

Renormalization of massive vector field theory coupled to scalar in curved spacetime

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We consider the renormalization of a massive vector field interacting with a charged scalar field in curved spacetime. Starting with the theory minimally coupled to external gravity and using the formulations with and without Stückelberg fields, we show that the longitudinal mode of the vector field is completely decoupled and the remaining theory of the transverse vector field is renormalizable by power counting. The formal arguments based on the covariance and power counting indicate that multiplicative renormalizability of the interacting theory may require introducing two nonminimal terms linear in the Ricci tensor in the vector sector. Nevertheless, a more detailed analysis shows that these nonminimal terms violate the decoupling of the longitudinal mode and are prohibited. As a verification of general arguments, we derive the one-loop divergences in the minimal massive scalar QED, using the Stückelberg procedure and the heat-kernel technique. The theory without nonminimal terms proves one-loop renormalizable and admits the renormalization group equations for all the running parameters in the scalar and vector sectors. One-loop beta functions do not depend on the gauge fixing and can be used to derive the effective potential.

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I. INTRODUCTION

The theory of a massive vector field (Proca model) describes a spin-1 massive vector particle. The mass term in the Lagrangian distinguishes it from a gauge vector field corresponding to massless particles with helicities ± 1 . Besides being the well-established subject of particle physics and formal quantum field theory (QFT) considerations, there is currently a growing interest in the study of the Proca model in curved spacetime, partially owing to the new cosmological applications (see e.g., [1–4] and many references therein). One can also mention the cosmological

models with the modified or generalized massive vector field actions (see, e.g., [5–7]). Also, for earlier applications of this model in the gravitational field, see [8–10].

Exploration of a theory at the quantum level is relevant, and not only because of the quantum corrections to the classical action. At least equally important is that the consistency of QFT enables one to restrict the form of the classical action. In this respect, the massive vector field A_μ and its extensions to the curved spacetime attracted some attention. In particular, Refs. [11–13] were devoted to the formulation of the theory in curved spacetime and evaluation of vacuum quantum effects. The problem to solve in these papers turned out to be very difficult and the corresponding calculations produced conflicting results. The reason was that, in these works, the free curved-space Proca model was formulated in a general form, with two nonminimal terms $R_{\mu\nu}A^\mu A^\nu$ and $RA^\mu A_\mu$ included. On the other hand, one should question why it is necessary to include these terms. Taking into account the well-known situation with the nonminimal curvature-scalar field interaction (see, e.g., [14]), the answer requires careful analysis of an interacting theory.

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An important feature of field theories in curved space-time is the possibility of a nonminimal coupling of the fields to gravity. A well-known example of such a coupling is the famous $\xi R\phi^2$ term in the scalar field Lagrangian. The need for this term becomes clear in the framework of interacting quantum theory, where the nonminimal term is a necessary element of renormalizable theory [14]. One can ask whether the same situation takes place in the theory of a curved-spacetime massive vector field coupled to matter. In principle, the interaction with matter may generate the nonminimal divergences of the form $R_{\mu\nu}A^\mu A^\nu$ and $RA^\mu A_\mu$ in the vector sector. In this case, the renormalizability would require us to introduce these nonminimal terms into the classical action as an ultraviolet completion.

The equations of motion in the free theory of a massive vector field A_μ yield the condition $\partial_\mu A^\mu = 0$, which means only the transverse field $A_\mu^\perp$ is propagating, while the longitudinal mode (equivalent to a scalar field) is non-physical. Therefore, including new interactions of the massive vector field with dynamical or external fields should not lead to a propagating longitudinal mode at both classical and quantum levels. This requirement imposes strong restrictions on the form of possible interaction. In particular, this condition restricts possible counterterms in the theory and is operational in ruling out the nonminimal terms that are formally acceptable by power counting.

The renormalizability of a massive vector field coupled to fermions in flat space has been studied by many authors (see, e.g., [15] for earlier references). A full analysis of renormalizability in flat space has been given in the paper [16] where it was proved by the direct transformations of the generating functional of Green functions that the longitudinal mode of the vector field decouples and the remaining theory of a transverse vector field is renormalizable in power counting. Let us also mention the recent review [17] of the related subject, where one can find further references.

In what follows, we prove the renormalizability of the Proca model coupled to a complex scalar field in curved space using two methods: one is a curved-space generalization of the analysis considered in [16] and the second one is an analysis of the functional integral for the corresponding gauge theory with a Stückelberg field. The power counting, in both approaches, indicates that the aforementioned nonminimal terms are formally necessary. However, these nonminimal terms violate the decoupling of a longitudinal mode of the vector field and therefore are prohibited. The same output follows from the analysis based on the gauge symmetry restored by means of the Stückelberg trick and the known gauge-fixing dependence in QFT. As a result, the multiplicative renormalizability in a curved spacetime background does not require us to take into account the curvature-dependent nonminimal terms.

The one-loop calculations in both the free massive vector theory and the interacting models, can be performed by

using the covariant Schwinger-DeWitt technique (see e.g., [18,19] and also the textbook [14] for a detailed introduction). The application of this technique to the free vector field theory in curved space can be found in [19,20] using two different approaches, but with equivalent results. The method of [19] is based on the introduction of an auxiliary operator eliminating the degeneracy of the bilinear form of the action [19]. The second approach relies on the Stückelberg procedure [20] (see also [12,13]), based on the introduction of an auxiliary scalar field, restoring the gauge symmetry violated by the vector mass. In this paper we follow the second approach, which we believe is the most elegant and simple enough for direct loop calculations.

The rest of the paper is organized as follows. In Sec. II, we briefly review the scalar-vector model with a massive vector field and introduce the Stückelberg procedure. Section III presents the proof of the power-counting renormalizability of the theory and completes the arguments in favor of the multiplicative renormalizability of the minimal theory. Section IV describes the calculation of the one-loop divergences through the use of the background field method, Stückelberg procedure and Schwinger-DeWitt technique. The main text contains sufficient technical details, but part of the intermediate formulas is separated in Appendix A. Furthermore, an alternative calculational approach based on the auxiliary operator and the subsequent difficulties are illustrated in Appendix B. The elements of the renormalization group, i.e., the beta and gamma functions are analyzed in Sec. V, where we also write down the expression for the effective potential of the scalar. Finally, in Sec. VI, we draw our conclusions and present the discussion of the validity of the one-loop results in the theory under discussion.

II. CURVED-SPACE SCALAR ELECTRODYNAMICS WITH MASSIVE VECTOR FIELD

Let us start from the general formulation of a massive vector field theory coupled to a complex scalar and external gravitational field. Unlike the conventional Abelian gauge-invariant scalar electrodynamics, the theory under consideration is not a gauge-invariant theory owing to the mass of the vector.

A. Classical action and notations

The action of massive scalar electrodynamics, minimally coupled to gravity in the vector sector, is written in the form

$$S[A, \Phi^*, \Phi] = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} M_v^2 A_\mu^2 + (\mathcal{D}^\mu \Phi)^* (\mathcal{D}_\mu \Phi) - V(\Phi^* \Phi) \right\}, \quad (1)$$

where

$$V(\Phi^*\Phi) = m^2\Phi^*\Phi + \lambda(\Phi^*\Phi)^2 - \xi R\Phi^*\Phi, \quad (2)$$

$F_{\mu\nu}^2 = F_{\mu\nu}F^{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $A_\mu^2 = A_\mu A^\mu$. The mass of the vector field is M_v , the mass of the scalar field is m and ξ is the parameter of nonminimal scalar-curvature interaction, which is well known to be relevant in curved spacetime [14]. Furthermore, the covariant derivatives are

$$\mathcal{D}_\mu\Phi = \nabla_\mu\Phi - ieA_\mu\Phi, \quad (\mathcal{D}_\mu\Phi)^* = \nabla_\mu\Phi^* + ieA_\mu\Phi^*. \quad (3)$$

Coupling constants include the scalar self-interaction λ and e .

B. Reformulation using the Stückelberg procedure

The peculiarity of the theory under consideration is that although the vector sector of the theory (1) as a whole is not gauge invariant, its kinetic term corresponds to the gauge-invariant theory, which creates some difficulties in quantum calculations. In particular, the covariant Schwinger-DeWitt technique for the effective action in an external gravitational field cannot be applied directly. For a free massive vector theory in an external gravitational field, these difficulties were overcome by the two different methods in [19,20]. Trying to generalize these two methods to the interacting model of a massive vector and scalar (1), we find the following situation. Different from the pure vector case, the two methods do not give the same result in the presence of a scalar field. One of the calculational procedures is described in the next section and the difficulties that emerge in another method are illustrated in Appendix B.

The Stückelberg procedure introduces an additional real scalar field θ according to

$$A_\mu \longrightarrow A_\mu - \frac{1}{M_v} \nabla_\mu \theta. \quad (4)$$

An equivalent expression is used in Sec. IV for identifying essential effective charges in the renormalization group running.

III. POWER COUNTING AND RENORMALIZATION IN CURVED SPACETIME

In this section, we generalize the analysis of power counting made in the paper [16] for a massive vector

In this way, the action of the theory becomes

$$\begin{aligned} S'[A, \theta, \Phi^*, \Phi] \\ = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} M_v^2 (A_\mu - \frac{1}{M_v} \nabla_\mu \theta)^2 \right. \\ \left. + g^{\mu\nu} (\mathcal{D}_\mu \Phi)^* (\mathcal{D}_\nu \Phi) - V(\Phi^*\Phi) \right\}. \end{aligned} \quad (5)$$

The last action is invariant under the gauge transformations

$$\begin{aligned} \Phi \longrightarrow \Phi' = e^{ie\zeta(x)} \Phi, \quad \Phi^* \longrightarrow \Phi'^* = e^{-ie\zeta(x)} \Phi^*, \\ A_\mu \longrightarrow A'_\mu = A_\mu + \nabla_\mu \zeta(x), \quad \text{and} \quad \theta \longrightarrow \theta' = \theta + M_v \zeta(x). \end{aligned} \quad (6)$$

Using the gauge invariance of the above action, we can impose the gauge condition $\theta = \text{const}$ which recovers the original action (1). This shows to what extent the theories (1) and (5) are classically equivalent. The advantage of the theory (5) is that it possesses the gauge invariance and enables one to apply the corresponding quantization scheme.

The equations of motion for all fields of the action (5) have the form

$$\begin{aligned} \mathcal{E}^* &= \mathcal{D}_\mu (\mathcal{D}^\mu \Phi)^* - (m^2 - \xi R + 2\lambda \Phi^* \Phi) \Phi^* = 0, \\ \mathcal{E} &= \mathcal{D}^\mu (\mathcal{D}_\mu \Phi) - (m^2 - \xi R + 2\lambda \Phi^* \Phi) \Phi = 0, \\ E_\theta &= -\nabla^\mu \nabla_\mu \theta + M_v \nabla_\mu A^\mu = 0, \\ \mathcal{E}^\nu &= \partial_\mu F^{\mu\nu} - M_v^2 \left(A^\nu - \frac{1}{M_v} \nabla^\nu \theta \right) + J^\nu = 0, \end{aligned} \quad (7)$$

where $J^\nu = ie[\Phi^* \mathcal{D}^\nu \Phi - \Phi (\mathcal{D}^\nu \Phi)^*]$.

It proves useful to consider a linear real combination of Eq. (7) with arbitrary constants α_1 , α_2 , and α_3 ,

$$\begin{aligned} \frac{\alpha_1}{2} (\Phi^* \mathcal{E} + \Phi \mathcal{E}^*) + \alpha_2 \mathcal{E}_\theta + \alpha_3 A_\nu \mathcal{E}^\nu &= \frac{\alpha_1}{2} [\Phi^* \mathcal{D}^\mu (\mathcal{D}_\mu \Phi) + \Phi \mathcal{D}^\mu (\mathcal{D}_\mu \Phi)^*] - \alpha_1 (m^2 - \xi R) \Phi^* \Phi - 2\alpha_1 \lambda (\Phi^* \Phi)^2 - \alpha_2 \theta \nabla^\mu \nabla_\mu \theta \\ &\quad + \alpha_2 M_v \theta \nabla_\mu A^\mu + \alpha_3 A_\nu \partial_\mu F^{\mu\nu} - \alpha_3 M_v^2 A_\nu^2 + \alpha_3 M_v A_\nu \nabla^\nu \theta + ie\alpha_3 A_\nu [\Phi^* \mathcal{D}^\nu \Phi - \Phi (\mathcal{D}^\nu \Phi)^*]. \end{aligned} \quad (8)$$

minimally coupled to fermions in flat space (see also the review [17]), for the theory in curved spacetime. Our main goal is to formulate the curved-space action which guarantees the renormalizability of the theory of a massive vector field coupled to charged scalars. Similar to [16], we show that the theory under consideration is renormalizable by power counting, but also highlight new aspects of its renormalization in curved spacetime. The analysis is

carried out in two ways: within the framework of a model with an auxiliary Stückelberg field (5) and directly using the original form of the model (1). The results regarding renormalizability obviously coincide, which is not surprising, since the models are classically equivalent and there are no sources of quantum anomalies. The consideration can be easily extended including the interaction with Dirac fermions or with both scalars and fermions in curved spacetime.

A. Model with auxiliary field

Our starting point is the gauge-invariant action (5). The gauge transformations have the form (6). We begin with the generating functional of the Green functions

$$Z[J_\mu, J, J^*] = \int \mathcal{D}A \mathcal{D}\theta \mathcal{D}\Phi^* \mathcal{D}\Phi \delta[\chi] \exp\{iS'[A, \theta, \Phi^*, \Phi] + i \int d^4x \sqrt{-g}(A_\mu J^\mu + \Phi^* J + \Phi J^*)\}, \quad (9)$$

where χ is a gauge-fixing function. Now we present the vector field as a sum of transverse and longitudinal parts, $A_\mu = A_\mu^\perp + \partial_\mu \varphi$, where $\nabla^\mu A_\mu^\perp = 0$. Then the action takes the form

$$S'[A^\perp, \varphi, \theta, \Phi^*, \Phi] = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^\perp F^{\perp\mu\nu} + \frac{1}{2} M_v^2 g^{\mu\nu} \left(A_\mu^\perp + \partial_\mu \varphi - \frac{1}{M_v} \partial_\mu \theta \right) \left(A_\nu^\perp + \partial_\nu \varphi - \frac{1}{M_v} \partial_\nu \theta \right) \right. \\ \left. + g^{\mu\nu} (\partial_\mu + ieA_\mu^\perp + ie\partial_\mu \varphi) \Phi^* (\partial_\nu - ieA_\nu^\perp - ie\partial_\nu \varphi) \Phi - V(\Phi^* \Phi) \right\}, \quad (10)$$

where the classical scalar potential $V(\Phi^* \Phi)$ is given by (2). For further consideration, it is convenient to fulfill a change of variables in the functional integral, $A_\mu \rightarrow (A_\mu^\perp, \varphi)$, with $F_{\mu\nu}^\perp = F_{\mu\nu}$. This is a linear change of variables, and hence the corresponding Jacobian depends only on the external metric and does not depend on the fields $A_\mu^\perp, \varphi$. Since we are interested in the renormalization of the matter field sector of the effective action (see, e.g., [20]), this Jacobian is omitted in the rest of this paper.¹ Then the generating functional takes the form

$$Z[J_\mu, J, J^*] = \int \mathcal{D}A^\perp \mathcal{D}\varphi \mathcal{D}\theta \mathcal{D}\Phi^* \mathcal{D}\Phi \delta[\chi] \exp\left\{iS'[A^\perp, \varphi, \theta, \Phi^*, \Phi] + i \int d^4x \sqrt{-g}(A_\mu J^\mu + \Phi^* J + \Phi J^*)\right\}, \quad (11)$$

where χ is the gauge-fixing condition.² It is convenient to take such a condition in the form

$$\chi = \frac{\theta}{M_v} - \varphi. \quad (12)$$

Owing to the factor of $\delta(\chi)$ in (11), we get

$$g^{\mu\nu} \left(A_\mu^\perp + \partial_\mu \varphi - \frac{1}{M_v} \partial_\mu \theta \right) \left(A_\nu^\perp + \partial_\nu \varphi - \frac{1}{M_v} \partial_\nu \theta \right) = g^{\mu\nu} A_\mu^\perp A_\nu^\perp. \quad (13)$$

As a result, we obtain the following action in the integrand:

$$S'[A^\perp, \Phi, \Phi^*, \Phi] = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^\perp F^{\perp\mu\nu} + \frac{1}{2} M_v^2 g^{\mu\nu} A_\mu^\perp A_\nu^\perp + g^{\mu\nu} (\partial_\mu + ieA_\mu^\perp + ie\partial_\mu \varphi) \Phi^* (\partial_\nu - ieA_\nu^\perp - ie\partial_\nu \varphi) \Phi - V(\Phi^* \Phi) \right\}. \quad (14)$$

Next, we perform the following change of variables in the functional integral:

$$\Phi \rightarrow e^{ie\varphi} \Phi, \quad \Phi^* \rightarrow e^{-ie\varphi} \Phi^*, \quad (15)$$

¹At one loop, the vacuum functional is a sum of the contributions of the free massive vector [19,20] given Eq. (51) and of the scalar field. Thus, its special evaluation in the interacting theory is not relevant.

²The corresponding Faddeev-Popov determinant depends only on the external metric and hence it does not affect the renormalizability in the matter fields sector. For this reason, it is omitted.

without changing the notations for the variables. After this, the field φ disappears completely from the action and we obtain the generating functional in the form,

$$Z[J_\mu, J, J^*] = \int \mathcal{D}A^\perp \mathcal{D}\varphi \mathcal{D}\theta \mathcal{D}\Phi^* \mathcal{D}\Phi \delta\left(\frac{\theta}{M_v} - \varphi\right) \exp\left\{iS'[A^\perp, \theta, \Phi^*, \Phi] + i \int d^4x \sqrt{-g}(A_\mu J^\mu + \Phi^* J + \Phi J^*)\right\}. \quad (16)$$

Integrating over φ yields

$$Z[J_\mu, J, J^*] = \int \mathcal{D}A^\perp \mathcal{D}\theta \mathcal{D}\Phi^* \mathcal{D}\Phi \exp\left\{iS'[A^\perp, \theta, \Phi^*, \Phi] + i \int d^4x \sqrt{-g}\left(A_\mu^\perp J^{\perp\mu} + \frac{1}{M_v} J^\mu \partial_\mu \theta + \Phi^* J + \Phi J^*\right)\right\}. \quad (17)$$

In the last expression, we introduced the transverse and longitudinal sources according to $J^\mu = J_\mu^\perp + \partial_\mu j$, where $\nabla^\mu J_\mu^\perp = 0$. Disregarding the total derivative, $J^\mu \partial_\mu \theta = -\theta \square j$. Now we can integrate over θ . According to the delta function property, this operation gives

$$\delta[\square j] = \frac{1}{\text{Det}\square} \delta[j]. \quad (18)$$

The expression $\text{Det}\square$ depends only on the external metric and can be ignored when analyzing the renormalizability in the matter sector.

Thus, we arrive at the expression

$$Z[J_\mu, J, J^*] = \delta[j] \int \mathcal{D}A^\perp \mathcal{D}\Phi^* \mathcal{D}\Phi \exp\left\{i[S[A^\perp, \theta, \Phi^*, \Phi] + i \int d^4x \sqrt{-g}(A_\mu^\perp J^{\perp\mu} + \Phi^* J + \Phi J^*)]\right\}. \quad (19)$$

We obtain the generating functional for the theory of the fields $A^\perp, \Phi^*, \Phi$ with the action

$$\begin{aligned} S'[A^\perp, \theta, \Phi^*, \Phi] &= \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M_v^2 g^{\mu\nu} A_\mu^\perp A_\nu^\perp \right. \\ &\quad \left. + g^{\mu\nu} (\partial_\mu \Phi^* + ie A_\mu^\perp \Phi^*) (\partial_\nu \Phi - ie A_\nu^\perp \Phi) - V(\Phi^* \Phi) \right\}. \end{aligned} \quad (20)$$

To analyze the superficial degree of divergences it is sufficient to represent the external metric in the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and expand all the terms containing $g_{\mu\nu}$ in power series in $h_{\mu\nu}$. All these terms can be treated as the perturbations. In this case the propagators of the field $A^\perp$ and of the fields Φ^*, Φ are, respectively,

$$\frac{1}{p^2 - M_v^2} \left(\delta_\mu^\nu - \frac{p_\mu p^\nu}{p^2} \right) \quad \text{and} \quad \frac{1}{p^2 - m^2}. \quad (21)$$

After the Wick rotation, the behavior of both propagators at $p \rightarrow \infty$ is of the p^{-2} type. Since the field $h_{\mu\nu}$ is dimensionless, its powers do not contribute to the superficial degree of divergence ω . Then the standard estimate is

$$\omega = 4 - N_A - N_\Phi, \quad (22)$$

where N_A is the number of external lines of the field $A^\perp$ and N_Φ is the number of external lines of the fields Φ^*, Φ . As a result, we conclude that the theory is renormalizable by power counting. However, as usual in curved space, this does not guarantee the multiplicative renormalizability of the theory if only minimal coupling to gravity is considered (see e.g., [14]). Let us analyze the possible covariant and local counterterms fitting the estimate (22).

First we consider the option with $N_\Phi = 2$ and $N_A = 0$. In this case, the unique nonminimal counterterm is the one of the form $R\Phi^*\Phi$, which is extensively discussed in the literature (see e.g., [14]). Let us note that this term is already included in the classical scalar potential $V(\Phi^*\Phi)$. It is worth noting that including this term in the classical action does not modify either the arguments presented below or the power counting.

Let us concentrate on another possible option, with $N_\Phi = 0$ and $N_A = 2$. In this case, $\omega = 2$. Hence, the counterterms in the sector of the fields $A^\perp$ may have the form

$$\begin{aligned} & -\frac{1}{4} \delta z_1 F_{\mu\nu} F^{\mu\nu} + \delta z_2 \frac{1}{2} M_v^2 A^{\perp\mu} A_\mu^\perp + \frac{1}{2} \delta z_3 R A^{\perp\mu} A_\mu^\perp \\ & + \frac{1}{2} \delta z_3' R^{\mu\nu} A_\mu^\perp A_\nu^\perp. \end{aligned} \quad (23)$$

Owing to the locality and covariance of divergences, one can safely replace $A_\mu^\perp$ by A_μ in the last expression, and hence we can expect, in the vector sector, the counterterms of the form

$$F_{\mu\nu} F^{\mu\nu}, \quad M_v^2 A_\mu A^\mu, \quad R A_\mu A^\mu, \quad R^{\mu\nu} A_\mu A_\nu. \quad (24)$$

The first two terms are present in the classical action with minimal coupling to gravity in the vector sector, which

corresponds to the renormalizability by power counting. On the other hand, the last two terms are nonminimal and were not included in the initial Lagrangian. According to the conventional approach used in the case of scalars (and some other cases, e.g., the theory with torsion [21] or other external fields [22]), to achieve the renormalizable theory we have to include in classical action these nonminimal terms with the coupling of a massive Abelian vector field to the external gravity. However, the present case is different. We leave the discussion of these terms to the last part of this section, after an additional analysis of the power counting by a different method.

Finally, there is one more type of terms, with $N_\Phi = 0$ and $N_A = 4$, such that $\omega = 0$. In this case, the counterterms in the sector of the fields $A^\perp$ should have the form $\delta z_4 (A^\perp A^\perp)^2$. In the covariant local form, the corresponding counterterm is $(A^\mu A_\mu)^2$. In principle, the multiplicative renormalizability requires us to include this kind of a term in the initial classical action, already in flat spacetime. As we know from the analysis (including explicit two-loop calculations) of the axial-vector model [23], this term may result in longitudinal divergences of the form $(\partial_\mu A^\mu)^2$ in higher loop orders. In its turn, this implies the violation of unitarity at the quantum level [24]. These arguments were operational and effectively used in [25] to construct the action of a propagating axial-vector dual to antisymmetric torsion. Power counting alone cannot provide protection

against this scenario, so it requires an additional detailed consideration. We shall see in the last part of this section, that in the case of the curved-space Proca model, there is a protection against this scenario.

B. The model without auxiliary field

For the sake of completeness, let us consider the power counting without using the Stückelberg trick. This approach is quite close to the one of [16]. The starting action is (1) and the generating functional of Green functions has the form

$$Z[J_\mu, J, J^*] = \int \mathcal{D}A \mathcal{D}\theta \mathcal{D}\Phi^* \mathcal{D}\Phi \exp \left\{ iS[A, \Phi^*, \Phi] + i \int d^4x \sqrt{-g} (A_\mu J^\mu + \Phi^* J + \Phi J^*) \right\}. \quad (25)$$

As in the previous version of the proof, one can make the change of variables $A_\mu = A_\mu^\perp + \partial_\mu \varphi$. The mass term in the vector sector transforms as follows:

$$\begin{aligned} g^{\mu\nu} A_\mu A_\nu &= g^{\mu\nu} (A_\mu^\perp + \partial_\mu \varphi) (A_\nu^\perp + \partial_\nu \varphi) \\ &= g^{\mu\nu} A_\mu^\perp A_\nu^\perp + g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \text{total derivatives}. \end{aligned} \quad (26)$$

After the change of variables (15) in the scalar sector, we arrive at the action

$$\begin{aligned} S[A^\perp, \varphi, \Phi^*, \Phi] &= \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^\perp F^{\perp\mu\nu} + \frac{M_v^2}{2} g^{\mu\nu} A_\mu^\perp A_\nu^\perp + \frac{M_v^2}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right. \\ &\quad \left. + g^{\mu\nu} (\partial_\mu + ieA_\mu^\perp) \Phi^* (\partial_\nu - ieA_\nu^\perp) \Phi - m^2 \Phi^* \Phi - \lambda (\Phi^* \Phi)^2 \right\}. \end{aligned}$$

The generating functional becomes

$$Z[J_\mu^\perp, j, J, J^*] = \int \mathcal{D}A \mathcal{D}\theta \mathcal{D}\varphi \mathcal{D}\Phi^* \mathcal{D}\Phi \exp \left\{ iS[A^\perp, \varphi, \Phi^*, \Phi] + i \int d^4x \sqrt{-g} (A_\mu^\perp J^\mu - \varphi \square j + \Phi^* J + \Phi J^*) \right\}. \quad (27)$$

The integral over φ gets factorized in the form of $Z_2[j]$ and does not depend on $A^\perp$ with

$$\begin{aligned} Z_2[j] &= \int \mathcal{D}\varphi \exp \left\{ i \int d^4x \sqrt{-g} \left(-\frac{M_v^2}{2} \varphi \square \varphi - \varphi \square j \right) \right\} \\ &= [Det \square]^{-1/2} \exp \left\{ \frac{i}{2M_v^2} \int d^4x \sqrt{-g} j \square j \right\}. \end{aligned} \quad (28)$$

As a result, we get

$$Z[J_\mu^\perp, j, J, J^*] = Z_1[J_\mu^\perp, J, J^*] Z_2[j], \quad (29)$$

where $Z_1[J_\mu^\perp, J, J^*]$ is the generating functional for the theory with the action

$$S[A^\perp, \Phi^*, \Phi] = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu}^\perp F^{\perp\mu\nu} + \frac{M_v^2}{2} g^{\mu\nu} A_\mu^\perp A_\nu^\perp + g^{\mu\nu} (\partial_\mu + ieA_\mu^\perp) \Phi^* (\partial_\nu - ieA_\nu^\perp) \Phi - m^2 \Phi^* \Phi - \lambda (\Phi^* \Phi)^2 \right\}. \quad (30)$$

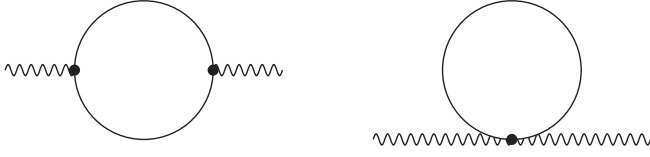


FIG. 1. Diagrams contributing to bilinear vector terms at the one-loop order.

It is easy to see that all relevant divergences are concentrated only in $Z_1[J_\mu^\perp, J, J^*]$. Further considerations are the same as in the previous section.

C. Renormalizability of the minimal vector theory

Now we are in a position to discuss whether the divergences of the nonminimal form $RA^\mu A_\mu$ and $R^{\mu\nu}A_\mu A_\nu$, and of the self-interacting form $(A^\mu A_\mu)^2$, can be expected at one- or higher-loop orders. At the one-loop level, in the Abelian theory, the diagrams with two external vector lines are those shown in Fig. 1. These diagrams are exactly the same as in the scalar QCD and, therefore, they will preserve the gauge invariance in the counterterms. This feature rules out the nonminimal terms since these terms violate the symmetry.

The main reason why the one-loop diagrams preserve gauge invariance is that there are no internal massive vector lines, violating the gauge invariance. Starting from the second loop, the situation is different. In the two-loop

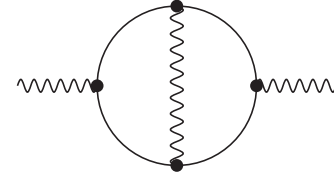


FIG. 2. One of the higher-loop diagrams expected to generate nonminimal terms.

diagram shown in Fig. 2, there is such an internal line and, therefore, according to the power counting, the nonminimal terms look possible at the second- or higher-loop orders.

The situation is quite similar in the case of $(A^\mu A_\mu)^2$, so we skip the details of this case. The final result is that this type of divergences may be expected at higher loops if we follow the dimensional arguments, without taking into account the symmetries.

Let us show that these arguments are not valid and, in fact, the minimal theory (1), without introducing the new curvature-dependent or $(A^\mu A_\mu)^2$ terms, is all-loop renormalizable. The starting point of our consideration is the symmetric formulation of the action (5). Then the general QFT theorems (see, e.g., [26] and references therein) tell us that, even in curved spacetime, the gauge symmetry under (6) holds in the counterterms and, on top of this, the counterterms are local.

The nonminimal and self-interacting terms in the original version (1), have the form

$$\Delta S[A, \Phi^*, \Phi] = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \zeta_1 R_{\mu\nu} A^\mu A^\nu + \frac{1}{2} \zeta_2 R A^\mu A_\mu - \frac{f}{4!} (A^\mu A_\mu)^2 \right\}. \quad (31)$$

On the other hand, the symmetric version of these terms, in the formulation (5), is more complicated,

$$\begin{aligned} \Delta S[A, \Phi^*, \Phi]_{\text{Stuck}} = \int d^4x \sqrt{-g} & \left\{ \frac{1}{2} \zeta_1 R^{\mu\nu} \left(A_\mu - \frac{1}{M_v} \nabla_\mu \theta \right) \left(A_\nu - \frac{1}{M_v} \nabla_\nu \theta \right) \right. \\ & \left. + \frac{1}{2} \zeta_2 R \left(A_\mu - \frac{1}{M_v} \nabla_\mu \theta \right)^2 - \frac{f}{4!} \left[\left(A_\mu - \frac{1}{M_v} \nabla_\mu \theta \right) \left(A^\mu - \frac{1}{M_v} \nabla^\mu \theta \right) \right]^2 \right\}. \end{aligned} \quad (32)$$

The main observation is that this expression is incompatible with the power counting of the theory, as it was established above. The point is that there are terms with the inverse powers of mass M_v and these factors are compensated by the extra derivatives of the scalar field θ . The terms with extra derivatives of the fields are typical for a theory which is nonrenormalizable by power counting theory. However, the emergence of these terms in our case, would be in contradiction with the analysis of the power counting presented above, in Sec. III A. We conclude that the counterterms of the form (32) cannot emerge in the theory with the restored gauge symmetry (5).

Does the last conclusion apply to the original theory (1)? To answer this question we note that this theory

corresponds to (5) under the particular gauge fixing. On the other hand, the general QFT theorems [27–30] tell us that the difference between the divergences in two different gauges is always proportional to the equations of motion. In our case, the divergences in the theory (1) differ from the ones in the theory (5) only by the terms proportional to the equations of motion of the theory with quantum corrections.

One can use this information to prove renormalizability. To this end, one can use the method based on iterations. Taking into account the locality of divergences and assuming multiplicative renormalizability of the minimal theory [i.e., starting from the action without the terms (31)] at the n th-loop order, we can see that the corresponding

equations of motion form the same combination (8) as we met at the classical level. The only change concerns the coefficients $\alpha_{1,2,3}$ in this combination. In view of the form of Eq. (7), we conclude that the terms (31) are ruled out as divergences in the original theory (1) at all loop orders.

There are also more simple arguments leading to the same conclusion. Let us decompose the vector field in transverse and longitudinal parts according to $A_\mu = A_\mu^\perp + \partial_\mu \varphi$ and replace it in (31). In this way, we get

$$\begin{aligned} \Delta S[A] = \int d^4x \sqrt{-g} \Big\{ & \frac{1}{2} \zeta_1 R^{\mu\nu} (A_\mu^\perp A_\nu^\perp + \partial_\mu \varphi \partial_\nu \varphi + 2A_\mu^\perp \partial_\nu \varphi) \\ & + \frac{1}{2} \zeta_2 R (A^{\perp\mu} A_\mu^\perp + \nabla_\mu \varphi \nabla^\mu \varphi) \\ & + \frac{f}{4!} (A_\mu^\perp A^{\perp\mu} + \nabla_\mu \varphi \nabla^\mu \varphi + 2A_\mu^\perp \nabla^\mu \varphi)^2 \Big\}. \end{aligned} \quad (33)$$

It is clear that the longitudinal mode φ in the terms containing $R_{\mu\nu}$ and A^4 does not decouple from transverse component $A^\perp$. On the other hand, in Sec. III B it was proved that the nonphysical longitudinal mode decouples in minimal theory. This means that, starting with the minimal theory, the aforementioned nonminimal terms cannot be generated. The only term in (33) where the longitudinal mode decouples from the transverse vector is the one with the coefficient ζ_2 .

To understand the difference between the terms with $M_v A^2$ and RA^2 , we note that for the massive term the decoupling of transverse and longitudinal modes is described by the expression

$$\frac{1}{2} M_v^2 A_\mu A^\mu = \frac{1}{2} M_v^2 [A_\mu^\perp A^{\perp\mu} + \nabla_\mu \varphi \nabla^\mu \varphi], \quad (34)$$

while for the curvature-dependent part we get

$$\frac{1}{2} \zeta_2 R A_\mu A^\mu = \frac{1}{2} R [A_\mu^\perp A^{\perp\mu} + \nabla_\mu \varphi \nabla^\mu \varphi]. \quad (35)$$

It is necessary to make the rescaling $\chi = M_v \varphi$, such that the new scalar χ gains the canonical dimension. After this, the sum of the scalar sectors in (34) and (35) becomes

$$\frac{1}{2} \left(1 + \frac{\zeta_2 R}{M_v^2} \right) \nabla_\mu \chi \nabla^\mu \chi. \quad (36)$$

In the case of $\zeta_2 \neq 0$, this relation contains the inverse mass coefficient that cannot be obtained as a divergence in the minimal theory because of the power-counting arguments. Therefore, the parameter ζ_2 must be equal to zero. As a result, we see that all the nonminimal terms in the vector sector are forbidden. We can state that the minimal theory is multiplicative renormalizable, including in the curved spacetime.

IV. STÜCKELBERG PROCEDURE AND ONE-LOOP DIVERGENCES

In this section we perform a calculation of the one-loop divergences in the interacting theory. The practical calculations are possible when using the Stückelberg trick, as it was suggested in the free Proca model in [20]. Thus, for the sake of covariant one-loop calculations, we work with the gauge-invariant action $S'[A, \theta, \Phi^*, \Phi]$ defined in (5). This action contains the Stückelberg field θ . Quantization is carried out using the Faddeev-Popov ansatz, involving the use of a gauge-fixing term and the ghost action.

The useful form of the covariant gauge-fixing action is

$$S_{\text{gf}} = -\frac{1}{2} \int d^4x \sqrt{-g} \chi^2, \quad \text{where } \chi = \nabla_\mu A^\mu - M_v \theta. \quad (37)$$

It is worth noting that this gauge condition is different from the $\theta = \text{const}$ that provides the equivalence of the original theory (1) and the theory with restored gauge symmetry (5). Of course, such a gauge is inconvenient for loop calculations. Luckily, there are general arguments that enable one to take into account the gauge-fixing dependence of the result. We shall present these arguments at the end of this section. This part is especially relevant because an alternative method of the heat-kernel calculations, as described in Appendix B, is difficult to apply in the present case.

The sum of the action (5) and the gauge-fixing term (37) has the form

$$\begin{aligned} S' + S_{\text{gf}} = \int d^4x \sqrt{-g} \Big\{ & \frac{1}{2} A^\nu (\delta_\nu^\mu \square - R^\mu_\nu - \delta_\nu^\mu M_v^2) A_\mu \\ & + \frac{1}{2} \theta (\square - M_v^2) \theta + g^{\mu\nu} (D_\mu \Phi)^* (D_\nu \Phi) \\ & - m^2 \Phi^* \Phi - \lambda (\Phi^* \Phi)^2 + \xi R \Phi^* \Phi \Big\}. \end{aligned} \quad (38)$$

To apply the background field method, we decompose the fields into classical and quantum $(\varphi, \varphi^*, B^\mu, \vartheta)$ counterparts,

$$\begin{aligned} \Phi^* &\rightarrow \Phi^* = \Phi^* + \varphi^*, & \Phi &\rightarrow \Phi = \Phi + \varphi, \\ A_\mu &\rightarrow A_\mu = A_\mu + B_\mu, & \theta &\rightarrow \theta = \theta + \vartheta \end{aligned} \quad (39)$$

and extract the bilinear in quantum fields form of the action (38)

$$S'^{(2)} + S_{\text{gf}} = \frac{1}{2} \int d^4x \sqrt{-g} \begin{pmatrix} \varphi^* & \varphi & \vartheta & B^\mu \end{pmatrix} \widehat{H} \begin{pmatrix} \varphi \\ \varphi^* \\ \vartheta \\ B_\nu \end{pmatrix}. \quad (40)$$

The differential operator $\widehat{H}'$ has a matrix form

$$\widehat{H}' = \begin{pmatrix} \widehat{H}'_{\varphi^* \varphi} & \widehat{H}'_{\varphi^* \varphi^*} & \widehat{H}'_{\varphi^* \vartheta} & \widehat{H}'_{\varphi^* B_\nu} \\ \widehat{H}'_{\varphi \varphi} & \widehat{H}'_{\varphi \varphi^*} & \widehat{H}'_{\varphi \vartheta} & \widehat{H}'_{\varphi B_\nu} \\ \widehat{H}'_{\vartheta \varphi} & \widehat{H}'_{\vartheta \varphi^*} & \widehat{H}'_{\vartheta \vartheta} & \widehat{H}'_{\vartheta B_\nu} \\ \widehat{H}'_{B^\mu \varphi} & \widehat{H}'_{B^\mu \varphi^*} & \widehat{H}'_{B^\mu \vartheta} & \widehat{H}'_{B^\mu B_\nu} \end{pmatrix}, \quad (41)$$

with the following nonvanishing elements:

$$\begin{aligned} \widehat{H}'_{\varphi^* \varphi} &= -\square - m^2 - 4\lambda\Phi^*\Phi + \xi R + 2ieA_\mu \nabla^\mu \\ &\quad + ie(\nabla_\mu A^\mu) + e^2 A_\mu^2, \\ \widehat{H}'_{\varphi \varphi^*} &= -\square - m^2 - 4\lambda\Phi^*\Phi + \xi R - 2ieA_\mu \nabla^\mu \\ &\quad - ie(\nabla_\mu A^\mu) + e^2 A_\mu^2, \\ \widehat{H}'_{B^\mu B_\nu} &= \delta_\mu^\nu \square - R_\mu{}^\nu + \delta_\mu^\nu M_v^2 + 2\delta_\mu^\nu e^2 \Phi^* \Phi, \\ \widehat{H}'_{\varphi^* B_\nu} &= ie\Phi \nabla^\nu + 2ie(\nabla^\nu \Phi) + 2e^2 A^\nu \Phi, \\ \widehat{H}'_{B^\mu \varphi} &= ie\Phi^* \nabla_\mu - ie(\nabla_\mu \Phi^*) + 2e^2 A_\mu \Phi^*, \\ \widehat{H}'_{\varphi B_\nu} &= -ie\Phi^* \nabla^\nu - 2ie(\nabla^\nu \Phi^*) + 2e^2 A^\nu \Phi^*, \\ \widehat{H}'_{B^\mu \varphi^*} &= -ie\Phi \nabla_\mu + ie(\nabla_\mu \Phi) + 2e^2 A_\mu \Phi, \\ \widehat{H}'_{\varphi^* \varphi^*} &= -2\lambda\Phi\Phi, \\ \widehat{H}'_{\varphi \varphi} &= -2\lambda\Phi^*\Phi^*, \\ \widehat{H}'_{\vartheta \vartheta} &= \square - M_v^2. \end{aligned}$$

The matrix operator (41) can be rewritten as follows:

$$\widehat{H}' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix} \widehat{H}, \quad (42)$$

where the new operator $\widehat{H}$ has the canonical form

$$\widehat{H} = \widehat{\mathbf{1}}\square + 2\widehat{h}^\alpha \nabla_\alpha + \widehat{\Pi}, \quad (43)$$

with $\widehat{\mathbf{1}} = \text{diag}(1, 1, 1, \delta_\mu^\nu)$,

$$\widehat{h}^\alpha = \begin{pmatrix} -ieA^\alpha & 0 & 0 & -\frac{1}{2}ieg^{\alpha\nu}\Phi \\ 0 & ieA^\alpha & 0 & \frac{1}{2}ieg^{\alpha\nu}\Phi^* \\ 0 & 0 & 0 & 0 \\ \frac{1}{2}ie\delta_\mu^\alpha\Phi^* & -\frac{1}{2}ie\delta_\mu^\alpha\Phi & 0 & 0 \end{pmatrix} \quad (44)$$

and

$$\widehat{\Pi} = \begin{pmatrix} m^2 + 4\lambda\Phi^*\Phi - \xi R & 2\lambda\Phi\Phi & 0 & -2ie(\mathcal{D}^\nu \Phi) \\ -ie(\nabla_\mu A^\mu) - e^2 A_\mu^2 & m^2 + 4\lambda\Phi^*\Phi - \xi R & 0 & 2ie(\mathcal{D}^\nu \Phi)^* \\ 2\lambda\Phi^*\Phi^* & +ie(\nabla_\mu A^\mu) - e^2 A_\mu^2 & 0 & 0 \\ 0 & 0 & -M_v^2 & 0 \\ -ie(\nabla_\mu \Phi^*) + 2e^2 A_\mu \Phi^* & +ie(\nabla_\mu \Phi) + 2e^2 A_\mu \Phi & 0 & -R_\mu{}^\nu + \delta_\mu^\nu M_v^2 \\ & & & +2\delta_\mu^\nu e^2 \Phi^* \Phi \end{pmatrix}. \quad (45)$$

The one-loop contribution to the effective action is given by

$$\overline{\Gamma}^{(1)} = \frac{i}{2} \text{Tr} \ln \widehat{H} - i \text{Tr} \ln \widehat{H}_{\text{gh}}, \quad (46)$$

where $\widehat{H}_{\text{gh}}$ the operator in the ghost action

$$\widehat{H}_{\text{gh}} = \square - M_v^2. \quad (47)$$

The divergent part of the one-loop effective action is given by the expression [18,19]

$$\begin{aligned} \overline{\Gamma}_{\text{div}}^{(1)} &= -\frac{\mu^{n-4}}{\epsilon} \int d^n x \sqrt{-g} \text{tr} \left\{ \frac{\widehat{\mathbf{1}}}{120} C^2 - \frac{\widehat{\mathbf{1}}}{360} E_4 \right. \\ &\quad \left. + \frac{1}{2} \widehat{P}^2 + \frac{1}{12} \widehat{S}_{\alpha\beta}^2 + \frac{1}{6} \square \widehat{P} + \frac{\widehat{\mathbf{1}}}{180} \square R \right\}, \quad (48) \end{aligned}$$

where $C^2 = C_{\mu\nu\alpha\beta}^2$ is the square of the Weyl tensor, E_4 is the integrand of the Gauss-Bonnet topological term,³

³The relations are $R_{\mu\nu\alpha\beta}^2 = 2C^2 - E_4 + \frac{1}{3}R^2$ and $R_{\mu\nu}^2 = \frac{1}{2}C^2 - \frac{1}{2}E_4 + \frac{1}{3}R^2$.

$\epsilon = (4\pi)^2(n-4)$ is the parameter of dimensional regularization and μ is the renormalization parameter. In the expression (48), the definitions are

$$\hat{P} = \hat{\Pi} + \frac{1}{6}R - \nabla_\alpha \hat{h} - \hat{h}_\alpha \hat{h}^\alpha, \quad (49)$$

$$\hat{S}_{\alpha\beta} = \hat{\mathcal{R}}_{\alpha\beta} + \nabla_\beta \hat{h}_\alpha - \nabla_\alpha \hat{h}_\beta + \hat{h}_\beta \hat{h}_\alpha - \hat{h}_\alpha \hat{h}_\beta, \quad (50)$$

where the commutator of geometric covariant derivatives is $\hat{\mathcal{R}}_{\alpha\beta} = \text{diag}(0, 0, 0, -R^\mu{}_{\nu\alpha\beta})$.

The contribution of the ghost action has the standard form [20]

$$\bar{\Gamma}_{\text{div,gh}}^{(1)} = -\frac{\mu^{n-4}}{\epsilon} \int d^n x \sqrt{-g} \left\{ \frac{1}{120} C^2 - \frac{1}{360} E_4 + \frac{1}{30} \square R + \frac{1}{72} R^2 - \frac{1}{6} M_v^2 R + \frac{1}{2} M_v^4 \right\}. \quad (51)$$

Using the general relation (48), after some algebra (the intermediate formulas can be found in Appendix A), the divergent part of the one-loop effective action is found in the form

$$\begin{aligned} \bar{\Gamma}_{\text{div}}^{(1)} = & -\frac{\mu^{n-4}}{\epsilon} \int d^n x \sqrt{-g} \left\{ \frac{1}{8} C^2 - \frac{13}{72} E_4 + \left[\left(\xi - \frac{1}{6} \right)^2 + \frac{1}{72} \right] R^2 - \frac{1}{3} \xi \square R - \left[\frac{1}{6} M_v^2 + 2m^2 \left(\xi - \frac{1}{6} \right) \right] R \right. \\ & + \left[2(e^2 - 4\lambda) \left(\xi - \frac{1}{6} \right) - \frac{1}{3} e^2 \right] R \Phi^* \Phi + (20\lambda^2 - 4e^2\lambda + 4e^4) (\Phi^* \Phi)^2 - 2[(e^2 - 4\lambda)m^2 - 3e^2 M_v^2] \Phi^* \Phi \\ & \left. + \frac{2}{3} (e^2 + 2\lambda) \square (\Phi^* \Phi) - 4e^2 (D_\mu \Phi^*) (D^\mu \Phi) - \frac{e^2}{6} F_{\mu\nu}^2 + \frac{3}{2} M_v^4 + m^4 \right\}. \end{aligned} \quad (52)$$

We note that the divergences form the three groups of terms. First of all, there are terms that reproduce the ones in the classical action (1). Furthermore, there are total derivative terms and, finally, the vacuum terms depending only on the external metric. This form of divergences confirms that the violation of gauge invariance is caused only by the mass of the Abelian vector field, i.e., the theory has only soft symmetry breaking and this feature holds at the quantum level. We can conclude that the renormalizability property is exactly like in the spinor massive electrodynamics, as explored in [16]. The general considerations of this work can be mapped to the scalar-massive vector theory, and hence the same structure of renormalization is expected to hold at higher loop orders up to the new nonminimal terms in the vector sector.

It is worth comparing the theory under discussion not only with the non-Abelian theory [16], but also with the Abelian theory of axial vector field [23] representing antisymmetric torsion. In this case, the gauge symmetry is violated by the spinor mass and, as a result, the axial vector mass is a necessary condition of renormalizability, including in the effective framework [25]. As a consequence, the longitudinal mode of the axial vector propagates starting from the second-loop corrections and there is a conflict between renormalizability and unitarity. Nothing of this sort occurs in the present case.

Last, but not least, one has to take special care about the possible gauge dependence. The reason is that, in the Stückelberg trick-based approach, it is only in the special gauge that the invariant theory reduces to the original

one (1). In the pure massive vector field model, even in a curved spacetime, the issue is trivial [20]. However, in the present case, the story is more complicated. What we know is that (see, e.g., [14,26,31] for formal proofs): (i) the divergent part of the effective action is a covariant local functional; (ii) the power-counting arguments hold in the theories with soft symmetry breaking [16,23]; and (iii) the differences between the one-loop divergences estimated in different gauges are proportional to the classical equations of motion [27]. Taken together, these arguments mean one can expect the gauge-dependent covariant local additions to (52) to be proportional to Eq. (7), i.e., there may be additional terms of the form

$$\begin{aligned} \Delta \bar{\Gamma}_{\text{div}}^{(1)} = & \frac{\mu^{n-4}}{\epsilon} \int d^n x \sqrt{-g} \left\{ \frac{f_\Phi}{2} (\mathcal{E} \Phi^* + \mathcal{E}^* \Phi) \right. \\ & \left. + f_\theta \mathcal{E}_\theta \theta + f_A \mathcal{E}^\nu A_\nu \right\}, \end{aligned} \quad (53)$$

where f_Φ , f_θ and f_A are arbitrary (gauge-fixing-dependent) real functions. These parameters are completely analogous to the constants α_1 , α_2 , and α_3 in Eq. (8). The inspection of Eq. (53) shows that it cannot produce dramatic changes in the divergences, e.g., generate a longitudinal mode of the vector field or the $(A_\mu A^\mu)^2$ -type term, as it occurs in the theory of the axial vector [23]. This means that there are good chances that the theory (1) may be renormalizable beyond one-loop order if the proper ultraviolet-completion terms are introduced.

Replacing the equations of motion (7) into (53), after some algebra we arrive at the difference between the one-loop divergences corresponding to two choices of the gauge fixing in the symmetric (Stückelberg) phase

$$\Delta\bar{\Gamma}_{\text{div}}^{(1)} = \bar{\Gamma}_{\text{div}}^{(1)}(\chi) - \bar{\Gamma}_{\text{div}}^{(1)}(\chi_0) = \frac{\mu^{n-4}}{\epsilon} \int d^n x \sqrt{-g} \left\{ f_\Phi [(D^\mu \Phi)^* (D_\mu \Phi) - (m^2 - \xi R) \Phi^* \Phi - 2\lambda (\Phi^* \Phi)^2] \right. \\ \left. + f_\theta [(\nabla_\mu \theta)^2 + M_v \theta \nabla_\mu A^\mu] + f_A \left[\frac{1}{2} F_{\mu\nu}^2 + M_v^2 \left(A_\nu^2 - \frac{1}{M_v} A_\nu \nabla^\nu \theta \right) - A_\nu J^\nu \right] \right\}. \quad (54)$$

In this expression, χ corresponds to an arbitrary gauge fixing while χ_0 represents a specific gauge condition of our choice. The one-loop divergence for gauge-fixing arbitrariness is given by sum of Eqs. (52) and (54). Let us note that the requirement of gauge invariance imposes the condition $f_\theta = f_A = 0$, while f_Φ is unconstrained.

V. ONE-LOOP RENORMALIZATION GROUP EQUATIONS

Consider renormalization in the theory (1). Since the calculations we performed were for a different theory (5) that emerges after the Stückelberg trick, the renormalization relations are subjects of ambiguity which was parametrized in (54).

The renormalized classical action is defined as $S_R = S + \Delta S$, where ΔS is the counterterm required to cancel the divergences. We can simply set $\Delta S = -\bar{\Gamma}_{\text{div}}^{(1)}$. For the scalar and vector fields, the renormalization relations are

$$\Phi_0 = \mu^{\frac{n-4}{2}} \left(1 - \frac{4e^2 + f_\Phi}{2\epsilon} \right) \Phi \quad \text{and} \quad A_\alpha^0 = \mu^{\frac{n-4}{2}} \left(1 + \frac{e^2}{3\epsilon} \right) A_\alpha, \quad (55)$$

which are explicitly ambiguous. For the two masses, we have

$$m_0^2 = m^2 + \frac{2}{\epsilon} [m^2(3e^2 - 4\lambda) - 3e^2 M_v^2] \quad (56)$$

and

$$M_{0,v}^2 = M_v^2 \left(1 - \frac{2e^2}{3\epsilon} \right). \quad (57)$$

The relations for the coupling constants have the form

$$e_0 = \mu^{\frac{4-n}{2}} \left(1 - \frac{e^2}{3\epsilon} \right) e, \\ \lambda_0 = \mu^{4-n} \left[\lambda - \frac{1}{\epsilon} (20\lambda^2 - 12e^2\lambda + 4e^4) \right], \quad (58)$$

and for the nonminimal parameter we have

$$\xi_0 = \xi + \frac{2}{\epsilon} \left[3e^2 \left(\xi - \frac{1}{9} \right) - 4\lambda \left(\xi - \frac{1}{6} \right) \right]. \quad (59)$$

It is remarkable that the contribution of the massive vector in this relation is *not* proportional to $\xi - 1/6$; regardless there is no mass dependence in this formula. The reason is that, before we take a strictly massless limit, there is an extra degree of freedom (equivalent to the Stückelberg field), which is nonconformal. The situation is analogous to the discontinuity described for the vacuum sector of the massive vector field theory in curved spacetime [20].

One can find the beta and gamma functions using the relations

$$\beta_P = \lim_{n \rightarrow 4} \mu \frac{dP}{d\mu} \quad \text{and} \quad \gamma_\Phi H = \lim_{n \rightarrow 4} \mu \frac{dH}{d\mu}, \quad (60)$$

where $P = (m, M_v, \xi, \lambda, e)$ are renormalized parameters and $H = (\Phi^*, \Phi, A_\mu)$ renormalized fields. Using the renormalization relations, we get

$$\beta_e = \frac{e^3}{3(4\pi)^2}, \\ \beta_\lambda = \frac{1}{(4\pi)^2} (20\lambda^2 - 12e^2\lambda + 4e^4), \\ \beta_\xi = \frac{2}{(4\pi)^2} \left[4\lambda \left(\xi - \frac{1}{6} \right) - 3e^2 \left(\xi - \frac{1}{9} \right) \right], \\ \beta_{m^2} = -\frac{2}{(4\pi)^2} [3e^2(m^2 - M_v^2) - 4m^2\lambda], \\ \beta_{M_v^2} = \frac{2e^2}{3(4\pi)^2} M_v^2. \quad (61)$$

The gamma functions have the form

$$\gamma_\Phi = \frac{2e^2}{(4\pi)^2} \quad \text{and} \quad \gamma_{A_\mu} = -\frac{e^2}{3(4\pi)^2}. \quad (62)$$

We note that γ_{A_μ} and β_e are exactly those of the usual QED. As we explained in Sec. III, this feature is supposed to hold

only at the one-loop order. On the other hand, γ_Φ can be a subject of ambiguity produced by the Stückelberg procedure at higher-loop orders.

The effective potential for the action (1) can be obtained using the renormalization group technique [32] adapted to curved spacetime [33] (see also [34]). The renormalization group equation for the effective potential has the form

$$\left\{ \mu \frac{\partial}{\partial \mu} + \beta_P \frac{\partial}{\partial P} + \gamma_\Phi \left(\Phi \frac{\partial}{\partial \Phi} + \Phi^* \frac{\partial}{\partial \Phi^*} \right) \right\} \times V_{\text{eff}}(g_{\alpha\beta}, \Phi^*, \Phi, P, \mu) = 0, \quad (63)$$

where μ is the renormalization parameter. The previous equation allows the effective potential to be rewritten in terms of beta and gamma functions. In our case, it is essential to account for the effect of the coupling between the scalar field and the gauge field, as it results in additional corrections to the potential. The procedure is analogous to that described in [34], and hence we just formulate the result for the effective potential,

$$\begin{aligned} V_{\text{eff}} = & V + \frac{1}{2}(\beta_m + 2m^2\gamma_\Phi)|\Phi|^2 \left[\ln\left(\frac{|\Phi|^2}{\mu^2}\right) - C_1 \right] \\ & + \frac{1}{2}(\beta_\xi + 2\xi\gamma_\Phi)R|\Phi|^2 \left[\ln\left(\frac{|\Phi|^2}{\mu^2}\right) - C_2 \right] \\ & + \frac{1}{2}(\beta_\lambda + 4\lambda\gamma_\Phi)|\Phi|^4 \left[\ln\left(\frac{|\Phi|^2}{\mu^2}\right) - C_3 \right], \end{aligned} \quad (64)$$

where $|\Phi|^2 = \Phi^*\Phi$, and the constants C_1 , C_2 and C_3 depend on the renormalization conditions. E.g., in the massless case and for the conditions used in [32–34], the values are $C_1 = 0$, $C_2 = -3$ and $C_3 = -25/6$.

VI. CONCLUSIONS

We have shown that the renormalizable curved-space theory of a massive vector field coupled to a scalar can be based on the minimal action (1), without the inclusion of nonminimal terms (31) in the vector field sector, including those proportional to the Ricci tensor. In this respect, the massive vector field is crucially different from the scalar field, where the nonminimal interaction with the Ricci scalar is the necessary condition for renormalizability. The difference is that the vector non-minimal terms are protected by the gauge symmetry, even regardless of the fact that this symmetry is softly broken in the original formulation of the theory. The statement about renormalizability was confirmed by the direct one-loop calculation.

The evaluation of one-loop divergences is an important part of a QFT model, as it provides information on the renormalization structure of the theory. We reported about the derivation of one-loop divergences in the massive charged scalar theory coupled to the massive vector theory. This model proved renormalizable if the vector field is Abelian and if it is not an axial vector. The non-Abelian vector theory or axial vector is not renormalizable, as it was discussed in [16] and proved by direct calculations in [23].

The beta functions (61) and the effective potential (64) were derived in the symmetric formulation (5). The general arguments concerning classification of parameters in curved spacetime [14] imply that the expressions (61) do not change under an arbitrary choice of the gauge fixing, and hence these expressions can be applied to the original theory (1).

Let us add the last observation concerning the choice of regularization and its influence on the results. The standard considerations about the power counting apply directly only assuming an invariant regularization. In our case, this means a procedure that preserves both general covariance and the gauge invariance in the Stückelberg formulation. The regularizations satisfying this condition include, e.g., dimensional, covariant Pauli-Villars and some others. Also there are invariant regularizations which are usually applied for the one-loop calculations, e.g., the cutoff in the proper-time integral, or zeta regularization. In the last case, one does not see the divergences, but the renormalization group beta functions can be restored by introducing the μ dependence artificially (see, e.g., [35]).

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: INTERMEDIATE EXPRESSIONS FOR ONE-LOOP DIVERGENCES

Let us collect the intermediate formulas for the derivation of divergences in Sec. III. According to Eqs. (49) and (50), we obtain

$$\hat{P} = (P)_{\mu}^{\nu} = \begin{pmatrix} \hat{P}_{\varphi^* \varphi} & \hat{P}_{\varphi^* \varphi^*} & \hat{P}_{\varphi^* \vartheta} & \hat{P}_{\varphi^* B_{\nu}} \\ \hat{P}_{\varphi \varphi} & \hat{P}_{\varphi \varphi^*} & \hat{P}_{\varphi \vartheta} & \hat{P}_{\varphi B_{\nu}} \\ \hat{P}_{\vartheta \varphi} & \hat{P}_{\vartheta \varphi^*} & \hat{P}_{\vartheta \vartheta} & \hat{P}_{\vartheta B_{\nu}} \\ \hat{P}_{B^{\mu} \varphi} & \hat{P}_{B^{\mu} \varphi^*} & \hat{P}_{B^{\mu} \vartheta} & \hat{P}_{B^{\mu} B_{\nu}} \end{pmatrix}, \quad (\text{A1})$$

with the following nonzero elements:

$$\begin{aligned} \hat{P}_{\varphi \varphi^*} &= \hat{P}_{\varphi^* \varphi} = m^2 + \left(\frac{1}{6} - \xi\right) R - \left(\frac{1}{4} \delta_{\mu}^{\nu} e^2 - 4\lambda\right) \Phi^* \Phi, \\ \hat{P}_{\vartheta \vartheta} &= -\left(M_v^2 - \frac{1}{6} R\right), \quad \hat{P}_{B^{\mu} B_{\nu}} = -R_{\mu}^{\nu} + \delta_{\mu}^{\nu} \left(M_v^2 + \frac{1}{6} R + \frac{3}{2} e^2 \Phi^* \Phi\right), \\ \hat{P}_{\varphi^* \varphi^*} &= \frac{1}{4} (\delta_{\mu}^{\nu} e^2 + 8\lambda) \Phi \Phi, \quad \hat{P}_{\varphi \varphi} = \frac{1}{4} (\delta_{\mu}^{\nu} e^2 + 8\lambda) \Phi^* \Phi^*, \\ \hat{P}_{\varphi^* B_{\nu}} &= -\frac{3}{2} i e (D^{\nu} \Phi), \quad \hat{P}_{B^{\mu} \varphi} = -\frac{3}{2} i e (D_{\mu} \Phi)^*, \\ \hat{P}_{\varphi B_{\nu}} &= \frac{3}{2} i e (D^{\nu} \Phi)^*, \quad \hat{P}_{B^{\mu} \varphi^*} = \frac{3}{2} i e (D_{\mu} \Phi). \end{aligned} \quad (\text{A2})$$

Similarly, the matrix $\hat{S}^{\alpha\beta}$ is given by

$$\hat{S}^{\alpha\beta} = (S^{\alpha\beta})_{\mu}^{\nu} = \begin{pmatrix} \hat{S}_{\varphi^* \varphi} & \hat{S}_{\varphi^* \varphi^*} & \hat{S}_{\varphi^* \vartheta} & \hat{S}_{\varphi^* B_{\nu}} \\ \hat{S}_{\varphi \varphi} & \hat{S}_{\varphi \varphi^*} & \hat{S}_{\varphi \vartheta} & \hat{S}_{\varphi B_{\nu}} \\ \hat{S}_{\vartheta \varphi} & \hat{S}_{\vartheta \varphi^*} & \hat{S}_{\vartheta \vartheta} & \hat{S}_{\vartheta B_{\nu}} \\ \hat{S}_{B^{\mu} \varphi} & \hat{S}_{B^{\mu} \varphi^*} & \hat{S}_{B^{\mu} \vartheta} & \hat{S}_{B^{\mu} B_{\nu}} \end{pmatrix}, \quad (\text{A3})$$

where the nonzero elements are

$$\begin{aligned} \hat{S}_{\varphi^* \varphi} &= \frac{1}{4} e^2 (\delta_{\mu}^{\alpha} g^{\beta\nu} - \delta_{\mu}^{\beta} g^{\alpha\nu}) \Phi^* \Phi + i e (\nabla^{\alpha} A^{\beta} - \nabla^{\beta} A^{\alpha}), \\ \hat{S}_{\varphi \varphi^*} &= \frac{1}{4} e^2 (\delta_{\mu}^{\alpha} g^{\beta\nu} - \delta_{\mu}^{\beta} g^{\alpha\nu}) \Phi^* \Phi - i e (\nabla^{\alpha} A^{\beta} - \nabla^{\beta} A^{\alpha}), \\ \hat{S}_{B^{\mu} B_{\nu}} &= R_{\mu}^{\nu\alpha\beta} + \frac{1}{2} e^2 (\delta_{\mu}^{\beta} g^{\alpha\nu} - \delta_{\mu}^{\alpha} g^{\beta\nu}) \Phi^* \Phi, \\ \hat{S}_{\varphi^* B_{\nu}} &= \frac{1}{2} i e [g^{\beta\nu} (D^{\alpha} \Phi) - g^{\alpha\nu} (D^{\beta} \Phi)], \\ \hat{S}_{B^{\mu} \varphi} &= -\frac{1}{2} i e [\delta_{\mu}^{\beta} (D^{\alpha} \Phi^*) - \delta_{\mu}^{\alpha} (D^{\beta} \Phi^*)], \\ \hat{S}_{\varphi B_{\nu}} &= -\frac{1}{2} i e [g^{\beta\nu} (D^{\alpha} \Phi^*) - g^{\alpha\nu} (D^{\beta} \Phi^*)], \\ \hat{S}_{B^{\mu} \varphi^*} &= \frac{1}{2} i e [\delta_{\mu}^{\beta} (D^{\alpha} \Phi) - \delta_{\mu}^{\alpha} (D^{\beta} \Phi)], \\ \hat{S}_{\varphi^* \varphi^*} &= \frac{1}{4} e^2 (\delta_{\mu}^{\beta} g^{\alpha\nu} - \delta_{\mu}^{\alpha} g^{\beta\nu}) \Phi \Phi, \\ \hat{S}_{\varphi \varphi} &= \frac{1}{4} e^2 (\delta_{\mu}^{\beta} g^{\alpha\nu} - \delta_{\mu}^{\alpha} g^{\beta\nu}) \Phi^* \Phi^*. \end{aligned}$$

Using these operators, the particular traces are

$$\frac{1}{6}\text{tr}\square\hat{P} = \frac{2}{3}(e^2 + 2\lambda)\square(\Phi^*\Phi) + \frac{1}{3}\left(\frac{1}{12} - \xi\right)\square R, \quad (\text{A4})$$

$$\begin{aligned} \frac{1}{2}\text{tr}\hat{P}^2 &= \frac{1}{2}R_{\mu\nu}^2 - \left[\frac{5}{72} - \left(\xi - \frac{1}{3}\right)\xi\right]R^2 + \left[\frac{M_v^2}{2} + 2m^2\left(\xi - \frac{1}{6}\right)\right]R + \frac{5}{2}M_v^4 + m^4 \\ &+ \left[2(e^2 - 4\lambda)\left(\xi - \frac{1}{6}\right) - \frac{1}{2}e^2\right]R(\Phi^*\Phi) + (20\lambda^2 - 4e^2\lambda + 5e^4)(\Phi^*\Phi)^2 \\ &- 2[m^2(e^2 - 4\lambda) - 3e^2M_v^2](\Phi^*\Phi) - \frac{9}{2}e^2(D^\mu\Phi)^*(D_\mu\Phi) \end{aligned} \quad (\text{A5})$$

and

$$\begin{aligned} \frac{1}{12}\text{tr}\hat{S}_{\alpha\beta}^2 &= -\frac{1}{12}R_{\mu\nu\alpha\beta}^2 + \frac{e^2}{6}R(\Phi^*\Phi) - e^4(\Phi^*\Phi)^2 \\ &+ \frac{e^2}{2}(D^\mu\Phi)^*(D_\mu\Phi) - \frac{e^2}{6}F_{\mu\nu}^2. \end{aligned} \quad (\text{A6})$$

APPENDIX B: USING THE AUXILIARY OPERATOR APPROACH

Let us consider an alternative approach for deriving the one-loop effective action, which was successfully applied to the pure theory of a massive vector field [19] and proved equivalent to the Stückelberg procedure-based approach. Here we use a similar method in the theory of a massive vector coupled to the scalar field.

First we have to determine the bilinear form of the action (1) without restoring the gauge symmetry. Using the background field method, we decompose the fields into classical and quantum counterparts as

$$\begin{aligned} \Phi^* \rightarrow \Phi^* = \Phi^* + \varphi^*, \quad \Phi \rightarrow \Phi = \Phi + \varphi, \\ A_\mu \rightarrow A_\mu = A_\mu + B_\mu \end{aligned} \quad (\text{B1})$$

and write the bilinear in quantum fields part of the action in the form

$$S^{(2)} = \frac{1}{2} \int d^4x \sqrt{-g} (\varphi^* \quad \varphi \quad B^\mu) \hat{H} \begin{pmatrix} \varphi \\ \varphi^* \\ B_\nu \end{pmatrix}, \quad (\text{B2})$$

$$\text{where } \hat{H} = \begin{pmatrix} \hat{H}_{\varphi^*\varphi} & \hat{H}_{\varphi^*\varphi^*} & \hat{H}_{\varphi^*B_\nu} \\ \hat{H}_{\varphi\varphi} & \hat{H}_{\varphi\varphi^*} & \hat{H}_{\varphi B_\nu} \\ \hat{H}_{B^\mu\varphi} & \hat{H}_{B^\mu\varphi^*} & \hat{H}_{B^\mu B_\nu} \end{pmatrix} \quad (\text{B3})$$

and the elements of the matrix are

$$\begin{aligned} \hat{H}_{\varphi^*\varphi} &= -\square - m^2 - 4\lambda\Phi^*\Phi + \xi R + 2ieA_\mu\nabla^\mu \\ &\quad + ie(\nabla_\mu A^\mu) + e^2A_\mu^2, \\ \hat{H}_{\varphi\varphi^*} &= -\square - m^2 - 4\lambda\Phi^*\Phi + \xi R - 2ieA_\mu\nabla^\mu \\ &\quad - ie(\nabla_\mu A^\mu) + e^2A_\mu^2, \\ \hat{H}_{B^\mu B_\nu} &= \delta_\mu^\nu \square - \nabla_\mu \nabla^\nu - R_\mu^\nu - \delta_\mu^\nu M_v^2 + 2\delta_\mu^\nu e^2\Phi^*\Phi, \\ \hat{H}_{\varphi^*B_\nu} &= ie\Phi\nabla^\nu + 2ie(\nabla^\nu\Phi) + 2e^2A^\nu\Phi, \\ \hat{H}_{B^\mu\varphi} &= ie\Phi^*\nabla_\mu - ie(\nabla_\mu\Phi^*) + 2e^2A_\mu\Phi^*, \\ \hat{H}_{\varphi B_\nu} &= -ie\Phi^*\nabla^\nu - 2ie(\nabla^\nu\Phi^*) + 2e^2A^\nu\Phi^*, \\ \hat{H}_{B^\mu\varphi^*} &= -ie\Phi\nabla_\mu + ie(\nabla_\mu\Phi) + 2e^2A_\mu\Phi, \\ \hat{H}_{\varphi^*\varphi^*} &= -2\lambda\Phi\Phi, \quad \hat{H}_{\varphi\varphi} = -2\lambda\Phi^*\Phi^*. \end{aligned} \quad (\text{B4})$$

The next step is to introduce the auxiliary operator

$$\hat{K} = K_\nu^\alpha = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\nabla_\nu \nabla^\alpha + \delta_\nu^\alpha M_v^2 \end{pmatrix}. \quad (\text{B5})$$

The operator in the corner of this matrix is the auxiliary operator introduced in [19]. However, in the present case, the form of the product is much more complicated,

$$\hat{H}^* = \hat{H} \hat{K} = \begin{pmatrix} \hat{H}_{\varphi^*\varphi}^* & \hat{H}_{\varphi^*\varphi^*}^* & \hat{H}_{\varphi^*B_\alpha}^* \\ \hat{H}_{\varphi\varphi}^* & \hat{H}_{\varphi\varphi^*}^* & \hat{H}_{\varphi B_\alpha}^* \\ \hat{H}_{B^\mu\varphi}^* & \hat{H}_{B^\mu\varphi^*}^* & \hat{H}_{B^\mu B_\alpha}^* \end{pmatrix}, \quad (\text{B6})$$

with the elements

$$\begin{aligned}
\hat{H}_{\varphi^*\varphi}^* &= \square + m^2 + 4\lambda\Phi^*\Phi - \xi R - 2ieA_\nu\nabla^\nu \\
&\quad - ie(\nabla_\nu A^\nu) - e^2 A_\nu^2, \\
\hat{H}_{\varphi\varphi^*}^* &= \square + m^2 + 4\lambda\Phi^*\Phi - \xi R + 2ieA_\nu\nabla^\nu + ie(\nabla_\nu A^\nu) - e^2 A_\nu^2, \\
\hat{H}_{B^\mu B_\alpha}^* &= M_v^2(\delta_\mu^\alpha \square - R_{\mu}{}^\alpha - \delta_\mu^\alpha M_v^2) - 2e^2(\Phi^*\Phi)(\nabla_\mu\nabla^\alpha - \delta_\mu^\alpha M_v^2), \\
\hat{H}_{\varphi^* B_\alpha}^* &= -ie\Phi(\square - M_v^2)\nabla^\alpha - 2ie(D^\nu\Phi)(\nabla_\nu\nabla^\alpha - \delta_\nu^\alpha M_v^2), \\
\hat{H}_{B^\mu\varphi}^* &= -ie\Phi^*\nabla_\mu + ie(\nabla_\mu\Phi^*) - 2e^2 A_\mu\Phi^*, \\
\hat{H}_{\varphi B_\alpha}^* &= ie\Phi^*(\square - M_v^2)\nabla^\alpha + 2ie(D^\nu\Phi)^*(\nabla_\nu\nabla^\alpha - \delta_\nu^\alpha M_v^2), \\
\hat{H}_{B^\mu\varphi^*}^* &= ie\Phi\nabla_\mu - ie(\nabla_\mu\Phi) - 2e^2 A_\mu\Phi, \\
\hat{H}_{\varphi^*\varphi^*}^* &= 2\lambda\Phi\Phi, \\
\hat{H}_{\varphi\varphi}^* &= 2\lambda\Phi^*\Phi^*.
\end{aligned}$$

The form of the operator $\hat{H}^*$ in Eq. (B6) is quite unusual because some of the nondiagonal elements of the matrix have operators that are third-order differential operators. The derivation of $\text{Tr} \log \hat{H}^*$ in this case is a nontrivial problem, which would be difficult to deal with even using the generalized Schwinger-DeWitt technique of [19]. Since we have another, much simpler, approach described in Sec. III, we do not describe the details of this functional trace. This appendix was included just to illustrate that the two methods may not be equivalent in the more sophisticated models with soft symmetry breaking.

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