

Next-to-leading-order electroweak correction to $H \rightarrow Z^0\gamma$ Wen-Long Sang^{1,*}, Feng Feng^{2,3,†} and Yu Jia^{3,4,‡}¹*School of Physical Science and Technology, Southwest University, Chongqing 400700, China*²*China University of Mining and Technology, Beijing 100083, China*³*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*⁴*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*

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Inspired by the recent observation of the Higgs boson radiative decay into Z^0 by ATLAS and CMS Collaborations, we investigate the next-to-leading-order (NLO) electroweak correction to this rare decay process in Standard Model (SM). Implementing the on shell renormalization scheme, we find that the magnitude of the NLO electroweak correction may reach 7% of the leading-order (LO) prediction, much more significant than that of the NLO QCD correction, which is merely about 0.3%. After incorporating the $\mathcal{O}(\alpha)$ correction, the predicted partial width from various α schemes tend to converge to each other. Including both NLO electroweak and QCD corrections, the SM prediction for the branching fraction shifts from the LO value of $(1.40\text{--}1.71) \times 10^{-3}$ to $(1.55 \pm 0.06) \times 10^{-3}$, considerably lower than the measured value $\mathcal{B}_{\text{exp}}[H \rightarrow Z^0\gamma] = (3.4 \pm 1.1) \times 10^{-3}$. Resolving this discrepancy clearly calls for further theoretical investigations, and, more importantly, experimental efforts from HL-LHC and the prospective Higgs factories such as CEPC and FCC-ee.

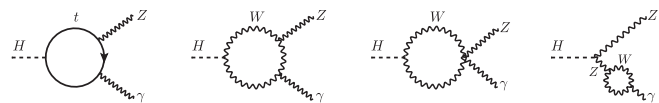
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Introduction. The groundbreaking discovery of a 125 GeV boson at LHC in 2012 heralds a new era of particle physics [1,2]. After decade long experimental endeavors, the couplings between this new particle and W , Z , heavy fermions have been confirmed to be consistent with the Standard Model (SM) predictions, provided that this new boson is identified with the Higgs boson [3–5]. Nevertheless, it is still important to test the SM predictions in some rare decay channels of Higgs boson, exemplified by $H \rightarrow Z^0\gamma$. As a loop-induced process, this decay channel serves a fruitful platform to look for the footprints of new particles predicted in many beyond Standard Model (BSM) scenarios.

Recently, the ATLAS and CMS Collaborations jointly reported the evidence of observing the rare decay $H \rightarrow Z^0\gamma$ [6–8], and found $\mu_{Z\gamma} = 2.2 \pm 0.7$, with $\mu \equiv (\sigma\mathcal{B})_{\text{meas}}/(\sigma\mathcal{B})_{\text{th}}$ being a parameter to gauge how well the measurement conforms to the SM prediction.

The measured branching fraction is $\mathcal{B}(H \rightarrow Z\gamma) = (3.4 \pm 1.1) \times 10^{-3}$, which is more than twice larger than the SM prediction $(1.5 \pm 0.1) \times 10^{-3}$ [9,10]. This should be contrasted with a similar channel $H \rightarrow \gamma\gamma$ with $\mu_{\gamma\gamma} = 1.04^{+0.10}_{-0.09}$ [11], which exhibits a satisfactory agreement between the measurement and the SM prediction. Recently, there has been a flurry of theoretical work attempting to alleviate the discrepancy between the measurements and SM predictions for $H \rightarrow Z^0\gamma$, by invoking various BSM models [12–19].

Needless to say, in order to ultimately resolve the discrepancy related to $H \rightarrow Z^0\gamma$, it is compulsive to first have a as precise as possible prediction in SM. At leading order (LO), the process $H \rightarrow Z^0\gamma$ is mediated via a heavy quark or a W boson loop, as depicted in Fig. 1. The LO prediction has been available four decades ago [20,21]. It turns out that the W -loop induced contribution dominates the t -loop induced one, while there exists a destructive interference between these two channels. The NLO QCD correction in numerical form has been known in early 1990s [22], and its analytical form has also been available

FIG. 1. Some representative LO diagrams for $H \rightarrow Z^0\gamma$.

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for about one decade [23,24]. The QCD correction appears to be rather insignificant. This may be partly understood by the fact that the gluons can be attached only to the top-quark loop, which by itself only yields a small portion of the contribution with respect to the W loop.

Therefore, the missing NLO electroweak correction is envisaged to constitute the major theoretical uncertainty for this decay channel, which has been estimated to reach 5% level of the LO contribution [10]. It is the goal of this work to conduct a detailed investigation on the NLO electroweak correction for $H \rightarrow Z^0\gamma$.

Partial width from form factors. Let us express the amplitude for $H \rightarrow Z^0\gamma$ as $\mathcal{A} = T_{\mu\nu} \epsilon_Z^{\mu*}(p_1) \epsilon_\gamma^{\nu*}(p_2)$. By Lorentz invariance, $T_{\mu\nu}$ can be decomposed into the following most general tensor structures:

$$T^{\mu\nu} = p_1^\mu p_1^\nu T_1 + p_2^\mu p_2^\nu T_2 + p_1^\mu p_2^\nu T_3 + p_2^\mu p_1^\nu T_4 + p_1 \cdot p_2 g^{\mu\nu} T_5 + \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} T_6, \quad (1)$$

where p_1 and p_2 signify the momenta of the outgoing Z boson and photon, respectively, and T_i ($i = 1, \dots, 6$) signify six scalar form factors. The transversity condition $p_1 \cdot \epsilon_Z = p_2 \cdot \epsilon_\gamma = 0$ implies that the terms entailing T_2 and T_3 do not contribute. The Ward identity $p_{2\nu} T^{\mu\nu} = 0$ implies that $T_1 = 0$ and $T_4 = -T_5$. The CP -violating T_6 term first arises at two-loop order in the SM, but its interference with the LO amplitude yields a vanishing contribution. Hence, we can neglect the T_6 term at the outset as far as the NLO electroweak correction is concerned. Consequently, only one independent form factor, $T_5 = -T_4$, finally survives. In practice, it is convenient to introduce a new dimensionless form factor $\tilde{T}_5$,

$$T_5 = \frac{M_H^2}{M_W s_W (M_H^2 - M_Z^2)} \tilde{T}_5, \quad (2)$$

where $c_W \equiv \frac{M_W}{M_Z}$, and $s_W = \sqrt{1 - c_W^2}$. The partial width then becomes

$$\Gamma(H \rightarrow Z^0\gamma) = \frac{1}{2M_H} \frac{1}{8\pi} \frac{2|\mathbf{p}_1|}{M_H} \frac{M_H^4}{2M_W^2 s_W^2} |\tilde{T}_5|^2, \quad (3)$$

where the three-momentum of the Z^0 boson in the Higgs boson rest frame is $|\mathbf{p}_1| = (M_H^2 - M_Z^2)/2M_H$. Our task is to deduce $\tilde{T}_5$ through NLO in α .

Outline of calculation. Throughout this work we adopt the Feynman gauge and employ the dimensional regularization to regularize UV divergences. We utilize the package FeynArts [25] to generate Feynman diagrams and the corresponding amplitudes for $H \rightarrow Z^0\gamma$ through NLO

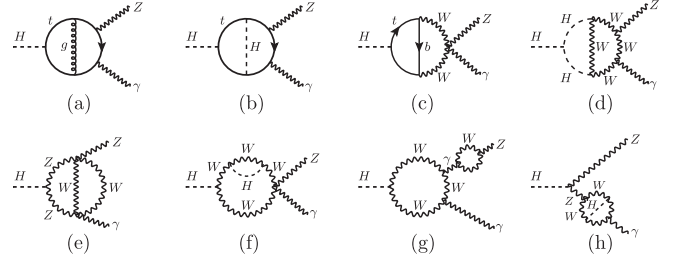


FIG. 2. Some representative two-loop diagrams for $H \rightarrow Z^0\gamma$. (a) Represents a sample diagram for NLO QCD correction, while (b)–(h) represent sample diagrams for NLO electroweak correction.

in α . This process entails approximately 50 LO (one-loop) diagrams, and more than 10^4 NLO (two-loop) diagrams (including the NLO QCD diagrams). Some representative diagrams relevant to the NLO QCD and electroweak corrections are depicted in Fig. 2. The form factors T_4 and T_5 are extracted from the amplitude with the aid of the covariant projectors [24]. We employ the packages FeynCalc [26] and FormLink [27] to perform Lorentz contraction and Dirac trace. We use the packages Apart [28] and FIRE [29] for integration-by-part reduction. We end up with eight one-loop master integrals (MIs) and over 700 two-loop MIs. The package AMFlow [30–32] is invoked to evaluate all these MIs with high numerical accuracy.

The on shell renormalization scheme¹ has been widely used in the field of electroweak radiative correction to tame the UV divergence [35,36]. The renormalized parameters are chosen to be those measured very precisely, such as Higgs boson mass, W/Z masses, top quark mass, and QED coupling at Thomson limit. Having implemented the mass and charge renormalization ($e_0 = Z_e e$), we find it convenient to stay with the bare field in the practical calculation. Following the Lehmann-Symanzik-Zimmermann reduction formula, we multiply the amputated amplitude of $H \rightarrow Z^0\gamma$ by the factor $\sqrt{Z_H Z_{ZZ} Z_{AA}}$, where the Z_H , Z_{ZZ} , and Z_{AA} denote the field renormalization constants associated with H , Z^0 , and γ , respectively. These field renormalization constants can be inferred from the unrenormalized propagators to one-loop accuracy, by identifying the residues in the on shell limit. It is important to explicitly include those diagrams where Z^0 converts via a loop into a outgoing

¹We stress that the widths of W/Z bosons have been neglected, because these virtual particles can not become on shell simultaneously. In principle, one may adopt the complex mass scheme [33,34] to incorporate the width effect in a gauge invariant way. The difference between the complex mass scheme and the conventional on shell renormalization scheme is anticipated to be immaterial at the level of accuracy concerned in this work.

photon in the external leg, or vice versa, as depicted in Fig. 1(d) and Fig. 2(g,h).²

After carrying out the mass and charge renormalization, we finally end up with the UV-finite results for T_4 and T_5 . We have confirmed the Ward identity requirement $T_4 = -T_5$ holds at $\mathcal{O}(\alpha)$ level, which serves a nontrivial crosscheck.

Three different recipes about charge renormalization. Implementing the charge renormalization in electroweak theory is not unique, which leads to several popular variants in the on shell renormalization scheme. In the so-called $\alpha(0)$ scheme, where α is taking with the Thomson-limit value. To one-loop order, δZ_e is expressed as

$$\delta Z_e|_{\alpha(0)} = \frac{1}{2} \Pi^{AA}(0) - \frac{s_W}{c_W} \frac{\Sigma_T^{AZ}(0)}{M_Z^2}, \quad (4)$$

where $\Pi(s) \equiv \Sigma_T^{AA}(s)/s$. The photon vacuum polarization in small momentum transfer is sensitive to the low-energy hadronic contribution, thereby an intrinsically nonperturbative quantity. Alternatively, one may rewrite δZ_e in $\alpha(0)$ scheme as

$$\delta Z_e|_{\alpha(0)} = \frac{1}{2} \Delta\alpha_{\text{had}}^{(5)}(M_Z) + \frac{1}{2} \text{Re}\Pi^{AA(5)}(M_Z^2) + \frac{1}{2} \Pi_{\text{rem}}^{AA}(0) - \frac{s_W}{c_W} \frac{\Sigma_T^{AZ}(0)}{M_Z^2}, \quad (5)$$

where $\Pi^{AA(5)}(M_Z^2)$ is the photon vacuum polarization from five massless quarks at momentum transfer M_Z^2 . $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$, as a nonperturbative parameter that encapsulate the hadronic contribution to photon vacuum polarization, can be determined from the measured R values in low-energy e^+e^- experiments. $\Pi_{\text{rem}}^{AA}(0)$ represents the vacuum polarization from the W bosons, charged leptons and top quarks at zero momentum transfer. Note all these

²Alternatively, one may choose to work with the renormalized gauge fields and introduce new counterterms in the renormalized SM Lagrangian. The renormalization constants linking the bare and renormalized Z^0 and γ fields have to be a 2×2 matrix with nonvanishing off-diagonal elements [37],

$$\begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2} \delta Z_{ZZ} & \frac{1}{2} \delta Z_{ZA} \\ \frac{1}{2} \delta Z_{AZ} & 1 + \frac{1}{2} \delta Z_{AA} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}.$$

By the on shell renormalization condition, in this case one no longer needs compute those topologically unamputated diagrams exemplified by Fig. 1(d) and Fig. 2(g,h). However, as an extra price to pay, one has to consider the $HZ\gamma$ vertex induced by the counterterm Lagrangian, with a Feynman rule i.e., $g_{\mu\nu} M_W \frac{1}{s_W c_W^2} \frac{1}{2} \delta Z_{ZA}$ [37]. It is envisaged that the renormalized perturbation theory must yield the identical results as the bare field approach as adopted in this work.

terms, except $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$, can be computed reliably in perturbation theory.

Two other popular variants of the on shell renormalization scheme are the $\alpha(M_Z)$ and G_μ schemes. The corresponding charge renormalization constant can be adapted from (4) into $\delta Z_e|_{\alpha(M_Z)} = \delta Z_e|_{\alpha(0)} - \frac{1}{2} \Delta\alpha(M_Z)$, and $\delta Z_e|_{G_\mu} = \delta Z_e|_{\alpha(0)} - \frac{1}{2} \Delta r$, respectively. $\Delta\alpha(M_Z) = \Pi_{f \neq t}^{AA}(0) - \text{Re}\Pi_{f \neq t}^{AA}(M_Z^2)$, and the expression for the oblique parameter Δr can be found in [37]. For latter convenience, we split the $\Delta\alpha(M_Z)$ into two different sources of contributions,

$$\Delta\alpha(M_Z) = \Delta\alpha_q(M_Z) + \Delta\alpha_\ell(M_Z), \quad (6)$$

where $\Delta\alpha_q$ and $\Delta\alpha_\ell$ represent the contributions from the five light quarks and three charged leptons, respectively. They are defined by

$$\Delta\alpha_q(M_Z) = \Pi_{f=u,d,s,c,b}^{AA}(0) - \text{Re}\Pi_{f=u,d,s,c,b}^{AA}(M_Z), \quad (7a)$$

$$\Delta\alpha_\ell(M_Z) = \Pi_{f=e,\mu,\tau}^{AA}(0) - \text{Re}\Pi_{f=e,\mu,\tau}^{AA}(M_Z). \quad (7b)$$

The QED coupling constants in the $\alpha(M_Z)$ and G_μ schemes become

$$\alpha(M_Z) = \frac{\alpha(0)}{1 - \Delta\alpha(M_Z)}, \quad (8a)$$

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right). \quad (8b)$$

In contrast to the $\alpha(0)$ scheme, these two subschemes effectively resum some universal large (non)logarithmic terms arising from the light fermion loop and top quark loop.

Numerical predictions. To make concrete predictions, we adopt the following values for the masses of various particles [38]: $M_H = 125.25$ GeV, $M_Z = 91.1876$ GeV, $M_W = 80.377$ GeV, $m_t = 172.69$ GeV, $m_e = 0.5109989$ MeV, $m_\mu = 105.65837$ MeV, $m_\tau = 1.77686$ GeV. The masses of the charged leptons are retained only when computing Z_e . In all other situations, we keep the top quark massive and treat the remaining five quarks and all charged leptons as massless.

We adopt the following values for various couplings; $\alpha(0) = 1/137.035999$, $G_\mu = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}$, and $\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02764$ [38]. The running strong coupling constant at the scale of the Higgs mass is set to $\alpha_s(M_H) = 0.115$ [24]. With the aid of (8b), we derive $\alpha_{G_\mu} = 1/132.168$. To calculate $\alpha(M_Z)$, we must know the values of $\Delta\alpha_q(M_Z)$ and $\Delta\alpha_\ell(M_Z)$. As mentioned before, the nonperturbative parameter $\Delta\alpha_q(M_Z)$ can be interchangeably

TABLE I. Predicted values of the partial width (in units of keV) for $H \rightarrow Z^0\gamma$ in various α schemes, at various levels of perturbative accuracy. Γ^{LO} represents the LO prediction to the decay width. The contributions from the NLO electroweak and QCD corrections are denoted by $\Gamma^{\mathcal{O}(\alpha)}$ and $\Gamma^{\mathcal{O}(\alpha_s)}$, respectively. We have taken the strong coupling constant $\alpha_s(M_H) = 0.115$. Γ^{Sum} is obtained by summing the LO contribution together with $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha_s)$ corrections, and the rightmost column is the branching fraction obtained via dividing Γ^{Sum} by the predicted full Higgs width, $\Gamma_H = 4.07^{+4.0\%}_{-3.9\%}$ MeV [10,38].

	α assignment at LO	Γ^{LO}	$\Gamma^{\mathcal{O}(\alpha)}$	$\Gamma^{\mathcal{O}(\alpha_s)}$	Γ^{Sum}	$\mathcal{B}(\times 10^{-3})$
$\alpha(0)$ scheme	$\alpha^3(0)$	5.921	0.313	0.019	6.252	1.54 ± 0.06
$\alpha(M_Z)$ scheme	$\alpha(0)\alpha^2(M_Z)$	6.689	-0.464	0.021	6.245	1.53 ± 0.06
G_μ scheme	$\alpha(0)\alpha_{G_\mu}^2$	6.365	-0.048	0.020	6.337	1.56 ± 0.06
Democratic scheme	$\alpha(0)\alpha(M_Z)\alpha_{G_\mu}$	6.525	-0.249	0.021	6.297	1.55 ± 0.06

used with the experimentally determined $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$. $\Delta\alpha_\ell(M_Z)$ can be calculated in perturbation theory, whose value is determined through four-loop accuracy [39–42]. We adopt the four-loop result $\Delta\alpha_\ell(M_Z) \approx 0.0314979$ [42]. Consequently, by employing (8a), we obtain $\alpha(M_Z) = 1/128.932$.

At present the measured full width of the Higgs boson is subject to large uncertainty, $\Gamma_H = 3.2^{+2.4}_{-1.7}$ MeV [38]. When predicting the branching fraction of $H \rightarrow Z^0\gamma$, we choose to use much more precise, theoretically predicted value of Higgs full width, $\Gamma_H = 4.07^{+4.0\%}_{-3.9\%}$ MeV [10,38], in order to reduce the theoretical error.

In Appendix we present the numerical results of the form factor $\tilde{T}_5$ at NLO accuracy in different α schemes. Plugging into (3), we are able to present the predictions for the partial width in various α schemes. It may look natural to take the QED coupling constant at the vertex attached to the outgoing photon as $\alpha(0)$. For the electromagnetic coupling constant that appears elsewhere, one has some freedom to choose different α scheme. In Table I we enumerate our predictions for $\Gamma(H \rightarrow Z^0\gamma)$ (in units of keV) based on four different α schemes at various levels of perturbative accuracy. For the reader's convenience, in Table I we also juxtapose the contribution of the NLO QCD correction to the partial width, which has been available long while ago [22–24].

As can be seen from Table I, we reconfirm that the NLO QCD correction is positive and tiny, which constitutes only 0.3% of the LO prediction of the partial width [24]. We also notice that, the NLO electroweak corrections is positive in the $\alpha(0)$ scheme, while negative in other three schemes. In addition, the $\mathcal{O}(\alpha)$ corrections appear to be sizable in magnitude in both the $\alpha(0)$ and $\alpha(M_Z)$ schemes, which may approach approximately $^{+5\%}_{-7\%}$ of the LO prediction. In contrast, the G_μ scheme as well as the democratic scheme yield a relatively modest $\mathcal{O}(\alpha)$ correction. Note the LO predictions of the partial width from four different α scheme are scattered in a wide range, from 5.921 keV to 6.689 keV. Including the error of the Higgs full width, we find that the LO SM prediction for the branching fraction lies in the range $(1.40\text{--}1.71) \times 10^{-3}$. Interestingly, after

including the NLO electroweak correction, the scheme dependence becomes substantially reduced, with the relative uncertainties among different schemes less than 2%. Taking the predictions from various α schemes as an estimation of the theoretical uncertainty, we then obtain the most accurate SM prediction to be $\mathcal{B}[H \rightarrow Z^0\gamma] = (1.55 \pm 0.06) \times 10^{-3}$. Note the error in the branching fraction mainly stem from the uncertainty associated with the full width of the Higgs boson.³

It is evident that the state-of-the-art SM prediction for the branching fraction is significantly lower than the measured value, $\mathcal{B}_{\text{exp}}[H \rightarrow Z^0\gamma] = (3.4 \pm 1.1) \times 10^{-3}$ [6]. It may be too early to claim that some sort of the BSM physics must be invoked to resolve this discrepancy. More accurate measurements from HL-LHC, and the prospective Higgs factories such as CEPC and FCC-ee, will play a crucial role to ultimately clear the smoke.

Summary. In this work we have calculated the NLO electroweak correction to the rare decay process $H \rightarrow Z^0\gamma$ within the on shell renormalization scheme. To assess the theoretical uncertainty, we present the predictions of the partial width and the branching fraction using several different α schemes. In contrast to the tiny NLO QCD correction, the magnitude of the NLO electroweak correction turns out to become sizable, reaching 7% of the LO result in the $\alpha(M_Z)$ schemes. After including the $\mathcal{O}(\alpha)$ correction, the predictions from various α schemes tend to converge to each other, which indicates that incorporating the NLO electroweak correction has significantly stabilized the

³We can estimate other potential sources of uncertainty. The contribution from the top quark loop constitutes approximately -10% of the LO decay width. Taking into account that the Yukawa coupling strength of $Hb\bar{b}$ is suppressed with respect to $Ht\bar{t}$ by a factor of m_b/m_t , we estimate that retaining the bottom quark mass may introduce a relative error of several per mille. Furthermore, uncertainties about the t , W masses may also introduce an uncertainty of several per mille in the partial width. All in all, these sorts of uncertainties are of the same order of magnitude as the NLO QCD correction, which are significantly smaller than the uncertainty stemming from four different α schemes.

SM prediction. Our most accurate prediction is then $\mathcal{B}[H \rightarrow Z^0\gamma] = (1.55 \pm 0.06) \times 10^{-3}$, with the uncertainty predominantly stemming from the error of the full width of Higgs. The relative error from varying the α schemes is less than 2%. Although this state-of-the-art SM prediction is significantly lower than the measured value, it is still premature to claim that one has to invoke some BSM scenarios to resolve this discrepancy. Likely the problem mainly lies on the experimental side. Improved measurements of this rare decay process at HL-LHC, and independent measurements at the prospective Higgs factories such as CEPC and FCC-ee, will be crucial.

Note added. While we were finalizing this work, a preprint by Chen, Chen, Qiao and Zhu has recently appeared [43]. Our results in $\alpha(M_Z)$ and G_μ schemes are compatible with their mixed 1 and mixed 2 schemes, provided that the same input parameters are used. However, our result in $\alpha(0)$ scheme slightly differs from theirs, probably due to the different treatment of the light quark contribution to Z_e .

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Appendix: The expression of $\tilde{T}_5$ from four different α schemes. Throughout this work we always retain the QED coupling associated with the photon emission vertex to be $\alpha(0)$. As aforementioned, there still exists some freedom in the on shell renormalization scheme to handle the charge renormalization. In this appendix, we present the expressions of the form factor $\tilde{T}_5$ in four different α schemes, which differ in choosing the values of the QED coupling constant in other vertices. Plugging these equations into (3), we then obtain the predicted partial widths associated with each scheme, as enumerated in Table I.

$\alpha(0)$ scheme: In this scheme, all the QED couplings in three vertices of the LO diagrams in Fig. 1 are chosen to be $\alpha(0)$, i.e., the fine structure constant in the Thomson limit. After including both $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha_s)$ corrections, the form factor $\tilde{T}_5$ reads

$$\tilde{T}_5 = 1.546\alpha^{3/2}(0) \left[1 + \Delta\alpha_{\text{had}}^{(5)}(M_Z) - 0.535 \frac{\alpha(0)}{\pi} + 0.043 \frac{\alpha_s}{\pi} \right], \quad (\text{A1})$$

which depends on both charged leptons and light quarks, with the effects of the latter encapsulated in $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$. As can be clearly seen, the prefactor accompanying $\alpha(0)$ is about two orders-of-magnitude greater than that accompanying α_s , which explains why the electroweak correction is much more important than the QCD correction.

$\alpha(M_Z)$ scheme: In this scheme, we choose the QED coupling constants associated with two vertices other than the photon emission vertex in Fig. 1 to be $\alpha(M_Z)$. With the aid of (6) and (8a), making the following substitution in (A1):

$$\begin{aligned} \alpha(0) &\rightarrow \alpha(M_Z)[1 - \Delta\alpha(M_Z)] \\ &\rightarrow \alpha(M_Z)[1 - \Delta\alpha_{\text{had}}^{(5)}(M_Z) - \Delta\alpha_\ell(M_Z)] \\ &= \alpha(M_Z) \left[1 - 0.02764 - 13.527 \frac{\alpha(M_Z)}{\pi} \right], \end{aligned} \quad (\text{A2})$$

we then obtain

$$\tilde{T}_5 = 1.546\alpha^{1/2}(0)\alpha(M_Z) \left[1 - 14.062 \frac{\alpha(M_Z)}{\pi} + 0.043 \frac{\alpha_s}{\pi} \right]. \quad (\text{A3})$$

To be compatible with the level of accuracy in the computation of $\delta Z_e|_{\alpha(0)}$ in $\alpha(0)$ scheme, we have utilized the one-loop expression of $\Delta\alpha_\ell(M_Z)$ in (A2), which reads

$$\Delta\alpha_\ell(M_Z) = \frac{\alpha(0)}{3\pi} \left(\ln \frac{M_Z^2}{m_e^2} + \ln \frac{M_Z^2}{m_\mu^2} + \ln \frac{M_Z^2}{m_\tau^2} - 5 \right). \quad (\text{A4})$$

It might be worth emphasizing that $\tilde{T}_5$ in (A3) is insensitive to the masses of all light quarks and charged leptons.

G_μ scheme: In this scheme, we choose the QED coupling constants affiliated with two vertices other than the photon emission vertex in Fig. 1 to be α_{G_μ} . Making the following substitution in (A1):

$$\begin{aligned} \alpha(0) &\rightarrow \alpha_{G_\mu} [1 - \Delta\alpha_{\text{had}}^{(5)}(M_Z) + \Delta\alpha_q(M_Z) - \Delta r] \\ &= \alpha_{G_\mu} \left[1 - 0.02764 - 1.030 \frac{\alpha_{G_\mu}}{\pi} \right], \end{aligned} \quad (\text{A5})$$

we then obtain

$$\tilde{T}_5 = 1.546\alpha^{1/2}(0)\alpha_{G_\mu} \left[1 - 1.565 \frac{\alpha_{G_\mu}}{\pi} + 0.043 \frac{\alpha_s}{\pi} \right]. \quad (\text{A6})$$

Note that in (A5) we have utilized the one-loop values for $\Delta\alpha_q(M_Z)$ and Δr . The explicit expression of Δr is too complicated to present here, which can instead be found in Eq. (8.14) of [37]. It is worth emphasizing that since both $\Delta\alpha_q(M_Z)$ and Δr possess identical sensitivity to the light quark masses, their difference in (A5) is not sensitive to nonperturbative hadronic effect, and is thus amenable to a perturbative treatment. As in $\alpha(M_Z)$ scheme, $\tilde{T}_5$ in G_μ

scheme is also insensitive to the masses of light quarks and charged fermions.

Democratic scheme: In the democratic scheme, we choose the QED couplings affiliated with three vertices in Fig. 1 to be α_{G_μ} , $\alpha(M_Z)$, and $\alpha(0)$, respectively. By plugging (A2) and (A5) into (A1), we obtain

$$\tilde{T}_5 = 1.546\alpha^{1/2}(0)\alpha^{1/2}(M_Z)\alpha_{G_\mu}^{1/2} \times \left[1 - 7.814 \frac{\alpha^{1/2}(M_Z)\alpha_{G_\mu}^{1/2}}{\pi} + 0.043 \frac{\alpha_s}{\pi} \right]. \quad (\text{A7})$$

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- [1] G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **716**, 1 (2012).
 - [2] S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **716**, 30 (2012).
 - [3] G. Aad *et al.* (ATLAS and CMS Collaborations), *J. High Energy Phys.* **08** (2016) 045.
 - [4] G. Aad *et al.* (ATLAS Collaboration), *Nature (London)* **607**, 52 (2022); **612**, E24 (2022).
 - [5] A. Tumasyan *et al.* (CMS Collaboration), *Nature (London)* **607**, 60 (2022); **623**, E4 (2023).
 - [6] G. Aad *et al.* (ATLAS and CMS Collaborations), *Phys. Rev. Lett.* **132**, 021803 (2024).
 - [7] G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **809**, 135754 (2020).
 - [8] A. Tumasyan *et al.* (CMS Collaboration), *J. High Energy Phys.* **05** (2023) 233.
 - [9] A. Djouadi, J. Kalinowski, and M. Spira, *Comput. Phys. Commun.* **108**, 56 (1998).
 - [10] D. de Florian *et al.* (LHC Higgs Cross Section Working Group), *10.23731/CYRM-2017-002* (2017).
 - [11] G. Aad *et al.* (ATLAS Collaboration), *J. High Energy Phys.* **07** (2023) 088.
 - [12] M. Herrero and R. A. Morales, *Phys. Rev. D* **102**, 075040 (2020).
 - [13] T. T. Hong, V. K. Le, L. T. T. Phuong, N. C. Hoi, N. T. K. Ngan, and N. H. T. Nha, *Prog. Theor. Exp. Phys.* **2024**, 033B04 (2024).
 - [14] S. Panghal and B. Mukhopadhyaya, *arXiv:2310.04136*.
 - [15] F. Buccioni, F. Devoto, A. Djouadi, J. Ellis, J. Quevillon, and L. Tancredi, *Phys. Lett. B* **851**, 138596 (2024).
 - [16] D. Barducci, L. Di Luzio, M. Nardecchia, and C. Toni, *J. High Energy Phys.* **12** (2023) 154.
 - [17] R. Boto, D. Das, J. C. Romao, I. Saha, and J. P. Silva, *Phys. Rev. D* **109**, 095002 (2024).
 - [18] K. Cheung and C. J. Ouseph, *arXiv:2402.05678*.
 - [19] X. G. He, Z. L. Huang, M. W. Li, and C. W. Liu, *arXiv:2402.08190*.
 - [20] R. N. Cahn, M. S. Chanowitz, and N. Fleishon, *Phys. Lett.* **82B**, 113 (1979).
 - [21] L. Bergstrom and G. Hulth, *Nucl. Phys.* **B259**, 137 (1985); **B276**, 744(E) (1986).
 - [22] M. Spira, A. Djouadi, and P. M. Zerwas, *Phys. Lett. B* **276**, 350 (1992).
 - [23] T. Gehrmann, S. Guns, and D. Kara, *J. High Energy Phys.* **09** (2015) 038.
 - [24] R. Bonciani, V. Del Duca, H. Frellesvig, J. M. Henn, F. Moriello, and V. A. Smirnov, *J. High Energy Phys.* **08** (2015) 108.
 - [25] T. Hahn, *Comput. Phys. Commun.* **140**, 418 (2001).
 - [26] R. Mertig, M. Bohm, and A. Denner, *Comput. Phys. Commun.* **64**, 345 (1991).
 - [27] F. Feng and R. Mertig, *arXiv:1212.3522*.
 - [28] F. Feng, *Comput. Phys. Commun.* **183**, 2158 (2012).
 - [29] A. V. Smirnov, *Comput. Phys. Commun.* **189**, 182 (2015).
 - [30] X. Liu, Y. Q. Ma, and C. Y. Wang, *Phys. Lett. B* **779**, 353 (2018).
 - [31] Z. F. Liu and Y. Q. Ma, *Phys. Rev. Lett.* **129**, 222001 (2022).
 - [32] X. Liu and Y. Q. Ma, *Comput. Phys. Commun.* **283**, 108565 (2023).
 - [33] A. Denner, S. Dittmaier, M. Roth, and D. Wackeroth, *Nucl. Phys.* **B560**, 33 (1999).
 - [34] A. Denner and S. Dittmaier, *Nucl. Phys. B, Proc. Suppl.* **160**, 22 (2006).
 - [35] D. A. Ross and J. C. Taylor, *Nucl. Phys.* **B51**, 125 (1973); **B58**, 643(E) (1973).
 - [36] W. F. L. Hollik, *Fortschr. Phys.* **38**, 165 (1990).
 - [37] A. Denner, *Fortschr. Phys.* **41**, 307 (1993).
 - [38] R. L. Workman *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
 - [39] S. Eidelman and F. Jegerlehner, *Z. Phys. C* **67**, 585 (1995).
 - [40] J. H. Kuhn and M. Steinhauser, *Phys. Lett. B* **437**, 425 (1998).
 - [41] M. Steinhauser, *Phys. Lett. B* **429**, 158 (1998).
 - [42] C. Sturm, *Nucl. Phys.* **B874**, 698 (2013).
 - [43] Z. Q. Chen, L. B. Chen, C. F. Qiao, and R. Zhu, *arXiv:2404.11441*.
 - [44] D. Binosi, J. Collins, C. Kaufhold, and L. Theussl, *Comput. Phys. Commun.* **180**, 1709 (2009).