

Unified study of nucleon and Δ baryon spectra and their strong decays with chiral dynamics

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In this work, we systematically study both the mass spectra and strong decays of the nucleon and Δ resonances up to the $N = 2$ shell within a unified quark model framework with chiral dynamics. In this framework we achieve a good description of the strong decay properties of the well-established nucleon and Δ resonances. Meanwhile, the mass reversal between $N(1440)1/2^+$ as the first radial excitation state and the $1P$ -wave nucleon resonances can be explained. We show that the three-body spin-orbit potential arising from the one-gluon exchange can cause a large configuration mixing between $N(1520)3/2^-$ and $N(1700)3/2^-$ and is also responsible for the large splitting between $\Delta(1600)1/2^-$ and $\Delta(1700)3/2^-$. Some of these baryon resonances tend to weakly couple to the $N\pi$, $N\eta$, $K\Lambda$, and $K\Sigma$ channels, which may answer the question of why they have not been established in these channels via the πN and γN scatterings. It shows that these “missing resonances” may have large potentials to be established in the $N\pi\pi$ final state due to their large decay rates into either the $\Delta(1232)$ or $1P$ -wave nucleon resonances via the pionic decays. Further experimental search for their signals in charmonium decays at BESIII is thus strongly recommended.

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I. INTRODUCTION

A better understanding of the nucleon as a bound state of quarks and gluons, as well as the spectrum and internal structure of excited baryons, remains a fundamental challenge and goal in hadronic physics [1,2]. The study of the excited nucleons can provide us with critical insights into the nature of QCD in the confinement domain [3]. In experiments, the excited states of nucleon and Δ (i.e., N^* and Δ^*) can be produced in various processes such as πN scattering, photo- or electroproduction. In the past two decades, experiments at ELSA, GRAAL, Jlab, MAMI, and SPring-8 (see the recent reviews [4–6]) have accumulated a large data sample that have provided more clues for more

nucleon resonances [7–9]. With the increase of the J/ψ and $\psi(3686)$ data sample accumulated at the Beijing Electron-Positron Collider, an alternative approach for the nucleon resonances is to study the baryon spectroscopy via the J/ψ and $\psi(3686)$ decays into baryon and antibaryon pairs [10], where the initial isoscalar of the charmonia provides an isospin filter for the produced baryon states. In Ref. [11] one new resonance $N(2040)3/2^+$ was reported by the BESIII Collaboration in $J/\psi \rightarrow \bar{p}p\pi^0$, and later, two new resonances $N(2300)3/2^+$ and $N(2570)5/2^-$ were observed in $\psi(3686) \rightarrow \bar{p}p\pi^0$ [12].

In theory, the quark model has achieved a great success in describing the baryon spectrum [1,2,13,14]. However, about the excited nucleon spectrum, there are several long-standing questions to be answered. The first one is why the roper resonance $N(1440)1/2^+$ belonging to the $N = 2$ shell lies so much lower than the first orbitally excited nucleon states, such as $N(1535)1/2^-$ and $N(1520)3/2^-$. This is the so-called mass reversal problem. In the quark potential model based on one-gluon exchange (OGE) [14], the $N(1440)1/2^+$ as a radial excitation is predicted to be ~ 80 MeV above the $N(1535)1/2^-$, rather than ~ 100 MeV

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below it. To solve this puzzle, models based on one-boson exchange (OBE) were proposed in the literature [15–18], and it may indicate that the chiral dynamics should play a role. Questions concerning these two different ways of describing the quark potentials, i.e., OGE and OBE, were raised and initiated a lot of debates in the literature [19,20].

Another relevant issue is the so-called missing resonance problem. Namely, why many excited nucleon states in the mass region of ~ 2.0 GeV predicted by the symmetric quark model have not been observed in experiments. Although the diquark picture was proposed as a possible solution for reducing the internal degrees of freedom, hence reducing the number of excited baryon states [21] (see [22] for review), the situation did not get improved even for the second orbital excitations. Nevertheless, since the quark-gluon interactions are flavor blind, there is no obvious reason that two quarks would be bound more tightly than the third one in a baryon state. Eventually, the diquark picture should have been ruled out by the results from the lattice QCD simulation. In Ref. [23] it is clearly shown that the $SU(6) \otimes O(3)$ symmetry holds well for the first orbital excitations.

To understand better the internal degrees of freedom of the excited nucleon resonances, different methods and approaches were developed and adopted in the literature, such as the quark model combining both OGE and OBE potentials [24–26], the relativistically covariant constituent quark model with instantaneous forces [27,28], the Dyson-Schwinger and Faddeev equations [29], lattice QCD [23,30–35], large N_c approach [36–42], QCD sum rules [43–45], the OGE model with higher order hyperfine interactions [46], and so on.

Although different approaches provide their different versions for the baryon spectra, these states can only be probed by their interactions with other particles in their productions and/or decays. Their strong decays are often investigated and compared with the experimental data, from which the structure information can be extracted. In the quark model scenario, one can find the elementary meson emission model [47,48], the quark pair creation model [49,50], the chiral quark model [51,52], the quark model with the N_c expansion [53], and the covariant quark model [54,55]. The status about the strong decay studies of the light baryon resonances can be found in Refs. [1,4–6,13,56].

In this work, we attempt to provide a unified study of both the mass spectra and strong decays of the N^* and Δ^* within a constituent quark model combining the chiral dynamics, i.e., the chiral quark model. The chiral quark model was broadly applied to the pseudoscalar meson photoproductions [57–67] and πN [68–70] and $\bar{K}N$ scatterings [71–73] in the resonance energy region. This method has also been successfully extended to study the strong decays of the singly heavy baryon resonances [52,74–81], strange baryon resonances [82–84], and heavy-light meson

states [85–91]. It clearly indicates that the light quark-meson interactions described with chiral dynamics should play a crucial role in dynamical processes where light quarks are involved. To be more specific, it shows that, in a light baryon system, in addition to the OGE potential for the quark-quark interactions, there are non-negligible contributions from the OBE. In this work, we include, in addition to the OGE potentials, the chiral potentials via the π , η , and σ exchanges between the constituent quarks to calculate the mass spectra of the nucleon and Δ baryons. The parameters of the chiral potentials are well determined by the study of the strong decays within the chiral quark model. This hybrid quark model including both the OGE and OBE potentials has been widely applied in the study of the hadron spectra and hadron-hadron interactions in the literature, e.g., Refs. [24,92–102].

The paper is organized as follows. The framework is given in Sec. II, where we give an introduction to the quark model classification for the excitations of nucleons, the potential quark model for the mass calculations, the chiral quark model for the evaluations of the strong decays, and the model parameters. In Sec. III, the masses and strong decay properties of N^* and Δ^* within the $N = 2$ shell are presented with discussions. A summary is given in Sec. IV.

II. FRAMEWORK

A. Quark model classification

In the following, we will give an introduction of the quark model classification for light baryons up to $N = 3$ shell. The wave function of a hadron includes four parts in the color, flavor, spin, and spatial spaces. The color wave function ψ^c should be a color singlet under $SU(3)$ symmetry.

For a light baryon system, the flavor wave function ϕ can be constructed by the light u , d , s quarks, which follows a flavor $SU(3)$ symmetry. The spin wave function χ can be constructed as the eigenstates of the quark spin and its projection along z axis, which follows the $SU(2)$ symmetry. Combining together the spin and flavor symmetry, the spin-flavor wave functions follow an $SU(6)$ symmetry and are usually denoted by $|N_6, {}^{2S+1}N_3\rangle$ in the literature, where N_6 and N_3 represent the dimensions of the $SU(6)$ and $SU(3)$ representations, respectively, and S stands for the quantum number of the total spin of a baryon state. The $SU(6)$ spin-flavor wave functions are given in Table I. More details of the flavor wave function ϕ and the spin wave function χ can be found in the previous work of our group [82].

The spatial wave functions satisfy the $O(3)$ symmetry under a rotation transformation. For a baryon system containing three quarks, the spatial wave function $\Psi_{NLM_L} = [\psi_{n_\rho l_\rho m_\rho}(\rho) \otimes \psi_{n_\lambda l_\lambda m_\lambda}(\lambda)]_{NLM_L}$ is composed of ρ - and λ -mode spatial wave functions. The Jacobi coordinates ρ and λ can be related to the quark coordinates $\mathbf{r}_j$ ($j = 1, 2, 3$). The explicit expressions are given by

TABLE I. The SU(6) spin-flavor wave functions of light baryons. χ and ϕ stand for the spin and flavor wave functions, respectively. The superscripts s and a , λ , and ρ stand for wave functions that are symmetric, antisymmetric, mixed symmetric, and mixed antisymmetric, respectively.

$ N_6, {}^{2S+1}N_3\rangle$	Wave function	$ N_6, {}^{2S+1}N_3\rangle$	Wave function
$ 56, {}^2 8\rangle^s$	$\frac{1}{\sqrt{2}}(\chi^\lambda \phi^\lambda + \chi^\rho \phi^\rho)$	$ 56, {}^4 10\rangle^s$	$\phi^s \chi^s$
$ 70, {}^2 8\rangle^\rho$	$\frac{1}{\sqrt{2}}(\phi^\rho \chi^\lambda + \phi^\lambda \chi^\rho)$	$ 70, {}^2 8\rangle^\lambda$	$\frac{1}{\sqrt{2}}(\phi^\rho \chi^\rho - \phi^\lambda \chi^\lambda)$
$ 70, {}^4 8\rangle^\rho$	$\phi^\rho \chi^s$	$ 70, {}^4 8\rangle^\lambda$	$\phi^\lambda \chi^s$
$ 70, {}^2 10\rangle^\rho$	$\phi^s \chi^\rho$	$ 70, {}^2 10\rangle^\lambda$	$\phi^s \chi^\lambda$
$ 70, {}^2 1\rangle^\rho$	$\phi^a \chi^\lambda$	$ 70, {}^2 1\rangle^\lambda$	$\phi^a \chi^\rho$
$ 20, {}^2 8\rangle^a$	$\frac{1}{\sqrt{2}}(\phi^\rho \chi^\lambda - \phi^\lambda \chi^\rho)$	$ 20, {}^4 1\rangle^a$	$\phi^a \chi^s$

$$\begin{aligned}\rho &= \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \\ \lambda &= \sqrt{\frac{2}{3}}\left(\frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} - \mathbf{r}_3\right), \\ \mathbf{R} &= \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + m_3 \mathbf{r}_3}{m_1 + m_2 + m_3}.\end{aligned}\quad (1)$$

The quantum numbers n_ρ , l_ρ , and m_ρ [or n_λ , l_λ , m_λ] stand for the radial excitation, relative orbital angular momentum, and its z component for the ρ -mode [or λ -mode] wave function, respectively. While N , L , and M_L stand for the principal quantum number, the quantum numbers of total orbital angular momentum, and its z component, respectively. They are defined by $N = 2n_\rho + 2n_\lambda + l_\rho + l_\lambda$, $|l_\rho - l_\lambda| \leq L \leq l_\rho + l_\lambda$, and $M_L = m_\rho + m_\lambda$. Furthermore, the superscript σ appearing in wave functions represents their permutation symmetries.

For the construction of spatial wave functions, in addition to the method suggested in Ref. [103], one can also construct them from the permutation symmetry directly. For a fully symmetric wave function φ^s , when exchanging any two quarks, it remains unchanged, that is, $\hat{P}_{ij}\varphi^s = \varphi^s$, where $\hat{P}_{ij}$ is a permutation operator with $i < j = 1, 2, 3$. For a completely antisymmetric wave function, one has $\hat{P}_{ij}\varphi^a = -\varphi^a$. For a mixed-symmetric wave function φ^λ and mixed-antisymmetric wave function φ^ρ , when exchanging quarks 1 and 2, one has $\hat{P}_{12}(\varphi^\lambda) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}(\varphi^\lambda)$. While for the permutation of quarks at positions 1 and 3, the wave function satisfies $\hat{P}_{13}(\varphi^\lambda) = U(\varphi^\lambda)$, where U is a unitary matrix. The explicit forms of the spatial wave function up to the $N = 3$ shell have been given in Table II.

The total wave function of a baryon can be expressed as

$$|qqq\rangle_A = |\text{color}\rangle_A \otimes |\text{spin, flavor, spatial}\rangle_S. \quad (2)$$

The color part is fully antisymmetric, thus, the spin-flavor-spatial part $|\text{spin, flavor, spatial}\rangle$ should be symmetric.

TABLE II. The spatial wave functions of the $N \leq 3$ shell $\Psi_{NLM_L}^\sigma(\rho, \lambda)$ as the linear combination of $\psi_{n_\rho l_\rho m_\rho}^\rho \psi_{n_\lambda l_\lambda m_\lambda}^\lambda$. In the table, $\{\psi_{02m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{LM_L}$, $\{\psi_{01m_\rho}^\rho \psi_{02m_\lambda}^\lambda\}_{LM_L}$, and $\{\psi_{01m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{LM_L}$ stand for coupling states with quantum numbers L, M_L . They are the linear combination of $\psi_{0l_\rho m_\rho}^\rho \psi_{0l_\lambda m_\lambda}^\lambda$ with the Clebsch-Gordon coefficients.

$\Psi_{000}^s =$	$\psi_{000}^\rho \psi_{000}^\lambda$
$\Psi_{11M_L}^s =$	$\psi_{000}^\rho \psi_{01M_L}^\lambda$
$\Psi_{11M_L}^\rho =$	$\psi_{01M_L}^\rho \psi_{000}^\lambda$
$\Psi_{200}^s =$	$\frac{1}{\sqrt{2}}(\psi_{100}^\rho \psi_{000}^\lambda + \psi_{000}^\rho \psi_{100}^\lambda)$
$\Psi_{200}^\lambda =$	$\frac{1}{\sqrt{2}}(-\psi_{100}^\rho \psi_{000}^\lambda + \psi_{000}^\rho \psi_{100}^\lambda)$
$\Psi_{200}^\rho =$	$\frac{1}{\sqrt{3}}(\psi_{011}^\rho \psi_{01-1}^\lambda - \psi_{010}^\rho \psi_{010}^\lambda + \psi_{01-1}^\rho \psi_{011}^\lambda)$
$\Psi_{21M_L}^a =$	$\frac{1}{\sqrt{2}}(\psi_{01m_\rho}^\rho \psi_{01m_\lambda}^\lambda - \psi_{01m_\lambda}^\rho \psi_{01m_\rho}^\lambda)$
$\Psi_{22M_L}^s =$	$\frac{1}{\sqrt{2}}(\psi_{02M_L}^\rho \psi_{000}^\lambda + \psi_{000}^\rho \psi_{02M_L}^\lambda)$
$\Psi_{22M_L}^\lambda =$	$\frac{1}{\sqrt{2}}(\psi_{02M_L}^\rho \psi_{000}^\lambda - \psi_{000}^\rho \psi_{02M_L}^\lambda)$
$\Psi_{22M_L}^\rho =$	$\{\psi_{01m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{2M_L}$
$\Psi_{33M_L}^s =$	$-\frac{1}{2}\psi_{000}^\rho \psi_{03M_L}^\lambda + \frac{\sqrt{3}}{2}\{\psi_{02m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{3M_L}$
$\Psi_{33M_L}^a =$	$\frac{1}{2}\psi_{03M_L}^\rho \psi_{000}^\lambda - \frac{\sqrt{3}}{2}\{\psi_{01m_\rho}^\rho \psi_{02m_\lambda}^\lambda\}_{3M_L}$
$\Psi_{33M_L}^\lambda =$	$\frac{\sqrt{3}}{2}\psi_{000}^\rho \psi_{03M_L}^\lambda + \frac{1}{2}\{\psi_{02m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{3M_L}$
$\Psi_{33M_L}^\rho =$	$\frac{\sqrt{3}}{2}\psi_{03M_L}^\rho \psi_{000}^\lambda + \frac{1}{2}\{\psi_{01m_\rho}^\rho \psi_{02m_\lambda}^\lambda\}_{3M_L}$
$\Psi_{32M_L}^\lambda =$	$-\{\psi_{02m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{2M_L}$
$\Psi_{32M_L}^\rho =$	$-\{\psi_{01m_\rho}^\rho \psi_{02m_\lambda}^\lambda\}_{2M_L}$
$\Psi_{31M_L}^s =$	$-\frac{\sqrt{15}}{6}\psi_{100}^\rho \psi_{01M_L}^\lambda + \frac{1}{2}\psi_{000}^\rho \psi_{11M_L}^\lambda - \frac{\sqrt{3}}{3}\{\psi_{02m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{1M_L}$
$\Psi_{31M_L}^a =$	$\frac{\sqrt{15}}{6}\psi_{01M_L}^\rho \psi_{100}^\lambda - \frac{1}{2}\psi_{11M_L}^\rho \psi_{000}^\lambda + \frac{\sqrt{3}}{3}\{\psi_{01m_\rho}^\rho \psi_{02m_\lambda}^\lambda\}_{1M_L}$
$\Psi_{31M_L}^\lambda =$	$-\frac{2}{3}\psi_{100}^\rho \psi_{01M_L}^\lambda + \frac{\sqrt{5}}{3}\{\psi_{02m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{1M_L}$
$\Psi_{31M_L}^\rho =$	$-\frac{2}{3}\psi_{01M_L}^\rho \psi_{100}^\lambda + \frac{\sqrt{5}}{3}\{\psi_{01m_\rho}^\rho \psi_{02m_\lambda}^\lambda\}_{1M_L}$
$\Psi_{31M_L}^\lambda =$	$-\frac{\sqrt{5}}{6}\psi_{100}^\rho \psi_{01M_L}^\lambda - \frac{\sqrt{3}}{2}\psi_{000}^\rho \psi_{11M_L}^\lambda - \frac{1}{3}\{\psi_{02m_\rho}^\rho \psi_{01m_\lambda}^\lambda\}_{1M_L}$
$\Psi_{31M_L}^\rho =$	$-\frac{\sqrt{5}}{6}\psi_{01M_L}^\rho \psi_{100}^\lambda - \frac{\sqrt{3}}{2}\psi_{11M_L}^\rho \psi_{000}^\lambda - \frac{1}{3}\{\psi_{01m_\rho}^\rho \psi_{02m_\lambda}^\lambda\}_{1M_L}$

Based on the requirement of the SU(6) $\otimes$ O(3) symmetry, one can obtain the configurations for the spin-flavor-spatial part. Our results for the light-flavor baryons up to the $N = 3$ shell have been given in Table III. For convenience, the different configurations are denoted by ${}^{2S+1}N_3[N_6, L_N^P]n^{2S+1}L_J^P$.

B. Potential model

To calculate the mass of a baryon, we adopt the following semirelativistic Hamiltonian

$$H = \sum_{i=1}^3 \sqrt{p_i^2 + m_i^2} + \sum_{i<j} V(r_{ij}) + C_0, \quad (3)$$

TABLE III. The total wave functions of N and Δ baryons on the $N \leq 3$ shell. The configurations are denoted by $^{2S+1}N_3[N_6, L_N^P]n^{2S+1}L_{J^P}$. $\Psi_{NLM_L}^\sigma$, χ^σ , and ϕ^σ stand for the spatial, spin, and flavor wave functions, respectively. The Clebsch-Gordan series for the spin and orbital angular-momentum addition $|JJ_z\rangle = \sum_{M_L+M_S=J_z} \langle L, M_L, S, M_S | JJ_z \rangle \chi(S, M_S) \Psi_{NLM_L}$.

States	I	L	S	J^P	Wave function
$28[56, 0_0^+]1^2S_{\frac{1}{2}^+}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	$\frac{1}{\sqrt{2}}\Psi_{000}^s(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda)$
$48[70, 1_1^-]1^4P_{J^-}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{11M_L}^\lambda\chi^s\phi^\lambda + \Psi_{11M_L}^\rho\chi^s\phi^\rho)$
$28[70, 1_1^-]1^2P_{J^-}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-, \frac{1}{2}^-$	$-\frac{1}{2}\Psi_{11M_L}^\lambda(\chi^\lambda\phi^\lambda - \chi^\rho\phi^\rho) + \frac{1}{2}\Psi_{11M_L}^\rho(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho)$
$28[56, 0_2^+]2^2S_{\frac{1}{2}^+}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	$\frac{1}{\sqrt{2}}\Psi_{200}^s(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda)$
$48[70, 0_2^+]2^4S_{\frac{3}{2}^+}$	$\frac{1}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}^+$	$\frac{1}{\sqrt{2}}(\Psi_{200}^\lambda\chi^s\phi^\lambda + \Psi_{200}^\rho\chi^s\phi^\rho)$
$28[70, 0_2^+]2^2S_{\frac{1}{2}^+}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	$-\frac{1}{2}\Psi_{200}^\lambda(\chi^\lambda\phi^\lambda - \chi^\rho\phi^\rho) + \frac{1}{2}\Psi_{200}^\rho(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho)$
$28[20, 1_2^+]1^2P_{J^+}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^+, \frac{1}{2}^+$	$\frac{1}{\sqrt{2}}\Psi_{21M_L}^a(\chi^\lambda\phi^\rho - \chi^\rho\phi^\lambda)$
$28[56, 2_2^+]1^2D_{J^+}$	$\frac{1}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+, \frac{3}{2}^+$	$\frac{1}{\sqrt{2}}\Psi_{22M_L}^s(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda)$
$48[70, 2_2^+]1^4D_{J^+}$	$\frac{1}{2}$	2	$\frac{3}{2}$	$\frac{7}{2}^+, \frac{5}{2}^+, \frac{3}{2}^+, \frac{1}{2}^+$	$\frac{1}{\sqrt{2}}(\Psi_{22M_L}^\lambda\chi^s\phi^\lambda + \Psi_{22M_L}^\rho\chi^s\phi^\rho)$
$28[70, 2_2^+]1^2D_{J^+}$	$\frac{1}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+, \frac{3}{2}^+$	$-\frac{1}{2}\Psi_{22M_L}^\lambda(\chi^\lambda\phi^\lambda - \chi^\rho\phi^\rho) + \frac{1}{2}\Psi_{22M_L}^\rho(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho)$
$28[56, 1_3^-]2^2P_{J^-}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}\Psi_{31M_L}^s(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda)$
$48[70, 1_3^-]2^4P_{J^-}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{31M_L}^\lambda\chi^s\phi^\lambda + \Psi_{31M_L}^\rho\chi^s\phi^\rho)$
$48[70, 1_3^-]2^4P_{J^-}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{31M_L}^\lambda\chi^s\phi^\lambda + \Psi_{31M_L}^\rho\chi^s\phi^\rho)$
$28[70, 1_3^-]2^2P_{J^-}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-, \frac{1}{2}^-$	$-\frac{1}{2}\Psi_{31M_L}^\lambda(\chi^\lambda\phi^\lambda - \chi^\rho\phi^\rho) + \frac{1}{2}\Psi_{31M_L}^\rho(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho)$
$28[70, 1_3^-]2^2P_{J^-}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-, \frac{1}{2}^-$	$-\frac{1}{2}\Psi_{31M_L}^\rho(\chi^\lambda\phi^\lambda - \chi^\rho\phi^\rho) + \frac{1}{2}\Psi_{31M_L}^\lambda(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho)$
$28[20, 1_3^-]2^2P_{J^-}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}\Psi_{31M_L}^a(\chi^\lambda\phi^\rho - \chi^\rho\phi^\lambda)$
$48[70, 2_3^-]1^4D_{J^-}$	$\frac{1}{2}$	2	$\frac{3}{2}$	$\frac{7}{2}^-, \frac{5}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{32M_L}^\lambda\chi^s\phi^\lambda + \Psi_{32M_L}^\rho\chi^s\phi^\rho)$
$28[70, 2_3^-]1^2D_{J^-}$	$\frac{1}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^-, \frac{3}{2}^-$	$-\frac{1}{2}\Psi_{32M_L}^\lambda(\chi^\lambda\phi^\lambda - \chi^\rho\phi^\rho) + \frac{1}{2}\Psi_{32M_L}^\rho(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho)$
$28[56, 3_3^-]1^2F_{J^-}$	$\frac{1}{2}$	3	$\frac{1}{2}$	$\frac{7}{2}^-, \frac{5}{2}^-$	$\frac{1}{\sqrt{2}}\Psi_{33M_L}^s(\chi^\rho\phi^\rho + \chi^\lambda\phi^\lambda)$
$48[70, 3_3^-]1^4F_{J^-}$	$\frac{1}{2}$	3	$\frac{3}{2}$	$\frac{9}{2}^-, \frac{7}{2}^-, \frac{5}{2}^-, \frac{3}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{33M_L}^\lambda\chi^s\phi^\lambda + \Psi_{33M_L}^\rho\chi^s\phi^\rho)$
$28[70, 3_3^-]1^2F_{J^-}$	$\frac{1}{2}$	3	$\frac{1}{2}$	$\frac{7}{2}^-, \frac{5}{2}^-$	$-\frac{1}{2}\Psi_{33M_L}^\lambda(\chi^\lambda\phi^\lambda - \chi^\rho\phi^\rho) + \frac{1}{2}\Psi_{33M_L}^\rho(\chi^\rho\phi^\lambda + \chi^\lambda\phi^\rho)$
$28[20, 3_3^-]1^2F_{J^-}$	$\frac{1}{2}$	3	$\frac{1}{2}$	$\frac{7}{2}^-, \frac{5}{2}^-$	$\frac{1}{\sqrt{2}}\Psi_{33M_L}^a(\chi^\lambda\phi^\rho - \chi^\rho\phi^\lambda)$
$410[56, 0_0^+]1^4S_{\frac{3}{2}^+}$	$\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}^+$	$\Psi_{000}^s\chi^s\phi^s$
$210[70, 1_1^-]1^2P_{J^-}$	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{11M_L}^\lambda\chi^\lambda + \Psi_{11M_L}^\rho\chi^\rho)\phi^s$
$410[56, 0_2^+]2^4S_{\frac{3}{2}^+}$	$\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}^+$	$\Psi_{200}^s\chi^s\phi^s$
$210[70, 0_2^+]2^2S_{\frac{3}{2}^+}$	$\frac{3}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	$\frac{1}{\sqrt{2}}(\Psi_{200}^\lambda\chi^\lambda + \Psi_{200}^\rho\chi^\rho)\phi^s$
$410[56, 2_2^+]1^4D_{J^+}$	$\frac{3}{2}$	2	$\frac{3}{2}$	$\frac{7}{2}^+, \frac{5}{2}^+, \frac{3}{2}^+, \frac{1}{2}^+$	$\Psi_{22M_L}^s\chi^s\phi^s$
$210[70, 2_2^+]1^2D_{J^+}$	$\frac{3}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+, \frac{3}{2}^+$	$\frac{1}{\sqrt{2}}(\Psi_{22M_L}^\lambda\chi^\lambda + \Psi_{22M_L}^\rho\chi^\rho)\phi^s$
$410[56, 1_3^-]2^4P_{J^-}$	$\frac{3}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-$	$\Psi_{31M_L}^s\chi^s\phi^s$
$210[70, 1_3^-]2^2P_{J^-}$	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{31M_L}^\lambda\chi^\lambda + \Psi_{31M_L}^\rho\chi^\rho)\phi^s$
$210[70, 1_3^-]2^2P_{J^-}$	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-, \frac{1}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{31M_L}^\lambda\chi^\lambda + \Psi_{31M_L}^\rho\chi^\rho)\phi^s$
$210[70, 2_3^-]1^2D_{J^-}$	$\frac{3}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^-, \frac{3}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{32M_L}^\lambda\chi^\lambda + \Psi_{32M_L}^\rho\chi^\rho)\phi^s$
$410[56, 3_3^-]1^4F_{J^-}$	$\frac{3}{2}$	3	$\frac{3}{2}$	$\frac{9}{2}^-, \frac{7}{2}^-, \frac{5}{2}^-, \frac{3}{2}^-$	$\Psi_{33M_L}^s\chi^s\phi^s$
$210[70, 3_3^-]1^2F_{J^-}$	$\frac{3}{2}$	3	$\frac{1}{2}$	$\frac{7}{2}^-, \frac{5}{2}^-$	$\frac{1}{\sqrt{2}}(\Psi_{33M_L}^\lambda\chi^\lambda + \Psi_{33M_L}^\rho\chi^\rho)\phi^s$

where $\sqrt{\mathbf{p}_i^2 + m_i^2}$ is the kinetic energy of the i th constituent quark with mass m_i and momentum $\mathbf{p}_i$, $V(r_{ij})$ is the effective potential between the i th and j th quarks with a distance $r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j|$, and C_0 is the zero point energy. For

convenience, in the calculations, one needs to express the single-partial coordinates $\mathbf{r}_i$ ($i = 1, 2, 3$) with the Jacobi coordinates, which has been defined by Eq. (1). By combining the Jacobi coordinates, one can obtain

$$\begin{aligned}
\mathbf{p}_1 &= \frac{m_1}{m_{\text{tot}}} \mathbf{P}_{\text{c.m.}} + \frac{1}{\sqrt{2}} \mathbf{p}_\rho + \frac{1}{\sqrt{6}} \mathbf{p}_\lambda, \\
\mathbf{p}_2 &= \frac{m_2}{m_{\text{tot}}} \mathbf{P}_{\text{c.m.}} - \frac{1}{\sqrt{2}} \mathbf{p}_\rho + \frac{1}{\sqrt{6}} \mathbf{p}_\lambda, \\
\mathbf{p}_3 &= \frac{m_3}{m_{\text{tot}}} \mathbf{P}_{\text{c.m.}} - \sqrt{\frac{2}{3}} \mathbf{p}_\lambda,
\end{aligned} \quad (4)$$

where $\mathbf{p}_\rho$ and $\mathbf{p}_\lambda$ represent the relative momenta corresponding to the Jacobi coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$, respectively, $m_{\text{tot}} = m_1 + m_2 + m_3$, while $\mathbf{P}_{\text{c.m.}}$ is the momentum of the center of mass. The contribution of the center of mass motion in the kinetic energy can be easily removed by using the relations given in Eq. (4).

The effective potential can be decomposed into the spin-independent and spin-dependent parts,

$$V(r_{ij}) = V^{\text{Corn}}(r_{ij}) + V^{sd}(r_{ij}). \quad (5)$$

The spin-independent part $V^{\text{Corn}}(r_{ij})$ has adopted the well-known Cornell form [104], i.e.,

$$V^{\text{Corn}}(r_{ij}) = \frac{b}{2} r_{ij} - \frac{2\alpha_S}{3 r_{ij}}, \quad (6)$$

which includes the linear confinement potential V^{Coul} and the Coulomb-like potential V^{Conf} derived from the OGE model [14,105], where, the slop parameter b represents the strength of the confinement potential, and α_S denotes the strong coupling constant. While, the spin-dependent potentials in the OGE model can be decomposed into

$$V_G^{sd}(r_{ij}) = V_G^{SS}(r_{ij}) + V_G^{LS}(r_{ij}) + V_G^T(r_{ij}), \quad (7)$$

where $V_G^{SS}(r_{ij})$, $V_G^T(r_{ij})$, and $V_G^{LS}(r_{ij})$ stand for the spin-spin, tensor, and the spin-orbit potentials, respectively. The spin-spin and tensor potentials are given by

$$V_G^{SS}(r_{ij}) = \frac{2\alpha_S}{3} \left\{ \frac{\pi}{2} \cdot \frac{\sigma_{ij}^3 e^{-\sigma_{ij}^2 r_{ij}^2}}{\pi^{3/2}} \cdot \frac{16}{3\tilde{m}_i \tilde{m}_j} (\mathbf{S}_i \cdot \mathbf{S}_j) \right\}, \quad (8)$$

$$V_G^T(r_{ij}) = \frac{2\alpha_S}{3} \frac{1}{\tilde{m}_i \tilde{m}_j r_{ij}^3} \left\{ \frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right\}. \quad (9)$$

The spin-orbit potential is given by [14]

$$V_G^{LS}(r_{ij}) = V_{ij}^{so(\nu)} + V_{ij}^{so(s)}, \quad (10)$$

which contains a color-magnetic piece

$$\begin{aligned}
V_{ij}^{so(\nu)} &= \frac{1}{r_{ij}} \frac{dV^{\text{Coul}}(r_{ij})}{dr_{ij}} \left(\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{S}_i}{2\tilde{m}_i^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{S}_j}{2\tilde{m}_j^2} \right. \\
&\quad \left. - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{S}_i - \mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{S}_j}{\tilde{m}_i \tilde{m}_j} \right),
\end{aligned} \quad (11)$$

and a Thomas-precession piece

$$V_{ij}^{so(s)} = -\frac{1}{r_{ij}} \frac{dV^{\text{Conf}}(r_{ij})}{dr_{ij}} \left(\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{S}_i}{2\tilde{m}_i^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{S}_j}{2\tilde{m}_j^2} \right). \quad (12)$$

In the spin-dependent potentials, the $\mathbf{S}_i$ and $\mathbf{p}_i$ are the spin and momentum operators of the i th quark, respectively. It should be mentioned that, in the spin-dependent potentials arising from the OGE, the light quark mass m_i has been replaced with an effective mass $\tilde{m}_i$ to include some relativistic corrections as adopted in the literature [89,106].

For a hadron system containing light quarks, the quark-quark interactions arising from the Goldstone boson exchanges may play important roles due to the spontaneous breaking of chiral symmetry. To better describe the mass spectrum of light Δ and N baryons, we include the chiral potentials by the exchanges of the light pseudoscalar mesons, π and η , and the scalar σ meson.

In the chiral quark model, the vertex for the tree-level quark-pseudoscalar-meson interaction is described by [51,57–60]

$$\mathcal{L}_{ps} = \frac{\delta}{\sqrt{2}f_{\mathbb{M}}} \bar{\psi}_j \gamma_\mu \gamma_5 \psi_j \hat{I} \cdot \partial^\mu \vec{\phi}_{\mathbb{M}}, \quad (13)$$

while the vertex for the quark- σ -meson interaction is described by [94]

$$\mathcal{L}_\sigma = -g_\sigma \bar{\psi}_j \sigma \psi_j. \quad (14)$$

In the above effective Lagrangians, ψ_j represents the j th light quark field in a hadron, $\phi_{\mathbb{M}}$ and σ represent the pseudoscalar meson fields (π, K, η, η') and scalar meson field σ , respectively. $\hat{I}$ is a flavor operator; its matrix representation can be related to the Gell-Mann matrices. $f_{\mathbb{M}}$ stands for the pseudoscalar meson decay constant. δ is a global parameter accounting for the strength of the quark-pseudoscalar-meson couplings, while g_σ is a coupling constant for the quark- σ -meson interaction. It should be mentioned that a suppressed factor,

$$F(q^2) = \sqrt{\frac{\Lambda^2}{\Lambda^2 + q^2}}, \quad (15)$$

is introduced to correct the interaction vertices given by Eqs. (13) and (14), where Λ is a cutoff parameter that is taken of the order of the chiral symmetry breaking scale, while $\mathbf{q}$ is the three momentum of the meson.

By using the effective chiral Lagrangians given by Eqs. (13) and (14) and performing a nonrelativistic reduction, one can obtain the chiral potentials arising from the pseudoscalar and scalar meson exchanges. The central part of these one-meson (π , K , η , σ) exchange potentials is given by [96]

$$V_{OBE}^C(r_{ij}) = V_\pi^C(r_{ij}) + V_K^C(r_{ij}) + V_\eta^C(r_{ij}) + V_\sigma^C(r_{ij}), \quad (16)$$

with

$$V_\pi^C(r_{ij}) = \frac{g_\pi^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_\pi^2} \frac{m_\pi^3}{3m_i m_j} \left[Y(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3} Y(\Lambda r_{ij}) \right] \cdot (\mathbf{S}_i \cdot \mathbf{S}_j) \sum_{a=1}^3 (\lambda_i^a \cdot \lambda_j^a), \quad (17)$$

$$V_K^C(r_{ij}) = \frac{g_K^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_K^2} \frac{m_K^3}{3m_i m_j} \left[Y(m_K r_{ij}) - \frac{\Lambda^3}{m_K^3} Y(\Lambda r_{ij}) \right] \cdot (\mathbf{S}_i \cdot \mathbf{S}_j) \sum_{a=4}^7 (\lambda_i^a \cdot \lambda_j^a), \quad (18)$$

$$V_\eta^C(r_{ij}) = \frac{g_\eta^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_\eta^2} \frac{m_\eta^3}{3m_i m_j} \left[Y(m_\eta r_{ij}) - \frac{\Lambda^3}{m_\eta^3} Y(\Lambda r_{ij}) \right] \cdot (\mathbf{S}_i \cdot \mathbf{S}_j) [(\lambda_i^8 \cdot \lambda_j^8) \cos \theta_p - (\lambda_i^0 \cdot \lambda_j^0) \sin \theta_p], \quad (19)$$

$$V_\sigma^C(r_{ij}) = -\frac{g_\sigma^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_\sigma^2} m_\sigma \left[Y(m_\sigma r_{ij}) - \frac{\Lambda}{m_\sigma} Y(\Lambda r_{ij}) \right]. \quad (20)$$

The noncentral tensor potentials are contributed by the π , K , and η exchanges, which are given by

$$V_{OBE}^T(r_{ij}) = V_\pi^T(r_{ij}) + V_K^T(r_{ij}) + V_\eta^T(r_{ij}), \quad (21)$$

with

$$V_\pi^T(r_{ij}) = \frac{g_\pi^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_\pi^2} \frac{m_\pi^3}{3m_i m_j} \left[H(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3} H(\Lambda r_{ij}) \right] \cdot S_{ij} \sum_{a=1}^3 (\lambda_i^a \cdot \lambda_j^a), \quad (22)$$

$$V_K^T(r_{ij}) = \frac{g_K^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_K^2} \frac{m_K^3}{3m_i m_j} \left[H(m_K r_{ij}) - \frac{\Lambda^3}{m_K^3} H(\Lambda r_{ij}) \right] \cdot S_{ij} \sum_{a=4}^7 (\lambda_i^a \cdot \lambda_j^a), \quad (23)$$

and

$$V_\eta^T(r_{ij}) = \frac{g_\eta^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_\eta^2} \frac{m_\eta^3}{3m_i m_j} \left[H(m_\eta r_{ij}) - \frac{\Lambda^3}{m_\eta^3} H(\Lambda r_{ij}) \right] \cdot S_{ij} [(\lambda_i^8 \cdot \lambda_j^8) \cos \theta_p - (\lambda_i^0 \cdot \lambda_j^0) \sin \theta_p]. \quad (24)$$

While the noncentral spin-orbit potentials are contributed by the σ exchange, which are given by

$$V_\sigma^{LS}(r_{ij}) = -\frac{g_\sigma^2}{4\pi} \frac{\Lambda^2 m_\sigma^3}{\Lambda^2 - m_\sigma^2} \left[G(m_\sigma r_{ij}) - \frac{\Lambda^3}{m_\sigma^3} G(\Lambda r_{ij}) \right] \cdot \left(\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{S}_i}{2m_i^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{S}_j}{2m_j^2} \right). \quad (25)$$

In the above equations, λ^a ($a = 1-8$) stands for the SU(3) Gell-Mann matrices in the flavor space, $Y(x) = \frac{1}{x} e^{-x}$, $H(x) = (1 + 3/x + 3/x^2)Y(x)$, $G(x) = (1/x + 1/x^2)Y(x)$, and $S_{ij} = 3(\mathbf{S}_i \cdot \hat{\mathbf{r}}_{ij})(\mathbf{S}_j \cdot \hat{\mathbf{r}}_{ij}) - \mathbf{S}_i \cdot \mathbf{S}_j$. m_π , m_K , m_η , and m_σ stand for the masses for the π , K , η , and σ mesons, respectively. In Eq. (24), θ_p is the mixing angle for the η meson in the SU(3) wave function basis. In Eqs. (17)–(19), the coupling constants $g_{\mathbb{M}}$ ($\mathbb{M} = \pi, K, \eta$) for the pseudoscalar mesons are defined by $g_{\mathbb{M}} = \delta \frac{m_u}{f_{\mathbb{M}}}$.

1. Numerical method

To calculate the matrix elements in coordinate space, we follow the same method adopted in our previous work [84]. The ρ - and λ -mode spatial wave functions $\psi_{n_\rho l_\rho m_\rho}$ and $\psi_{n_\lambda l_\lambda m_\lambda}$ can be expressed by

$$\psi_{n_\xi l_\xi m_\xi}(\xi) = R_{n_\xi l_\xi}(\xi) Y_{l_\xi m_\xi}(\xi), \quad (26)$$

where $Y_{l_\xi m_\xi}(\xi)$ ($\xi = \rho, \lambda$) is the spherical harmonic function. The radial part, $R_{n_\xi l_\xi}(\xi)$, is expanded with a series of harmonic oscillator functions

$$R_{n_\xi l_\xi}(\xi) = \sum_{\ell=1}^n C_{\xi \ell} \Phi_{n_\xi l_\xi}(\alpha_{\xi \ell} \xi), \quad (27)$$

where

$$\Phi_{n_\xi l_\xi}(\alpha_{\xi \ell} \xi) = \alpha_{\xi \ell}^{\frac{3}{2}} \left[\frac{2^{l_\xi+2-n_\xi} (2l_\xi + 2n_\xi + 1)!!}{\sqrt{\pi} n_\xi! [(2l_\xi + 1)!!]^2} \right]^{\frac{1}{2}} \times (\alpha_{\xi \ell} \xi)^{l_\xi} e^{-\frac{1}{2} \alpha_{\xi \ell}^2 \xi^2} F\left(-n_\xi, l_\xi + \frac{3}{2}, \alpha_{\xi \ell}^2 \xi^2\right). \quad (28)$$

The $F(-n_\xi, l_\xi + \frac{3}{2}, \alpha_{\xi \ell}^2 \xi^2)$ in Eq. (28) is the confluent hypergeometric function, the parameter $\alpha_{\xi \ell}$ can be related to the harmonic oscillator frequency $\omega_{\xi \ell}$ with $\alpha_{\xi \ell} \equiv 1/d_{\xi \ell} = \sqrt{M_\xi \omega_{\xi \ell}}$. The reduced masses $M_{\rho, \lambda}$ are defined by $M_\rho \equiv \frac{2m_1 m_2}{m_1 + m_2}$, $M_\lambda \equiv \frac{3(m_1 + m_2)m_3}{2(m_1 + m_2 + m_3)}$. On the other hand, the

harmonic oscillator frequencies $\omega_{\xi\ell}$ can be related to the harmonic oscillator stiffness factor K_ℓ with $\omega_{\xi\ell} = \sqrt{3K_\ell/M_\xi}$. For a baryon system containing light u/d quarks, one has $d_{\rho\ell} = d_{\lambda\ell} = d_\ell = (3m_u K_\ell)^{-1/4}$. With this relation, the spatial wave function can be simply expanded as

$$\Psi_{NLM_L}^\sigma(\boldsymbol{\rho}, \boldsymbol{\lambda}) = \sum_{\ell}^n C_\ell \Psi_{NLM_L}^\sigma(d_\ell, \boldsymbol{\rho}, \boldsymbol{\lambda}). \quad (29)$$

Then, the Schrödinger equation can be solved by dealing with the generalized eigenvalue problem,

$$\sum_{\ell'=1}^n (H_{\ell\ell'} - EN_{\ell\ell'}) C_{\ell'} = 0, \quad (30)$$

where $H_{\ell\ell'} \equiv \langle \zeta(d'_\ell) | H | \zeta(d_\ell) \rangle$ and $N_{\ell\ell'} \equiv \langle \zeta(d'_\ell) | \zeta(d_\ell) \rangle$, while the $\zeta(d_\ell)$ function is given by

$$\zeta(d_\ell) = \sum_{M_L + M_S = M} \langle LM_L, SM_S | JM_J \rangle \Psi_{NLM_L}^\sigma(d_\ell, \boldsymbol{\rho}, \boldsymbol{\lambda}) \chi_{M_S}^\sigma \phi^\sigma. \quad (31)$$

The variational method is applied to solve the few-body problems. In the calculations, following the method suggested in Ref. [107], the variational parameter d_ℓ is selected to form a geometric progression,

$$d_\ell = d_1 a^{\ell-1} (\ell = 1, \dots, n), \quad (32)$$

where n represents the number of Gaussian basis functions, and a is the ratio coefficient. There are three parameters $\{d_1, d_n, n\}$ to be determined with the variation method. It is found that when taking $d_1 = 0.1$ fm, $d_n = 2$ fm, and $n = 10$, we can obtain stable solutions for the Δ and N baryons.

Finally, it should be mentioned that the spin-dependent terms can cause mixing between the different configurations with the same J^P quantum numbers. Thus, in this work, we also consider the configuration mixing induced by the spin-spin, spin-orbit, and tensor potentials.

C. Strong decay

In this work, we also study the Okubo-Zweig-Iizuka (OZI)-allowed two-body strong decays of the baryon resonances by emitting a single pseudoscalar meson (π , K , η , or η'). The strong decay amplitudes can be evaluated with the chiral quark model [51]. Within the same framework, the chiral potentials have been derived for our mass spectrum study. In this model, the low-energy quark-pseudoscalar-meson interactions are described by the effective Lagrangian given by Eq. (13).

To match the nonrelativistic wave functions of the initial and final baryon states, it is crucial to adopt the nonrelativistic form of the effective Lagrangian in the calculations. Performing a nonrelativistic reduction, at the tree level, one has [90]

$$H_I = \mathcal{H}^{NR} + \mathcal{H}^{RC}, \quad (33)$$

with

$$\mathcal{H}^{NR} = g \sum_j \left(\mathcal{G} \boldsymbol{\sigma}_j \cdot \mathbf{q} + \frac{\omega_{\mathbb{M}}}{2\mu_q} \boldsymbol{\sigma}_j \cdot \mathbf{p}_j \right) F(\mathbf{q}^2) I_j \varphi_{\mathbb{M}}, \quad (34)$$

and

$$\mathcal{H}^{RC} = -\frac{g}{32\mu_q^2} \sum_j [m_{\mathbb{M}}^2 (\boldsymbol{\sigma}_j \cdot \mathbf{q}) + 2\boldsymbol{\sigma}_j \cdot (\mathbf{q} - 2\mathbf{p}_j) \times (\mathbf{q} \times \mathbf{p}_j)] F(\mathbf{q}^2) I_j \varphi_{\mathbb{M}}. \quad (35)$$

In the above equations, $\boldsymbol{\sigma}_j$ and $\mathbf{p}_j$ are the spin operator and internal momentum operator of the j th light quark within a hadron. $\varphi_{\mathbb{M}} = e^{-iq \cdot \mathbf{r}_j}$ is the plane wave part of the emitted light meson. The factors g and $\mathcal{G}$ are defined by $g = \delta \sqrt{(E_i + M_i)(E_f + M_f)} / (\sqrt{2} f_{\mathbb{M}})$ and $\mathcal{G} = -(\frac{\omega_{\mathbb{M}}}{E_f + M_f} + 1 + \frac{\omega_{\mathbb{M}}}{2m'_j})$, in which (E_i, M_i) and (E_f, M_f) are the energy and mass of the initial baryon and final baryon, respectively. $\omega_{\mathbb{M}}$, $\mathbf{q}$, and $m_{\mathbb{M}}$ are the energy, three momentum, and mass of the final state pseudoscalar meson. The reduced mass μ_q is expressed as $1/\mu_q = 1/m_j + 1/m'_j$ with m_j and m'_j for the masses of the j th quark in the initial and final baryons, respectively. $F(\mathbf{q}^2)$ as a factor for suppressing the unphysical contributions in the high momentum region has been given in Eq. (15). The flavor operator $\hat{I}_j$ for each pseudoscalar meson is given by [52,59,85]

$$\hat{I}_j = \begin{cases} a_j^\dagger(s) a_j(u) & \text{for } K^+, \\ a_j^\dagger(u) a_j(s) & \text{for } K^-, \\ a_j^\dagger(s) a_j(d) & \text{for } K^0, \\ a_j^\dagger(d) a_j(s) & \text{for } \bar{K}^0, \\ a_j^\dagger(u) a_j(d) & \text{for } \pi^-, \\ a_j^\dagger(d) a_j(u) & \text{for } \pi^+, \\ \frac{1}{\sqrt{2}} [a_j^\dagger(u) a_j(u) - a_j^\dagger(d) a_j(d)] & \text{for } \pi^0, \\ \frac{1}{\sqrt{2}} [a_j^\dagger(u) a_j(u) + a_j^\dagger(d) a_j(d)] \cos \phi_P, & \\ -a_j^\dagger(s) a_j(s) \sin \phi_P & \text{for } \eta, \\ \frac{1}{\sqrt{2}} [a_j^\dagger(u) a_j(u) + a_j^\dagger(d) a_j(d)] \sin \phi_P, & \\ +a_j^\dagger(s) a_j(s) \cos \phi_P & \text{for } \eta', \end{cases} \quad (36)$$

where $a_j^\dagger(u, d, s)$ and $a_j(u, d, s)$ are the creation and annihilation operators for the u , d , and s quarks, and ϕ_p is the mixing angle for the η and η' mesons in the flavor basis, which can be related to the mixing angle θ_p appearing in Eq. (24) by $\phi_p = \theta_p + 54.7^\circ$. In this work, we take $\phi_p = 41.2^\circ$ as that determined in Ref. [65]. It should be mentioned that, in this work, with the flavor bases

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (37)$$

the matrix representation of the flavor operator $\hat{I}$ for each pseudoscalar meson can be related to the Gell-Mann matrices with $I_{\mathbb{M}} = \lambda_{\mathbb{M}}/\sqrt{2}$, where $\lambda_{\pi^\pm} = \frac{1}{\sqrt{2}}(\lambda^1 \mp i\lambda^2)$, $\lambda_{\pi^0} = \lambda^3$, $\lambda_{K^\pm} = \frac{1}{\sqrt{2}}(\lambda^4 \mp i\lambda^5)$, $\lambda_{K^0} = \frac{1}{\sqrt{2}}(\lambda^6 - i\lambda^7)$, $\lambda_{\bar{K}^0} = \frac{1}{\sqrt{2}}(\lambda^6 + i\lambda^7)$, $\lambda_\eta = \cos\phi_p(\frac{2}{3}\lambda^0 + \frac{1}{\sqrt{3}}\lambda^8) - \sin\phi_p(\frac{\sqrt{2}}{3}\lambda^0 - \sqrt{\frac{2}{3}}\lambda^8)$, and $\lambda_{\eta'} = \sin\phi_p(\frac{2}{3}\lambda^0 + \frac{1}{\sqrt{3}}\lambda^8) + \cos\phi_p(\frac{\sqrt{2}}{3}\lambda^0 - \sqrt{\frac{2}{3}}\lambda^8)$.

The chiral quark model has been widely adopted to study the strong decays of excited hadrons [52,74–91] and pseudoscalar meson production processes [57–73]. In these works, only the $\mathcal{H}^{NR}$ term is kept in the calculations, while the so-called relativistic correction term $\mathcal{H}_I^{RC}$ has been overlooked. Recently, the $\mathcal{H}_I^{RC}$ term was first included in the investigations of the strong decays of heavy baryon resonances and multistrangeness baryon resonances [108,109]. It is found that the agreement with the data is significantly improved, in particular, the decay widths of the Roper-like states are greatly increased by 1 order of magnitude as compared to the results without the relativistic correction. Lately, the long-standing puzzle of the broad width for the radially excited heavy-light mesons, such as $D_0(2550)/D_{s0}(2590)$ and $D_1^*(2600)/D_{s1}^*(2700)$, was overcome naturally by including the $\mathcal{H}_I^{RC}$ term in Ref. [90].

Within the chiral quark model, the two-body OZI-allowed strong decay amplitude for the $\mathcal{B} \rightarrow \mathcal{B}'\mathbb{M}$ process can be worked out by

$$\mathcal{M}[\mathcal{B} \rightarrow \mathcal{B}'\mathbb{M}] = \langle \mathcal{B}' | H_I | \mathcal{B} \rangle, \quad (38)$$

where $\mathcal{B}$ and $\mathcal{B}'$ stand for the initial and final baryon states, respectively, and $\mathbb{M}$ is the emitting pseudoscalar meson. With derived decay amplitudes from Eq. (38), the partial decay width for the $\mathcal{B} \rightarrow \mathcal{B}'\mathbb{M}$ process can be obtained with

$$\Gamma = \frac{1}{8\pi} \frac{|\mathbf{q}|}{M_i^2} \frac{1}{2J_i + 1} \sum_{J_{iz}J_{fz}} |\mathcal{M}_{J_{iz}J_{fz}}|^2, \quad (39)$$

TABLE IV. Quark model parameters used in this work.

m_u (GeV)	$\tilde{m}_u$ (GeV)	b (GeV ²)	σ (GeV)	C_0 (GeV)	r_c (fm)
0.42	0.62	0.135	0.344	-0.829	0.23
α_s	δ	f_π (GeV)	$f_{K/\eta}$ (GeV)	Λ (GeV)	r'_c (fm)
0.40	0.576	0.093	0.113	0.66	0.70

where J_i is the total angular-momentum quantum number of the initial baryon, J_{iz} and J_{fz} represent the third components of the total angular momenta of the initial and final baryons, respectively.

D. Model parameters

The model parameters adopted in this work have been presented in Table IV. The decay constants for π , K , and η mesons are fixed with $f_\pi = 93$ and $f_{K/\eta} = 113$ MeV. The cutoff parameter Λ in the factor $F(q^2) = \sqrt{\frac{\Lambda^2}{\Lambda^2 + q^2}}$ is determined to be $\Lambda = 0.66$ GeV to consist with the behavior of the exponential form $F(q^2) = e^{-\frac{q^2}{2\Lambda^2}}$ with $\Lambda = 0.78$ GeV (see Fig. 1), which is determined from our recent study of the heavy-light meson spectrum [90]. The cut-off parameter $\Lambda = 0.66$ GeV adopted in the present work is slightly smaller than ~ 0.83 GeV often adopted in the literature, e.g., Refs. [94,96,97]. The typical value of the constituent quark mass for the u/d quark is in the range of 400 ± 100 MeV; in this work, it is fixed with $m_{u/d} = 420$ MeV, while the effective quark mass in the one-gluon-exchange potentials is fixed with $\tilde{m}_u = 0.62$ GeV as that adopted in our previous work [89]. The dimensionless parameter δ accounting for the strength of the quark-meson couplings

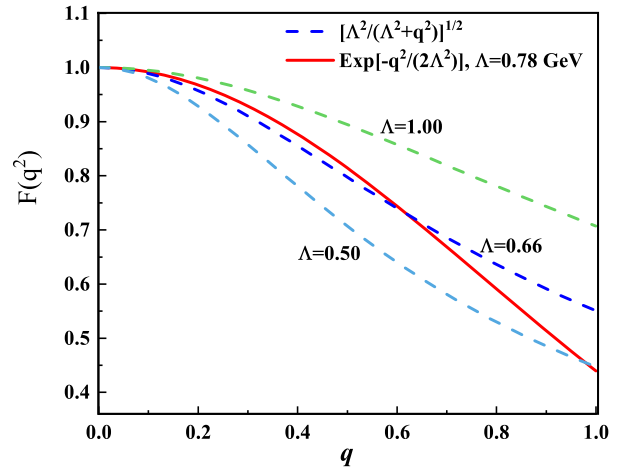


FIG. 1. The shape factor in exponential form (solid line) and fractional form (dashed line) as a function of the momentum q .

is taken as the same value $\delta = 0.576$ as that determined by the strong decays of light strange baryon resonances in the previous studies of our group [82–84]. With the relation $g_{\mathbb{M}} = \delta \frac{m_u}{f_{\mathbb{M}}}$, we obtain $g_{\pi} \simeq 2.6$ and $g_K = g_{\eta} \simeq 2.14$, which are comparable with the chiral coupling constant $g_{ch} \simeq 2.6$ – 2.7 determined from the πNN coupling in the literature, e.g., Refs. [24,94,96–99,101,102]. In this work, we take $g_{\sigma} = g_{\pi}$ as an approximation.

The other four parameters $\{b, C_0, \alpha_S, \sigma\}$ in the potential model are determined by an overall description of the masses of the eight well-established states $N(938)$, $\Delta(1232)$, $\Delta(1600)3/2^+$, $N(1520)1/2^-$, $N(1535)1/2^-$, $N(1650)1/2^-$, $\Delta(1620)1/2^-$, and $N(1675)5/2^-$ listed in the Review of Particle Physics (RPP) by the Particle Data Group (PDG) [110]. The determined value of the slope parameter, $b = 0.135 \text{ GeV}^2$, is compatible with $b = 0.10$ – 0.14 GeV^2 adopted in the previous works of our group [84,91,111–113]. The parameters $\alpha_S = 0.40$ and $\sigma = 0.344 \text{ GeV}$ are mainly constrained by the mass splitting between $N(938)1/2^+$ and $\Delta(1232)3/2^+$, since this splitting is sensitive to the OGE spin-spin potentials.

It should be noted that we cannot obtain stable solutions for some states due to the divergent behavior of $1/r^3$ in the OGE spin-orbit and tensor potentials, if we do not treat them as perturbative terms. In order to overcome this problem, following the method adopted in the previous works [113–115], we introduce a cutoff distance r_c in the calculation. In a small range $(0, r_c)$, we let $1/r^3 = 1/r_c^3$ in the spin-orbit and tensor potentials. It is found that the mass of the nucleon excitation configuration $48[70, 1_1]1^4P_{1/2^-}$ is more sensitive to the r_c due to its relatively larger factor $\langle \mathbf{L}_{ij} \cdot (\mathbf{S}_i + \mathbf{S}_j) \rangle$ than the other states. Thus, the cutoff parameter $r_c = 0.23 \text{ fm}$ is determined by fitting the mass of $48[70, 1_1]1^4P_{1/2^-}$ obtained with the method of perturbation. In this method, we treat the spin-orbit and tensor potentials as perturbative terms. First, by neglecting the contributions of these terms we obtain the mass m_0 and spatial wave function, then by using the wave function we further calculate the mass correction term Δm from the perturbative terms; finally, we obtained the whole mass $M = m_0 + \Delta m$ for our fitting.

The OBE spin-orbit and tensor potentials are less important for our understanding the mass spectrum, thus, they are often neglected in the literature. However, they can affect our determining the configuration mixing between $N(1535)1/2^-$ and $N(1650)1/2^-$. Note that the singular behavior also exists in the OBE spin-orbit and tensor potentials. To overcome this problem, similar to the OGE case, in a small range $(0, r'_c)$, we let $1/r^3 = 1/r_c^3$ and $1/r^2 = 1/r_c^2$ in OBE spin-orbit and tensor potentials. Taking $r'_c \simeq 0.70 \text{ fm}$, the mixing angle between $N(1535)$ and $N(1650)$ is determined to be $\theta_S \simeq 25^\circ$, which is

consistent with that extracted from the π and η production processes [66,67]. In this case, the contributions from the $1/r^3$ and $1/r^2$ terms in the one-pion-exchange tensor potential are strongly suppressed, the role of the potential is governed by the $1/r$ term.

In the calculation of the strong decays, we adopt the numerical wave functions for the N^* and Δ^* baryons to calculate the transition amplitudes. The details of the wave functions for the ground states $N(938)$ and $\Delta(1232)$ are crucial for understanding the strong decay properties of the N^* and Δ^* baryons because all of these excited states should decay into $N(938)$ and/or $\Delta(1232)$. Considering the uncertainty of the wave functions of the $N(938)$ and $\Delta(1232)$, we adjust their size appropriately to more reasonably describe the strong decay properties of the well-established states. For the convenience of adjustment, the wave functions of the ground states adopt a simple harmonic oscillator form with an effective size parameter α . The size parameters for $N(938)$ and $\Delta(1232)$ are determined to be $\alpha = 0.49$ and 0.40 GeV , respectively. The high N^* and Δ^* baryons can decay into the Λ , Σ , and/or Σ^* . Their wave functions adopt the simple harmonic oscillator form as well. The harmonic oscillator size parameter α_ρ for the ρ oscillator in the spatial wave function is taken as $\alpha_\rho = 0.4 \text{ GeV}$, while the parameter α_λ for the λ oscillator is related to α_ρ as $\alpha_\lambda = \sqrt[4]{3m_u/(2m_s + m_u)}\alpha_\rho$ [82].

In the calculations of the strong decays, we also need the masses of initial and final hadrons. For the well-established states, the masses adopt the average values from the PDG [110]. While for the unestablished states, the masses are taken from our quark model predictions.

III. RESULTS AND DISCUSSION

The obtained mass spectra for the nucleon and Δ baryons up to the $N = 2$ shell compared with the data and some other works are given in Tables V and VI, respectively. For clarity, the mass spectra for the nucleon and Δ baryons compared with the data are plotted in Figs. 2 and 3, respectively. To better know about the dynamical mechanism of the strong interactions in the nucleon and Δ baryon systems, we also analyze the contributions of each part of the Hamiltonian to their masses. The results are given in Tables VII and VIII. Furthermore, by combining the spectra obtained within the potential model, the strong decay properties are studied within the chiral quark model which has been adopted to derive the chiral potentials for the mass spectrum calculations. The decay properties are given in Tables IX, X, XI. From the tables, one can see that the mass spectra and decay properties for the well-established nucleon and Δ baryons can be reasonably described within a unified framework.

TABLE V. Mass (MeV) spectrum of the nucleon baryon compared with the experimental data from the PDG (labeled with Expt.) [110], the results obtained from the OGE model [14], the OGE model with higher order hyperfine interactions (labeled with OGEh) [46], the quark model combining both OGE and OBE potentials (labeled with Hyb) [26], the quark-diquark model (Diq) [21], the relativistically covariant constituent quark model with instantaneous forces (labeled with Inst) [28], the Dyson-Schwinger and Faddeev equations [29], and the large $1/N_c$ expansion approach (labeled with $1/N_c$) [36–42]. If a resonance is a mixed state dominated by a certain quark model configuration, it is marked with bold type.

State	Ours	OGE [14]	OGEh [46]	OBE [18]	Hyb [26]	Diq [21]	Inst [28]	$1/N_c$ [36–42]	Expt [110]
${}^2_8[56, 0_0^+]{1}^2S_{\frac{1}{2}^+}$	938	960	939	939	939	939	939		938
${}^2_8[56, 0_2^+]{2}^2S_{\frac{3}{2}^+}$	1438	1540	1462	1459	1445	1513	1518	1450	1410–1470
${}^2_8[70, 0_2^+]{2}^2S_{\frac{1}{2}^+}$	1824	1770	1748	1776	1833	1768	1729	1712	1680–1740
${}^4_8[70, 2_2^+]{1}^4D_{\frac{1}{2}^+}$	1948	1880	1887			1893	1950		1830–1930
${}^2_8[20, 1_2^+]{1}^2P_{\frac{1}{2}^+}$	2010	1975	2060				1996	1983	2050–2150
${}^2_8[56, 2_2^+]{1}^2D_{\frac{3}{2}^+}$	1817	1795	1734		1690	1768	1688	1674	1680–1750
${}^4_8[70, 0_2^+]{2}^4S_{\frac{3}{2}^+}$	1886	1870	1857			1808	1809	1885	
${}^2_8[70, 2_2^+]{1}^2D_{\frac{3}{2}^+}$	1922	1910	1975				1936		
${}^4_8[70, 2_2^+]{1}^4D_{\frac{3}{2}^+}$	1953	1950	1952				1969		$\begin{cases} 1890\text{--}1950 \\ 2015\text{--}2065 \end{cases}$
${}^2_8[20, 1_2^+]{1}^2P_{\frac{3}{2}^+}$	2015	2030	2058						
${}^2_8[56, 2_2^+]{1}^2D_{\frac{5}{2}^+}$	1775	1770	1738		1690	1808	1723	1689	1680–1690
${}^2_8[70, 2_2^+]{1}^2D_{\frac{5}{2}^+}$	1944	1980	1953				1934		1800–1980
${}^4_8[70, 2_2^+]{1}^4D_{\frac{5}{2}^+}$	1962	1995	2007				1959	1850	1942–2090
${}^4_8[70, 2_2^+]{1}^4D_{\frac{7}{2}^+}$	1936	2000	1943				1989	1872	1950–2100
${}^2_8[70, 1_1^-]{1}^2P_{\frac{1}{2}^-}$	1549	1460	1497	1519	1568	1527	1435	1541	1515–1545
${}^4_8[70, 1_1^-]{1}^4P_{\frac{1}{2}^-}$	1606	1535	1650	1647	1658	1671	1660	1660	1635–1665
${}^2_8[70, 1_1^-]{1}^2P_{\frac{3}{2}^-}$	1560	1495	1548	1519	1568	1527	1476	1532	1510–1520
${}^4_8[70, 1_1^-]{1}^4P_{\frac{3}{2}^-}$	1666	1625	1731	1647	1658	1671	1606	1699	1650–1800
${}^4_8[70, 1_1^-]{1}^4P_{\frac{5}{2}^-}$	1646	1630	1655	1647	1658	1671	1655	1671	1665–1680

A. Nucleon baryons

1. $N(1535, 1650)1/2^-$ and tensor interactions

In the constituent quark model, the $N(1535)1/2^-$ and $N(1650)1/2^-$ resonances are often considered to be two mixed states via ${}^2_8[70, 1_1^-]{1}^2P_{1/2^-} - {}^4_8[70, 1_1^-]{1}^4P_{1/2^-}$ mixing mainly due to the tensor interactions [118,119]. The mixing scheme is expressed as

$$\begin{pmatrix} N(1535) \\ N(1650) \end{pmatrix} = \begin{pmatrix} \cos \theta_S & -\sin \theta_S \\ \sin \theta_S & \cos \theta_S \end{pmatrix} \begin{pmatrix} {}^2_8[70, 1_1^-]{1}^2P_{1/2^-} \\ {}^4_8[70, 1_1^-]{1}^4P_{1/2^-} \end{pmatrix}, \quad (40)$$

where θ_S is the mixing angle. The configuration mixing in $N(1535)1/2^-$ and $N(1650)1/2^-$ seems to be inevitable for correctly describing the reactions $\gamma p \rightarrow \eta p$ [62,66] and $\gamma p \rightarrow \pi^0 p$ [67] in the resonances region. By fitting the

reaction data, the mixing angle is determined to be $\theta_S \simeq 26^\circ$ [66,67], which is consistent with $\theta_S \simeq 32^\circ$ obtained with the OGE tensor potentials, other than the negative mixing angle determined with the OBE tensor potentials [117,120]. About the OGE and OBE models, there were heated debates between Isgur and Glozman in 1999 [19,20].

Do both the OGE and OBE potential exist in the light baryon states at the same time? Can we explain the mixing angle $\theta_S \simeq 26^\circ$ determined by the scattering data when we include both the OGE and OBE tensor potentials? To answer these questions, we carry out some analyses as follows.

In Fig. 4, the tensor potentials of the OGE and OBE for the ${}^4_8[70, 1_1^-]{1}^4P_{1/2^-}$ configuration are plotted with red dashed and blue solid curves, respectively. It should be mentioned that the OBE tensor potential is mainly contributed by the one-pion exchange. The total tensor potential is shown with green dash-dotted curve. It is seen

TABLE VI. Mass (MeV) spectrum of the Δ baryon compared with the experimental data from the PDG (Expt.) [110], the results obtained from the OGE model [14], the OGE model with higher order hyperfine interactions (OGEh) [46], the quark model combining both OGE and OBE potentials (Hyb) [26], the quark-diquark model (Diq) [21], the relativistically covariant constituent quark model with instantaneous forces (Inst) [28], the Dyson-Schwinger and Faddeev equations [29], and the large $1/N_c$ expansion approach (labeled with $1/N_c$) [36–42]. If a resonance is a mixed state dominated by a certain quark model configuration, it is marked with bold type.

State	Ours	OGE [14]	OGEh [46]	OBE [18]	Hyb [26]	Diq [21]	Inst [28]	$1/N_c$ [36–42]	Expt [110]
$^4 10[56, 0_0^+] 1^4 S_{\frac{3}{2}^+}$	1232	1230	1231	1240	1232	1233	1260		1230–1234
$^4 10[56, 0_2^+] 2^4 S_{\frac{3}{2}^+}$	1694	1795	1790	1718	1659	1602	1810	1625	1500–1640
$^2 10[70, 0_2^+] 2^2 S_{\frac{1}{2}^+}$	1874	1835	1867		1874	1858	1866	1746	1660–1782
$^4 10[56, 2_2^+] 1^4 D_{\frac{1}{2}^+}$	1913	1875	1900			1952	1906	1897	1850–1950
$^4 10[56, 2_2^+] 1^4 D_{\frac{3}{2}^+}$	1906	1915	1904		2090	1952	1871	1906	1870–1970
$^2 10[70, 2_2^+] 1^2 D_{\frac{3}{2}^+}$	1973	1985	1972				1950		1855–1910
$^4 10[56, 2_2^+] 1^4 D_{\frac{5}{2}^+}$	1864	1910	1931		1874	1952	1897	1921	
$^2 10[70, 2_2^+] 1^2 D_{\frac{5}{2}^+}$	1975	1990	1967				1985	1756	1991–2039
$^4 10[56, 2_2^+] 1^4 D_{\frac{7}{2}^+}$	1867	1940	1902		1874	1952	1956	1942	1915–1950
$^2 10[70, 1_1^-] 1^2 P_{\frac{1}{2}^-}$	1595	1555	1668	1642	1667	1554	1654	1645	1590–1630
$^2 10[70, 1_1^-] 1^2 P_{\frac{3}{2}^-}$	1655	1620	1668	1642	1667	1554	1628	1720	1690–1730

that the sign of the OGE tensor potential is opposite to that of OBE; the two kinds of potentials have a strong cancelation. If we adopt this potential directly, the fairly large mixing angle $\theta_S \simeq 26^\circ$ cannot be explained. Note that the singular behavior in the tensor potentials will bring unphysical contributions in the short ranges, which should be reasonably eliminated. Following the method adopted in Refs. [113–115], we cut off the OGE and OBE tensor

potentials at $r_c = 0.23$ and $r'_c = 0.70$ fm, respectively. In this case, a reasonable mixing angle $\theta_S \simeq 25^\circ$ can be obtained.

The OGE and OBE tensor potentials after truncations are shown with blue dotted and short dashed curves in Fig. 4, respectively. From the figure, one can find that the average depth of the OGE tensor potential in the range of $r \leq 0.23$ fm is about -100 MeV. The behavior of the OBE tensor potential after a truncation changes notably in the short region, which is dominated by the $1/r$ term. It is

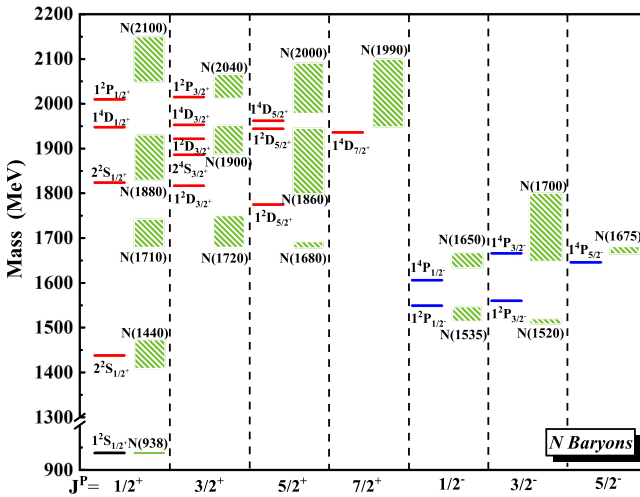


FIG. 2. Nucleon spectrum with principal quantum number $N \leq 2$. The short solid lines stand for the theoretical results, while the shaded boxes stand for the experimental data from the PDG [110].

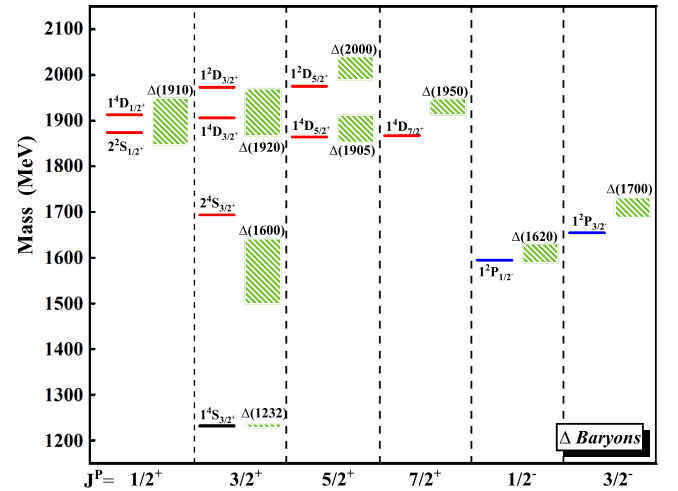


FIG. 3. Δ spectrum with principal quantum number $N \leq 2$. The short solid lines stand for the theoretical results, while the shaded boxes stand for the experimental data from the PDG [110].

TABLE VII. Predicted masses of nucleon resonances with principal quantum number $N \leq 2$ and the average contributions of each part of the Hamiltonian (in MeV). T stands for the contribution of the kinetic energy term. V^{Conf} and V^{Coul} stand for the contributions from the linear confinement and Coulomb-like potentials, respectively. V_G^{SS} , V_G^{LS} , and V_G^T denote the contributions of the spin-spin, spin-orbit, and tensor terms, respectively, in a single gluon-exchange potential. V_π^C , V_η^C , and V_σ^C represent the contributions of the central potentials of one-boson exchanges, while V_π^T , V_η^T , V_σ^{LS} denote the corresponding contributions of the tensor and spin-orbit terms, respectively. The Δm_{mix} in the last column represents the mass shift due to configuration mixing. The zero-point energy for each of them is -829 MeV.

State	Mixing matrix	Mass	T	V^{Conf}	V^{Coul}	V_G^{SS}	V_G^{LS}	V_G^T	V_π^C	V_η^C	V_σ^C	V_π^T	V_η^T	V_σ^{LS}	Δm_{mix}
$^2 8[56, 0_0^+] 1^2 S_{\frac{1}{2}^+}$		938	2716	400	-597	-20	0	0	-571	38	-198	0	0	0	0
$^2 8[56, 0_2^+] 2^2 S_{\frac{1}{2}^+}$		1438	2747	656	-508	-14	0	0	-499	34	-149	0	0	0	0
$^2 8[70, 0_2^+] 2^2 S_{\frac{1}{2}^+}$		1824	2148	884	-246	-7	0	0	-62	5	-68	0	0	0	0
$^4 8[70, 2_2^+] 1^4 D_{\frac{1}{2}^+}$	$\begin{pmatrix} 0.96 & 0.29 \\ 0.29 & -0.96 \end{pmatrix}$	1948	2209	882	-231	6	-14	-13	7	-3	-63	-5	-1	12	-9
$^2 8[20, 1_2^+] 1^2 P_{\frac{1}{2}^+}$		2010	2071	983	-193	-4	-2	-1	18	0	-45	0	0	4	9
$^2 8[56, 2_2^+] 1^2 D_{\frac{3}{2}^+}$		1817	2280	823	-263	-8	1	0	-111	5	-85	0	0	4	0
$^2 8[70, 2_2^+] 1^2 D_{\frac{3}{2}^+}$		1922	2137	918	-219	-6	0	0	-29	2	-57	4	0	0	0
$^4 8[70, 0_2^+] 2^4 S_{\frac{3}{2}^+}$		1886	2076	933	-232	7	0	0	-3	-4	-62	0	0	0	0
$^4 8[70, 2_2^+] 1^4 D_{\frac{3}{2}^+}$	$\begin{pmatrix} 0.88 & 0.47 \\ 0.47 & -0.88 \end{pmatrix}$	1953	2092	954	-208	4	2	0	7	-2	-51	0	0	6	-21
$^2 8[20, 1_2^+] 1^2 P_{\frac{3}{2}^+}$		2015	2049	995	-191	-2	1	0	14	0	-44	0	0	1	21
$^2 8[56, 2_2^+] 1^2 D_{\frac{5}{2}^+}$	$\begin{pmatrix} 0.90 & -0.43 \\ -0.43 & -0.90 \end{pmatrix}$	1775	2194	870	-243	-7	-3	0	-84	4	-73	0	0	-3	-51
$^2 8[70, 2_2^+] 1^2 D_{\frac{5}{2}^+}$		1944	2116	930	-218	-6	-2	0	-40	2	-57	0	0	-3	51
$^4 8[70, 2_2^+] 1^4 D_{\frac{5}{2}^+}$		1962	2044	984	-202	5	1	6	2	-2	-49	-2	1	1	0
$^4 8[70, 2_2^+] 1^4 D_{\frac{7}{2}^+}$		1936	2038	988	-201	5	-10	-2	2	-2	-48	1	-1	-7	0
$^2 8[70, 1_1^-] 1^2 P_{\frac{1}{2}^-}$	$\begin{pmatrix} 0.90 & -0.43 \\ 0.43 & 0.90 \end{pmatrix}$	1549	2179	708	-300	-6	-21	-5	-69	4	-94	-5	0	4	-17
$^4 8[70, 1_1^-] 1^4 P_{\frac{1}{2}^-}$		1606	2253	679	-318	8	-46	-20	-15	-6	-102	-23	1	7	17
$^2 8[70, 1_1^-] 1^2 P_{\frac{3}{2}^-}$	$\begin{pmatrix} 0.94 & -0.34 \\ 0.34 & 0.94 \end{pmatrix}$	1560	2075	759	-275	-7	4	1	-66	4	-83	0	0	-1	-22
$^4 8[70, 1_1^-] 1^4 P_{\frac{3}{2}^-}$		1666	1998	807	-256	6	-3	9	-11	-3	-74	-3	1	2	22
$^4 8[70, 1_1^-] 1^4 P_{\frac{5}{2}^-}$		1646	1983	816	-253	8	6	-3	-3	-4	-73	1	0	-4	0

reasonable to take a larger cutoff parameter $r'_c = 0.70$ fm for the OBE tensor potential. As we know, in the short range, the quark-quark interactions are mainly due to the OGE, while the nonperturbative interactions due to OBE

should be strongly suppressed. However, the singular behavior of the OBE potentials notably enlarges the unphysical contributions in the short range. Thus, we need a larger cutoff parameter $r'_c = 0.70$ fm to suppress

TABLE VIII. Predicted masses of Δ resonances with principal quantum number $N \leq 2$ and the average contributions of each part of the Hamiltonian (in MeV). The average contributions of each part of the Δ resonances. The caption is the same as that of Table VII.

State	Mixing matrix	Mass	T	V^{Conf}	V^{Coul}	V_G^{SS}	V_G^{LS}	V_G^T	V_π^C	V_η^C	V_σ^C	V_π^T	V_η^T	V_σ^{LS}	Δm_{mix}
$^4 10[56, 0_0^+] 1^4 S_{\frac{3}{2}^+}$		1232	2066	569	-391	15	0	0	-52	-15	-132	0	0	0	0
$^4 10[56, 0_2^+] 2^4 S_{\frac{3}{2}^+}$		1694	2208	816	-344	11	0	0	-48	-15	-105	0	0	0	0
$^2 10[70, 0_2^+] 2^2 S_{\frac{1}{2}^+}$	$\begin{pmatrix} 0.97 & 0.25 \\ 0.25 & -0.97 \end{pmatrix}$	1874	2090	923	-234	-3	1	0	-4	-63	-3	0	0	1	-4
$^4 10[56, 2_2^+] 1^4 D_{\frac{1}{2}^+}$		1913	2165	886	-242	7	14	-7	-17	-67	-4	-5	-2	10	4
$^2 10[70, 2_2^+] 1^2 D_{\frac{3}{2}^+}$	$\begin{pmatrix} 0.92 & 0.38 \\ -0.38 & 0.92 \end{pmatrix}$	1973	2083	953	-212	-3	14	0	0	-53	-1	0	0	4	17
$^4 10[56, 2_2^+] 1^4 D_{\frac{3}{2}^+}$		1906	2124	914	-232	5	13	0	-14	-62	-3	0	0	6	-17
$^2 10[70, 2_2^+] 1^2 D_{\frac{5}{2}^+}$	$\begin{pmatrix} 0.78 & 0.63 \\ -0.63 & 0.78 \end{pmatrix}$	1975	2080	950	-215	0	-6	2	-4	-55	-2	2	0	-1	53
$^4 10[56, 2_2^+] 1^4 D_{\frac{5}{2}^+}$		1864	2086	942	-220	2	-3	3	-8	-57	-2	3	1	0	-53
$^4 10[56, 2_2^+] 1^4 D_{\frac{7}{2}^+}$		1867	2080	938	-224	6	-18	-2	-15	-3	-59	-2	-1	-6	0
$^2 10[70, 1_1^-] 1^2 P_{\frac{1}{2}^-}$		1595	2061	767	-272	-4	-38	0	-2	-3	-82	0	0	-4	0
$^2 10[70, 1_1^-] 1^2 P_{\frac{3}{2}^-}$		1655	2000	805	-257	-3	17	0	-3	-2	-75	0	0	2	0

TABLE IX. Partial decay widths (MeV) of nucleon resonances compared with the data and the other theoretical results. For the ΣK decay channel, the calculated partial widths of the nucleon resonances are less than 1 MeV, therefore they are not listed in the table. The unopened decay channels for a resonance are labeled by $\dots$. If a resonance is a mixed state dominated by a certain quark model configuration, it is marked with bold type. The number of the stars indicate the experimental status of the resonance assessed by the PDG.

State	$N\pi$	$\Delta\pi$	$N\eta$	ΛK	$N(1440)\pi$	$N(1520)\pi$	$N(1535)\pi$	Sum	
$N(1535)_{\frac{1}{2}}^{-}$	40–91	1–7	38–96					125–175	PDG [110]
${}^2_8[70, 1_1^-]$	$62.3^{+0.2}_{-0.4}$	$2.8^{+0.9}_{-0.8}$	$79.5^{+9.6}_{-12.9}$	$\dots$	$\dots$	$\dots$	$\dots$	$144.6^{+10.7}_{-14.1}$	Ours
1530^{+15}_{-15}	63	16	75					154	$U(7)$ [116]
****	84	6	50					140	hQM [116]
	51 ± 21		121 ± 15					172 ± 36	CQM [117]
	57	0.9	73					130.9	$1/N_c$ [53]
$N(1650)_{\frac{1}{2}}^{-}$	50–105	6–27	15–53	5–23	6–39			100–150	PDG [110]
${}^4_8[70, 1_1^-]$	$96.5^{+1.5}_{-1.2}$	$25.5^{+4.7}_{-4.1}$	$13.2^{+0.0}_{-0.1}$	$4.3^{+0.5}_{-0.7}$	$0.9^{+0.1}_{-0.1}$	$\dots$	$\dots$	$140.4^{+3.6}_{-3.6}$	Ours
1650^{+15}_{-15}	41	18	72					131	$U(7)$ [116]
****	51	4	29					84	hQM [116]
	81 ± 22		28 ± 22	9 ± 6				118 ± 50	CQM [117]
	133	5.1	12.5	11.5				162.1	$1/N_c$ [53]
$N(1520)_{\frac{3}{2}}^{-}$	55–78	22–41	0.07–0.1					100–120	PDG [110]
${}^2_8[70, 1_1^-]$	$48.2^{+1.7}_{-1.6}$	$8.4^{+0.7}_{-0.6}$	$0.1^{+0.1}_{-0.1}$	$\dots$	$\dots$	$\dots$	$\dots$	$56.7^{+2.5}_{-2.3}$	Ours
1515^{+5}_{-5}	134	207	0.0					341	$U(7)$ [116]
****	111	206	0.0					317	hQM [116]
	66 ± 7		0.19					66.2 ± 7	CQM [117]
	72	19	0.26					91.3	$1/N_c$ [53]
$N(1700)_{\frac{3}{2}}^{-}$	7–51	55–255	1–6	1–6	3–33	< 12		100–300	PDG [110]
${}^4_8[70, 1_1^-]$	$46.4^{+19.1}_{-13.8}$	$181.2^{+4.0}_{-15.8}$	0.0	$0.4^{+1.1}_{-0.4}$	$0.5^{+1.6}_{-0.4}$	$3.9^{+15.0}_{-3.9}$	$2.2^{+10.7}_{-1.9}$	$234.6^{+51.5}_{-36.2}$	Ours
1720^{+80}_{-70}	9	561	3					573	$U(7)$ [116]
***	13 ± 10		0.5 ± 0.5	0.1 ± 0.1					CQM [117]
	12	297.3	≤ 0.15	≤ 0.03	0.0			309.3	$1/N_c$ [53]
$N(1675)_{\frac{5}{2}}^{-}$	49–67	30–59	< 2	< 0.06				130–160	PDG [110]
${}^4_8[70, 1_1^-]$	$25.3^{+0.6}_{-0.6}$	$59.2^{+3.1}_{-3.0}$	$7.6^{+0.5}_{-0.5}$	0.0	0.0	0.0	0.0	$92.1^{+4.2}_{-4.1}$	Ours
1675^{+5}_{-5}	47	108	11					166	$U(7)$ [116]
****	41	85	9					135	hQM [116]
	51	75	6.3	≤ 0.1				132.3	$1/N_c$ [53]
$N(1440)_{\frac{1}{2}}^{+}$	138–338	15–122						250–450	PDG [110]
${}^2_8[56, 0_2^+]$	$363.1^{+49.6}_{-48.7}$	$25.8^{+24.1}_{-16.2}$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$388.9^{+73.7}_{-64.9}$	Ours
1440^{+30}_{-30}	85	13						98	$U(7)$ [116]
****	105	12						117	hQM [116]
$N(1710)_{\frac{1}{2}}^{+}$	4–40	2–18	8–100	4–50			7–42	80–200	PDG [110]
${}^2_8[70, 0_2^+]$	$10.2^{+1.2}_{-1.0}$	$8.2^{+0.3}_{-0.0}$	$4.2^{+0.1}_{-0.2}$	$0.5^{+0.1}_{-0.1}$	$92.7^{+36.9}_{-32.1}$	$0.3^{+0.7}_{-0.2}$	$51.7^{+39.2}_{-32.5}$	$167.8^{+75.5}_{-64.1}$	Ours
1710^{+30}_{-30}	5	56	9	3				73	$U(7)$ [116]
****	18	70	12	14.1				114.1	hQM [116]
$N(1720)_{\frac{3}{2}}^{+}$	12–56	71–356	2–20	6–76	< 8	2–20		150–400	PDG [110]
${}^2_8[56, 2_2^+]$	$21.0^{+2.1}_{-2.2}$	$6.7^{+2.8}_{-2.1}$	$0.9^{+0.2}_{-0.2}$	$1.3^{+0.5}_{-0.4}$	$88.7^{+30.2}_{-27.7}$	$17.7^{+8.8}_{-7.7}$	0.0	$136.3^{+44.6}_{-40.3}$	Ours
1720^{+30}_{-40}	111	36	7	14				168	$U(7)$ [116]
****	141	30	8	12				191	hQM [116]
$N(1680)_{\frac{5}{2}}^{+}$	69–91	13–30	< 1					115–130	PDG [110]

(Table continued)

TABLE IX. (Continued)

State	$N\pi$	$\Delta\pi$	$N\eta$	ΛK	$N(1440)\pi$	$N(1520)\pi$	$N(1535)\pi$	Sum	
$^2\mathbf{8}[56,2_2^+]$	$43.8^{+1.5}_{-1.4}$	$1.7^{+0.1}_{-0.1}$	$0.5^{+0.0}_{-0.0}$	$0.1^{+0.0}_{-0.0}$	$0.1^{+0.0}_{-0.0}$	0.0	0.0	$46.2^{+1.6}_{-1.5}$	Ours
1685^{+5}_{-5}	121	100	1					222	$U(7)$ [116]
****	91	92	0					183	hQM [116]

TABLE X. Partial decay widths (MeV) of nucleon resonances compared with the data and the other theoretical results. The calculated partial widths for the $N\eta'$ and Σ^*K channels are usually small, less than 2 MeV, thus, they are not presented in the table. For the $N^*\pi/\Delta^*\pi$ column, the partial widths for the $N(1520)\pi$, $N(1535)\pi$, $N(1650)\pi$, $N(1675)\pi$, $N(1680)\pi$, and $\Delta(1620)\pi$, and $\Delta(1700)\pi$ channels are given; to save space, the less important decay channels with a tiny partial width (less than 5 MeV) are not listed in the table. If a resonance is a mixed state dominated by a certain quark model configuration, it is marked with bold type. The number of the stars indicate the experimental status of the resonance assessed by the PDG.

State	$N\pi$	$\Delta\pi$	$N\eta$	$\Lambda K/\Sigma K$	$N(1440)\pi$	$N^*\pi/\Delta^*\pi$			Sum	
						$N(1520)\pi$	$N(1680)\pi$	$\Delta(1700)\pi$		
$N(1900)_{\frac{3}{2}}^{3+}$	1–64	30–224	2–45	$2 \sim 64/3 \sim 22$		7–74			100–320	PDG [110]
$^2\mathbf{8}[70,2_2^+]$	$9.6^{+0.2}_{-0.3}$	$25.2^{+5.8}_{-5.0}$	$3.3^{+0.1}_{-0.2}$	$2.0^{+0.1}_{-0.1}/0.2^{+0.0}_{-0.0}$	$61.7^{+4.0}_{-4.9}$	$102.4^{+5.9}_{-7.6}$	$5.5^{+3.5}_{-2.6}$	$11.6^{+6.1}_{-5.0}$	$221.5^{+25.7}_{-25.7}$	Ours
1920^{+30}_{-30}	11	63	12	13/1					100	$U(7)$ [116]
****	15	70	12	13/1					111	hQM [116]
						$N(1520)\pi$	$N(1535)\pi$	$\Delta(1620)\pi$		
$N(1880)_{\frac{1}{2}}^{1+}$	6–124	10–168	2–220	$2 \sim 12/20 \sim 96$		8–48			200–400	PDG [110]
$^4\mathbf{8}[70,2_2^+]$	$2.4^{+0.1}_{-0.1}$	$293.7^{+47.3}_{-47.6}$	$3.2^{+0.1}_{-0.2}$	$0.0/3.0^{+0.3}_{-0.5}$	$19.7^{+1.5}_{-2.3}$	$3.5^{+2.3}_{-1.6}$	$99.8^{+19.3}_{-20.8}$	$210.4^{+50.2}_{-56.5}$	$635.7^{+121.1}_{-129.6}$	Ours
1948^{+50}_{-50}										

						$N(1535)\pi$	$N(1650)\pi$	$\Delta(1620)\pi$		
$N(2100)_{\frac{1}{2}}^{1+}$	16–102	12–45	10–144	$< 3/\dots$		52–109			200–320	PDG [110]
$^2\mathbf{8}[20,1_2^+]$	$0.2^{+0.0}_{-0.0}$	$32.1^{+4.1}_{-4.3}$	$0.3^{+0.0}_{-0.0}$	$0.0/0.3^{+0.0}_{-0.0}$	$2.0^{+0.0}_{-0.1}$	$46.4^{+5.4}_{-6.6}$	$41.4^{+10.6}_{-11.0}$	$68.3^{+11.7}_{-13.7}$	$191.0^{+8.4}_{-35.7}$	Ours
2010^{+50}_{-50}										

						$N(1520)\pi$	$N(1675)\pi$	$N(1680)\pi$		
$N(1860)_{\frac{5}{2}}^{5+}$	13–87	64–234	0–26	$< 0.17/\dots$					318–434	PDG [110]
$^2\mathbf{8}[70,2_2^+]$	$0.8^{+0.1}_{-0.1}$	$40.6^{+11.6}_{-9.9}$	$2.2^{+0.8}_{-0.6}$	$0.3^{+0.2}_{-0.1}/0.4^{+0.3}_{-0.2}$	$0.2^{+0.2}_{-0.1}$	$12.6^{+7.8}_{-5.5}$	$4.2^{+2.0}_{-1.8}$	$8.6^{+9.0}_{-5.5}$	$69.9^{+32.0}_{-23.8}$	Ours
1944^{+50}_{-50}										
**										
						$N(1520)\pi$	$N(1650)\pi$	$N(1675)\pi$		
$N(2000)_{\frac{5}{2}}^{5+}$	20–45	100–356	< 17.8						335–445	PDG [110]
$^4\mathbf{8}[70,2_2^+]$	$1.6^{+0.3}_{-0.3}$	$44.7^{+13.0}_{-11.0}$	$1.0^{+0.3}_{-0.3}$	$0.0/0.5^{+0.3}_{-0.2}$	$0.3^{+0.2}_{-0.1}$	$3.7^{+2.2}_{-1.6}$	$2.3^{+2.8}_{-1.5}$	$89.3^{+32.0}_{-32.5}$	$143.4^{+51.1}_{-47.5}$	Ours
1962^{+50}_{-50}										
**										
						$N(1520)\pi$	$N(1675)\pi$	$N(1680)\pi$		
$N(1990)_{\frac{7}{2}}^{7+}$	4–24		< 12	$12 \sim 24/\dots$					200–400	PDG [110]
$^4\mathbf{8}[70,2_2^+]$	$6.3^{+1.4}_{-1.2}$	$31.7^{+12.4}_{-9.8}$	$3.7^{+1.3}_{-1.1}$	$0.0/1.6^{+1.2}_{-0.8}$	$1.0^{+0.7}_{-0.5}$	$15.0^{+10.3}_{-7.0}$	$2.5^{+4.9}_{-2.0}$	$1.1^{+1.2}_{-0.7}$	$62.9^{+33.4}_{-23.1}$	Ours
1936^{+50}_{-50}										
**										
						$N(1520)\pi$	$N(1535)\pi$	$\Delta(1700)\pi$		
$^4\mathbf{8}[70,0_2^+]$	$0.3^{+0.2}_{-0.2}$	$3.4^{+2.3}_{-2.7}$	$1.3^{+0.4}_{-0.4}$	$0.0/0.4^{+0.2}_{-0.1}$	$36.4^{+8.5}_{-8.4}$	$69.9^{+17.2}_{-17.7}$	$3.7^{+3.5}_{-2.1}$	$48.6^{+68.8}_{-34.6}$	$164.0^{+94.9}_{-59.4}$	Ours

(Table continued)

TABLE X. (Continued)

State	$N\pi$	$\Delta\pi$	$N\eta$	$\Lambda K/\Sigma K$	$N(1440)\pi$	$N^*\pi/\Delta^*\pi$			Sum	
1886^{+50}_{-50}						$N(1520)\pi$	$N(1535)\pi$	$\Delta(1700)\pi$		
$N(1953)_{\frac{3}{2}}^{+}$	$0.9^{+0.0}_{-0.0}$	$17.5^{+4.7}_{-4.1}$	$1.3^{+0.0}_{-0.1}$	$0.0/1.3^{+0.1}_{-0.2}$	$6.0^{+0.4}_{-0.6}$	$7.1^{+1.5}_{-1.5}$	$6.7^{+4.7}_{-3.2}$	$9.3^{+4.7}_{-4.4}$	$50.1^{+16.1}_{-14.1}$	Ours
$48[70, 2_2^+]$										
1953^{+50}_{-50}						$N(1520)\pi$	$\Delta(1620)\pi$	$\Delta(1700)\pi$		
$N(2015)_{\frac{3}{2}}^{+}$	$0.3^{+0.0}_{-0.0}$	$7.2^{+1.3}_{-1.1}$	$0.4^{+0.0}_{-0.0}$	$0.0/0.4^{+0.0}_{-0.0}$	$1.8^{+0.0}_{-0.1}$	$5.2^{+0.3}_{-0.0}$	$13.1^{+9.8}_{-6.5}$	$63.2^{+25.5}_{-22.8}$	$91.6^{+36.3}_{-30.5}$	Ours
$28[20, 1_2^+]$										
2015^{+50}_{-50}										

TABLE XI. Partial decay widths (MeV) of Δ resonances compared with the data and the other theoretical results. For the $N^*\pi/\Delta^*\pi$ column, the partial widths for the $N(1520)\pi$, $N(1535)\pi$, $N(1650)\pi$, $N(1675)\pi$, $N(1680)\pi$, and $\Delta(1620)\pi$, $\Delta(1700)\pi$ channels are given; to save space, the less important decay channels with a tiny partial width are not listed in the table. If a resonance is a mixed state dominated by a certain quark model configuration, it is marked with bold type. The number of the stars indicate the experimental status of the resonance assessed by the PDG.

State	$N\pi$	$\Delta\pi$	ΣK	$\Delta\eta$	$N(1440)\pi$	$N^*\pi/\Delta^*\pi$			Sum	
$\Delta(1232)_{\frac{3}{2}}^{+}$	113–119								114–120	PDG [110]
$4^{10}[56, 0_0^+]$	$49.8^{+1.3}_{-1.3}$	...	...	...	...	...			$49.8^{+1.3}_{-1.3}$	Ours
1232_{-2}^{+2}	71								71	$U(7)$ [116]
****	63								63	hQM [116]
$\Delta(1600)_{\frac{3}{2}}^{+}$	16–72	116–246			34–81				200–300	PDG [110]
$2^{10}[70, 2_2^+]$	$21.6^{+0.0}_{-1.6}$	$79.3^{+53.5}_{-43.3}$	...	...	$0.1^{+4.0}_{+0.5}$	...			$101.0^{+57.5}_{-44.4}$	Ours
1570_{-70}^{+70}										

$\Delta(1620)_{\frac{1}{2}}^{-}$	28–53	48–108			< 14				110–150	PDG [110]
$2^{10}[70, 1_1^-]$	$16.1^{+0.2}_{+0.2}$	$29.9^{+8.8}_{-7.3}$	...	...	0.0	...			$46.0^{+8.6}_{-7.1}$	Ours
1610_{-20}^{+20}	5	76							81	$U(7)$ [116]
****	9	59							68	hQM [116]
						$N(1520)\pi$	$N(1535)\pi$	$N(1650)\pi$		
$\Delta(1700)_{\frac{3}{2}}^{-}$	22–76	20–266				2–19	1–6		220–380	PDG [110]
$2^{10}[70, 1_1^-]$	$25.2^{+2.4}_{-2.3}$	$107.1^{+9.3}_{-8.7}$	$0.0^{+0.1}_{-0.0}$	...	$0.2^{+0.1}_{-0.1}$	$5.7^{+4.1}_{-3.0}$	$0.2^{+0.2}_{-0.2}$	...	$138.4^{+16.2}_{-14.3}$	Ours
1710_{-20}^{+20}	46	311							357	$U(7)$ [116]
****	40	333							373	hQM [116]
						$N(1535)\pi$	$N(1650)\pi$	$\Delta(1620)\pi$		
$\Delta(1910)_{\frac{1}{2}}^{+}$	20–120	68–264	8–56	10–52	6–180				200–400	PDG [110]
$4^{10}[56, 2_2^+]$	$17.1^{+1.2}_{-1.5}$	$3.7^{+0.6}_{+0.3}$	$4.9^{+1.1}_{-1.3}$	$1.3^{+0.4}_{-0.6}$	$164.6^{+22.5}_{-28.2}$	$4.3^{+0.2}_{-0.6}$	$57.5^{+32.6}_{-28.0}$	$115.8^{+39.6}_{-41.0}$	$369.2^{+97.0}_{-100.9}$	Ours
1900_{-50}^{+50}	26	32	38	4					100	$U(7)$ [116]
****	49	34	38	4					125	hQM [116]
						$N(1675)\pi$	$N(1680)\pi$	$\Delta(1620)\pi$		
$\Delta(1905)_{\frac{5}{2}}^{+}$	24–60	> 130		5–24			14–60		270–400	PDG [110]
$4^{10}[56, 2_2^+]$	$12.9^{+1.9}_{-1.4}$	$34.8^{+9.0}_{-6.4}$	$0.4^{+0.3}_{-0.2}$	$0.4^{+0.4}_{-0.2}$	$0.2^{+0.1}_{+0.0}$	$2.5^{+1.3}_{-0.8}$	$2.5^{+3.1}_{-1.6}$	$2.6^{+2.4}_{-1.3}$	$56.3^{+18.3}_{-11.9}$	Ours
1880_{-25}^{+30}	31	188	1	19					239	$U(7)$ [116]
****	26	182	0	15					223	hQM [116]

(Table continued)

TABLE XI. (Continued)

State	$N\pi$	$\Delta\pi$	ΣK	$\Delta\eta$	$N(1440)\pi$	$N^*\pi/\Delta^*\pi$			Sum	
						$N(1520)\pi$	$N(1675)\pi$	$N(1680)\pi$		
$\Delta(1950)_{\frac{7}{2}}^{+}$	82–151	2–30	1–2	< 2				7–30	235–335	PDG [110]
$^4_{10}[56,2_2^{+}]$	$50.8^{+4.4}_{-3.1}$	$36.7^{+5.5}_{-3.8}$	$2.8^{+0.8}_{-0.5}$	$0.8^{+0.4}_{-0.2}$	$4.7^{+1.2}_{-0.8}$	$3.2^{+0.8}_{-0.5}$	$0.7^{+0.5}_{-0.2}$	$6.3^{+2.5}_{-1.6}$	$106.0^{+16.1}_{-10.7}$	Ours
1930^{+20}_{-15}	172	92	5	1					270	$U(7)$ [116]
****	146	70	3	1					220	hQM [116]
						$N(1520)\pi$	$N(1680)\pi$	$\Delta(1700)\pi$		
$\Delta(1920)_{\frac{3}{2}}^{+}$	12–72	> 110	5–22	12–61	10–310	< 18			240–360	PDG [110]
$^4_{10}[56,2_2^{+}]$	$6.0^{+0.2}_{-0.3}$	$38.0^{+13.9}_{-11.2}$	$1.9^{+0.3}_{-0.4}$	$1.8^{+1.4}_{-1.0}$	$41.6^{+4.2}_{-5.9}$	$48.7^{+10.0}_{-10.6}$	$14.4^{+17.3}_{-10.3}$	$3.4^{+2.2}_{-2.1}$	$155.8^{+49.5}_{-41.8}$	Ours
1920^{+50}_{-50}	7	132	23	22					184	$U(7)$ [116]
***	17	137	22	20					196	hQM [116]
						$N(1535)\pi$	$N(1675)\pi$	$N(1680)\pi$		
$\Delta(1975)_{\frac{5}{2}}^{+}$	$0.2^{+0.0}_{-0.0}$	$17.4^{+0.0}_{-0.9}$	0.0	$5.6^{+0.9}_{-1.4}$	$3.5^{+0.8}_{-0.7}$	$13.8^{+8.5}_{-6.0}$	$70.7^{+21.3}_{-23.0}$	$16.3^{+12.9}_{-8.9}$	$127.5^{+44.4}_{-40.9}$	Ours
$^2_{10}[70,2_2^{+}]$										
1975^{+50}_{-50}										
						$N(1520)\pi$	$N(1535)\pi$	$N(1650)\pi$		
$\Delta(1874)_{\frac{1}{2}}^{+}$	$2.8^{+0.5}_{-0.4}$	$2.5^{+1.8}_{-2.3}$	$0.8^{+0.0}_{-0.1}$	$2.1^{+0.4}_{-1.1}$	$6.3^{+2.2}_{-1.9}$	$19.2^{+18.0}_{-10.9}$	$269.2^{+80.4}_{-79.8}$	$28.9^{+23.7}_{-17.6}$	$331.8^{+122.4}_{-108.7}$	Ours
$^2_{10}[70,0_2^{+}]$										
1874^{+50}_{-50}										
						$N(1520)\pi$	$N(1680)\pi$	$\Delta(1700)\pi$		
$\Delta(1973)_{\frac{3}{2}}^{+}$	$4.8^{+0.0}_{-0.1}$	$24.3^{+7.1}_{-6.0}$	$1.8^{+0.1}_{-0.2}$	$2.1^{+1.0}_{-0.8}$	$31.9^{+1.3}_{-2.7}$	$56.4^{+5.2}_{-7.3}$	$10.5^{+8.1}_{-5.7}$	$127.4^{+54.7}_{-53.7}$	$259.2^{+77.5}_{-76.5}$	Ours
$^2_{10}[70,2_2^{+}]$										
1973^{+50}_{-50}										

the unphysical contributions. The whole tensor potential (OBE + OGE) after truncations is shown with a green dotted curve in Fig. 4, which is very similar to the OGE tensor potential. This may be an answer for the question why the OGE tensor potential can give a mixing angle consistent with the data.

With the mixing angle $\theta_S \simeq 25^\circ$, the strong decay properties of the $N(1535)1/2^-$ and $N(1650)1/2^-$ are evaluated within the chiral quark model. Our results together with the experimental data and some other model predictions are given in Table IX. It is seen that our predictions are in reasonably good agreement with the data. The anomalously large $N\eta$ branching ratio of the $N(1535)1/2^-$ and the anomalously small $N\eta$ branching ratio of the $N(1650)1/2^-$ can be well explained. The partial width ratios between the $N\eta$ and $N\pi$ channels for the $N(1535)1/2^-$ and $N(1650)1/2^-$ are predicted to be

$$R_1^{1/2^-} = \frac{\Gamma[N(1535) \rightarrow N\eta]}{\Gamma[N(1535) \rightarrow N\pi]} \simeq 1.28, \quad (41)$$

$$R_2^{1/2^-} = \frac{\Gamma[N(1650) \rightarrow N\eta]}{\Gamma[N(1650) \rightarrow N\pi]} \simeq 0.14. \quad (42)$$

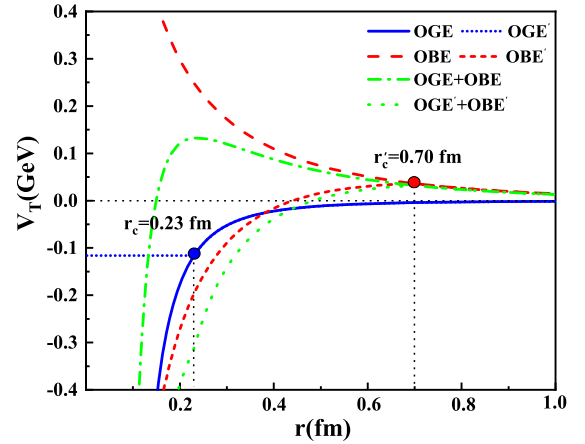


FIG. 4. The tensor potentials as functions of the distance r between two quarks for the $^4s[70, 1_1^-]1^4P_{1/2^-}$ configuration, which is the dominant component of the $N(1650)1/2^-$. The solid and dashed curves represent the potentials arising from the one-gluon and one-pion exchanges (labeled with OGE and OBE), respectively. The short dotted and short dashed curves represent the one-gluon- and one-pion-exchange potentials when a truncation is carried out; they are labeled with OGE' and OBE', respectively. The dash-dotted curve represents the whole potential without truncations, which is labeled with OGE + OBE in the figure, while the dotted curve represents the whole potential with truncations, which is labeled with OGE' + OBE'.

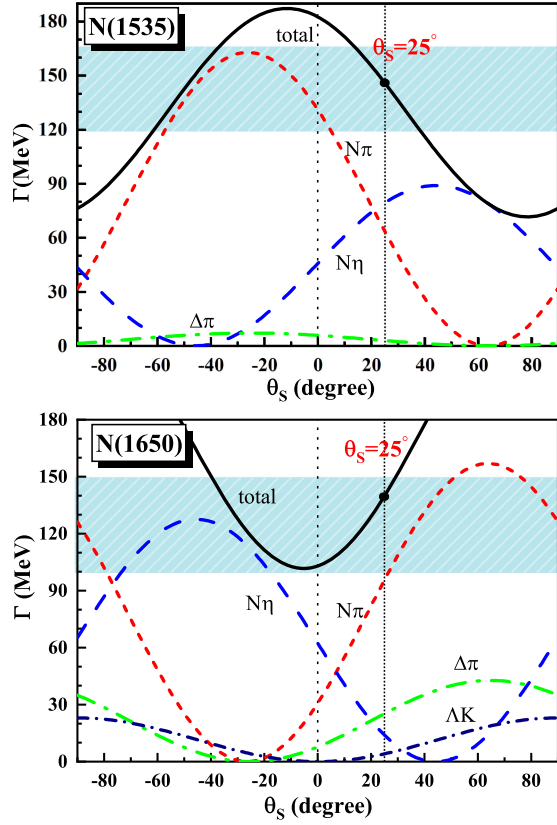


FIG. 5. Partial decay widths for the $N(1535)1/2^-$ and $N(1650)1/2^-$ resonances as functions of the mixing angle θ_S .

The predicted ratio $R_1^{1/2^-}$ for $N(1535)1/2^-$ is consistent with the ratio 0.95 ± 0.3 extracted from the data of π and η electroproduction off the proton by the CLAS Collaboration [121] and the new solution $1.09^{+0.37}_{-0.28}$ extracted from the $\pi N \rightarrow \pi N, \eta N$ data by the Zagreb group [122]. The predicted ratio $R_2^{1/2^-} = 0.14$ for $N(1650)1/2^-$ is consistent with the new solution $0.17^{+0.07}_{-0.08}$ extracted from the $\pi N \rightarrow \pi N, \eta N$ data by the Zagreb group [122].

To see the mixing angle dependency of the decay properties, the partial decay widths of the $N(1535)1/2^-$ and $N(1650)1/2^-$ as functions of the mixing angle θ_S are plotted in Fig. 5. It is found that, without configuration mixing ($\theta_S = 0$), the $N(1535)1/2^-$ dominantly decays into the $N\pi$ and $N\eta$ channels with branching fractions of $\sim 70\%$ and $\sim 25\%$, while $N(1650)1/2^-$ decays into the $N\eta$ and $N\pi$ channels with branching fractions of $\sim 60\%$ and $\sim 30\%$, respectively. Taking $\theta_S = 25^\circ$, the $N\eta$ becomes the dominant decay mode of $N(1535)1/2^-$ with a decay rate of $\sim 55\%$, while for $N(1650)1/2^-$, the decay rate of the $N\eta$ is suppressed to $\sim 9\%$, and the $N\pi$ and $\Delta\pi$ become the dominant decay modes with a decay rate of $\sim 69\%$ and $\sim 18\%$.

As a whole, within the constituent quark model, both the mass and strong decay properties of the $N(1535)1/2^-$ and $N(1650)1/2^-$ can be reasonably understood. Including the

OBE tensor potential together with that of OGE, one can explain the mixing angle extracted from the $\gamma p \rightarrow \eta p, \pi^0 p$ data. To better understand the tensor potentials, more accurate measurements of the partial width decay ratios between $N\eta$ and $N\pi$ for the $N(1535)1/2^-$ and $N(1650)1/2^-$ are needed in future experiments.

2. $N(1520, 1700)3/2^-$ and three-body spin-orbit potentials

In the constituent quark model, the $N(1520)3/2^-$ and $N(1700)3/2^-$ resonances are assigned to the ${}^2_8[70, 1^-]1^4P_{3/2^-}$ and ${}^4_8[70, 1^-]1^4P_{3/2^-}$ states, respectively. From Table V, it is seen that the masses of both $N(1520)3/2^-$ and $N(1700)3/2^-$ can be well described within various quark models. Configuration mixing between these two states may also exist. Taking the following mixing scheme,

$$\begin{pmatrix} N(1520) \\ N(1700) \end{pmatrix} = \begin{pmatrix} \cos \theta_D & -\sin \theta_D \\ \sin \theta_D & \cos \theta_D \end{pmatrix} \begin{pmatrix} {}^2_8[70, 1^-]_{\frac{3}{2}} \\ {}^4_8[70, 1^-]_{\frac{3}{2}} \end{pmatrix}, \quad (43)$$

the mixing angle is predicted to be $\theta_D \simeq 8^\circ$ based on the tensor interactions arising from the OGE [118,119].

In the present work, it is found that the mixing angle may reach up to a fairly large value $\theta_D \simeq 20^\circ$, if the three-body components of the spin-orbit interactions arising from the OGE are included. In the following, we give a review of the three-body components of the spin-orbit potential since it may be unfamiliar to someone. For example, considering the spin-orbit interaction between two light u/d quarks, from Eq. (4) one has

$$V_G^{LS}(r_{12}) = \frac{1}{2\sqrt{2}\tilde{m}_u^2} \left(\frac{\alpha_S}{\rho^3} - \frac{b}{2\rho} \right) \mathbf{L}_\rho \cdot (\mathbf{S}_1 + \mathbf{S}_2) - \frac{1}{4\sqrt{6}\tilde{m}_u^2} \left(\frac{2\alpha_S}{\sqrt{3}\rho^3} + \frac{b}{\rho} \right) \boldsymbol{\rho} \times \mathbf{p}_\lambda \cdot (\mathbf{S}_1 - \mathbf{S}_2). \quad (44)$$

In the above equation, the first term is the two-body spin-orbit potential which is widely adopted in the literature, while the second term proportional to $\boldsymbol{\rho} \times \mathbf{p}_\lambda$ just corresponds to the three-body spin-orbit potential [14,119]. Unfortunately, this term was often neglected in the previous study of the baryon spectrum. In our calculations, it is found that the contributions from the OGE two-body spin-orbit potentials nearly cancel out those from the OGE tensor potentials, thus, the configuration mixing between the two low-lying $J^P = 3/2^-$ states is mainly caused by the three-body spin-orbit potentials.

The sizable configuration mixing between $N(1520)3/2^-$ and $N(1700)3/2^-$ seems to be needed for reasonably describing the reactions $\gamma p \rightarrow \eta p$ in the resonances region [66]. By fitting the reaction data, the mixing angle is

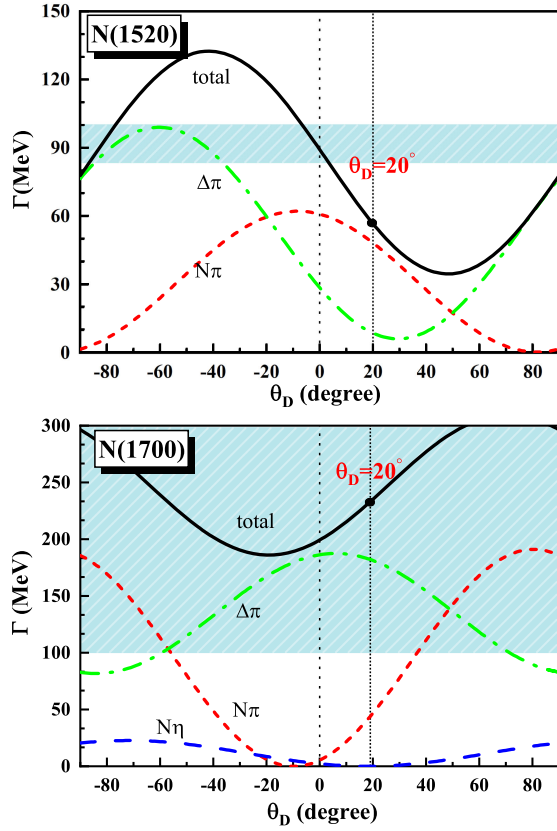


FIG. 6. Partial decay widths for the $N(1520)3/2^-$ and $N(1700)3/2^-$ resonances as functions of the mixing angle θ_D .

determined to be $\theta_D \simeq 21^\circ$ [66], which is consistent with $\theta_D \simeq 20^\circ$ obtained with the three-body OGE spin-orbit potentials in the present work.

With the mixing angle $\theta_D \simeq 20^\circ$, the strong decay properties of the $N(1520)3/2^-$ and $N(1700)3/2^-$ are evaluated within the chiral quark model. Our results together with the experimental data and some other model predictions are given in Table IX. It is seen that our predictions are in reasonable agreement with the data. Our predicted width of $N(1520)3/2^-$, $\Gamma \simeq 57$ MeV, is much narrower (about a factor of 3) smaller than $\Gamma \simeq 180$ MeV predicted for $N(1700)3/2^-$. The $N\pi$ and $\Delta\pi$ are the two important decay channels of both $N(1520)3/2^-$ and $N(1700)3/2^-$. The partial width ratios between the $N\pi$ and $\Delta\pi$ for these two states are predicted to be

$$R_1^{3/2-} = \frac{\Gamma[N(1520) \rightarrow N\pi]}{\Gamma[N(1520) \rightarrow \Delta\pi]} \simeq 5.7, \quad (45)$$

$$R_2^{3/2-} = \frac{\Gamma[N(1700) \rightarrow N\pi]}{\Gamma[N(1700) \rightarrow \Delta\pi]} \simeq 0.26, \quad (46)$$

which are close to the upper limit of the estimated ratios ~ 1.6 – 3.0 , ~ 0.08 – 0.30 , respectively, from the PDG [110]. It

should be mentioned that these ratios predicted from various works are very different, which can be seen from Table IX. From the Fig. 6, it is also found that the ratio $R_2^{3/2-}$ is sensitive to the mixing angle. Thus, a precise measurement of $R_2^{3/2-}$ is very useful for not only determining the mixing angle, but also testing the various predictions in the literature.

As a whole, within the constituent quark model, both the mass and strong decay properties of the $N(1520)3/2^-$ and $N(1700)3/2^-$ can be reasonably understood. The three-body spin-orbit potential arising from the OGE can cause a fairly large mixing angle $\theta_D \simeq 20^\circ$ for these two resonance states. To test the large mixing angle due to the three-body spin-orbit potentials, an accurate measurement of the partial width decay ratio between $N\pi$ and $\Delta\pi$ for the $N(1700)3/2^-$ may be helpful.

3. $N(1440)1/2^+$ and mass reversal problem

In the quark model, the Roper resonance $N(1440)1/2^+$ is usually assigned as the first radially excited state $2^8[56, 0_2^+][2^2S_{1/2}^+]$. The measured mass of $N(1440)1/2^+$ is about 90 MeV smaller than the first orbitally excited state $N(1535)1/2^-$. However, in the common quark potential model based on OGE, the predicted mass of the $N(1440)1/2^+$ is about 80 MeV larger than the $N(1535)1/2^-$. This is the well-known “mass reversal problem.”

In the present work, it is found that the so-called mass reversal problem can be naturally overcome by including the OBE interactions together with the OGE potentials. It should be emphasized that the OBE interactions adopted for the mass spectrum calculations are described within the same framework, i.e., the chiral quark model, for dealing

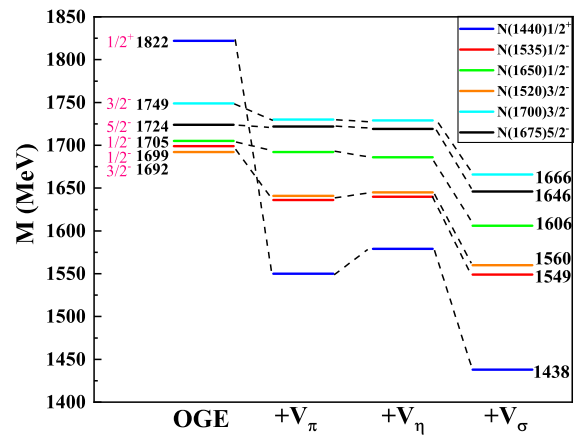


FIG. 7. Illustration of the dynamic mechanism for the mass reversal between $N(1440)1/2^+$ and the $1P$ -wave nucleon states. In the figure, four scenarios are shown. From left to right, OGE stands for the case with the OGE potentials only, while $+V_\pi$, $+V_\eta$, and $+V_\sigma$ stand for the cases in which the π -, η -, and σ -exchange potentials are further included one by one.

with the strong decays. The parameters in the OBE potentials have been well determined by the strong decay properties of the baryon resonances. The dynamic mechanism of mass reversal is explained as follows.

From Fig. 7, it is clearly seen that the one-pion-exchange (OPE) potential is responsible for the mass reversal between the $1P$ states with negative parity and the Roper resonance $N(1440)1/2^+$. Our conclusion is consistent with that based on the Goldstone-boson-exchange interaction models [15,18,26] and the lattice calculation of valence QCD [123–125]. In Table VII, the contributions of each part of the Hamiltonian to the baryon mass are given. One can see that the central part of the OPE potential decreases the mass of $N(1440)1/2^+$ with a fairly large value of ~ 500 MeV, however, for the $1P$ states, the mass reduction due to OPE interactions is no more than 70 MeV. This leads to a mass reversal between the $N(1440)1/2^+$ and the $1P$ states. On the other hand, one can find that in the $1P$ states there is an obvious gap of about 100 MeV between the $|70,^2 8\rangle$ (dominant) states $N(1535)1/2^-$ and $N(1520)3/2^-$ and the $|70,^4 8\rangle$ (dominant) states $N(1650)1/2^-$, $N(1700)3/2^-$, and $N(1675)5/2^-$ due to the central part of the OPE interactions. Furthermore, the η - and σ -exchanged potentials also have some contributions. From Fig. 7 and Table VII, one can see that the central part of the η -exchanged potential increases the mass of $N(1440)1/2^+$ with a sizable value of ~ 30 MeV, while its contributions to the $1P$ states are tiny. The central part of the σ -exchanged potential causes an overall shift for $N(1440)1/2^+$ and the $1P$ states to the low-mass region with a value of about 60–150 MeV.

To better understand the large contribution of the OPE interactions to the mass of $^2 8[56, 0_2^+]2^2 S_{1/2^+}$ [$N(1440)1/2^+$], as an example we further carry out an analysis of the matrix elements of the OBE potentials for the $2S$ states $^2 8[56, 0_2^+]2^2 S_{1/2^+}$, $^2 8[70, 0_2^+]2^2 S_{1/2^+}$, and $^4 8[70, 0_2^+]2^2 S_{3/2^+}$. The matrix elements of the central parts of OBE potentials can be decomposed into the product of the spatial integral part and the spin-isospin factor contributed by the $(\mathbf{S}_i \cdot \mathbf{S}_j)(\lambda_i^a \cdot \lambda_j^a)$ operator in the spin-flavor space. For the $2S$ states, in the spatial integral part there are five terms,

$$\begin{aligned} \langle o \rangle_1 &\equiv \langle \psi_{100}^\rho(d_\ell) \psi_{000}^\lambda(d_\ell) | V_{\pi/\eta/\sigma}^C | \psi_{100}^\rho(d_{\ell'}) \psi_{000}^\lambda(d_{\ell'}) \rangle, \\ \langle o \rangle_2 &\equiv \langle \psi_{100}^\lambda(d_\ell) \psi_{000}^\rho(d_\ell) | V_{\pi/\eta/\sigma}^C | \psi_{100}^\lambda(d_{\ell'}) \psi_{000}^\rho(d_{\ell'}) \rangle, \\ \langle o \rangle_3 &\equiv \langle \psi_{100}^\lambda(d_\ell) \psi_{000}^\rho(d_\ell) | V_{\pi/\eta/\sigma}^C | \psi_{100}^\rho(d_{\ell'}) \psi_{000}^\lambda(d_{\ell'}) \rangle, \\ \langle o \rangle_4 &\equiv \langle \psi_{100}^\rho(d_\ell) \psi_{000}^\lambda(d_\ell) | V_{\pi/\eta/\sigma}^C | \psi_{100}^\lambda(d_{\ell'}) \psi_{000}^\rho(d_{\ell'}) \rangle, \\ \langle o \rangle_5 &\equiv \langle \psi_{01m}^\lambda(d_\ell) \psi_{01m'}^\rho(d_\ell) | V_{\pi/\eta/\sigma}^C | \psi_{01m''}^\rho(d_{\ell'}) \psi_{01m'''}^\lambda(d_{\ell'}) \rangle. \end{aligned}$$

The spin-isospin factors corresponding to these integral terms are given in Table XII. From the table, it is seen that

TABLE XII. The spin-isospin factors of the matrix elements contributed by the chiral potentials for the $2S$ nucleon states.

	V_π^C			V_η^C			V_σ^C		
	$\langle \hat{o} \rangle_{1,2}$	$\langle \hat{o} \rangle_{3,4}$	$\langle \hat{o} \rangle_5$	$\langle \hat{o} \rangle_{1,2}$	$\langle \hat{o} \rangle_{3,4}$	$\langle \hat{o} \rangle_5$	$\langle \hat{o} \rangle_{1,2}$	$\langle \hat{o} \rangle_{3,4}$	$\langle \hat{o} \rangle_5$
$ 56,^2 8, \rangle_{1/2^+}^+$	5/2	5/2	0	-1/2	-1/2	0	1/2	1/2	0
$ 70,^2 8, \rangle_{1/2^+}^+$	5/4	-5/4	-3/2	1/4	-1/4	-1/2	1/4	-1/4	1/2
$ 70,^4 8, \rangle_{3/2^+}^+$	1/4	-1/4	-3/2	-1/4	1/4	1/2	1/4	-1/4	1/2

large contribution of the OPE potential to the $N(1440)1/2^+$ is due to the two aspects: (i) the spin-isospin factors of π exchange are much larger than those for the other two states; (ii) the different integral terms contributed to the matrix element are constructive due to the same sign of the spin-isospin factors; however, for the other two $2S$ states the matrix element contributed by $\langle o \rangle_{1,2}$ has a large cancellation with the $\langle o \rangle_{3,4}$ term due to the opposite signs of the spin-isospin factors.

It should be mentioned that, in Ref. [25], the authors also studied the nucleon spectrum within a similar potential model including both the OGE and OBE potentials. Unfortunately, they did not obtain a successful explanation of the low-mass nature of $N(1440)1/2^+$. The reason is because, in their calculations, the cross terms $\langle o \rangle_{3,4}$ for $N(1440)1/2^+$ were neglected as an approximation. These terms are found to be important here and can account for the mysterious mass reversal problem between the $2S$ and $1P$ states.

The strong decay properties of the $N(1440)1/2^+$ resonance can be well understood with the chiral dynamics as well. Within the chiral quark model, the decay width of $N(1440)1/2^+$ is predicted to be

$$\Gamma \simeq 389 \text{ MeV}, \quad (47)$$

which is consistent with the PDG value $\Gamma_{\text{exp}} = 350 \pm 100$ MeV [110]. The decays are governed by the $N\pi$ channel with a branching fraction of $\sim 90\%$, which is a factor of ~ 1.2 larger than the upper limit of the observation $55\%–75\%$ [110]. Furthermore, there is a sizable decay rate into the $\Delta\pi$ channel, and the branching fraction is predicted to be $\sim 7\%$, which is comparable with the experimental data $6\%–27\%$ [110]. It should be mentioned that the relativistic correction term $\mathcal{H}^{RC}$ plays a crucial role in the $N(1440) \rightarrow N\pi$ decay process. If only with the nonrelativistic term $\mathcal{H}^{NR}$, the partial width of $\Gamma[N(1440) \rightarrow N\pi]$ will be notably (a factor of ~ 2) underestimated. In the recent studies, it is found that the $\mathcal{H}^{RC}$ is crucial not only for describing the strong decays of the Roper(-like) baryon states [108,109], but also for describing the strong decays of the first radially excited heavy-light meson states [90].

As a whole with the chiral dynamics, one can not only explain the mass reversal between $N(1440)1/2^+$ and the

1 P -wave nucleon states, but also understand their strong decay properties (see Table IX). The central part of the one-pion-exchange interaction is responsible for the mass reversal.

4. Positive parity nucleon resonances around 1.7 GeV

In the mass region of ~ 1.7 GeV, there are three positive parity nucleon resonances $N(1710)1/2^+$, $N(1720)3/2^+$, and $N(1680)5/2^+$ listed in the RPP [110]. According to the observed mass and parity, they may be candidates of some low-lying states in the $N = 2$ shell predicted in the quark model.

Commonly, the four-star resonance $N(1710)1/2^+$ is assigned to the $2S$ -wave state ${}^2_8[70, 0_2^+]2^2S_{1/2^+}$ classified in the quark model. This state represents a new feature: its wave function contains components of both orbital and radial excitations, while in the orbital excitation component the two angular momenta l_λ and l_ρ are both one and couple to zero. With this assignment, our predicted mass

$$M \simeq 1824 \text{ MeV} \quad (48)$$

is about 90 MeV larger than the upper limit of the observed value $M_{\text{exp}} = 1710 \pm 30$ MeV [110]. From Table V, it is seen that the mass of $N(1710)1/2^+$ is also overestimated by ~ 50 – 90 MeV in most of the quark models, e.g., Refs. [14, 18, 21, 26, 46]. On the other hand, the strong decay properties are also studied; our results have been given in Table IX. Our predicted width,

$$\Gamma \simeq 168 \text{ MeV}, \quad (49)$$

is comparable with the width $\Gamma_{\text{exp}} = 140 \pm 60$ MeV extracted from the reaction data [110]. From Table IX, one can see that the ${}^2_8[70, 0_2^+]2^2S_{1/2^+}$ state mainly decays into the $N(1440)\pi$, $N(1535)\pi$, $N\pi$, and $\Delta\pi$ channels with branching fractions $\sim 55\%$, $\sim 31\%$, 6% , and 5% , respectively. Our predicted branching fractions for the $N(1535)\pi$, $N\pi$, and $\Delta\pi$ channels are consistent with the observations [110]. The large decay rate of $N(1710)1/2^+ \rightarrow N\pi\pi$ (14–48%) observed in experiments may be mainly contributed by the $N(1440)\pi$ and $N(1535)\pi$ channels via cascade decays. Our predicted branching fraction for the $K\Lambda$ channel is a tiny value $\sim 0.3\%$, which is consistent with the result $1.8 \pm 1.5\%$ extracted by using an updated multichannel energy-dependent partial-wave analysis method [126]. However, our predicted branching fraction for the ηN channel, $\sim 3\%$, is notably smaller than the experimental data 10%–50% [110].

The four-star resonance $N(1720)3/2^+$ is usually assigned to the $1D$ -wave state ${}^2_8[56, 2_2^+]1^2D_{3/2^+}$ classified in the quark model. With this assignment, our predicted mass,

$$M \simeq 1817 \text{ MeV}, \quad (50)$$

is about 70 MeV larger than the upper limit of the observed mass $M_{\text{exp}} = 1715 \pm 35$ MeV. The mass of $N(1720)3/2^+$ is often overestimated in the literature, e.g., Refs. [14, 21]. Furthermore, our predicted width,

$$\Gamma \simeq 136 \text{ MeV}, \quad (51)$$

is close to the lower limit of the measured range of ~ 150 – 400 MeV [110]. The details of the strong decay properties have been given in Table IX. According to our predictions, the ${}^2_8[56, 2_2^+]1^2D_{3/2^+}$ mainly decays into the $N(1440)\pi$, $N(1520)\pi$, and $N\pi$ channels with branching fractions $\sim 65\%$, $\sim 13\%$, and 15% , respectively. While the decay rates into the $\Delta\pi$, $N\eta$, and $K\Lambda$ channels are on the order of 1%. Our predicted branching fractions for the $N\pi$, $N\eta$, and $K\Lambda$ channels are in agreement with the observations [110]. However, the observed branching fractions, 47%–89% and $< 2\%$, for both the $\Delta\pi$ and $N(1440)\pi$ channels [110] are inconsistent with our predictions. The large decay rate of $N(1720)3/2^+ \rightarrow N\pi\pi$ ($> 50\%$) observed in experiments may be mainly contributed by the $N(1440)\pi$ channel via cascade decays.

The four-star resonance $N(1680)5/2^+$ listed in RPP might correspond to the low-mass mixed state via the ${}^2_8[56, 2_2^+]1^2D_{5/2^+}$ – ${}^2_8[70, 2_2^+]1^2D_{5/2^+}$ mixing (see Table VII). Their mixing is mainly induced by the spin-spin interactions. Our predicted mass,

$$M \simeq 1775 \text{ MeV}, \quad (52)$$

is ~ 90 MeV larger than the PDG average data $M_{\text{exp}} = 1685 \pm 5$ MeV [110]. When neglecting the contributions from the $N\rho$, $N\sigma$, and $N\pi\pi$ channels, our predicted width,

$$\Gamma \simeq 46 \text{ MeV}, \quad (53)$$

is about one-third of the observed one $\Gamma_{\text{exp}} \simeq 115$ – 130 MeV [110]. The details of the strong decay properties have been given in Table IX. According to our predictions, the $N\pi$ should be the dominant channel of the $N(1680)5/2^+$, while its decay rates in the other channels, such as $\Delta\pi$, $N\eta$, and $K\Lambda$ are tiny. These predicted decay properties are consistent with the observations [110].

As a whole, assigning the $N(1710)1/2^+$, $N(1720)3/2^+$, and $N(1680)5/2^+$ resonances to the low-lying nucleon excitations in the $N = 2$ shell, only a very rough description of their properties can be obtained in theory. Our predicted masses are systematically larger than the center values of PDG [110] by ~ 100 MeV. Some other dynamic mechanisms, such as coupled-channel effects, may be responsible for their low-mass nature. The $N(1710)1/2^+$ and $N(1720)3/2^+$ resonances may be better established in

the three-body final state $N\pi\pi$ via the cascade decays, since they may dominantly decay into the $N(1440)\pi$ channel.

5. Positive parity nucleon resonances around 1.9 GeV

In the higher mass region around 1.9 GeV, there are several positive parity nucleon resonances, such as $N(1860)5/2^+$, $N(1880)1/2^+$, $N(1900)3/2^+$, and $N(1990)7/2^+$, listed in the RPP [110]. These states may be candidates of the nucleon excitations in the $N = 2$ shell predicted within the constituent quark models (see Table V).

The four-star resonance $N(1900)3/2^+$ is usually assigned to the second orbital excitation of nucleon ${}^8[70, 2_2^+]1^2D_{3/2^+}$. With this assignment, our predicted mass and width,

$$M \simeq 1922, \quad \Gamma \simeq 222 \text{ MeV}, \quad (54)$$

are consistent with the PDG values $M_{\text{exp}} \simeq 1920 \pm 30$ and $\Gamma_{\text{exp}} \simeq 210 \pm 110$ MeV, respectively [110]. The details of our predicted decay properties have been given in Table X. It is found that the $N(1900)3/2^+$ as the ${}^8[70, 2_2^+]1^2D_{3/2^+}$ assignment should dominantly decay into the $N(1520)\pi$, $N(1440)\pi$, and $\Delta\pi$ with branching fractions $\sim 46\%$, $\sim 28\%$, and $\sim 11\%$, respectively. Large decay rates into $\Delta\pi$ and $N(1520)\pi$ have been observed in experiments, however, the extracted branching fractions for the $\Delta\pi$ and $N(1520)\pi$ channels, $\sim 30\%$ – 70% and $\sim 7\%$ – 23% [127], are notably different from our predictions. The large decay rate of $N(1900)3/2^+ \rightarrow N\pi\pi$ ($> 56\%$) observed in experiments [110] may be mainly contributed by the $N(1440)\pi$ and $N(1520)\pi$ channels via cascade decays. Our predicted branching fractions for the $N\pi$ and $N\eta$ channels, $\sim 4.3\%$ and $\sim 1.5\%$, are consistent with the values extracted from the reaction data [126,127]. It should be mentioned that the decay rates of $N(1900)3/2^+$ into the $K\Lambda$ and $K\Sigma$ channels are predicted to be on the order of $\mathcal{O}(10^{-3})$, which is about 1–2 orders of magnitude smaller than the PDG values [110].

The three-star resonance $N(1880)1/2^+$ may be assigned to the excited nucleon state ${}^8[70, 2_2^+]1^4D_{1/2^+}$ classified in the quark model. According to our predictions, the ${}^8[70, 2_2^+]1^4D_{1/2^+}$ slightly mixes with the ${}^8[20, 1_2^+]1^2P_{1/2^+}$ (see Table VII). The mass and width are predicted to be

$$M \simeq 1948, \quad \Gamma \simeq 636 \text{ MeV}, \quad (55)$$

which are in good agreement with the determinations $M_{\text{exp}} \simeq 1967 \pm 20$ and $\Gamma_{\text{exp}} \simeq 500 \pm 70$ MeV from an updated multichannel energy-dependent partial-wave analysis of πN scattering in Ref. [126]. It should be mentioned that our predicted mass is about 60 MeV larger than the PDG averaged mass, ~ 1880 MeV [110], while our

predicted width is much broader than the PDG value ~ 200 – 400 MeV [110]. From Table X, one find that the $N(1880)1/2^+$, as the ${}^8[70, 2_2^+]1^4D_{1/2^+}$ dominant state, mainly decays into the $\Delta\pi$, $\Delta(1620)\pi$, and $N(1535)\pi$ channels with branching fractions $\sim 46\%$, $\sim 33\%$, and $\sim 16\%$, respectively, while the branching fractions for the $N\pi$, $N\eta$, and $K\Sigma$ channels are predicted to be in the range of $\sim 0.5\%$ – 1.0% . Most of the predicted branching fractions are in the PDG ranges.

The two-star resonance $N(1860)5/2^+$ may be experimental evidence for the mixed state dominated by the ${}^8[70, 2_2^+]1^2D_{5/2^+}$ (see Table VII). For this state, the mass and width are predicted to be

$$M \simeq 1944 \text{ MeV}, \quad \Gamma \simeq 70 \text{ MeV}, \quad (56)$$

which are comparable with the mass $M_{\text{exp}} \simeq 1882 \pm 10$ and $\Gamma_{\text{exp}} \simeq 95 \pm 20$ MeV determined by the partial-wave analysis of the πN scattering data [128]. Our predicted width is also close to the lower limit of the width $\Gamma_{\text{exp}} \simeq 122 \pm 41$ MeV extracted by using the Laurent + Pietarinen method from the $\pi N \rightarrow \pi N$ reaction [129]. It should be mentioned that there exist large uncertainties in the extracted masses and widths from different Partial Wave Analysis (PWA) groups. A very broad width $\Gamma_{\text{exp}} \simeq 376 \pm 58$ MeV was extracted by the PWA group of Kent State University [126]. According to our predicted decay properties given in Table X, the $N(1860)5/2^+$ as the ${}^8[70, 2_2^+]1^2D_{5/2^+}$ assignment should dominantly decay into the $\Delta\pi$ channel with a branching fraction $\sim 58\%$; furthermore, there are sizable decay rates into the $N\pi$, $N\eta$, $N(1520)\pi$, $N(1675)\pi$, and $N(1680)\pi$ with a comparable branching fraction $\sim 1\%$ – 12% . Our predicted branching fractions for $N(1860)5/2^+ \rightarrow \Delta\pi, N\pi, N\eta$ ($\sim 58\%$, $\sim 1\%$, and $\sim 3\%$) are consistent with the PDG values [110].

The two-star resonance $N(2000)5/2^+$ may be experimental evidence for the ${}^8[70, 2_2^+]1^4D_{5/2^+}$ state. Assigning the $N(2000)5/2^+$ to this state, the pole mass $M_{\text{exp}} \simeq 1900$ and width $\Gamma_{\text{exp}} \simeq 123$ MeV extracted from the pion- and photon-induced reactions with the coupled-channel PWA method [130] are comparable with our predicted mass and width

$$M \simeq 1962, \quad \Gamma \simeq 143 \text{ MeV}, \quad (57)$$

respectively. It should be mentioned that a larger mass, 2030 ± 30 MeV, and a broader width, 380 ± 60 MeV, were extracted by the CBELSA/TAPS Collaboration from the $\gamma p \rightarrow p\pi^0\pi^0$ reaction [127]. According to our predicted decay properties given in Table X, the $N(2000)5/2^+$ as the ${}^8[70, 2_2^+]1^4D_{5/2^+}$ assignment should dominantly decay into the $\Delta\pi$ and $N(1675)\pi$ channels with branching fractions

$\sim 31\%$ and $\sim 62\%$, respectively. The decay rates into the $N\pi$ and $N\eta$ are on the order of magnitude $\sim 1\%$. The branching fractions for $N(2000)5/2^+ \rightarrow \Delta\pi, N\eta, N\pi$ predicted in theory are comparable with the PDG values [110]. The large decay rate (35%–90%) of $N(2000)5/2^+ \rightarrow N\pi\pi$ may be contributed by the cascade decays $N(2000)5/2^+ \rightarrow \Delta\pi, N(1675)\pi \rightarrow N\pi\pi$.

The two-star resonance $N(1990)7/2^+$ may correspond to the second orbital excitation of nucleon $^48[70, 2_2^+]1^4D_{7/2^+}$, which is the only nucleon state with $J^P = 7/2^+$ in the $N = 2$ shell classified in the quark model. This $J^P = 7/2^+$ state should be a fairly narrow state. Its mass and width are predicted to be

$$M \simeq 1936, \quad \Gamma \simeq 63 \text{ MeV}, \quad (58)$$

respectively. As a two-star resonance rated by the PDG, the mass and width of $N(1990)7/2^+$ are still not well constrained by experiments [110]. Recently, the pole position of the $N(1990)7/2^+$ determined with the Jülich-Bonn dynamical coupled-channel model [131] changes notably compared with the PDG values [110]. The newly determined mass $M_{\text{exp}} = 1861 \pm 9 \text{ MeV}$ is much smaller than the PDG value $2020 \pm 80 \text{ MeV}$ [110], while the newly determined width $\Gamma_{\text{exp}} = 72 \pm 5 \text{ MeV}$ is much more narrow than the PDG value $300 \pm 100 \text{ MeV}$ [110]. Considering the $N(1990)7/2^+$ as the $^48[70, 2_2^+]1^4D_{7/2^+}$ assignment, both the mass and the narrow width determined in Ref. [131] are reasonably consistent with our predictions. According to our predicted strong decay properties given in Table X, the $N(1990)7/2^+$ should dominantly decay into the $\Delta\pi$ channel with a branching fraction $\sim 50\%$. Furthermore, the branching fractions for the $N(1990)7/2^+ \rightarrow N\pi, N\eta$ decays are estimated to be $\sim 10\%$ and $\sim 6\%$, respectively, which are slightly larger than the PDG values 2%–6% and $< 3\%$ [110].

As a whole, the $N(1880)1/2^+$, $N(1900)3/2^+$, $N(1860)5/2^+$, $N(2000)5/2^+$, and $N(1990)7/2^+$ may favor the assignments of excited nucleon states in the $N = 2$ shell. However, in this shell, four quark model states $^48[70, 0_2^+]2^4S_{3/2^+}$, $^48[70, 2_2^+]1^4D_{3/2^+}$, $^28[20, 1_2^+]1^2P_{1/2^+}$, and $^28[20, 1_2^+]1^2P_{3/2^+}$ are still missing. These missing states may have a mass of $\sim 1.9 \text{ GeV}$ and a relatively narrow width of approximately in the range of tens to 100 MeV. Their decay properties have been given in Table X. These missing nucleon states very weakly couple to the $N\pi, N\eta, K\Lambda$, and $K\Sigma$ channels, which can naturally explain why they have not been observed in the $N\pi, N\eta, K\Lambda$, and $K\Sigma$ final states via the πN and γN scattering. It is interesting to find that these missing nucleon states couple strongly to the $\Delta\pi$ channel; they may have large potentials to be established in the $N\pi\pi$ final state by using the charmonium cascade decays, such as J/ψ and $\psi(2S) \rightarrow \bar{p}(\Delta\pi)_{N^*} \rightarrow \bar{p}(p\pi)_{\Delta\pi}$, at BESIII.

B. Δ baryons

1. $\Delta(1232)$

The $\Delta(1232)3/2^+$ resonance is the ground state in the Δ baryon spectrum. Its mass can be well described in theory, however, its partial decay width of $\Gamma[\Delta(1232) \rightarrow N\pi]$ is notably underestimated by a factor of ~ 2 in various models (see Table XI). This puzzle may arise from the phase-space factor that we adopt. By adopting an effective phase-space factor (EPSF) as suggested in Ref. [132], the predicted width can be consistent with the data [116]. A similar puzzle also exists in the meson sector. For example, it is found that with the standard relativistic phase space the predicted partial width of $\Gamma[\phi(1020) \rightarrow K\bar{K}] = 2.5 \text{ MeV}$ in a previous work of our group is notably smaller than the measured value 3.5 MeV; with the EPSF this discrepancy is overcome [111].

2. $\Delta(1620)1/2^-$ and $\Delta(1700)3/2^-$

Commonly, the $\Delta(1620)1/2^-$ and $\Delta(1700)3/2^-$ resonances are assigned to the two $1P$ -wave states $^210[70, 1_1^-]1^2P_{1/2^-}$ and $^210[70, 1_1^-]1^2P_{3/2^-}$, respectively. From Table VI, it is found that the masses of these two states were predicted to be degenerate in many works, e.g., Refs. [18, 21, 26, 46]. However, the mass splitting between $\Delta(1620)1/2^-$ and $\Delta(1700)3/2^-$ is a fairly large value $\Delta M \simeq 80 \text{ MeV}$.

The three-body spin-orbit potentials arising from the OGE may be responsible for the mass splitting between $\Delta(1620)1/2^-$ and $\Delta(1700)3/2^-$. It is found that the masses of the two $1P$ -wave states with $J^P = 1/2^-$ and $J^P = 3/2^-$ are exactly degenerated when only the two-body spin-orbit potentials [i.e., the first term of Eq. (44)] are included. In Ref. [14], Capstick and Isgur realized the importance of the three-body spin-orbit potentials that are proportional to $\rho \times \mathbf{p}_\lambda$ [i.e., the second term of Eq. (44)]. Including the three-body spin-orbit potentials, the masses of the two $1P$ -wave states $J^P = 1/2^-$ and $J^P = 3/2^-$ are predicted to be

$$M \simeq 1595, \quad 1655 \text{ MeV}, \quad (59)$$

respectively. Our predicted masses together with the mass splitting, $\Delta M \simeq 60 \text{ MeV}$, are consistent with the experimental data [110]. Similarly, the three-body spin-orbit potentials should cause a notable mass splitting between the two $1P$ -wave Ω states $\Omega(1P)1/2^-$ and $\Omega(1P)3/2^-$, which can be further tested in future experiments.

Furthermore, our predicted decay properties have been given in Table XI. It is seen that the $\Delta(1620)1/2^-$ dominantly decays into the $\Delta\pi$ channel, while the decay rate into the $N\pi$ channel is also sizable. Only including the contributions from the $N\pi$ and $\Delta\pi$ channels, our predicted width of $\Delta(1620)1/2^-$,

$$\Gamma \simeq 46 \text{ MeV}, \quad (60)$$

is comparable with those predictions with the 3P_0 model in Ref. [116]; however, it is a factor of ~ 3 smaller than the PDG averaged data $\Gamma_{\text{exp}} = 130 \pm 20 \text{ MeV}$ [110]. This disparity may be notably reduced if the contributions from the $N\rho$ channel are included. For the $\Delta(1620)1/2^-$, the partial width ratio between the $N\pi$ and $\Delta\pi$ channels is predicted to be

$$\frac{\Gamma[\Delta(1620)1/2^- \rightarrow \Delta\pi]}{\Gamma[\Delta(1620)1/2^- \rightarrow N\pi]} \simeq 1.9, \quad (61)$$

which is consistent with the ratio ~ 2.0 extracted from the reaction data by the Kent State University (KSU) group [126] and CBELSA/TAPS Collaboration [127].

The $\Delta(1700)3/2^-$ resonance mainly decays into the $\Delta\pi$ and $N\pi$ channels. Its width is predicted to be

$$\Gamma \simeq 138 \text{ MeV}, \quad (62)$$

which is consistent with the value $\Gamma_{\text{exp}} = 119 \pm 70 \text{ MeV}$ extracted from πN data by the Pitt-ANL group [133], however, it is much narrower than the PDG value $\Gamma_{\text{exp}} = 300 \pm 80 \text{ MeV}$ [110] and the predictions within the 3P_0 model in Ref. [116] (see Table XI). The partial width ratio between the $N\pi$ and $\Delta\pi$ channels is predicted to be

$$\frac{\Gamma[\Delta(1700)3/2^- \rightarrow \Delta\pi]}{\Gamma[\Delta(1700)3/2^- \rightarrow N\pi]} \simeq 4.2, \quad (63)$$

which is consistent with the value extracted by the KSU group in their recent work [126].

As a whole, the masses and widths of the $\Delta(1620)1/2^-$ and $\Delta(1700)3/2^-$ can be reasonably understood by assigning them to the $1P$ -wave Δ baryons. The three-body spin-orbit potentials arising from the OGE, which are often neglected in the literature, may be responsible for the splitting between the two $1P$ -wave states.

3. $\Delta(1600)3/2^+$

The $\Delta(1600)3/2^+$ is often assigned to the first radially excited Δ resonance with $J^P = 3/2^+$, $^410[56, 0_2^+2^4S_{1/2}^+]$. The mass of this state is predicted to be

$$M \simeq 1694 \text{ MeV}, \quad (64)$$

which is close to the upper limit of the PDG value $M_{\text{exp}} = 1570 \pm 70 \text{ MeV}$ [110].

Taking our theoretically predicted mass of $M = 1694 \text{ MeV}$, within our chiral quark model the width of $\Delta(1600)3/2^+$ is predicted to be

$$\Gamma \simeq 214 \text{ MeV}, \quad (65)$$

which is close to the lower limit of the PDG estimated range of $\Gamma_{\text{exp}} = 250 \pm 50 \text{ MeV}$ [110]. While taking the mass in the range of $1570 \pm 70 \text{ MeV}$ observed from experiments, the width of the $\Delta(1600)3/2^+$ is estimated to be

$$\Gamma \simeq 101_{-44}^{+58} \text{ MeV}, \quad (66)$$

which is consistent with the recently extracted data $\Gamma_{\text{exp}} \simeq 136 \text{ MeV}$ from a PWA by including $K\Sigma$ photo-production [131] within uncertainties; however, it is about a factor of 2 narrower than the PDG averaged width $\Gamma_{\text{exp}} = 250 \pm 50 \text{ MeV}$ [110]. The $\Delta(1600)3/2^+$, as the first radially excited Δ resonance, should mainly decay into the $N\pi$ and $\Delta\pi$ channels. The partial width of the $\Delta\pi$ channel strongly depends on the mass of $\Delta(1600)3/2^+$ that we adopt. In the observed mass range of $M_{\text{exp}} = 1570 \pm 70 \text{ MeV}$, the partial width ratio between the $N\pi$ and $\Delta\pi$ channels is predicted to be

$$\frac{\Gamma[\Delta(1600)3/2^+ \rightarrow \Delta\pi]}{\Gamma[\Delta(1600)3/2^+ \rightarrow N\pi]} \simeq 2 - 6, \quad (67)$$

which is consistent with the PDG estimated range of 2.4–10.3 [110]. The precise measurement of this ratio may be helpful to accurately determine the mass of $\Delta(1600)3/2^+$ from the scattering data.

There may also exist a mass reversal problem between the radially excited state $\Delta(1600)3/2^+$ and the orbitally excited state $\Delta(1620)1/2^-$. Some PWA groups extracted a relatively small mass, $\sim 1500 \text{ MeV}$, for the $\Delta(1600)3/2^+$, which is about 100 MeV smaller than that extracted for the $\Delta(1620)1/2^-$ [7, 127]. However, in most studies in theory the radially excited Δ state should have a larger mass than $\Delta(1620)1/2^-$ (see Table VI). In the present work, it is predicted that the $\Delta(1600)3/2^+$ as the radially excited state $^410[56, 0_2^+2^4S_{1/2}^+]$ should lie $\sim 80 \text{ MeV}$ above $\Delta(1620)1/2^-$. It should be mentioned that our result is consistent with that extracted by the KSU group in their recent work [126].

As a whole, there still exist large uncertainties in the determined mass and width for the $\Delta(1600)3/2^+$ resonance. To better understand the properties of the $\Delta(1600)3/2^+$, more accurate measurements of the resonance parameters and the partial width ratio between the $N\pi$ and $\Delta\pi$ channels are needed.

4. Positive parity Δ resonances around 1.9 GeV

In the higher mass region of $\sim 1.9 \text{ GeV}$, there are several positive parity resonances $\Delta(1910)1/2^+$, $\Delta(1920)3/2^+$, $\Delta(1905)5/2^+$, $\Delta(2000)5/2^+$, and $\Delta(1950)7/2^+$ listed in the RPP [110]. These states just lie in the mass region of the $1D$ -wave Δ states predicted within the constituent quark models (see Table VI).

The four-star resonance $\Delta(1910)1/2^+$ has mass and width of $M_{\text{exp}} = 1900 \pm 50$ and $\Gamma_{\text{exp}} = 300 \pm 100$ MeV, respectively [110]. It may be a candidate of the $^4_{10}[56, 2_2^+]1^4D_{1/2^+}$ state classified in the quark model. With this assignment, both our predicted mass and width,

$$M \simeq 1913, \quad \Gamma \simeq 369 \text{ MeV}, \quad (68)$$

are consistent with the observations [110]. The branching fractions for the $N\pi$ and $N(1440)\pi$ channels are predicted to be $\sim 5\%$ and $\sim 45\%$, which are close to the observed ranges 10%–30% and 3%–45%, respectively [110]. However, our predicted branching fraction, $\sim 1\%$, for the $\Delta\pi$ channel is too tiny to be comparable with the observed value 34%–66% [110]. Furthermore, according to our predictions (see Table XI), the $\Delta(1910)1/2^+$ may have large decay rates into the $N(1650)\pi$ and $\Delta(1620)\pi$ channels with branching fractions $\sim 16\%$ and $\sim 31\%$, respectively. These branching fractions are waiting to be tested in future experiments.

The four-star resonance $\Delta(1950)7/2^+$ listed in RPP [110] should be assigned to the quark model state $^4_{10}[56, 2_2^+]1^4D_{7/2^+}$. With this assignment, both our predicted mass and width,

$$M \simeq 1867, \quad \Gamma \simeq 106 \text{ MeV}, \quad (69)$$

are consistent with the pole mass $M_{\text{exp}} \simeq 1875$ and width $\Gamma_{\text{exp}} \simeq 166$ MeV determined by the πN and γN reaction data with the Jülich-Bonn dynamical coupled-channel approach [131]. However, our predicted width is about a factor of 2 narrower than the measured width $\Gamma_{\text{exp}} = 285 \pm 50$ MeV and the predicted values within the 3P_0 model [116]. The strong decay properties have been given in Table XI. It is seen that the $\Delta(1950)7/2^+$ as the $^4_{10}[56, 2_2^+]1^4D_{7/2^+}$ state may mainly decay into the $N\pi$ and $\Delta\pi$ channels with branching fractions $\sim 48\%$ and $\sim 35\%$, respectively. Our predicted branching fraction for the $N\pi$ channel is in the range of 35%–45% estimated by the PDG [110]. The large decay rate into the $N\pi\pi$ channel, $57\% \pm 20\%$, determined from the $\gamma N \rightarrow \pi^+\pi^-p$ reaction [134] may be mainly contributed by the $\Delta\pi$ channel via a cascade decay.

The four-star resonance $\Delta(1905)5/2^+$ and two-star resonance $\Delta(2000)5/2^+$ listed in the RPP [110] may relate to the $^4_{10}[56, 2_2^+]1^4D_{5/2^+}$ and $^2_{10}[70, 2_2^+]1^2D_{5/2^+}$ states. Our study shows that these two quark model configurations highly mix with each other with a mixing angle $\theta_{1D} \simeq 38^\circ$ mainly due to the spin-dependent interactions, which is consistent with the prediction in Ref. [14]. For the low-mass mixed state, the mass and width are predicted to be

$$M \simeq 1864, \quad \Gamma \simeq 56 \text{ MeV}. \quad (70)$$

It dominantly decays into the $\Delta\pi$ and $N\pi$ channel with branching fractions $\sim 62\%$ and $\sim 23\%$, respectively. While for the high-mass mixed state, the mass and width are predicted to be

$$M \simeq 1975, \quad \Gamma \simeq 127 \text{ MeV}. \quad (71)$$

This high-mass state mainly decays into the $\Delta\pi$ and $N(1675)\pi$ channels with branching fractions $\sim 14\%$ and $\sim 55\%$, respectively.

Assigning the $\Delta(1905)5/2^+$ to the high mixed state, the measured mass $M_{\text{exp}} \simeq 1880$ MeV [110] is consistent with the theoretical prediction; moreover, the measured branching fractions of the $\Delta\pi$ and $N\pi$ channels, $> 48\%$ and $\sim 9\%$ – 15% [110], are also comparable with our predictions. However, the average width $\Gamma_{\text{exp}} \simeq 300$ MeV from the PDG [110] is much broader than our prediction. It should be pointed out that there are large differences in the resonance parameters of $\Delta(1905)5/2^+$ extracted from various PWA groups. To well determine the resonance parameters of $\Delta(1905)5/2^+$, more measurements are needed in future experiments.

The $\Delta(2000)5/2^+$ as a two-star resonance is still not well established in experiments. Its mass and width determined from different PWA groups scatter in fairly large ranges $M_{\text{exp}} \sim 1700$ – 2300 and $\Gamma_{\text{exp}} \sim 100$ – 600 MeV, respectively [110]. Based on the poor observations of $\Delta(2000)5/2^+$, it is difficult to determine whether it corresponds to the high-mass mixed state or not. The high mixed $J^P = 5/2^+$ state couples weakly to the $N\pi$ channel, which might be its missing reason in the observations. To establish the $J^P = 5/2^+$ states, the observation of the $N\pi\pi$ final state via the cascade decay $\Delta^* \rightarrow \Delta\pi \rightarrow N\pi\pi$ may be helpful.

For the three-star resonance $\Delta(1920)3/2^+$, the extracted mass and width from the reaction data are $M_{\text{exp}} = 1920 \pm 50$ and $\Gamma_{\text{exp}} = 300 \pm 60$ MeV, respectively [110]. The $\Delta(1920)3/2^+$ may be a good candidate of the $^4_{10}[56, 2_2^+]1^4D_{3/2^+}$ state classified in the quark model. With this assignment, our predicted mass and width,

$$M \simeq 1906, \quad \Gamma \simeq 156 \text{ MeV}, \quad (72)$$

are roughly comparable with the observations. According to our predicted decay properties given in Table XI, it is seen that the $^4_{10}[56, 2_2^+]1^4D_{3/2^+}$ dominantly decays into the $N(1440)\pi$ channel with a branching fraction of $\sim 27\%$, which is in the range of measurements, 4%–86% [110]. Furthermore, the $\Delta\pi$, $N(1520)\pi$, and $N(1680)\pi$ channels play an important role in the decays: they have a comparable branching fraction of $\sim 24\%$, $\sim 31\%$, and $\sim 9\%$. The large decay rate of the $\Delta(1920)3/2^+ \rightarrow N\pi\pi$ observed in experiments may be contributed by the $\Delta\pi$, $N(1520)\pi$, and $N(1680)\pi$ channels. Our predicted sizable

branching fraction of 4% for the $\Delta(1920)3/2^+ \rightarrow N\pi$ is also in the measured range of 5%–20% [110]. Finally, it should be mentioned that there are large discrepancies in both the mass and width of $\Delta(1920)3/2^+$ extracted by various PWA groups [110].

The other $J^P = 3/2^+$ state ${}^2_{10}[70, 2_2^+]1^2D_{3/2^+}$ is still missing. The mass and width are predicted to be

$$M \simeq 1973, \quad \Gamma \simeq 259 \text{ MeV}, \quad (73)$$

respectively. More details about its decay properties have been given in Table XI. This state dominantly decays into the $\Delta(1700)\pi$, $\Delta\pi$, and $N(1520)\pi$ with branching fractions $\sim 49\%$, $\sim 9\%$, and $\sim 22\%$, respectively. However, the decay rate into the $N\pi$ is very tiny ($\sim 2\%$), which can naturally explain why the ${}^2_{10}[70, 2_2^+]1^2D_{3/2^+}$ state has not been established in the $\pi N/\gamma N \rightarrow \pi N$ reactions. This missing state is likely to be established in the $N\pi\pi$ final state via the cascade decays $\Delta^* \rightarrow \Delta\pi/N(1520)\pi \rightarrow N\pi\pi$.

In addition to the $1D$ -wave states, in the $N = 2$ shell there is a $2S$ -wave state ${}^2_{10}[70, 0_2^+]2^2S_{1/2^+}$. This state is still missing. The mass and width is predicted to be

$$M \simeq 1874, \quad \Gamma \simeq 332 \text{ MeV}, \quad (74)$$

respectively. More details about the mass and decay properties have been given in Tables VIII and XI. Our predicted mass is consistent with the predictions in the literature [14, 21, 26, 28, 46]. This state mainly decays into the $N(1535)\pi$, $N(1650)\pi$, and $N(1520)\pi$ with branching fractions $\sim 81\%$, $\sim 9\%$, and $\sim 6\%$, respectively. The decay rate into $N\pi$ is rather small, $\sim 1\%$. Thus, this state is hard to be established by using the conventional $\pi N/\gamma N \rightarrow \pi N$ reactions. This missing state is most likely to be established in the $N\pi\pi$ final state via the cascade decay $\Delta^* \rightarrow N(1535)\pi \rightarrow N\pi\pi$.

As a whole, the masses and decay properties of $\Delta(1910)1/2^+$, $\Delta(1920)3/2^+$, and $\Delta(1905)7/2^+$ can be reasonably understood within the quark model. However, for the $\Delta(1950)5/2^+$ resonance, there is a discrepancy between our predicted width and the observed one. The existence of the $\Delta(2000)5/2^+$ should be further confirmed in experiments. Around the mass region of 1.9 GeV, three states, i.e., the $2S$ -wave state ${}^2_{10}[70, 0_2^+]2^2S_{1/2^+}$, the $1D$ -wave state ${}^2_{10}[70, 0_2^+]1^2D_{3/2^+}$, and the high-mass mixed $1D$ -wave state with $J^P = 5/2^+$, are still missing due to their very weak couplings to the $N\pi$ channel. The observations of the three-body final state $N\pi\pi$ via the cascade decays might be very helpful to not only establish the missing resonances, but also better understand the nature of the resonances observed in the previous experiments.

IV. SUMMARY

In this work, we systematically study both the mass spectrum and strong decay of the N^* and Δ^* within the quark model framework by combining the chiral dynamics. A reasonable description of the masses and strong decay properties for the well-established states has been achieved. Some key points are emphasized as follows.

The chiral dynamics play an important role in the light baryon spectra. With the chiral dynamics, the mass reversal between $N(1440)1/2^+$ and the $1P$ -wave nucleon resonances together with their strong decay properties can be naturally understood. The central part of the one-pion-exchange interaction is responsible for the mass reversal.

The mixing between the $N(1535)1/2^-$ and $N(1650)1/2^-$ resonances is mainly caused by the OGE tensor potentials in the short distance range. The singular behavior of the $1/r^3$ term in the OBE tensor potentials (mainly contributed by the one-pion exchange) enlarges the unphysical contributions in the short distance region, which cannot be effectively suppressed by the spatial wave functions and form factors. These unphysical contributions should be reasonably removed to reproduce the correct mixing angle.

The OGE three-body spin-orbit potentials, which are often neglected in the literature, play an important role in the baryon spectrum. They not only cause a large configuration mixing between $N(1520)3/2^-$ and $N(1700)3/2^-$, but also are responsible for the $\Delta(1600)1/2^- - \Delta(1700)3/2^-$ splitting.

The $N(1880)1/2^+$, $N(1900)3/2^+$, $N(1860)5/2^+$, $N(2000)5/2^+$, $N(1990)7/2^+$, $\Delta(1910)1/2^+$, $\Delta(1920) \times 3/2^+$, and $\Delta(1905)7/2^+$ listed in the RPP can be assigned to the excitations in the $N = 2$ shell. For the $\Delta(1950)5/2^+$, there is a discrepancy between our predicted width and the observations. The existence of the $\Delta(2000)5/2^+$ should be further confirmed in future experiments.

In the $N = 2$ shell, four isospin $1/2$ states and three isospin $3/2$ states are still missing. These missing states may have a mass of ~ 1.9 GeV and a width approximately in the range of tens to 200 MeV. They very weakly couple to the $N\pi$, $N\eta$, $K\Lambda$, and $K\Sigma$ channels, which can naturally explain why they have not been observed in these channels via the πN and γN scatterings.

Most of the nucleon and Δ baryons in the $N = 2$ shell have large decay rates into the $\Delta(1232)$ or the $1P$ -wave excitations of nucleons. The missing resonances in this shell may have large potentials to be established in the $N\pi\pi$ final state by using the charmonium cascade decays at BESIII.

The low-mass nature of the low-lying positive parity states $N(1710)1/2^+$, $N(1720)3/2^+$, and $N(1680)5/2^+$ cannot be well understood in theory. Our predicted masses are systematically larger than the center values of PDG by ~ 100 MeV. Some other dynamic mechanisms, such as

coupled-channel effects, may be responsible for their low-mass nature.

For lack of space, our results and discussions for the nucleon and Δ baryons in the $N = 3$ shell will be given in another work.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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