu^-}$ comes about. Unlike the total cross section for $e^+e^- \rightarrow \text{hadrons}$, which can be well-approximated as the sum over squares of Breit-Wigner amplitudes into each distinct and interfering final state, in $e^+e^- \rightarrow \mu^+\mu^-$ we have virtual intermediate states which are not in principle observable. The quantum interference of the different amplitudes is captured through the vacuum polarization and dispersion integral as given in Eqs. (1) and (2) rather than with the sum of Breit-Wigner amplitudes used in Eq. (3) of Ref. [3]. The 1 in the denominator of Eq. (1) is the leading single photon contribution, thus the real part of the vacuum polarization plays a more important role in the interference than does its imaginary part. From

Eq. (4) expressing the real part, one sees how the contribution of a resonance to $\text{Re}\Pi_{nl}(s)$ changes sign with respect to the Born amplitude which evolves continuously, producing the distinctive dip-bump structure in the $e^+e^- \rightarrow \mu^+\mu^-$ cross section at a resonance. Since Ref. [3] does not use the correct expression for the contribution of resonances to the $e^+e^- \rightarrow \mu^+\mu^-$ cross section, their conclusions concerning hadronic resonances from fitting that data are not valid.

Fitting $e^+e^- \rightarrow \mu^+\mu^-$ from R_{had} . We define χ^2 to assess the quality of a fit as

$$\begin{aligned} \chi^2 &= \sum_i^N \left(\frac{R_{\mu^+\mu^-}^i - \hat{R}_{\mu^+\mu^-}^i / (f_0 + f_1(E_i - E_c))}{\sigma_{R_{\mu^+\mu^-}^i}} \right)^2 \\ &+ \frac{\frac{1}{N} \sum_i^N (1 - f_0 - f_1(E_i - E_c))^2}{(0.0291)^2}, \end{aligned} \quad (9)$$

where $R_{\mu^+\mu^-}^i$ and $\hat{R}_{\mu^+\mu^-}^i$ are the predicted and measured values of $R_{\mu^+\mu^-}^i$ at the i th data point, $\sigma_{R_{\mu^+\mu^-}^i}$ is the statistical error on the i th data point, 0.0291 is the systematic overall normalization error estimate from BESIII for these measurements, and f_0, f_1 provide a linear, energy-dependent rescaling to the overall normalization; $E_c = 4.2$ GeV is the mean energy of the dataset.¹

The best fit to the $R_{\mu^+\mu^-}$ data, using Eq. (1) and Π_{nl} calculated in [2] from the measured R_{had} [5] using Eq. (3), yields the fit shown in the left and center panels of Fig. 1, with normalization rescaling parameters $f_0 = 1.00286 \pm 0.00052$ and $f_1 = 0.01811 \pm 0.00301$ GeV⁻¹. The normalization rescaling ranges from -0.004 to $+0.01$ over the 3.8–4.6 GeV range of the data—well within the estimated absolute normalization uncertainty of 0.0291.²

However even after this rescaling, the data is significantly different from the prediction based on the measured $\sigma_{e^+e^- \rightarrow \text{had}}$, as can be seen by eye and as reflected in the goodness-of-fit parameter $\chi^2/ndf = 170.47/(133 - 2)$, corresponding to a confidence level of only 0.012. Thus in the following we consider the possibility that some contributions to the vacuum polarization could have been missed.

The right panel of Fig. 1 shows the fit with one and two additional resonances. The fit parameters are listed in Table I. The fit with one additional resonance improves the χ^2 by 27.6 with 3 more free parameters giving $\chi^2/ndf = 142.87/(133 - 5)$. The statistical significance of this resonance is 4.6σ . However the confidence level is 0.17, indicating possible room for improvement in the fit to the data.

The fit with two additional resonances has a $\chi^2/ndf = 127.53/(133 - 8)$, corresponding to a confidence level of 0.42, indicating a very good fit to the data. The masses of the two resonances are too far apart relative to

TABLE I. Fit results adding contribution from constant-width Breit-Wigner resonances.

Parameters	One-BW fit	Two-BW fit
m_1 (MeV)	...	$4211.4^{+1.8}_{-0.8}$
Γ_1 (MeV)	...	0.18 ± 0.56
$\Gamma_{ee,1}$ (keV)	...	0.077 ± 0.028
m_2 (MeV)	$4421.5^{+6.2}_{-1.4}$	$4421.8^{+7.0}_{-1.6}$
Γ_2 (MeV)	15.9 ± 10.9	17.3 ± 12.1
$\Gamma_{ee,2}$ (keV)	0.63 ± 0.22	0.68 ± 0.23
f_0	1.0019 ± 0.0004	1.0017 ± 0.0005
f_1 (GeV ⁻¹)	0.0175 ± 0.0023	0.0184 ± 0.0023
χ^2/ndf	$142.87/(133 - 5)$	$127.53/(133 - 8)$

their narrow widths to interfere, so we drop $\mathcal{B}$ and ϕ for the final fitting. Allowing for a second resonance improves the χ^2 by 15.3 with three more free parameters compared to the fit with one resonance, as can be seen in the left panel of Fig. 2 which shows the variation in χ^2 when scanning over the mass m_1 of a possible additional resonance taking the parameters of R2 to be fixed. The statistical significance of the second resonance is 3.2σ . Figure 2 (right) shows the contribution of R1 and R2 to the R_{had} value, where R_{had} here means the total e^+e^- cross section to states other than charged lepton pairs, divided by $\sigma_{\mu^+\mu^-}^B$. Inclusion of a third resonance does not improve the fit significantly, as seen in the left panel of Fig. 2; the most significant (but only 0.8σ) contribution is at around 4020 MeV.

Implications for the vacuum polarization. The presence of resonances R1 and R2 decaying into ‘invisible’ final states modifies the vacuum polarization, as discussed above in connection with $R_{\mu^+\mu^-}$. If the missed final states are hadronic, they modify the hadronic vacuum polarization (HVP) relative to that inferred from R_{had} data and they contribute to lattice QCD measurements of the HVP. Whether or not the missed states are hadronic, the modified VP changes predictions for precision observables such as the muonic g-2 and the running of the QED coupling. For brevity, in the following we subsume both cases under the label HVP as the analysis is identical. The HVP contribution to the muon g-2 is

$$a_\mu^{\text{HVP,LO}} = \frac{\alpha^2}{3\pi^2} \int ds \frac{R(s)K(s)\Theta(s)}{s}, \quad (10)$$

where s is the c.m. energy squared, $R(s) \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma_B(e^+e^- \rightarrow \mu^+\mu^-)$ and the function $K(s)$ is given in [6]. The function $\Theta(s) \rightarrow 1$ when calculating $a_\mu^{\text{HVP,LO}}$, but we include it for more detailed comparison to lattice QCD calculations below. With a different weighting function an analogous calculation yields the running of the QED coupling to higher scales.

¹To test for the possible impact of potential correlated and uncorrelated systematic uncertainties among data from different running years, we also perform the analysis using only data taken in winter and spring 2013/2014; there is no significant difference. In the restricted sample, 104 energy scan datasets were taken from 3.84 GeV to 4.59 GeV, each with an integrated luminosity of about 8 pb⁻¹. The scan data were taken from Dec. 9, 2013 to Jan. 24, 2014. For each point in the scan data-taking, the accelerator energy was set to a specific value, data was taken for about 7 hours, then the energy was increased one step and data was taken again. These data were taken with a monotonically increasing energy. Five additional high-luminosity datasets were taken in the remainder of the same data-taking period, at c.m. energies 4.600, 4.467, 4.527, 4.575, and 4.416 GeV, respectively.

²As an alternative to the linear-in-energy systematic shift in normalization, we considered whether the effect could be due to the presence of a wide, high energy resonance which was not well-constrained by experiments. However the parameters of such a resonance would have to be totally far-fetched, with a width of 1.7 GeV, and the improvement in χ^2 is less than for the linear shift. This then validates the baseline treatment.

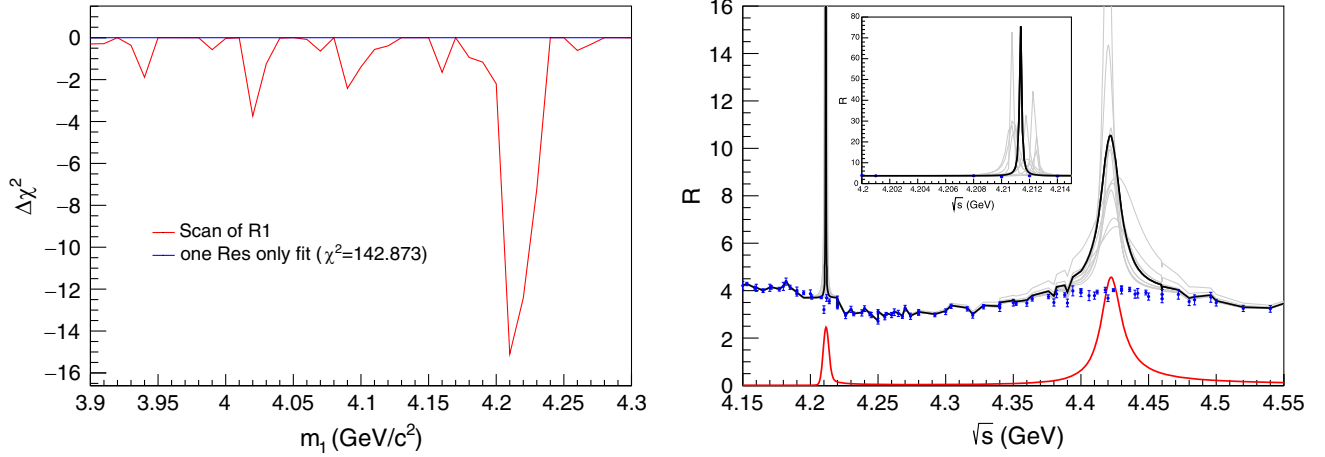


FIG. 2. Left: Variation of χ^2 when coarsely scanning over the mass of a second contributing resonance. Right: PDG data on R_{had} (blue) with the contribution of R1 and R2 added shown in black, connected with a line for readability. The gray curves are 10 randomly drawn solutions from the covariance matrix. The red curves show the resonances as they would be observed, due to beam energy spread and ISR smearing.

We calculate the contributions of the new resonances to a_μ and $\Delta\alpha(Z)$, by sampling the fit parameters according to the covariance matrix. The contributions of resonances R1 and R2 to a_μ^{HVP} and $\Delta\alpha(Z)$ are closely correlated to one another, increasing them by $\delta a_\mu = 3.14^{+0.99}_{-0.83} \times 10^{-11}$ and $\delta\Delta\alpha(Z) = 0.71^{+0.22}_{-0.19} \times 10^{-4}$. For more direct comparison to the predictions of lattice QCD, we use window functions to calculate the R1 and R2 contributions to the intermediate and short distance window functions [7], finding: $\delta a_\mu^W = 0.45^{+0.14}_{-0.12} \times 10^{-11}$ and $\delta a_\mu^{SD} = 2.69^{+0.84}_{-0.72} \times 10^{-11}$. As of now, all these modifications are smaller than the uncertainties of the measurements.

Interpretation. The dependence of $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ on energy seen in the high-precision measurements by the BESIII Collaboration [3] between $E_{\text{cm}} = 3.8$ to 4.6 GeV, is poorly described using dispersion relations and the measured $\sigma(e^+e^- \rightarrow \text{hadrons})$. The shape of the spectrum has a confidence level of only 0.012, after allowing for a linear, energy-dependent shift in absolute normalization within the systematic uncertainty estimate. Adding additional resonances, R1 and R2, results in an excellent fit. The contributions of R1 and R2 to R_{had} are shown in the right panel of Fig. 2, with the mean value in black and 10 random realizations from the covariance matrix in gray. The R_{had} data in blue shows no such prominent structures. This means that if the R1 and R2 signals in $e^+e^- \rightarrow \mu^+\mu^-$ are real, their final states somehow elude detection.

The upper state R2, whose statistical significance is 4.6σ , has mass $4421.5^{+7.0}_{-1.6}$ MeV, width 17.3 ± 12.1 MeV and $\Gamma_{ee} = 0.68 \pm 0.23$ keV. The mass is virtually identical to that of $\psi(4415)$ whose PDG value is 4421 ± 4 MeV. The PDG lists many determinations of the width of $\psi(4415)$, ranging from 33 ± 10 MeV [8] to

$122.5 \pm 7.5 \pm 8.1$ MeV [9]; such a diversity of results naturally arises when the widths are measured for different final states, if there are multiple resonances with multiple decay channels interfering with different phases. A possible interpretation of the combined $R_{\mu^+\mu^-}$ and R_{had} data is that R2 is in fact the $\psi(4415)$, with a width $\lesssim 30$ MeV to account for the structure in $R_{\mu^+\mu^-}$, and a substantial fraction of its final states not being detected in the R_{had} measurements. The relatively unstructured R_{had} in this energy range could result from the various hadronic final states interfering with those of neighboring resonances. In this scenario, the Γ_{ee} of R2 should be interpreted as $\Gamma_{ee} \times \mathcal{B}(\psi(4415) \rightarrow \text{invisible})$ and that of the $\psi(4415)$ from PDG (0.58 ± 0.07 keV) [10] as $\Gamma_{ee} \times \mathcal{B}(\psi(4415) \rightarrow \text{visible})$ final states. Then Γ_{ee} of the $\psi(4415)$ would be about 1.3 keV. It should be noted that interpretation of the R_{had} measurements is quite model dependent [11] and that the resonance parameters of the $\psi(4415)$ in PDG are estimates [10].

The lower mass resonance R1, at $4211.4^{+1.8}_{-0.8}$ MeV, with $\Gamma = 0.18 \pm 0.56$ MeV and $\Gamma_{ee} = 0.077 \pm 0.028$ keV, has a statistical significance of 3.2σ in the $R_{\mu^+\mu^-}$ data. It is very different in both mass and width from the known states in this energy region, the $\psi(4160)$ and $\psi(4230)$ [10]. Although intrinsically narrow enough to be missed between scan points, initial state radiation would spread its contribution over several scan points as indicated in the red curve in the right panel of Fig. 2. Thus if R1 is not just a statistical or systematic artifact, its final states must not meet event selection criteria.

Summary and conclusions. Following up on the remark that certain types of hadronic final states would not meet the event selection criteria for $e^+e^- \rightarrow \text{hadrons}$ [1], we have performed the first search for evidence of missed states via their virtual effect in $e^+e^- \rightarrow \mu^+\mu^-$. In the dispersive

representation of the vacuum polarization, a narrow resonance in the hadronic cross section produces a characteristic dip-peak shape in the $e^+e^- \rightarrow \mu^+\mu^-$ cross section. This striking structure appears in two places in the BESIII energy range. The one at higher mass has a statistical significance of 4.6σ . If real, its position and leptonic width suggests it is due to the known resonance $\psi(4415)$, whose true width is narrower and its production amplitude greater than apparent in the R_{had} data. This could be due to a significant portion of its final states going undetected. There is also an indication of the same shape structure produced by a very narrow state at 4211 MeV, albeit with only 3.2σ statistical significance in the present data.

We explored many different types of systematic errors, indirect effects from radiative corrections or other artifacts, and found that only a narrow resonancelike contribution to the HVP produces the characteristic narrow, dip-bump structure observed in $R_{\mu^+\mu^-}$. It cannot be excluded that these structures arise from statistical fluctuations or systematic glitches in the $e^+e^- \rightarrow \mu^+\mu^-$ or $e^+e^- \rightarrow \text{hadrons}$ data, but the agreement of fits using different datasets taken at different times and in both scan and hi-luminosity modes, argues for their robustness.

There are several paths to investigate and extend these results. Recorded but not-selected events in BESIII data can be reexamined, e.g., searching for pair-produced, undetected neutrals via interactions of their decay products in the detector or via asymmetric energy deposits which caused the event class to be rejected previously. An analysis

of inclusive ISR data from *BABAR*, *Belle*, and *Belle II* would in principle be complementary to this approach, but given the present energy resolution of ISR photons such a search could not detect states as narrow as those which are accessible with our technique. The states we find have only a small impact on a_μ and $\Delta\alpha(Z)$, but similar states at lower energies would have a greater impact. If sufficiently precise $e^+e^- \rightarrow \mu^+\mu^-$ data were available at lower energy, they could be used as here to see if undetected final states may contribute to the vacuum polarization in those energy regions. If hadronic, such states might contribute to removing the 3.8σ discrepancy between lattice QCD and dispersive R_{had} determinations of the intermediate-window-function-weighted HVP [12–16].

BESIII is currently taking data up to 5.0 GeV and will be able to extend the energy coverage to 5.6 GeV, with improved peak luminosity, after the upgrade in 2024; these measurements should significantly improve the sensitivity to undetected states in e^+e^- annihilation in the 3.6–6 GeV regime.

Acknowledgments. We have benefited from helpful discussions with A. Bondar, L. Dixon, M. Karliner and N. Weiner. This work is supported in part by the National Key Research and Development Program of China under Contract No. 2020YFA0406300, National Natural Science Foundation of China (NSFC) under Contracts No. 11835012 and No. 12335004. The research of G. R. F. was supported by the Simons Foundation.

-
- [1] G. R. Farrar, [arXiv:2206.13460](#).
 - [2] X.-K. Dong, X.-H. Mo, P. Wang, and C.-Z. Yuan, *Chin. Phys. C* **44**, 083001 (2020).
 - [3] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **102**, 112009 (2020).
 - [4] M. Ablikim *et al.* (BESIII Collaboration), *Chin. Phys. C* **40**, 063001 (2016).
 - [5] J. Z. e. a. Bai (BES Collaboration), *Phys. Rev. Lett.* **88**, 101802 (2002).
 - [6] T. Aoyama *et al.*, *Phys. Rep.* **887**, 1 (2020).
 - [7] G. Colangelo, A. X. El-Khadra, M. Hoferichter, A. Keshavarzi, C. Lehner, P. Stoffer, and T. Teubner, *Phys. Lett. B* **833**, 137313 (2022).
 - [8] J. Siegrist *et al.*, *Phys. Rev. Lett.* **36**, 700 (1976).
 - [9] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **131**, 151903 (2023).
 - [10] R. L. Workman *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
 - [11] X. H. Mo, C. Z. Yuan, and P. Wang, *Phys. Rev. D* **82**, 077501 (2010).
 - [12] S. Borsanyi *et al.*, *Nature (London)* **593**, 51 (2021).
 - [13] M. Cè *et al.*, *Phys. Rev. D* **106**, 114502 (2022).
 - [14] C. Alexandrou *et al.* (Extended Twisted Mass Collaboration), *Phys. Rev. D* **107**, 074506 (2023).
 - [15] T. Blum *et al.* (RBC, UKQCD Collaborations), *Phys. Rev. D* **108**, 054507 (2023).
 - [16] A. Boccaletti *et al.*, [arXiv:2407.10913](#).