

From P_ψ^N and $P_{\psi s}^\Lambda$ to $\bar{T}_{cc}^f$: Symmetry analysis of the interactions in the $(\bar{c}q)(\bar{c}q)$, $(ccq)(\bar{c}q)$, and $(ccq)(ccq)$ dihadron systems

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We investigate the interactions of $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ dihadron systems based on a contact Lagrangian possessing the SU(3) flavor and SU(2) spin symmetries. Under the assumptions of two scenarios for the J^P quantum numbers of $P_\psi^N(4440)$ and $P_\psi^N(4457)$ states, we obtain the parameters $(\tilde{g}_s, \tilde{g}_a)$ introduced from this contact Lagrangian. Then we include the SU(3) breaking effect by introducing a factor g_x , this quantity can be constrained by the experimental mass of $P_{\psi s}^\Lambda(4338)$ state. We can reproduce the mass of $T_{cc}^f(3875)$ state with the parameters extracted from the observed P_ψ^N states, the fairly good description of the masses of the observed $P_\psi^N/P_{\psi s}^\Lambda/T_{cc}^f$ states motivates us to investigate the possible molecular candidates in the $\bar{T}_{cc}^f/P_{\psi c}^\Lambda/H_{\Omega_{ccc}}^\Lambda$ systems within the same framework. Then we proceed to discuss the existences of $\bar{T}_{cc\bar{s}}^\theta/P_{\psi cs}^N/H_{\Omega_{ccc\bar{s}}}^N$ states by investigating the SU(3) breaking effects. Our results show that the states in the $\bar{T}_{cc\bar{s}}^\theta/P_{\psi cs}^N$ systems can hardly form bound states, while the states in the $H_{\Omega_{ccc\bar{s}}}^N$ system can form bound states due to their larger reduced masses.

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I. INTRODUCTION

The discoveries of P_ψ^N [1,2], $P_{\psi s}^\Lambda$ [3,4], and $T_{cc}^f(3875)$ [5,6] states¹ significantly expand our understanding of exotic hadronic states. To elucidate their underlying structures, several theoretical frameworks were proposed, including the molecular interpretations, compact pentaquark/tetraquark states, hadrocharmonium states, and kinematical effects [8–24]. The proximity of the masses of P_ψ^N , $P_{\psi s}^\Lambda$, and $T_{cc}^f(3875)$ states to their respective dihadron thresholds strongly suggests their molecular

configurations. Consequently, the molecular picture has emerged as the most widely accepted interpretation for these manifestly exotic states.

With the accumulation of experimental data, we can anticipate the discoveries of more molecular candidates in the near future. In the molecular picture, the residual strong interaction, primarily driven by the exchanges of light mesons, facilitates the binding of a dihadron system. Specifically, for an S-wave dihadron system consisting of two ground-state heavy-light hadrons, a possible molecular picture is that the characteristic of the residual strong interaction is contingent upon the correlations between the light quark components in each hadron. Consequently, the interactions among different dihadron systems can be interconnected through SU(3) flavor and SU(2) spin symmetries [SU(3)_f ⊗ SU(2)_s symmetry], such symmetry depends on the flavor and spin structures arising from the light degrees of freedom (d.o.f.) within each dihadron system.

Under this picture, the light quark components in dihadron system determine the property of interaction

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¹We adopt the nomenclature proposed by the LHCb collaboration [7] throughout this paper.

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among these two hadrons, while the heavy quark components are important in stabilizing this system. For example, in our previous work [25], by considering the heavy diquark-antiquark symmetry (HDAS) [26–29], we related the $\bar{D}^{(*)}$ and $\bar{D}_s^{(*)}$ mesons to the $\Xi_{cc}^{(*)}$ and $\Omega_{cc}^{(*)}$ baryons, respectively. With the same attractive force, the dibaryon $H_{\Omega_{ccc}}^N$ and $H_{\Omega_{ccc}s}^N$ systems bind deeper than that of the baryon-meson (B-M) P_ψ^N and $P_{\psi s}^N$ systems, respectively.

Thus, such a possible picture suggests that for an S -wave dihadron system composed of two ground heavy-light hadrons, the light quark components and heavy quark components play distinct roles in forming molecules. The $SU(3)_f \otimes SU(2)_s$ symmetry relates the interactions of dihadron systems with different light quark components, i.e., relating the P_ψ^N ($cnn + \bar{c}n$) ($n = u, d$) and $P_{\psi s}^N$ ($cnn + \bar{c}s$ or $cns + \bar{c}n$) states to the $\bar{T}_{cc}^f$ ($\bar{c}n + \bar{c}n$) and $\bar{T}_{cc\bar{s}}^\theta$ ($\bar{c}n + \bar{c}s$) states, respectively. While the HDAS relates the interactions of dihadron systems with different heavy quark components, i.e., relating the $\bar{T}_{cc}^f$ and $\bar{T}_{cc\bar{s}}^\theta$ states to the $P_{\psi c}^N$ ($ccn + \bar{c}n$) and $P_{\psi cs}^N$ ($ccn + \bar{c}s$ or $ccs + \bar{c}n$) states, or relating the $\bar{T}_{cc}^f$ and $\bar{T}_{cc\bar{s}}^\theta$ states to the $H_{\Omega_{ccc}}^f$ ($ccn - ccn$) and $H_{\Omega_{ccc}s}^\theta$ ($ccn - ccs$) states.

In this work, we investigate the interactions of $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^\theta$, $P_{\psi c}^N/P_{\psi cs}^N$, and $H_{\Omega_{ccc}}^N/H_{\Omega_{ccc}s}^N$ systems from the perspective of symmetry. These three sets of systems have identical light quarks, and they are expected to have very similar interactions. Among these systems, the $\bar{T}_{cc}^f$ system attracted most attentions. Based on the molecule or compact tetraquark assumptions, the properties of $\bar{T}_{cc}^f$ states are widely discussed in a large amount of theoretical works [30–55]. As the strange partners of $\bar{T}_{cc}^f$ states, the existences, productions, possible mass spectra, and other relevant topics of $\bar{T}_{cc\bar{s}}^\theta$ states have also been discussed in the literature [49,54–69]. Besides, the triple-charm $P_{\psi c}^N$ molecular pentaquarks have been investigated within the framework of one-boson-exchange model [70,71]. However, to our knowledge, the theoretical studies on the interactions of $P_{\psi cs}^N$, $H_{\Omega_{ccc}}^N$, $H_{\Omega_{ccc}s}^N$ systems are missing at present. This work is devoted to study and relate the interactions of $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^\theta$, $P_{\psi c}^N/P_{\psi cs}^N$, and $H_{\Omega_{ccc}}^N/H_{\Omega_{ccc}s}^N$ systems in a possible unified framework.

This work is organized as follows, we present our framework in Sec. II. In Sec. III, we present our numerical results and discussions for the $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^\theta$, $P_{\psi c}^N/P_{\psi cs}^N$, and $H_{\Omega_{ccc}}^N/H_{\Omega_{ccc}s}^N$ systems. Section IV is devoted to a summary.

II. FRAMEWORK

We list the considered multi-heavy dihadron systems with the strangeness numbers 0 and -1 in Table I. We also list their corresponding thresholds in Table I. Here, we

TABLE I. The considered multiheavy $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ dihadron systems with the strangeness numbers 0 and -1 . To list the thresholds of the considered systems, we adopt the experimental mass of Ξ_{cc}^{++} baryon measured from the LHCb collaboration [72,73], and adopt the theoretical masses of Ξ_{cc}^* and $\Omega_{cc}^{(*)}$ baryons calculated from Ref. [74]. All values are in units of MeV.

$S = 0$	$\bar{D}\bar{D}$	$\bar{D}^*\bar{D}$	$\bar{D}^*\bar{D}^*$
	3734.5	3875.6	4017.1
	$\Xi_{cc}\bar{D}$	$\Xi_{cc}^*\bar{D}$	$\Xi_{cc}^*\bar{D}^*$
	5488.6	5594.2	5630.0
	$\Xi_{cc}\Xi_{cc}$	$\Xi_{cc}^*\Xi_{cc}$	$\Xi_{cc}^*\Xi_{cc}^*$
	7242.8	7348.4	7454.0
$S = -1$	$\bar{D}\bar{D}_s$	$\bar{D}^*\bar{D}_s$	$\bar{D}^*\bar{D}_s^*$
	3836.2	3977.6	3979.4
	$\Xi_{cc}\bar{D}_s$	$\Xi_{cc}^*\bar{D}_s$	$\Xi_{cc}^*\bar{D}_s^*$
	5590.4	5696.0	5733.6
	$\Omega_{cc}\bar{D}$	$\Omega_{cc}^*\bar{D}$	$\Omega_{cc}^*\bar{D}^*$
	5645.2	5739.2	5786.6
	$\Xi_{cc}\Omega_{cc}$	$\Xi_{cc}^*\Omega_{cc}$	$\Xi_{cc}^*\Omega_{cc}^*$
	7399.4	7505.0	7493.4
			7599.0

adopt the experimental mass of Ξ_{cc}^{++} baryon from the LHCb collaboration [72,73], and adopt the theoretical masses of Ξ_{cc}^* and $\Omega_{cc}^{(*)}$ baryons calculated within a relativistic quark model [74].

A. The flavor-spin wave functions for all the considered systems

To describe the flavor-spin wave functions of heavy-light dihadron systems listed in Table I, we introduce three types of notations to denote the total wave functions of the considered dihadron states. Explicitly, we use the

$$|[H_1 H_2]_J^I\rangle = \sum_{m_{I_1} m_{I_2}} C_{I_1, m_{I_1}; I_2, m_{I_2}}^{I, I_z} \phi_{I_1, m_{I_1}}^{H_1} \phi_{I_2, m_{I_2}}^{H_2} \otimes \sum_{m_{S_1} m_{S_2}} C_{S_1, m_{S_1}; S_2, m_{S_2}}^{J, J_z} \phi_{S_1, m_{S_1}}^{H_1} \phi_{S_2, m_{S_2}}^{H_2} \quad (1)$$

to denote the $S = 0$ dihadron state with a definite total isospin I and a total angular momentum J . We use the

$$|[H_1 H_2]_J^{\frac{1}{2}(\pm)}\rangle = \sqrt{\frac{1}{2}} (\phi_{I_1, m_{I_1}}^{H_1} \phi_{I_2, m_{I_2}}^{H_2} \pm \phi_{I_2, m_{I_2}}^{H_2} \phi_{I_1, m_{I_1}}^{H_1}) \otimes \sum_{m_{S_1} m_{S_2}} C_{S_1, m_{S_1}; S_2, m_{S_2}}^{J, J_z} \phi_{S_1, m_{S_1}}^{H_1} \phi_{S_2, m_{S_2}}^{H_2} \quad (2)$$

to denote the $S = -1$ dihadron state that has a definite symmetry in the flavor space and with a total angular momentum J . Finally, we use the

$$|[H_1 H_2]_J\rangle = \phi_{I_1, m_{I_1}}^{H_1} \phi_{I_2, m_{I_2}}^{H_2} \otimes \sum_{m_{S_1}, m_{S_2}} C_{S_1, m_{S_1}; S_2, m_{S_2}}^{J, J_z} \phi_{S_1, m_{S_1}}^{H_1} \phi_{S_2, m_{S_2}}^{H_2} \quad (3)$$

to denote the $H_1 H_2$ state that does not have a definite symmetry in the flavor space but with a definite total angular momentum J . $C_{I_1, m_{I_1}; I_2, m_{I_2}}^{J, J_z}$ and $C_{S_1, m_{S_1}; S_2, m_{S_2}}^{J, J_z}$ are the Clebsch-Gordan coefficients. $\phi_{I_1, m_{I_1}}^{H_1}$ ($\phi_{I_2, m_{I_2}}^{H_2}$) and $\phi_{S_1, m_{S_1}}^{H_1}$ ($\phi_{S_2, m_{S_2}}^{H_2}$) are the flavor and spin wave functions of H_1 (H_2) hadron, respectively.

B. The effective potentials

Since we are interested in the effective potentials near the thresholds, we assume that the considered dihadron systems exchange light mesons via S -wave interactions, and we omit the terms proportional to the transferred momenta q^2 . Then the effective potentials of $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ dihadron systems have the following form

$$V_{\text{Total}} = C_s + C_a S(S). \quad (4)$$

Here, $S(S)$ is the spin factor, this factor is related to the dihadron system from the spin structure of its light d.o.f.. C_s and C_a are the two coupling constants that should be extracted from experiments. According to the structures of these two terms, we tentatively interpret the C_s and $C_a S(S)$ as the contributions from the exchanges of fictitious scalar and axial-vector light mesons, respectively. When we adopt Eq. (4) to describe the effective potentials of $(\bar{c}q)(\bar{c}q)$, $(ccq)(\bar{c}q)$, and $(ccq)(ccq)$ dihadron systems, we need to separately determine the parameters C_s and C_a for each system.

To reduce the number of undetermined coupling constants, we further introduce the flavor factor $\mathcal{I}(I)$ to relate each set of C_s and C_a among these three systems, this factor can be determined from the flavor structures of light d.o.f. in the $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ dihadron systems. Then the effective potentials have the following form

$$V_{\text{Total}} = \tilde{g}_s \mathcal{I}(I) + \tilde{g}_a \mathcal{I}(I) S(S). \quad (5)$$

Here, $\tilde{g}_s$ and $\tilde{g}_a$ are the two new coupling constants obtained by dividing the flavor factor $\mathcal{I}(I)$ with C_s and C_a , respectively.

In principle, the numerical values of $\tilde{g}_s$ and $\tilde{g}_a$ should be slightly different among the considered three systems due to their different reduced masses and low energy dynamics. At present, it is difficult to quantify these effects. Nevertheless, for the considered $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ systems, due to the identical binding mechanism in the molecular picture and their similar configurations, i.e., dihadron systems composed of two heavy-light hadrons, we assume that the values of $\tilde{g}_s$ and $\tilde{g}_a$ are the

same for the $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ systems. With this approximation, we can present a preliminary symmetry analysis to the interactions of the considered three dihadron systems.

As presented from the above discussions, for each dihadron state with specific isospin and total angular momentum quantum numbers, we only need to calculate the isospin factor $\mathcal{I}(I)$ and spin factor $S(S)$ to derive its effective potential. Instead of introducing the contact Lagrangian at hadron level, we adopt an equivalent way [for example, for the pentaquark $(cq q)(\bar{c}q)$ and hexaquark $(cq q)(ccq)$ states, one can find that the spin factors $S(S)$ calculated from our method [25] are consistent with the spin factors calculated from the contact Lagrangian constructed at hadron level [75]], i.e., we introduce a quark-level Lagrangian

$$\mathcal{L} = g_s \bar{q} S q + g_a \bar{q} \gamma_\mu \gamma^5 \mathcal{A}^\mu q \quad (6)$$

to calculate $\mathcal{I}(I)$ and $S(S)$ [76–79]. Here S and $\mathcal{A}^\mu$ are the fictitious scalar octet and axial-vector octet fields, respectively. They can be expanded as

$$S = S_3 \lambda^i + S_2 \lambda^j + S_1 \lambda^8, \quad (7)$$

$$\mathcal{A}^\mu = \mathcal{A}_3^\mu \lambda^i + \mathcal{A}_2^\mu \lambda^j + \mathcal{A}_1^\mu \lambda^8, \quad (8)$$

where λ^i ($i = 1, 2, 3$), λ^j ($j = 4, 5, 6, 7$), and λ^8 are the Gell-Mann matrices. S_3 ($\mathcal{A}_3^\mu$), S_2 ($\mathcal{A}_2^\mu$), and S_1 ($\mathcal{A}_1^\mu$) denote the scalar (axial-vector) isospin triplet, doublet, and singlet fields, respectively. Besides the quark-level Lagrangian introduced in this work, Refs. [80,81] also suggested that the contact interaction between the $\bar{D}^{(*)}$ meson and $\Sigma_c^{(*)}$ baryon can be described in terms of light-quark subfield q_L within heavy mesons and light-diquark subfield d_L within heavy baryons.

Since we include the exchanges of different light scalar (axial-vector) mesons with different isospins, in principle, we should adopt different coupling parameters for the exchanges of light scalar (axial-vector) mesons with different isospins. Here, to reduce the number of undetermined parameters, we constrain them with the SU(3) flavor symmetry and adopt the approximations

$$\tilde{g}_s \propto \frac{g_s^2}{m_S^2} = \frac{g_{ss}^2}{m_{S_s}^2} = \frac{g_{st}^2}{m_{S_t}^2} = \frac{g_{sd}^2}{m_{S_d}^2}, \quad (9)$$

$$\tilde{g}_a \propto \frac{g_a^2}{m_A^2} = \frac{g_{as}^2}{m_{A_s}^2} = \frac{g_{at}^2}{m_{A_t}^2} = \frac{g_{ad}^2}{m_{A_d}^2}. \quad (10)$$

Here, m_{S_s} (m_{A_s}), m_{S_t} (m_{A_t}), and m_{S_d} (m_{A_d}) are the masses of scalar (axial-vector) singlet, triplet, and doublet mesons, respectively. g_{ss} (g_{as}), g_{st} (g_{at}), and g_{sd} (g_{ad}) are the coupling parameters of scalar (axial-vector) singlet, triplet, and doublet mesons, respectively. m_S and m_A are the SU(3)

approximated masses of the scalar and axial-vector light mesons, respectively. With these SU(3) approximations, we can introduce only two redefined parameters $\tilde{g}_s$ and $\tilde{g}_a$ for the couplings of fictitious scalar and axial-vector fields, respectively. The $\tilde{g}_s$ and $\tilde{g}_a$ are proportional to g_s^2/m_S^2 and g_a^2/m_A^2 , respectively, their signs should be determined from the masses of the observed P_c states [1,2].

The effective potential derived from Eq. (6) is

$$V = \tilde{g}_s \lambda_1 \cdot \lambda_2 + \tilde{g}_a \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2. \quad (11)$$

Here, $\lambda_{1(2)}$ and $\sigma_{1(2)}$ are the Gell-Mann and Pauli matrices in the flavor and spin spaces, respectively. The advantage of our approach is that we can relate the coupling constants of different dihadron systems with different isospins and angular momenta through isospin and spin factors, respectively.

The effective potential in Eq. (11) suggests the $SU(3)_f \otimes SU(2)_s$ symmetry among the molecule community composed of two heavy-light hadrons. This symmetry is based on the strong assumption that in the low energy scattering processes, the effective potentials near thresholds could be described by the Lagrangian in Eq. (6). If this Lagrangian could gain the main features of effective potentials for the considered dihadron systems, then the $SU(3)_f \otimes SU(2)_s$ symmetry can relate the mass arrangements of different dihadron molecular systems. This symmetry could be a unique property for the molecule community that are composed of two heavy-light hadrons.

Since in the low energy dihadron scattering processes, it is possible to interpret the same experiment with different effective potentials by selecting appropriate parameters. For example, the OBE model [80,82], effective field theory [83,84], and resonance saturation model [85,86] were also suggested to describe the interactions of dihadron molecular candidates. Thus, the effective potential introduced in Eq. (11) is also a possible option among many other phenomenological models.

When we adopt the Lagrangian in Eq. (6) to describe the interactions of the considered $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ systems, we implicitly introduce the HQS and HADS to render the heavy quark and heavy diquark as spectators in the charmed mesons and double-charm baryons, respectively. In each dihadron system, the interaction only depends on the correlation among the two heavy-light hadrons from their light d.o.f.

For the charmed mesons, the HQS is a good approximation with its violation of the order of Λ_{QCD}/m_Q , while for the double-charm baryons, the HADS manifests well only if when the spatial extent of diquark of size $1/(m_Q v)$ is much smaller than the wavelength of light d.o.f. $1/\Lambda_{QCD}$. Since the violation for HADS is of the order of $\Lambda_{QCD}/(m_Q v)$, this quantity is expected to be 30% for the double-charm baryon system [27,29]. Thus, the HADS is a more approximate symmetry than the HQS.

We can obtain the effective potential for a $[H_1 H_2]_J^I$ (the same for the $[H_1 H_2]_J^{\frac{1}{2}(\pm)}$ or $[H_1 H_2]_J$ case) dihadron state as

$$V_{|[H_1 H_2]_J^I} = \langle [H_1 H_2]_J^I | V | [H_1 H_2]_J^I \rangle. \quad (12)$$

Here, we need to emphasize that the operators $\lambda_1 \cdot \lambda_2$ and $\sigma_1 \cdot \sigma_2$ only act on the light quark components of total wave function $|[H_1 H_2]_J^I\rangle$ (the same for the $[H_1 H_2]_J^{\frac{1}{2}(\pm)}$ or $[H_1 H_2]_J$ case) in the flavor and spin spaces, respectively. Then we can extract the flavor ($\langle \mathcal{O}_{\text{tot}}^f \rangle$) and flavor-spin ($\langle \mathcal{O}_{\text{tot}}^{\text{fs}} \rangle$) matrix elements by calculating the

$$\langle \mathcal{O}_{\text{tot}}^f \rangle = \langle [H_1 H_2]_J^I | \lambda_1 \cdot \lambda_2 | [H_1 H_2]_J^I \rangle \quad (13)$$

and

$$\langle \mathcal{O}_{\text{tot}}^{\text{fs}} \rangle = \langle [H_1 H_2]_J^I | \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2 | [H_1 H_2]_J^I \rangle, \quad (14)$$

respectively. Note that the operators $\mathcal{O}_{\text{tot}}^f$ and $\mathcal{O}_{\text{tot}}^{\text{fs}}$ can be expanded as

$$\mathcal{O}_{\text{tot}}^f = \mathcal{O}_1^f + \mathcal{O}_2^f + \mathcal{O}_3^f, \quad (15)$$

$$\mathcal{O}_{\text{tot}}^{\text{fs}} = \mathcal{O}_1^{\text{fs}} + \mathcal{O}_2^{\text{fs}} + \mathcal{O}_3^{\text{fs}}, \quad (16)$$

with

$$\begin{aligned} \mathcal{O}_1^f &= \sum_{i=1}^3 \lambda_1^i \lambda_2^i, & \mathcal{O}_1^{\text{fs}} &= \sum_{i=1}^3 \lambda_1^i \lambda_2^i \sigma_1 \cdot \sigma_2, \\ \mathcal{O}_2^f &= \sum_{j=4}^7 \lambda_1^j \lambda_2^j, & \mathcal{O}_2^{\text{fs}} &= \sum_{j=4}^7 \lambda_1^j \lambda_2^j \sigma_1 \cdot \sigma_2, \\ \mathcal{O}_3^f &= \lambda_1^8 \lambda_2^8, & \mathcal{O}_3^{\text{fs}} &= \lambda_1^8 \lambda_2^8 \sigma_1 \cdot \sigma_2. \end{aligned}$$

Thus, the matrix elements $\langle \mathcal{O}_1^f \rangle$ ($\langle \mathcal{O}_1^{\text{fs}} \rangle$), $\langle \mathcal{O}_2^f \rangle$ ($\langle \mathcal{O}_2^{\text{fs}} \rangle$), and $\langle \mathcal{O}_3^f \rangle$ ($\langle \mathcal{O}_3^{\text{fs}} \rangle$) represent the proportions of contributions from the exchanges of isospin triplet, doublet, and singlet scalar (axial-vector) mesons, respectively.

As discussed in this section, in the SU(3) limit, the scalar (axial-vector) coupling parameters $\tilde{g}_s$ ($\tilde{g}_a$) for the exchanges of isospin triplet, doublet, and singlet light mesons are the same. However, to take into account the SU(3) breaking effect, in Ref. [87], we suggested that the masses of the exchanged isospin doublet light strange mesons are heavier than that of the exchanged isospin triplet nonstrange light mesons, in the nonrelativistic limit, the propagator from the exchange of isospin doublet light meson is suppressed by the factor $1/m_{\text{ex}}^2$, where m_{ex} is the mass of the exchanged strange meson. Thus, we collectively multiply a SU(3) breaking factor g_x in front of the matrix elements $\langle \mathcal{O}_2^f \rangle$ and $\langle \mathcal{O}_2^{\text{fs}} \rangle$ to suppress the contributions from the exchanges of the isospin doublet scalar and axial-vector light mesons, respectively.

Besides our framework, we notice that Refs. [85] and [86] also attempted to systematize heavy-heavy and heavy-antiheavy hadronic molecules within a unified description. In their framework, the couplings between heavy (antiheavy) hadrons and light pseudoscalar/vector mesons were introduced by using the hidden gauge symmetry model. Since they were only interested in the effective potentials near the thresholds, so they omitted the q^2 -dependent pseudoscalar exchanges and reduced the vector exchange potential into a contact term by omitting the terms that were suppressed by q^2/m_{ex}^2 . They did not include the spin splitting structures in the dihadron spectra since the vector exchange potentials did not incorporate spin information. Thus, the contact terms providing the main driving forces of forming bound states were interpreted as vector exchanges in their framework. Another systematic work based on the idea of resonance saturation model can be found in Ref. [81].

To find the bound state solutions of considered systems, we encounter both the single-channel and two-channel Lippmann-Schwinger (LS) equations. Specifically, in the single-channel case, we search for the bound state solutions by solving the following LS equation

$$T(E) = V + VG(E)T(E). \quad (17)$$

Correspondingly, in the two-channel case, the corresponding LS equation has the following matrix form

$$\mathbb{T}(E) = \mathbb{V} + \mathbb{V}\mathbb{G}(E)\mathbb{T}(E), \quad (18)$$

with

$$\mathbb{V} = \begin{pmatrix} V^{H_1 H_2 \rightarrow H_1 H_2} & V^{H_1 H_2 \rightarrow H_3 H_4} \\ V^{H_3 H_4 \rightarrow H_1 H_2} & V^{H_3 H_4 \rightarrow H_3 H_4} \end{pmatrix}, \quad (19)$$

$$\mathbb{T}(E) = \begin{pmatrix} t_{11}(E) & t_{12}(E) \\ t_{21}(E) & t_{22}(E) \end{pmatrix}, \quad (20)$$

and

$$\mathbb{G}(E) = \text{diag}\{G_1(E), G_2(E)\}, \quad (21)$$

the function G_i is defined as

$$G_i = \frac{1}{2\pi^2} \int dq \frac{q^2}{E - \sqrt{m_{i1}^2 + q^2} - \sqrt{m_{i2}^2 + q^2}} u^2(\Lambda). \quad (22)$$

Here, m_{i1} and m_{i2} are the masses of two hadrons in the i -th channel, we adopt a dipole form factor $u(\Lambda) = (1 + q^2/\Lambda^2)^{-2}$ [87–89] to suppress the contributions from higher momenta, we will discuss the value of cutoff Λ in Sec. II C.

C. Parameters

In Ref. [25], the parameters introduced in our model ($\tilde{g}_s$, $\tilde{g}_a$, g_x , and Λ) were determined from the inputs of observed P_{ψ}^N and $P_{\psi s}^{\Lambda}$ states. Here, we briefly review the scheme we pin down their numerical values and the scheme we estimate the uncertainties introduced from the experimental errors.

First, we used the experimental masses of $P_{\psi}^N(4440)$ ($4440.0^{+4.0}_{-5.0}$ MeV) and $P_{\psi}^N(4457)$ ($4457.3^{+4.0}_{-1.8}$ MeV) [1,2] as inputs to fix $\tilde{g}_s$ and $\tilde{g}_a$. The quantum numbers of these two states have not been measured yet, thus, the assignments of these two states may have the following two scenarios

$$\text{Scenario 1: } P_{\psi}^N(4440) \left| \Sigma_c \bar{D}^*; \frac{1}{2}^- \right\rangle, \quad P_{\psi}^N(4457) \left| \Sigma_c \bar{D}^*; \frac{3}{2}^- \right\rangle, \quad (23)$$

$$\text{Scenario 2: } P_{\psi}^N(4457) \left| \Sigma_c \bar{D}^*; \frac{1}{2}^- \right\rangle, \quad P_{\psi}^N(4440) \left| \Sigma_c \bar{D}^*; \frac{3}{2}^- \right\rangle. \quad (24)$$

These two assignments have been discussed in various models. For example, the scenario 1 was suggested within the framework of OBE model [82] or chiral effective field theory [83]. While the scenario 2 was suggested in Ref. [90] by considering the observed P_{ψ}^N states as the superpositions of meson-baryon channels coupled to a compact $uudc\bar{c}$ core. The $J/\Psi p$ invariant mass distributions were fitted in the both scenarios in Ref. [84], they found that the $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ with $\frac{3}{2}^-$ and $\frac{1}{2}^-$ were preferred once the full OPE (one-pion-exchange) potential was included. In Refs. [80,81], the authors further discussed the Dirac-delta contributions in the OBE model and suggested that to consistently describe the masses of the three observed [1,2] P_{ψ}^N states with similar cutoffs, the Dirac-delta terms should be removed, then the preferred J^P numbers for the $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ are $\frac{3}{2}^-$ and $\frac{1}{2}^-$, respectively. The mass splittings induced from the Dirac-delta terms were also studied by varying the strength of the $\delta(\vec{r})$ potential [91], their results also preferred the scenario 2, they also used the same framework [92] to investigate the mass ordering of the anticharmed strange pentaquark candidates.

The above discussions show that at present, determining the assignments of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states theoretically is rather model-dependent and relies on different theoretical treatments or approximations. Thus, in the following, we will consider both scenarios.

Under these two scenarios, in Ref. [25], we performed a coupled-channel calculation by considering the S -wave channels that were composed of the $(\Lambda_c/\Sigma_c/\Sigma_c^*) + (\bar{D}/\bar{D}^*)$ hadrons. We fixed the cutoff Λ introduced from

the dipole form factor at 1.0 GeV, and solved the coupled-channel LS equation to obtain $\tilde{g}_s$ and $\tilde{g}_a$. In the scenario 1 and scenario 2, we found

$$\text{Scenario 1: } \tilde{g}_s = 8.28 \text{ GeV}^{-2}, \quad \tilde{g}_a = -1.46 \text{ GeV}^{-2}, \quad (25)$$

$$\text{Scenario 2: } \tilde{g}_s = 9.12 \text{ GeV}^{-2}, \quad \tilde{g}_a = 1.25 \text{ GeV}^{-2}. \quad (26)$$

Here, we need to emphasis that if we only include the contact terms, the effective potentials derived from our quark-level description are consistent with the effective potentials derived from the hadron-level description [76]. We present more details on this issue in the Appendix.

We also selected the other Λ values in the (0.5, 1.5) GeV region, and we can correspondingly obtain two sets of $\tilde{g}_s$ and $\tilde{g}_a$ solutions based on these two scenarios. We found that our predictions of the masses to the other P_ψ^N states had very weak Λ -dependences in both of the scenarios. Thus, we will adopt $\Lambda = 1.0$ GeV throughout this work.

To estimate the uncertainties introduced from the experimental masses of $P_\psi^N(4440)$ and $P_\psi^N(4457)$ states, we combined the upper or lower limit of the mass of $P_\psi^N(4440)$ state with the upper or lower limit of the mass of $P_\psi^N(4457)$ state. In each scenario, we used these four sets of inputs to solve four sets of coupling parameters $\tilde{g}_s$ and $\tilde{g}_a$ correspondingly. We used these four sets of coupling parameters as inputs to give the lower and upper limits of the other predicted states.

We further used the mass of $P_{\psi s}^\Lambda(4338)$ to constrain the value of the SU(3) breaking factor g_x . First, this factor was introduced to suppress the contributions from the exchanges of the isospin doublet scalar and axial-vector light mesons, the value of this factor should be smaller than 1.

Second, within the same framework [25], we found that in the single channel case, the mass of $P_{\psi s}^\Lambda(4338)$ was obtained as 4328.1 and 4323.1 MeV in the scenario 1 and scenario 2, respectively. But after including the $\Lambda_c \bar{D}_s - \Xi_c \bar{D}$ coupling, we found that as the $\tilde{g}_x$ increases from 0 to 0.62, the attractive force in the $\Xi_c \bar{D}$ channel became weak, and the mass of $P_{\psi s}^\Lambda(4338)$ moved forward to the $\Xi_c \bar{D}$ threshold. When $g_x = 0.62$, the mass of $P_{\psi s}^\Lambda$ became 4335.9 and 4335.7 MeV in the scenario 1 and scenario 2, respectively. When $g_x > 0.62$, in the both scenarios, the attractive force between the Ξ_c and $\bar{D}$ was not enough to form a bound state. Thus, due to the observation of $P_{\psi s}^\Lambda(4338)$, we can further constrain the SU(3) breaking factor to be $g_x \leq 0.62$.

Here, we want to mention that the LHCb collaboration also reported another structure $P_{\psi s}^\Lambda(4459)$ [3]. Since the $P_{\psi s}^\Lambda(4338)$ and $P_{\psi s}^\Lambda(4459)$ are close to the $\Xi_c \bar{D}$ and $\Xi_c \bar{D}^*$ thresholds, respectively, from our contact Lagrangian, the 2×2 effective potential matrix of $P_{\psi s}^\Lambda(4338)$

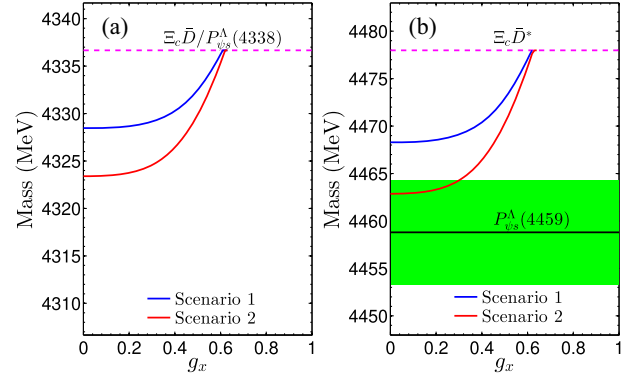


FIG. 1. The g_x -dependences of the masses of $P_{\psi s}^\Lambda(4338)$ state (a) and $P_{\psi s}^\Lambda(4459)$ state (b). We take the threshold of $\Xi_c \bar{D}$ as the experimental mass of $P_{\psi s}^\Lambda(4338)$. We use the dotted lines to illustrate the thresholds of $\Xi_c \bar{D}$ and $\Xi_c \bar{D}^*$. The blue and red lines are the results calculated with the inputs from the scenario 1 and 2, respectively. In (b), we further use the black line to label the central value of the experimental mass of $P_{\psi s}^\Lambda(4459)$ state, its uncertainty is illustrated with green band.

$$V_{P_{\psi s}^\Lambda(4338)} = \begin{pmatrix} v^{\Lambda_c \bar{D}_s \rightarrow \Lambda_c \bar{D}_s} & v^{\Lambda_c \bar{D}_s \rightarrow \Xi_c \bar{D}} \\ v^{\Xi_c \bar{D} \rightarrow \Lambda_c \bar{D}_s} & v^{\Xi_c \bar{D} \rightarrow \Xi_c \bar{D}} \end{pmatrix} \quad (27)$$

is identical [25] to the effective potential matrix of $P_{\psi s}^\Lambda(4459)$

$$V_{P_{\psi s}^\Lambda(4459)} = \begin{pmatrix} v^{\Lambda_c \bar{D}_s^* \rightarrow \Lambda_c \bar{D}_s^*} & v^{\Lambda_c \bar{D}_s^* \rightarrow \Xi_c \bar{D}^*} \\ v^{\Xi_c \bar{D}^* \rightarrow \Lambda_c \bar{D}_s^*} & v^{\Xi_c \bar{D}^* \rightarrow \Xi_c \bar{D}^*} \end{pmatrix}. \quad (28)$$

Here, the spin-spin interacting terms for the $\Xi_c \bar{D}^* - \Xi_c \bar{D}^*$, $\Xi_c \bar{D}^* - \Lambda_c \bar{D}_s^*$, and $\Lambda_c \bar{D}_s^* - \Lambda_c \bar{D}_s^*$ couplings vanish when the total angular momentum is $\frac{1}{2}$ or $\frac{3}{2}$. Thus, we can assume that the total angular momentum for the $P_{\psi s}^\Lambda(4459)$ could be $\frac{1}{2}$ or $\frac{3}{2}$ since they have identical effective potentials [87].

We perform a calculation to present the g_x -dependences of the masses of $P_{\psi s}^\Lambda(4338)$ and $P_{\psi s}^\Lambda(4459)$ states. We present our results in Fig. 1. The central value of the experimental mass [4] of $P_{\psi s}^\Lambda(4338)$ state is slightly above the $\Xi_c^+ \bar{D}^-$ or $\Xi_c^0 \bar{D}^0$ thresholds. Here, we restrict our scope to the molecular interpretation of $P_{\psi s}^\Lambda(4338)$ state, we adopt the isospin average value of $\Xi_c \bar{D}$ threshold as the experimental mass of $P_{\psi s}^\Lambda(4338)$ state. From Fig. 1(a), we find that at $g_x = 0.62$, the mass of $P_{\psi s}^\Lambda(4338)$ equals to the threshold of $\Xi_c \bar{D}$ in the scenario 1 or 2. Besides, in both of the scenarios, when g_x is in the region $[0, 0.62]$, we can find bound state solutions below the $\Xi_c \bar{D}$ threshold, and we illustrate them with blue and red lines for the scenario 1 and 2 cases, respectively.

Now we discuss the results in Fig. 1(b). As shown in Fig. 1(b), when we investigate the possible bound state below the $\Xi_c \bar{D}^*$ threshold, we can also obtain bound state

solutions in both of the scenarios, but only the result in the scenario 2 (red line) has small overlap with the experimental mass of $P_{\psi_s}^{\Lambda}(4459)$ after including its relatively large experimental uncertainty. Note that the present framework can give a fairly good unified description to the observed $P_{\psi}^N(4312)$, $P_{\psi}^N(4440)$, $P_{\psi}^N(4457)$, and $P_{\psi_s}^{\Lambda}(4338)$ states, the inconsistency of $P_{\psi_s}^{\Lambda}(4459)$ state in our model implies that this state is not a good molecular candidate and needs further confirmation.

D. Reduce the number of the considered dihadron states

The dihadron systems listed in Table I can couple to various dihadron states with different total isospins (I) and total angular momenta (J). We can further remove some dihadron states that cannot form bound states according to the following two rules.

First, we remove the states that are forbidden according to the selection rule. For the general identical fermion and boson systems, the quantum numbers of the H_1 and H_2 hadrons satisfy the following selection rule

$$L + S_{\text{tot}} + I_{\text{tot}} + 2i + 2s = \begin{cases} \text{Even, for bosons,} \\ \text{Odd, for fermions.} \end{cases} \quad (29)$$

Second, we remove the states that apparently cannot form bound states. On the one hand, as can be seen from the obtained parameters $\tilde{g}_s$ and $\tilde{g}_a$ in Eqs. (25) and (26), in both scenarios, the $\tilde{g}_s$ that is related to the flavor operator $\mathcal{O}_{\text{tot}}^f$ is much larger than the $\tilde{g}_a$ that is related to the flavor-spin operator $\mathcal{O}_{\text{tot}}^{\text{fs}}$. Besides, the $\tilde{g}_s$ obtained in these two scenarios have the same sign and are comparable to each other. Thus, whether the considered dihadron state can form a bound state is flavor dominant, i.e., mainly depends on the matrix element $\langle \mathcal{O}_{\text{tot}}^f \rangle$.

On the other hand, for the dihadron state with only one light quark in each hadron, since each hadron belongs to the flavor 3 representation, we have

$$3 \otimes 3 = 6 \oplus \bar{3}. \quad (30)$$

Thus, all the considered dihadron states should belong to either flavor antisymmetric $\bar{3}$ multiplet or flavor 6 symmetric multiplet. From Eq. (13), the matrix elements $\langle \mathcal{O}_{\text{tot}}^f \rangle$ for the $\bar{3}$ and 6 are obtained as $-\frac{8}{3}$ and $\frac{4}{3}$, respectively. In our convention, a negative potential and a positive potential will lead to an attractive force and a repulsive force, respectively. Thus, we only consider the dihadron states that belong to the flavor $\bar{3}$ multiplet. This is the reason that we do not list the $S = -2$ dihadron systems in Table I, since they cannot form bound states according to their symmetric flavor wave functions.

According to the above two rules, We remove some of the dihadron states that are composed of the systems listed in Table I, then we collect the rest of dihadron states and

TABLE II. The remaining dihadron states selected by the discussed two rules. We use the notations defined in Eqs. (1)–(3) to describe the flavor-spin wave functions of the considered dihadron states. Since all the considered dihadron states belong to the $\bar{3}$ flavor representation, we implicitly omit the “−” sign on the superscript of the $[H_1 H_2]_J^{\frac{1}{2}-}$ state defined in Eq. (2).

$\bar{T}_{cc}^f$		$[\bar{D}\bar{D}^*]_1^0$	$[\bar{D}^*\bar{D}^*]_1^0$	
$\bar{T}_{cc\bar{s}}^{\theta}$		$[\bar{D}\bar{D}^*]_1^1$	$[\bar{D}^*\bar{D}^*]_1^1$	
		$[\bar{D}_s\bar{D}^*]_1^1$		
$P_{\psi c}^{\Lambda}$	$[\Xi_{cc}\bar{D}]_{\frac{1}{2}}^0$	$[\Xi_{cc}^*\bar{D}]_{\frac{1}{2}}^0$	$[\Xi_{cc}\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^0$	$[\Xi_{cc}^*\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^0$
$P_{\psi cs}^N$	$[\Xi_{cc}\bar{D}_s]_{\frac{1}{2}}^1$	$[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^1$	$[\Xi_{cc}\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^1$	$[\Xi_{cc}^*\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^1$
	$[\Omega_{cc}\bar{D}]_{\frac{1}{2}}^1$	$[\Omega_{cc}^*\bar{D}]_{\frac{1}{2}}^1$	$[\Omega_{cc}\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^1$	$[\Omega_{cc}^*\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^1$
$H_{\Omega_{ccc}}^{\Lambda}$	$[\Xi_{cc}\Xi_{cc}]_1^0$	$[\Xi_{cc}\Xi_{cc}^*]_1^0$	$[\Xi_{cc}\Xi_{cc}^*]_2^0$	$[\Xi_{cc}^*\Xi_{cc}]_{1,3}^0$
$H_{\Omega_{ccc}s}^N$	$[\Xi_{cc}\Omega_{cc}]_1^{\frac{1}{2}}$	$[\Xi_{cc}\Omega_{cc}^*]_1^{\frac{1}{2}}$	$[\Xi_{cc}\Omega_{cc}^*]_2^{\frac{1}{2}}$	$[\Xi_{cc}^*\Omega_{cc}]_{1,3}^{\frac{1}{2}}$
		$[\Xi_{cc}^*\Omega_{cc}]_1^{\frac{1}{2}}$	$[\Xi_{cc}^*\Omega_{cc}]_2^{\frac{1}{2}}$	

present them in Table II. Note that in Table II, we adopt the wave function notations defined in Eqs. (1)–(3). Especially, we implicitly omit the minus sign on the superscript of the wave function $[H_1 H_2]_J^{\frac{1}{2}-}$ since we only consider the states belong to the $\bar{3}$ flavor multiplet. In the following, we will only investigate the states listed in Table II.

III. NUMERICAL RESULTS

We calculate the matrix elements of $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^{\theta}$, $P_{\psi c}^{\Lambda}/P_{\psi cs}^N$, and $H_{\Omega_{ccc}}^{\Lambda}/H_{\Omega_{ccc}s}^N$ states collected in Table II and present the obtained $[\langle \mathcal{O}_{\text{tot}}^f \rangle, \langle \mathcal{O}_{\text{tot}}^{\text{fs}} \rangle]$ in Table III.

As presented in Table III, since the mass of $\bar{T}_{cc}^f(3875)$ is very close to the $\bar{D}^0 D^{*-}$ and $D^- \bar{D}^{*0}$ thresholds, thus, we take into account mass difference between the $\bar{D}^0 D^{*-}$ and $D^- \bar{D}^{*0}$ thresholds and mix these two components through a 2×2 matrix. We can further obtain the strangeness partner of $\bar{T}_{cc}^f$ states by replacing a d quark to an s quark. Note that the effective potentials of $\bar{T}_{cc\bar{s}}^{\theta}$ states are slightly different from that of the $\bar{T}_{cc}^f$ states by a SU(3) breaking factor g_x , in the SU(3) limit with $g_x = 1$, the $\bar{T}_{cc}^f$ states and their strangeness partner $\bar{T}_{cc\bar{s}}^{\theta}$ states share identical effective potentials.

The eigenvalues of $\bar{T}_{cc}^f$ matrices calculated from the operators $\mathcal{O}_{\text{tot}}^f$ and $\mathcal{O}_{\text{tot}}^{\text{fs}}$ are also presented in the fifth and sixth columns of Table III, respectively. Besides, one can easily find that in the SU(3) limit with $g_x = 1$, the $\bar{T}_{cc}^f$ states and its strangeness partners $\bar{T}_{cc\bar{s}}^{\theta}$ states have identical effective potential matrices, and thus have identical $\langle \mathcal{O}_{\text{Eig}}^f \rangle$ and $\langle \mathcal{O}_{\text{Eig}}^{\text{fs}} \rangle$ matrix elements.

For the $P_{\psi c}^{\Lambda}$ states, we adopt the isospin limit and calculate the effective potentials of $\Xi_{cc}^* \bar{D}^{(*)}$ states in the single-channel formalism. However, as demonstrated in

TABLE III. The matrix elements $[\langle \mathcal{O}_{\text{tot}}^f \rangle, \langle \mathcal{O}_{\text{tot}}^s \rangle]$ of $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^\theta$, $P_{\psi c}^f/P_{\psi cs}^N$, and $H_{\Omega_{ccc}c}^\Lambda/H_{\Omega_{ccc}cs}^N$ states collected in Table II. We consider the isospin breaking for the $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^\theta$ states, and adopt the isospin limit but consider the SU(3) breaking for the $P_{\psi c}^f/P_{\psi cs}^N$ and $H_{\Omega_{ccc}c}^\Lambda/H_{\Omega_{ccc}cs}^N$ states.

$\bar{T}_{cc}^f$	$[\langle \mathcal{O}_{\text{tot}}^f \rangle, \langle \mathcal{O}_{\text{tot}}^s \rangle]$	$\bar{T}_{cc\bar{s}}^\theta$	$[\langle \mathcal{O}_{\text{tot}}^f \rangle, \langle \mathcal{O}_{\text{tot}}^s \rangle]$	$\langle \mathcal{O}_{\text{Eig}}^f \rangle$	$\langle \mathcal{O}_{\text{Eig}}^s \rangle$
$([\bar{D}^0 D^{*-}]_1, [D^- \bar{D}^{*0}]_1)$	$\begin{pmatrix} [-\frac{2}{3}, 0] & [2, 0] \\ [-\frac{2}{3}, 0] \end{pmatrix}$	$([\bar{D}^0 D_s^{*-}]_1, [D_s^- \bar{D}^{*0}]_1)$	$\begin{pmatrix} [-\frac{2}{3}, 0] & [2g_x, 0] \\ [-\frac{2}{3}, 0] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$[\bar{D}^{*0} D^{*-}]_1^0$	$[-\frac{8}{3}, \frac{8}{3}]$	$[\bar{D}^{*0} D_s^{*-}]_1^{\frac{1}{2}}$	$[-2g_x - \frac{2}{3}, 2g_x + \frac{2}{3}]$	$(-\frac{8}{3})$	$(\frac{8}{3})$
$P_{\psi c}^f$	$[\langle \mathcal{O}_{\text{tot}}^f \rangle, \langle \mathcal{O}_{\text{tot}}^s \rangle]$	$P_{\psi cs}^N$	$[\langle \mathcal{O}_{\text{tot}}^f \rangle, \langle \mathcal{O}_{\text{tot}}^s \rangle]$	$\langle \mathcal{O}_{\text{Eig}}^f \rangle$	$\langle \mathcal{O}_{\text{Eig}}^s \rangle$
$[\Xi_{cc} \bar{D}]_{\frac{1}{2}}^0$	$[-\frac{8}{3}, 0]$	$([\Xi_{cc} \bar{D}_s]_{\frac{1}{2}}, [\Omega_{cc} \bar{D}]_{\frac{1}{2}})$	$\begin{pmatrix} [-\frac{2}{3}, 0] & [2g_x, 0] \\ [-\frac{2}{3}, 0] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$[\Xi_{cc} \bar{D}^*]_{\frac{1}{2}}^0$	$[-\frac{8}{3}, -\frac{16}{9}]$	$([\Xi_{cc} \bar{D}_s^*]_{\frac{1}{2}}, [\Omega_{cc} \bar{D}^*]_{\frac{1}{2}})$	$\begin{pmatrix} [-\frac{2}{3}, -\frac{4}{9}] & [2g_x, \frac{4}{3}g_x] \\ [-\frac{2}{3}, -\frac{4}{9}] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{16}{9} \\ \frac{8}{9} \end{pmatrix}$
$[\Xi_{cc}^* \bar{D}^*]_{\frac{1}{2}}^0$	$[-\frac{8}{3}, \frac{40}{9}]$	$([\Xi_{cc}^* \bar{D}_s^*]_{\frac{1}{2}}, [\Omega_{cc}^* \bar{D}^*]_{\frac{1}{2}})$	$\begin{pmatrix} [-\frac{2}{3}, \frac{10}{9}] & [2g_x, -\frac{10}{3}g_x] \\ [-\frac{2}{3}, \frac{10}{9}] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{40}{9} \\ -\frac{20}{9} \end{pmatrix}$
$[\Xi_{cc} \bar{D}]_{\frac{3}{2}}^0$	$[-\frac{8}{3}, 0]$	$([\Xi_{cc}^* \bar{D}_s]_{\frac{3}{2}}, [\Omega_{cc}^* \bar{D}]_{\frac{3}{2}})$	$\begin{pmatrix} [-\frac{2}{3}, 0] & [2g_x, 0] \\ [-\frac{2}{3}, 0] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$[\Xi_{cc} \bar{D}^*]_{\frac{3}{2}}^0$	$[-\frac{8}{3}, \frac{8}{9}]$	$([\Xi_{cc} \bar{D}_s^*]_{\frac{3}{2}}, [\Omega_{cc} \bar{D}^*]_{\frac{3}{2}})$	$\begin{pmatrix} [-\frac{2}{3}, \frac{2}{9}] & [2g_x, -\frac{2}{3}g_x] \\ [-\frac{2}{3}, \frac{2}{9}] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{8}{9} \\ -\frac{4}{9} \end{pmatrix}$
$[\Xi_{cc}^* \bar{D}^*]_{\frac{3}{2}}^0$	$[-\frac{8}{3}, \frac{16}{9}]$	$([\Xi_{cc}^* \bar{D}_s^*]_{\frac{3}{2}}, [\Omega_{cc}^* \bar{D}^*]_{\frac{3}{2}})$	$\begin{pmatrix} [-\frac{2}{3}, \frac{4}{9}] & [2g_x, -\frac{4}{3}g_x] \\ [-\frac{2}{3}, \frac{4}{9}] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{16}{9} \\ -\frac{8}{9} \end{pmatrix}$
$[\Xi_{cc} \bar{D}^*]_{\frac{5}{2}}^0$	$[-\frac{8}{3}, -\frac{8}{3}]$	$([\Xi_{cc} \bar{D}_s^*]_{\frac{5}{2}}, [\Omega_{cc} \bar{D}^*]_{\frac{5}{2}})$	$\begin{pmatrix} [-\frac{2}{3}, -\frac{2}{3}] & [2g_x, 2g_x] \\ [-\frac{2}{3}, -\frac{2}{3}] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$
$H_{\Omega_{ccc}c}^\Lambda$	$[\langle \mathcal{O}_{\text{tot}}^f \rangle, \langle \mathcal{O}_{\text{tot}}^s \rangle]$	$H_{\Omega_{ccc}cs}^N$	$[\langle \mathcal{O}_{\text{tot}}^f \rangle, \langle \mathcal{O}_{\text{tot}}^s \rangle]$	$\langle \mathcal{O}_{\text{Eig}}^f \rangle$	$\langle \mathcal{O}_{\text{Eig}}^s \rangle$
$[\Xi_{cc} \Xi_{cc}]_1^0$	$[-\frac{8}{3}, -\frac{8}{27}]$	$[\Xi_{cc} \Omega_{cc}]_1^{\frac{1}{2}}$	$[-2g_x - \frac{2}{3}, -\frac{2}{9}g_x - \frac{2}{27}]$	$(-\frac{8}{3})$	$(-\frac{8}{27})$
$[\Xi_{cc} \Xi_{cc}^*]_1^0$	$[-\frac{8}{3}, -\frac{40}{27}]$	$([\Xi_{cc} \Omega_{cc}^*]_1, [\Omega_{cc} \Xi_{cc}^*]_1)$	$\begin{pmatrix} [-\frac{2}{3}, -\frac{10}{27}] & [2g_x, \frac{10}{9}g_x] \\ [-\frac{2}{3}, -\frac{10}{27}] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{40}{27} \\ \frac{20}{27} \end{pmatrix}$
$[\Xi_{cc}^* \Xi_{cc}^*]_1^0$	$[-\frac{8}{3}, \frac{88}{27}]$	$[\Xi_{cc}^* \Omega_{cc}^*]_1^{\frac{1}{2}}$	$[-2g_x - \frac{2}{3}, \frac{22}{9}g_x + \frac{22}{27}]$	$(-\frac{8}{3})$	$(\frac{88}{27})$
$[\Xi_{cc} \Xi_{cc}^*]_2^0$	$[-\frac{8}{3}, \frac{8}{9}]$	$([\Xi_{cc} \Omega_{cc}^*]_2, [\Omega_{cc} \Xi_{cc}^*]_2)$	$\begin{pmatrix} [-\frac{2}{3}, \frac{2}{9}] & [2g_x, -\frac{2}{3}g_x] \\ [-\frac{2}{3}, \frac{2}{9}] \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{8}{9} \\ -\frac{4}{9} \end{pmatrix}$
$[\Xi_{cc}^* \Xi_{cc}^*]_3^0$	$[-\frac{8}{3}, -\frac{8}{3}]$	$[\Xi_{cc}^* \Omega_{cc}^*]_3^{\frac{1}{2}}$	$[-2g_x - \frac{2}{3}, -2g_x - \frac{2}{3}]$	$(-\frac{8}{3})$	$(-\frac{8}{3})$

Table III, for the $P_{\psi cs}^N$ states, the strange quark belongs to the baryon or meson will lead to different final states, this is another source of the SU(3) breaking. As presented in Table III, we include this effect by distinguishing these two components and calculating the effective potentials of $P_{\psi cs}^N$ states through a 2×2 matrices. In the SU(3) limit with $g_x = 1$, the mixture of $\Xi_{cc}^{(*)} \bar{D}_s^{(*)}$ and $\Omega_{cc}^{(*)} \bar{D}^{(*)}$ components will lead to two different flavor matrix elements $-\frac{8}{3}$ and $\frac{4}{3}$, they belong to the $\bar{3}$ and 6 flavor multiplets, respectively. As discussed in Sec. II D, only the $P_{\psi cs}^N$ states with flavor matrix element $-\frac{8}{3}$ have attractive forces and thus may form bound states.

The effective potentials of $H_{\Omega_{ccc}c}^\Lambda/H_{\Omega_{ccc}cs}^N$ states are also presented in the lower panel of Table III. Similar to the $P_{\psi c}^f$

states, for the $H_{\Omega_{ccc}c}^\Lambda$ states, we adopt the isospin limit to calculate the effective potentials of $\Xi_{cc}^{(*)} \Xi_{cc}^{(*)}$ states. But for the $H_{\Omega_{ccc}cs}^N$ states, the $\Xi_{cc} \Omega_{cc}^*$ and $\Omega_{cc} \Xi_{cc}^*$ components have different thresholds, we mix these two components to include the SU(3) breaking effect induced from the constituent strange quark mass.

With the effective potentials for all the considered $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^\theta/P_{\psi c}^f/P_{\psi cs}^N/H_{\Omega_{ccc}c}^\Lambda/H_{\Omega_{ccc}cs}^N$ states listed in Table III, we find the possible bound states via solving the corresponding LS equations. Here, with the parameters from the scenario 1 and scenario 2 in Eqs. (25) and (26), we first discuss the possible bound states and their spectra among the $\bar{T}_{cc}^f/P_{\psi c}^f/H_{\Omega_{ccc}c}^\Lambda$ states. Then we discuss the existences of

TABLE IV. The binding energies of the nonstrange $\bar{T}_{cc}^f$, $P_{\psi c}^{\Lambda}$, and $H_{\Omega_{ccc}}^{\Lambda}$ bound states. The “*” denotes that the bound state solution no longer exists at its upper limit. All the results are in units of MeV.

System	Scenario 1		Scenario 2	
	Mass	BE	Mass	BE
$[\bar{D}\bar{D}^*]_1^0$	$3875.0_{-0.3}^{+*}$	$-0.1_{-0.3}^{+*}$	$3874.1_{-1.2}^{+1.0}$	$-1.0_{-1.2}^{+1.0}$
$[\bar{D}^*\bar{D}^*]_1^0$	$4013.1_{-2.6}^{+1.4}$	$-4.1_{-2.6}^{+1.4}$	$4016.9_{-0.5}^{+*}$	$-0.2_{-0.5}^{+*}$
$[\Xi_{cc}\bar{D}]_{\frac{1}{2}}^0$	$5484.2_{-1.8}^{+2.6}$	$-4.4_{-1.8}^{+2.6}$	$5480.8_{-2.5}^{+2.9}$	$-7.9_{-2.5}^{+2.9}$
$[\Xi_{cc}\bar{D}^*]_{\frac{1}{2}}^0$	$5627.7_{-1.4}^{+1.8}$	$-2.3_{-1.4}^{+1.8}$	$5616.1_{-3.1}^{+3.7}$	$-13.9_{-3.1}^{+3.7}$
$[\Xi_{cc}^*\bar{D}]_{\frac{1}{2}}^0$	$5716.7_{-3.0}^{+5.1}$	$-18.8_{-3.0}^{+5.1}$	$5733.8_{-1.5}^{+1.3}$	$-1.8_{-1.5}^{+1.3}$
$[\Xi_{cc}^*\bar{D}^*]_{\frac{1}{2}}^0$	$5589.6_{-1.8}^{+2.7}$	$-4.7_{-1.8}^{+2.7}$	$5586.0_{-2.6}^{+3.0}$	$-8.2_{-2.6}^{+3.0}$
$[\Xi_{cc}\bar{D}^*]_{\frac{3}{2}}^0$	$5622.2_{-2.4}^{+3.1}$	$-7.7_{-2.4}^{+3.1}$	$5622.5_{-2.3}^{+3.2}$	$-7.5_{-2.3}^{+3.2}$
$[\Xi_{cc}^*\bar{D}^*]_{\frac{3}{2}}^0$	$5725.1_{-3.0}^{+2.9}$	$-10.5_{-3.0}^{+2.9}$	$5729.6_{-2.0}^{+3.1}$	$-6.0_{-2.0}^{+3.1}$
$[\Xi_{cc}^*\bar{D}^*]_{\frac{3}{2}}^0$	$5734.3_{-1.5}^{+*}$	$-1.2_{-1.5}^{+*}$	$5718.8_{-4.5}^{+3.6}$	$-16.7_{-4.5}^{+3.6}$
$[\Xi_{cc}\Xi_{cc}]_1^0$	$7227.0_{-3.0}^{+5.5}$	$-15.8_{-3.0}^{+5.5}$	$7219.0_{-4.1}^{+4.8}$	$-23.8_{-4.1}^{+4.8}$
$[\Xi_{cc}\Xi_{cc}^*]_1^0$	$7336.2_{-2.4}^{+5.8}$	$-12.2_{-2.4}^{+5.8}$	$7319.7_{-4.7}^{+4.4}$	$-28.7_{-4.7}^{+4.4}$
$[\Xi_{cc}^*\Xi_{cc}]_1^0$	$7422.7_{-7.9}^{+4.6}$	$-31.3_{-7.9}^{+4.6}$	$7440.5_{-2.5}^{+5.5}$	$-13.5_{-2.5}^{+5.5}$
$[\Xi_{cc}\Xi_{cc}^*]_{\frac{1}{2}}^0$	$7327.6_{-3.7}^{+5.0}$	$-20.8_{-3.7}^{+5.0}$	$7328.0_{-3.6}^{+5.2}$	$-20.4_{-3.6}^{+5.2}$
$[\Xi_{cc}^*\Xi_{cc}^*]_{\frac{1}{2}}^0$	$7445.1_{-3.2}^{+5.7}$	$-8.9_{-3.2}^{+5.7}$	$7420.1_{-6.0}^{+4.9}$	$-33.9_{-6.0}^{+4.9}$

the bound states in the $\bar{T}_{cc\bar{s}}^{\theta}/P_{\psi cs}^N/H_{\Omega_{ccc}s}^N$ systems by including the SU(3) breaking effects. We proceed to discuss their possible mass spectra if they do exist.

Note that in this work, we use the exactly identical theoretical framework and parameters to that of our previous work [25], where we present a detailed investigation on the flavor-spin symmetry properties of the interactions among the $P_{\psi}^N/P_{\psi s}^{\Lambda}/H_{\Omega_{ccc}}^N/H_{\Omega_{ccc}s}^N$ systems. Then the $SU(3)_f \otimes SU(2)_s$ symmetry allows us to relate the interactions of $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^{\theta}/P_{\psi c}^{\Lambda}/P_{\psi cs}^N/H_{\Omega_{ccc}c}^{\Lambda}/H_{\Omega_{ccc}s}^N$ systems studied in this work to that of the $P_{\psi}^N/P_{\psi s}^{\Lambda}/H_{\Omega_{ccc}}^N/H_{\Omega_{ccc}s}^N$ systems discussed in Ref. [25].

A. Nonstrange $\bar{T}_{cc}^f$, $P_{\psi c}^{\Lambda}$, and $H_{\Omega_{ccc}}^{\Lambda}$ states

With the parameters $\tilde{g}_s$ and $\tilde{g}_a$ from the scenario 1 and scenario 2 listed in Eqs. (25) and (26), we search for the bound state solutions from the considered dihadron $\bar{T}_{cc}^f/P_{\psi c}^{\Lambda}/H_{\Omega_{ccc}}^{\Lambda}$ states, and collect their binding energies in Table IV. We use the experimental uncertainties from the masses of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states to give the lower and upper limits of our predictions. In Table IV, the “+*” denotes that the bound state solution does not exist at its upper limit.

As presented in Table III, for the $[\bar{D}\bar{D}^*]_1^0$ state, we consider the isospin breaking effect by including the mixture of the $[\bar{D}^0D^{*-}]_1$ and $[D^-\bar{D}^{*0}]_1$ components. The threshold of the $\bar{D}^0D^{*-}$ (3875.1 MeV) is slightly below

the threshold of the $D^-\bar{D}^{*0}$ (3876.5 MeV). We search for the bound state solution below the $\bar{D}^0D^{*-}$ threshold and we also check the quasibound state solution in the energy region $E_{\bar{D}^0D^{*-}} < E < E_{D^-\bar{D}^{*0}}$. We can only find one bound state which is below the lowest $\bar{D}^0D^{*-}$ channel.

The obtained binding energies of $[\bar{D}\bar{D}^*]_1^0$ and $[\bar{D}^*\bar{D}^*]_1^0$ states in these two scenarios are very interesting. As presented in Table IV, in the scenario 1, the absolute value of binding energy for the $[\bar{D}^*\bar{D}^*]_1^0$ state is larger than that of the $[\bar{D}\bar{D}^*]_1^0$ state. This result suggests that if the J^P of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states were measured to be $1/2^-$ and $3/2^-$, respectively, then this measurement would give a strong support to the existence of a $[\bar{D}^*\bar{D}^*]_1^0$ state with its absolute value of binding energy larger than that of the $T_{cc}^f(3875)$ state.

Similarly, as presented in Table IV, in the scenario 2, we find that the absolute value of binding energy for the $[\bar{D}^*\bar{D}^*]_1^0$ state is smaller than that of the $[\bar{D}\bar{D}^*]_1^0$ state. This result suggests that if the J^P of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ were measured to be $3/2^-$ and $1/2^-$, respectively, then this measurement would suggest that the $[\bar{D}^*\bar{D}^*]_1^0$ should have a very minor binding energy or could not exist.

Or vice versa, the observation/nonobservation of $[\bar{D}^*\bar{D}^*]_1^0$ bound state and the measurement of its binding energy would give strong hints to the J^P quantum numbers of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states. Then the J^P quantum numbers of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states can be further related to the J^P quantum numbers of the other predicted P_{ψ}^N states according to the $SU(3)_f \otimes SU(2)_s$ symmetry discussed in our previous work [25].

The binding energies of $P_{\psi c}^{\Lambda}$ and $H_{\Omega_{ccc}c}^{\Lambda}$ calculated from the scenario 1 and scenario 2 are also presented in Table IV (See also the calculations in Refs. [93,94]). As listed in Table III, the effective potentials of $[\Xi_{cc}^*\bar{D}^*]_{\frac{1}{2}}^0$ and $[\Xi_{cc}^*\Xi_{cc}^*]_{\frac{1}{2}}^0$ are the same, thus, the binding energies given in Table IV show that with the same effective potential, the state with larger reduced mass binds deeper.

We plot the mass spectra of $P_{\psi c}^N$ bound states obtained from the scenario 1 and scenario 2 on the left and right sides of Fig. 2(a), respectively. Note that the flavor-spin coupling $\tilde{g}_a$ determined in the scenario 1 and scenario 2 have opposite signs, thus, the scenario 1 and scenario 2 give different mass arrangements to the mass spectra of $P_{\psi c}^N$ bound states. As can be checked from Fig. 2(a), in the scenario 1, the masses of bound states composed of the $\Xi_{cc}\bar{D}^*$ decrease as the total angular momenta J increase, while the masses of bound states composed of the $\Xi_{cc}^*\bar{D}^*$ increase as the total angular momenta J increase. These tendencies are opposite in the scenario 2 case.

We also plot the mass spectra of $H_{\Omega_{ccc}c}^{\Lambda}$ bound states obtained from the scenario 1 and scenario 2 on the left and right sides of Fig. 2(b), respectively. As can be checked

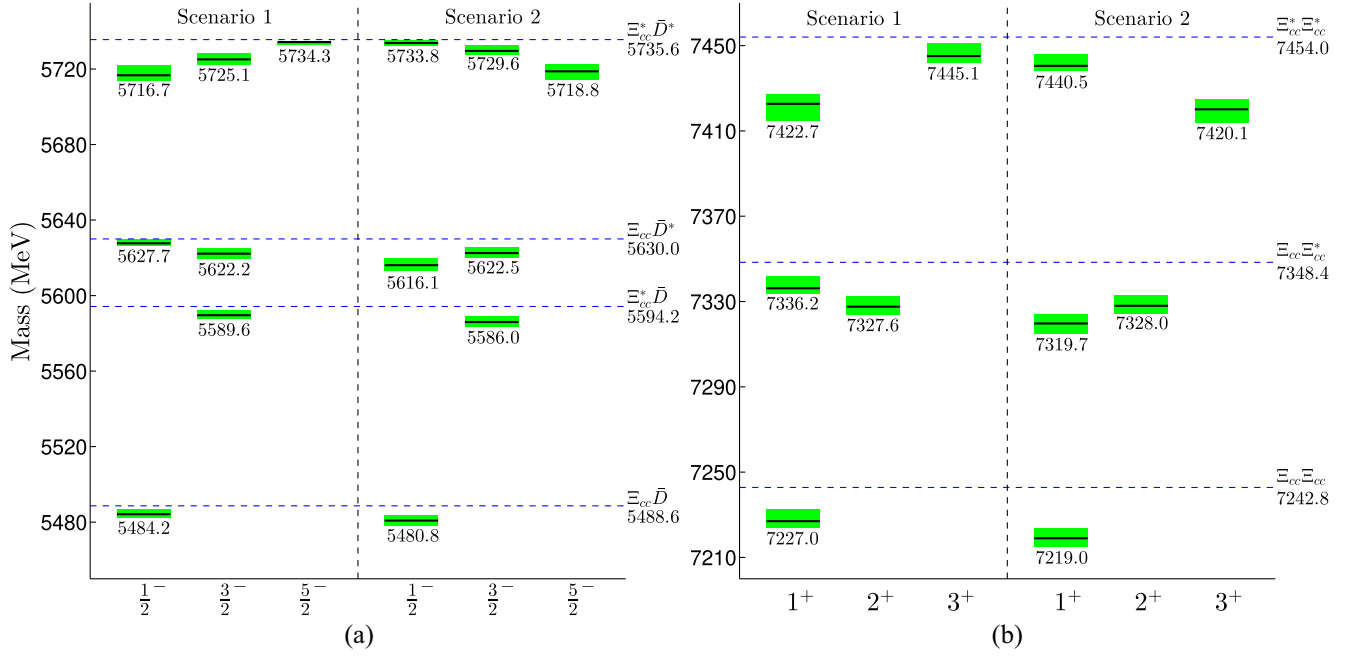


FIG. 2. The mass spectra of $P_{\psi c}^\Lambda$ and $H_{\Omega_{ccc}c}^\Lambda$ bound states. The results of $P_{\psi c}^\Lambda$ bound states for the scenario 1 and scenario 2 are illustrated on the left and right sides of (a), respectively. The results of $H_{\Omega_{ccc}c}^\Lambda$ bound states for the scenario 1 and scenario 2 are illustrated on the left and right sides of (b), respectively. We plot the central values of $P_{\psi c}^\Lambda$ and $H_{\Omega_{ccc}c}^\Lambda$ bound states with black lines and label the corresponding masses. The theoretical errors are introduced by considering the experimental errors from the masses of $P_\psi^N(4440)$ and $P_\psi^N(4457)$ states, they are labeled with green bands.

from Fig. 2(b), similar to the mass spectra of P_ψ^N states, different scenarios also lead to different mass arrangements of the $\Xi_{cc}\Xi_{cc}^*$ or $\Xi_{cc}^*\Xi_{cc}^*$ bound states with different total angular momenta J .

The Ξ_{cc} was observed in the $\Xi_c^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ channel [72,73], besides, the $\Xi_c^+ \pi^+$ mode was also suggested as its favorable decay mode with comparable branching fraction [95]. Thus, the possible weak decay mode for the triply-heavy pentaquark $[\Xi_{cc}^{++} \bar{D}^-]_{\frac{1}{2}}^0$ could be $\Lambda_c^+ K^- \pi^+ \pi^+ D^-$ and $\Xi_c^+ \pi^+ D^-$ channels, similarly, the quadruply-heavy hexaquark $[\Xi_{cc}\Xi_{cc}]_1^0$ could be observed in the $2(\Xi_c^+ \pi^+ D^-)$ decay mode.

B. The existences of the bound states in the $\bar{T}_{cc\bar{s}}^\theta/P_{\psi cs}^N/H_{\Omega_{ccc}cs}^N$ systems

In Sec. III A, with the parameters extracted from the inputs of $P_\psi^N(4440)$ and $P_\psi^N(4457)$ states, we successfully reproduce the mass of $T_{cc}^f(3875)$ state, indicating the similar binding mechanism for the P_ψ^N and T_{cc}^f bound states. Then we proceed to use the parameters extracted from the inputs of the observed $P_\psi^N(4440)$, $P_\psi^N(4457)$, and $P_{\psi s}^\Lambda(4338)$ states to discuss the existences of the bound states in the $\bar{T}_{cc\bar{s}}^\theta/P_{\psi cs}^N/H_{\Omega_{ccc}cs}^N$ systems.

Since the observed $\bar{T}_{cc}^f(3875)$ has a very tiny binding energy, implying that the interaction between the $\bar{D}$ and $\bar{D}^*$

is just enough to bind them together. Thus, to clarify the existences of its strangeness partner $\bar{T}_{cc\bar{s}}^\theta$ as well as its HDAS partners $P_{\psi cs}^N$ and $H_{\Omega_{ccc}cs}^N$, the corrections from the SU(3) breaking effects to their effective potentials become important.

We introduce the SU(3) breaking effects through two ways. First, due to the differences between the constituent (u, d) quark masses and s quark mass, the physical masses of $\bar{D}_s^{(*)}$ and $\Omega_{cc}^{(*)}$ hadrons are heavier than that of the $\bar{D}^{(*)}$ and $\Xi_{cc}^{(*)}$ hadrons, respectively. We adopt their physical masses to partly introduce the SU(3) breaking effect.

Second, the effective potentials of $\bar{T}_{cc}^f/P_{\psi c}^\Lambda/H_{\Omega_{ccc}c}^\Lambda$ states are induced from the exchanges of the isospin singlet and triplet light mesons, while the effective potentials of $\bar{T}_{cc\bar{s}}^\theta/P_{\psi cs}^N/H_{\Omega_{ccc}cs}^N$ states are induced from the exchanges of the isospin singlet and doublet light mesons. Thus, in the effective potentials of $\bar{T}_{cc}^f/P_{\psi c}^\Lambda/H_{\Omega_{ccc}c}^\Lambda$ and $\bar{T}_{cc\bar{s}}^\theta/P_{\psi cs}^N/H_{\Omega_{ccc}cs}^N$ states, we introduce a SU(3) breaking factor g_x to describe the different contributions from the exchanges of isospin triplet and doublet light mesons, respectively.

1. The existences of $\bar{T}_{cc\bar{s}}^\theta$ bound states

The effective potentials of $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ and $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ states have already been presented in Table III. As given in

Table III, for the $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ state, we consider the mass difference between the $D_s^- \bar{D}^{*0}$ (3975.9 MeV) and $\bar{D}^0 D_s^{*-}$ (3977.0 MeV) components and search for the bound state solution and quasibound state solution in the $E < E_{D_s^- \bar{D}^{*0}}$ and $E_{D_s^- \bar{D}^{*0}} < E < E_{\bar{D}^0 D_s^{*-}}$ regions, respectively.

In the both scenarios, we run g_x value in the $[0, 1]$ region and find that at small g_x value, the attractive force is not enough to form the $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ bound state. The attractive force increases as the g_x increases. When g_x is close to 1, the $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ state start to form bound state, its mass lies below the $D_s^- \bar{D}^{*0}$ threshold. As presented in Table III, when g_x is close to 1, the effective potential of $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ state is almost identical to that of the $[D \bar{D}^*]_1^0$ state. Similar to the $[D \bar{D}^*]_1^0$ state, when g_x is close to 1, the binding energy of $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ bound state is below and at the edge of the $D_s^- \bar{D}^{*0}$ threshold. However, since g_x value is further constrained by the mass of $P_{\psi s}^{\Lambda}(4338)$, i.e., $g_x \leq 0.62$. To give a more serious conclusion on whether the $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ bound state can exist, we need to consider both the experimental uncertainties from the masses of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states, and the g_x -dependence on the pole position of $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ state.

Now we proceed to include the experimental uncertainties from the masses of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states. The scheme we give the lower and upper limits of our predictions has been discussed in detail in Sec. II C, i.e., we combine the lower or upper limit of the mass of $P_{\psi}^N(4440)$ state with the lower or upper limit of the mass of $P_{\psi}^N(4457)$ state, we use these four sets masses as inputs to solve four sets of $\tilde{g}_s$ and $\tilde{g}_a$ coupling parameters correspondingly. Then we select the lower limit (with the maximum attractive force within the experimental error) and upper limit (with minimum attractive force within the experimental error) of the mass of $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ state.

We illustrate the g_x -dependence of the lower limit for the bound state solution of $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ state in Fig. 3(a). We use the blue-dotted and red-dotted lines to plot the results from the scenario 1 and scenario 2, respectively. As can be seen from Fig. 3(a), in the scenario 1 and scenario 2 cases, the lower limits of the masses of $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ state are far away from the physical region of the SU(3) breaking factor g_x . The results from the both scenarios show that the strangeness partner of the $T_{cc}^f(3875)$ cannot exist after including the SU(3) breaking effect.

Similarly, in both scenarios, we plot the g_x -dependences of the lower limits for the bound state solutions of $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ state in Fig. 3(b). As illustrated in Fig. 3(b), in the scenario 1, the lower limit of the mass of $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ bound state labeled with blue-dotted line has a very tiny overlap with

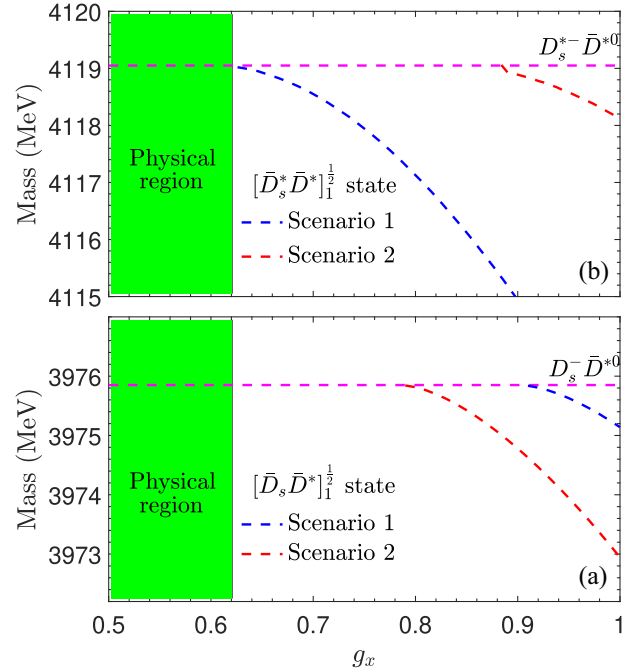


FIG. 3. As the g_x increases, we illustrate the lower limits of the masses of $[\bar{D}_s \bar{D}^*]_1^{\frac{1}{2}}$ and $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ states in (a) and (b), respectively. The results from the scenario 1 and scenario 2 are plotted with blue-dotted and red-dotted lines, respectively. The green area with $g_x \leq 0.62$ is the parameter region that the $P_{\psi s}^{\Lambda}(4338)$ state can exist.

the physical g_x region. We cannot entirely exclude the existence of $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ bound state, but our result prefers the conclusion that the $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ does not exist.

In the scenario 2, the lower limit of the pole position for $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ bound state is labeled with red-dotted line, as presented Fig. 3(b), our result shows that the lower limit of the pole position for $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ bound state is far away from the physical region of the SU(3) breaking factor g_x , in this scenario, the existence of $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ bound state can be excluded. Or vise versa, the result of $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ state in the scenario 2 suggests that once the J^P numbers of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ were measured to be $\frac{3}{2}^-$ and $\frac{1}{2}^-$, respectively, then we can conclude that the $[\bar{D}^* \bar{D}_s]_1^{\frac{1}{2}}$ bound state does not exist.

2. The existences of $P_{\psi cs}^N$ bound states

Now we proceed to discuss the existences of $P_{\psi cs}^N$ bound states. As presented in Table I, due to the large constituent strange quark mass, for a triple-charm baryon-meson (B-M) system, the strange quark belongs to the double-charm baryon or charmed meson will lead to different B-M systems. Thus, we investigate the possible $P_{\psi cs}^N$ bound

states via a mixing scheme. We mix the $(ccn) - (\bar{c}s)$ and $(ccs) - (\bar{c}n)$ components to derive the effective potentials of the considered $P_{\psi cs}^N$ states.

As listed in Table III, we need to consider the 7 pairs of B-M systems, i.e., the $([\Xi_{cc}\bar{D}_s]_{\frac{1}{2}}, [\Omega_{cc}\bar{D}]_{\frac{1}{2}})$, $([\Xi_{cc}\bar{D}_s]_{\frac{1}{2}}, [\Omega_{cc}\bar{D}^*]_{\frac{1}{2}})$, $([\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{1}{2}})$, $([\Xi_{cc}^*\bar{D}_s]_{\frac{3}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{3}{2}})$, $([\Xi_{cc}^*\bar{D}_s]_{\frac{3}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{1}{2}})$, $([\Xi_{cc}^*\bar{D}_s]_{\frac{3}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{3}{2}})$, and $([\Xi_{cc}^*\bar{D}_s]_{\frac{3}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{5}{2}})$ systems. Correspondingly, their SU(3) flavor partners in the isospin limit are the $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^0$, $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^0$, $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^0$, $[\Xi_{cc}^*\bar{D}_s]_{\frac{3}{2}}^0$, and $[\Xi_{cc}^*\bar{D}_s]_{\frac{3}{2}}^0$, respectively. Here, we need to emphasize that these nonstrange $P_{\psi c}^\Lambda$ states all have bound state solutions as discussed in Sec. III A.

From Table I, the threshold of $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ system is lower than the threshold of $\Omega_{cc}^{(*)}\bar{D}^{(*)}$ system, thus, in each pair of B-M system, we search for the possible bound state solution and quasibound state solution in the energy region $E < E_{\Xi_{cc}^{(*)}\bar{D}_s^{(*)}}$ and $E_{\Xi_{cc}^{(*)}\bar{D}_s^{(*)}} < E < E_{\Omega_{cc}^{(*)}\bar{D}^{(*)}}$, respectively. We find that the bound state below the $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ threshold may exist in the SU(3) limit with $g_x = 1$, but there is no quasibound state in the energy region between the $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ and $\Omega_{cc}^{(*)}\bar{D}^{(*)}$ thresholds.

We test the g_x -dependences of the bound state solutions that are below the corresponding $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ thresholds for the considered 7 pairs of B-M systems. However, unlike the $P_{\psi c}^\Lambda$ states, we find that all these 7 pairs of B-M systems can hardly form bound states when the $g_x \leq 0.62$, i.e., in the physical g_x region that the $P_{\psi s}^\Lambda(4338)$ can exist. Especially, according to our calculation, among these 7 pairs of B-M systems, the $([\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{1}{2}})$ system in the scenario 1 has the most attractive force. However, even for this extreme case, our result still prefers the nonexistence of $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^{\frac{1}{2}}$ bound state.

In Fig. 4, we plot the g_x -dependence of the $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^{\frac{1}{2}}$ bound state solution calculated from the effective potential of $([\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{1}{2}})$ system. In Fig. 4, the blue-dotted lines on the left and right are the lower and upper limits of the bound state solutions for the $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^{\frac{1}{2}}$ state. The blue solid line is obtained with the inputs from the central values of the masses of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states.

As can be seen from Fig. 4, the lower limit of the mass of $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^{\frac{1}{2}}$ bound state has a tiny overlap with the physical g_x region. From this result, we can conclude that the $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^{\frac{1}{2}}$ bound state can hardly exist. We do not further plot the g_x dependences for the rest of the 6 pairs of the $([\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^{\frac{1}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{1}{2}})$ systems, since the attractive forces in these 6 systems in both scenarios are all weaker than that of the

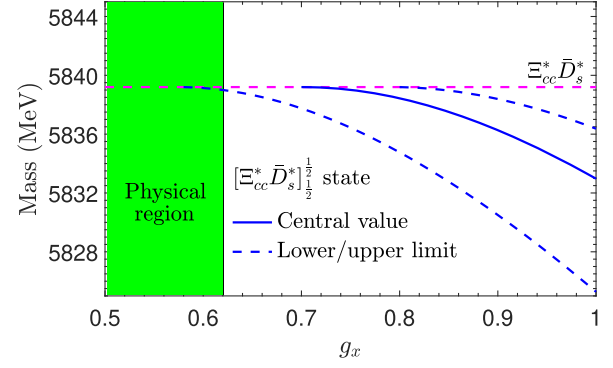


FIG. 4. From the left to right are the g_x -dependences of *lower limit* (blue-dotted line), *central value* (blue solid line), and *upper limit* (blue-dotted line) of the bound state solutions for the $[\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}^{\frac{1}{2}}$ state with the inputs from the scenario 1. The green area with $g_x \leq 0.62$ is the parameter region that the $P_{\psi s}^\Lambda(4338)$ can exist.

discussed $([\Xi_{cc}^*\bar{D}_s]_{\frac{1}{2}}, [\Omega_{cc}^*\bar{D}^*]_{\frac{1}{2}})$ system with the parameters from scenario 1 and thus cannot form bound states. Collectively, there exist seven bound states in the $P_{\psi c}^\Lambda$ system, while all their SU(3) strange $P_{\psi cs}^N$ partners can hardly exist. This conclusion is very similar to the $\bar{T}_{cc}^f$ and $\bar{T}_{cc\bar{s}}^\theta$ cases.

The significant differences between the $P_{\psi c}^\Lambda$ and $P_{\psi cs}^N$ systems are mainly attributed to the SU(3) breaking effects. Note that in our framework, the SU(3) breaking effects are mainly introduced from two sources. First, the threshold of $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ is different from the threshold of $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ or $\Omega_{cc}^{(*)}\bar{D}^{(*)}$ due to the large constituent strange quark mass. Second, the effective potentials of $P_{\psi c}^\Lambda$ states consist of the interactions introduced from the exchanges of the isospin singlet and triplet light mesons, while the effective potentials of $P_{\psi cs}^N$ states consist of the interactions introduced from the exchanges of the isospin singlet and doublet light mesons, thus, comparing to the contributions from the exchanges of the isospin triplet light mesons, the contributions from the exchanges of the isospin doublet light mesons are expected to be suppressed by the factor $1/m_{ex}^2$.

Here, we present more discussions to clarify how these two SU(3) breaking sources influence the formation of the $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ bound state. The possible $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ bound state is investigated from the effective potential of the $(\Xi_{cc}^{(*)}\bar{D}_s^{(*)}, \Omega_{cc}^{(*)}\bar{D}^{(*)})$ system listed in Table III, to construct the direct relation between the $\Xi_{cc}^{(*)}\bar{D}^{(*)}$ and $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ states, we demonstrate the following analogy

$$V_{\Xi_{cc}^{(*)}\bar{D}^{(*)}} = \begin{pmatrix} v_{\Xi_{cc}^{(*)++}D^{(*)-} \rightarrow \Xi_{cc}^{(*)++}D^{(*)-}} & v_{\Xi_{cc}^{(*)++}D^{(*)-} \rightarrow \Xi_{cc}^{(*)+}\bar{D}^{(*)0}} \\ v_{\Xi_{cc}^{(*)+}\bar{D}^{(*)0} \rightarrow \Xi_{cc}^{(*)++}D^{(*)-}} & v_{\Xi_{cc}^{(*)+}\bar{D}^{(*)0} \rightarrow \Xi_{cc}^{(*)+}\bar{D}^{(*)0}} \end{pmatrix}, \quad (31)$$

$$V_{(\Xi_{cc}^{(*)}\bar{D}_s^{(*)}, \Omega_{cc}^{(*)}\bar{D}^{(*)})} = \begin{pmatrix} \psi_{\Xi_{cc}^{(*)++}D_s^{(*)-} \rightarrow \Xi_{cc}^{(*)++}D_s^{(*)-}} & \psi_{\Xi_{cc}^{(*)++}D_s^{(*)-} \rightarrow \Omega_{cc}^{(*)+}\bar{D}^{(*)0}} \\ \psi_{\Omega_{cc}^{(*)+}\bar{D}^{(*)0} \rightarrow \Xi_{cc}^{(*)++}D_s^{(*)-}} & \psi_{\Omega_{cc}^{(*)+}\bar{D}^{(*)0} \rightarrow \Omega_{cc}^{(*)+}\bar{D}^{(*)0}} \end{pmatrix}. \quad (32)$$

Here, the $\Xi_{cc}^{(*)++}D_s^{(*)-}$ and $\Xi_{cc}^{(*)+}\bar{D}^{(*)0}$ components in Eq. (31) correspond to the $\Xi_{cc}^{(*)++}D_s^{(*)-}$ and $\Omega_{cc}^{(*)+}\bar{D}^{(*)0}$ components in Eq. (32), respectively. Note that the magnitude of the mass correction from the SU(3) breaking is much larger than that of the isospin breaking, thus, the mass gap between the $\Xi_{cc}^{(*)++}D_s^{(*)-}$ and $\Xi_{cc}^{(*)+}\bar{D}^{(*)0}$ components are much smaller than that of the $\Xi_{cc}^{(*)++}D_s^{(*)-}$ and $\Omega_{cc}^{(*)+}\bar{D}^{(*)0}$ components. Thus, comparing to the off-diagonal $\Xi_{cc}^{(*)++}D_s^{(*)-} - \Xi_{cc}^{(*)+}\bar{D}^{(*)0}$ coupling, the off-diagonal $\Xi_{cc}^{(*)++}D_s^{(*)-} - \Omega_{cc}^{(*)+}\bar{D}^{(*)0}$ coupling is suppressed by the large mass gap between the $\Xi_{cc}^{(*)++}D_s^{(*)-}$ and $\Omega_{cc}^{(*)+}\bar{D}^{(*)0}$ components. Thus, the SU(3) breaking effect induced from the mass gap of the $\Xi_{cc}^{(*)++}D_s^{(*)-} - \Omega_{cc}^{(*)+}\bar{D}^{(*)0}$ system will suppress the attractive force in the attractive $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ channel.

On the other hand, since the off-diagonal $\Xi_{cc}^{(*)++}D_s^{(*)-} - \Omega_{cc}^{(*)+}\bar{D}^{(*)0}$ channel can exchange strange isospin doublet light mesons, comparing to the off-diagonal $\Xi_{cc}^{(*)++}D_s^{(*)-} - \Xi_{cc}^{(*)+}\bar{D}^{(*)0}$ channel, the $\Xi_{cc}^{(*)++}D_s^{(*)-} - \Omega_{cc}^{(*)+}\bar{D}^{(*)0}$ coupling should be further suppressed by the mass of the exchanged strange light mesons. The double-suppression from these two SU(3) breaking sources are the main reasons that the $\Xi_{cc}^{(*)}\bar{D}_s^{(*)}$ bound state can hardly exist.

3. The existences of $H_{\Omega_{ccc}s}^N$ bound states

Now we discuss the possible $H_{\Omega_{ccc}s}^N$ bound states. As presented in Table III, we search for the possible bound states in the $\Xi_{cc}\Omega_{cc}$ and $\Xi_{cc}^*\Omega_{cc}^*$ systems via solving the single-channel LS equation. But for the $(\Xi_{cc}\Omega_{cc}^*, \Xi_{cc}^*\Omega_{cc})$ system, we perform a two-channel calculation and search for the bound state solutions and quasibound state solutions in the energy regions $E < E_{\Xi_{cc}\Omega_{cc}^*}$ and $E_{\Xi_{cc}\Omega_{cc}^*} < E < E_{\Xi_{cc}^*\Omega_{cc}}$, respectively. We find that there are no quasibound states in the $E_{\Xi_{cc}\Omega_{cc}^*} < E < E_{\Xi_{cc}^*\Omega_{cc}}$ region, but in the $E < E_{\Xi_{cc}\Omega_{cc}^*}$ region, the $\Xi_{cc}\Omega_{cc}^*$ bound states may exist.

We investigate the existences of $H_{\Omega_{ccc}s}^N$ bound states by including the SU(3) breaking effects. In Fig. 5, we plot the g_x -dependences of the bound state solutions for the considered $H_{\Omega_{ccc}s}^N$ states. We use the parameters solved from the experimental central values of the masses of $P_\psi^N(4440)$ and $P_\psi^N(4457)$ states to calculate the possible bound states in the $H_{\Omega_{ccc}s}^N$ system. Then we use the blue and red solid lines to label the results calculated with the inputs from the scenario 1 and scenario 2, respectively. As illustrated in Figs. 5(a)–5(e), we find that at $g_x = 0.62$, the $[\Xi_{cc}\Omega_{cc}]_1^{\frac{1}{2}}$,

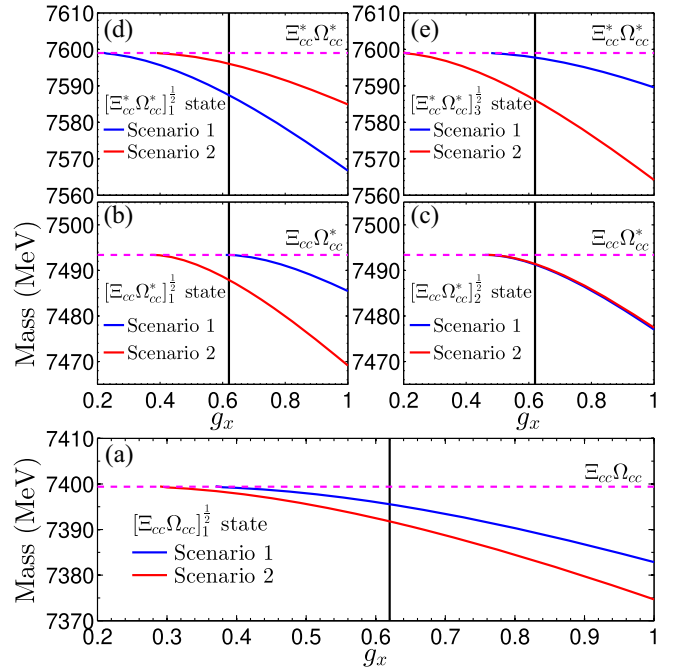


FIG. 5. The g_x -dependences of the bound state solutions for the (a)–(e): $[\Xi_{cc}\Omega_{cc}]_1^{\frac{1}{2}}$, $[\Xi_{cc}\Omega_{cc}]_2^{\frac{1}{2}}$, $[\Xi_{cc}\Omega_{cc}]_3^{\frac{1}{2}}$, $[\Xi_{cc}^*\Omega_{cc}^*]_1^{\frac{1}{2}}$, and $[\Xi_{cc}^*\Omega_{cc}^*]_3^{\frac{1}{2}}$ states. The bound state solutions are calculated with the parameters solved from the experimental *central values* of the masses of $P_\psi^N(4440)$ and $P_\psi^N(4457)$ states. The blue solid and red solid lines are used to label the results calculated from the scenario 1 and scenario 2.

$[\Xi_{cc}\Omega_{cc}^*]_1^{\frac{1}{2}}$, $[\Xi_{cc}\Omega_{cc}^*]_2^{\frac{1}{2}}$, $[\Xi_{cc}^*\Omega_{cc}]_1^{\frac{1}{2}}$, and $[\Xi_{cc}^*\Omega_{cc}]_3^{\frac{1}{2}}$ all have bound state solutions in the both scenarios.

Note that comparing to the states in the $\bar{T}_{ccs}^N$ and $P_{\psi cs}^N$ systems, the states in the $H_{\Omega_{ccc}s}^N$ system have larger reduced masses. Thus, our results for the $H_{\Omega_{ccc}s}^N$ bound states show the important roles of the reduced masses in stabilizing heavy flavor molecular states.

Besides, we can also obtain the different mass arrangements of $H_{\Omega_{ccc}s}^N$ bound states in the scenario 1 and scenario 2. Explicitly, in the scenario 1, as plotted in Figs. 5(b) and 5(c), the masses of the bound states that are mainly composed of the $\Xi_{cc}\Omega_{cc}^*$ decrease as the total angular momenta increase, while as plotted in Figs. 5(d) and 5(e), the masses of the bound states that are mainly composed of the $\Xi_{cc}^*\Omega_{cc}^*$ increase as the total angular momenta increase. On the contrary, the above tendencies are opposite in the scenario 2 case.

IV. SUMMARY

The special formation mechanism of the molecular states composed of two heavy-light hadrons motivates us to introduce a symmetric framework to relate the interactions of different dihadron systems composed of two heavy-light hadrons.

We suggest that the SU(3) flavor symmetry together with the SU(2) spin symmetry can be used to relate the dihadron systems with different light quark components, i.e.,

$$P_{\psi}^N(cnn)(\bar{c}n) \leftrightarrow \bar{T}_{cc}^f(\bar{c}n)(\bar{c}n), \quad (33)$$

$$P_{\psi s}^{\Lambda} \begin{cases} (cns)(\bar{c}n) \\ (cnn)(\bar{c}s) \end{cases} \leftrightarrow \bar{T}_{cc\bar{s}}^{\theta}(\bar{c}n)(\bar{c}s), \quad (34)$$

while the HDAS together with the SU(2) spin symmetry can be used to relate the dihadron systems with different heavy quark components, i.e.,

$$\begin{aligned} \bar{T}_{cc}^f(\bar{c}n)(\bar{c}n) &\leftrightarrow P_{\psi c}^{\Lambda}(ccn)(\bar{c}n) \leftrightarrow H_{\Omega_{ccc}c}^{\Lambda}(ccn)(ccn), \\ \bar{T}_{cc\bar{s}}^{\theta}(\bar{c}n)(\bar{c}s) &\leftrightarrow P_{\psi cs}^N \begin{cases} (ccs)(\bar{c}n) \\ (ccn)(\bar{c}s) \end{cases} \leftrightarrow H_{\Omega_{ccc}cs}^N(ccn)(ccs). \end{aligned}$$

This work is devoted to give a possible unified description to the mass spectra of $\bar{T}_{cc}^f/\bar{T}_{ccs}^{\theta}$, $P_{\psi c}^{\Lambda}/P_{\psi cs}^N$, and $H_{\Omega_{ccc}c}^{\Lambda}/H_{\Omega_{ccc}cs}^N$ systems. We introduce a contact Lagrangian possessing $SU(3)_f \otimes SU(2)_s$ symmetry to describe the interactions of the considered dihadron systems. When we adopt the Lagrangian to the considered $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ systems, we implicitly introduce the HQS and more approximate HADS to render the heavy quark and heavy diquark as spectators in the charmed mesons and double-charm baryons, respectively.

The suggested $SU(3)_f \otimes SU(2)_s$ symmetry depends on the following two strong assumptions. First, we only introduce the S -wave contact terms to describe the interactions of $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ systems. This is the most economic way to reproduce the masses of the observed $P_{\psi}^N/P_{\psi s}^{\Lambda}/\bar{T}_{cc}^f$ states. Indeed, one can also include other momentum-dependent terms to describe the interactions of the observed molecular candidates. However, at present, we cannot identify the relative contributions from these terms. Measuring the J^P numbers of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ could help us to distinguish the potentials derived from different models, and would also give important clues to the interactions of $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ systems.

Second, we uniformly use two parameters $\tilde{g}_s$ and $\tilde{g}_a$ for the $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ systems. In principle, the effective couplings $\tilde{g}_s$ and $\tilde{g}_a$ should be slightly different for the considered dimeson, baryon-meson, and dibaryon systems due to their low energy dynamics. Thus, the exact values of our predictions for the $(\bar{c}q)(\bar{c}q)/(ccq)(\bar{c}q)/(ccq)(ccq)$ systems would deviate from the physical values by a few MeV, but this will not change the relative positions of the spin multiplets in these systems.

We include the SU(3) breaking effects from two sources. First, the large violation of SU(3) symmetry is reflected by the physical masses of strange double-charm baryons and

single-charm mesons, we adopt their physical masses to perform our calculations. Second, the $\bar{T}_{cc\bar{s}}^{\theta}/P_{\psi cs}^N/H_{\Omega_{ccc}cs}^N$ systems can exchange the isospin doublet light mesons, which should be suppressed by the large masses of exchanged strange light mesons, we describe this suppression by introducing an SU(3) breaking factor g_x .

We introduce four parameters in our model, the Λ , $\tilde{g}_s$, $\tilde{g}_a$, and g_x . Here, the cutoff Λ in our dipole form factor is fixed at 1.0 GeV throughout this work. The J^P quantum numbers of $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ are assumed in the two scenarios, correspondingly, the parameters $\tilde{g}_s$ and $\tilde{g}_a$ are solved in the both scenarios. Then we further constrain the SU(3) breaking factor g_x by considering the mass of the observed $P_{\psi s}^{\Lambda}(4338)$ state.

In both scenarios, we calculate the mass spectra of $\bar{T}_{cc}^f/P_{\psi c}^{\Lambda}/H_{\Omega_{ccc}c}^{\Lambda}$ bound states. We demonstrate that according to the flavor-spin symmetry, the measurements of the J^P quantum numbers for the $P_{\psi}^N(4440)$ and $P_{\psi}^N(4457)$ states would provide important information to the binding energies of the bound states in the $\bar{T}_{cc}^f$ system. We also present that these two scenarios will lead to different mass arrangements in the $P_{\psi c}^{\Lambda}$ and $H_{\Omega_{ccc}c}^{\Lambda}$ mass spectra. All these discussed signatures can be used to test the $SU(3)_f \otimes SU(2)_s$ symmetry among the interactions of the heavy flavor molecule community.

We also investigate the existences of $\bar{T}_{cc\bar{s}}^{\theta}/P_{\psi cs}^N/H_{\Omega_{ccc}cs}^N$ bound states. By considering the SU(3) breaking effects from the two aforementioned sources, we demonstrate that for the $\bar{T}_{cc\bar{s}}^{\theta}/P_{\psi cs}^N/H_{\Omega_{ccc}cs}^N$ states that have attractive effective potentials, these two sources of SU(3) breaking effects both suppress the attractive forces in the $\bar{T}_{cc\bar{s}}^{\theta}/P_{\psi cs}^N/H_{\Omega_{ccc}cs}^N$ states. Thus, we find that unlike the $\bar{T}_{cc}^f/P_{\psi c}^{\Lambda}$ systems, the states in the $\bar{T}_{cc\bar{s}}^{\theta}/P_{\psi cs}^N$ systems can hardly form bound states. However, due to the large reduced masses, the states in the $H_{\Omega_{ccc}cs}^N$ systems can still form bound states although the double-suppression from the SU(3) breaking effects will significantly decrease the absolute values of their binding energies.

Finally, we need to emphasize that the $\bar{T}_{cc}^f/\bar{T}_{cc\bar{s}}^{\theta}$, $P_{\psi c}^{\Lambda}/P_{\psi cs}^N$, $H_{\Omega_{ccc}c}^{\Lambda}/H_{\Omega_{ccc}cs}^N$ systems discussed in this work and the $P_{\psi}^N/P_{\psi s}^{\Lambda}$, $H_{\Omega_{ccc}}^N/H_{\Omega_{ccc}s}^{\Lambda}$ systems discussed in our previous work [25] are calculated within the same framework and identical parameters, this allow us to build direct relations from one dihadron system to another dihadron system through the SU(3) flavor symmetry or HDAS symmetry together with the SU(2) spin symmetry. Thus, the experimental measurements from one of the above systems may give valuable informations to the rest of discussed systems. We hope that some of our predictions could be confirmed by the lattice QCD simulations or the BESIII, LHCb, and BELLE II collaborations in the future.

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APPENDIX: A BRIEF DISCUSSION ON THE QUARK-LEVEL AND HADRON-LEVEL DESCRIPTIONS

We take the $P_\psi^N(4440)$ and $P_\psi^N(4457)$ states as examples to discuss the consistencies of their effective potentials derived from the quark-level description and hadron-level description.

The contact potentials of $[\Sigma_c \bar{D}]_{\frac{1}{2}}^{\frac{1}{2}}$ and $[\Sigma_c \bar{D}]_{\frac{3}{2}}^{\frac{1}{2}}$ states (the superscript and subscript denote the total isospin and total angular momentum, respectively) were also derived from a hadron-level contact Lagrangian [76]. In the Appendix A of Ref. [76], we found that from the hadron-level description, the contact potential of $\Sigma_c \bar{D}^*$ system with $I = \frac{1}{2}$ can be written as

$$V_{\Sigma_c \bar{D}^*} = -\mathbb{D}_1 - \frac{2}{3}\mathbb{D}_2(\boldsymbol{\sigma}_B \cdot \mathbf{T}). \quad (\text{A1})$$

Here, $\boldsymbol{\sigma}_B$ [we use this notation to distinguish it from the light quark spin operator $\boldsymbol{\sigma}_{1(2)}$ in this work] and $\mathbf{T}$ are related to the spin of spin- $\frac{1}{2}$ Σ_c baryon and spin-1 $\bar{D}^*$ meson, respectively. The matrix element $\langle \boldsymbol{\sigma}_B \cdot \mathbf{T} \rangle$ can be calculated as

$$\langle \boldsymbol{\sigma}_B \cdot \mathbf{T} \rangle = \begin{cases} 2, & J = \frac{1}{2}, \\ -1, & J = \frac{3}{2}. \end{cases} \quad (\text{A2})$$

The $\mathbb{D}_1$ and $\mathbb{D}_2$ are the two low energy constants (LECs) that should be determined from the experiments. Based on the assignments assumed in the scenario 1, the signs of $\mathbb{D}_1$ and $\mathbb{D}_2$ are found to be both positive [76]. The determined $\mathbb{D}_1$ and $\mathbb{D}_2$ are only valid for the $I = \frac{1}{2}$ $\Sigma_c \bar{D}^*$ system. If one wants to investigate the $I = \frac{3}{2}$ $\Sigma_c \bar{D}^*$ system, one needs to further determine these two LECs from the experiments (which is not available at present).

With the matrix elements of the $\langle \boldsymbol{\sigma}_B \cdot \mathbf{T} \rangle$, we can write the explicit expressions of the effective potentials for $[\Sigma_c \bar{D}]_{\frac{1}{2}}^{\frac{1}{2}}$ and $[\Sigma_c \bar{D}]_{\frac{3}{2}}^{\frac{1}{2}}$ states as

$$V_{[\Sigma_c \bar{D}]_{\frac{1}{2}}^{\frac{1}{2}}} = -\mathbb{D}_1 - \frac{4}{3}\mathbb{D}_2, \quad (\text{A3})$$

$$V_{[\Sigma_c \bar{D}]_{\frac{3}{2}}^{\frac{1}{2}}} = -\mathbb{D}_1 + \frac{2}{3}\mathbb{D}_2. \quad (\text{A4})$$

The effective potentials of $\Sigma_c \bar{D}^*$ can also be described by the quark-level description

$$V_{\Sigma_c \bar{D}^*} = \tilde{g}_s \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2 + \tilde{g}_a \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2. \quad (\text{A5})$$

By comparing Eq. (A1) with Eq. (A5), we can infer that Eq. (A5) implicitly assumes that the LECs with different total isospins and total angular momenta are related via the matrix elements $\langle \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2 \rangle$ and $\langle \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \rangle$. This assumption ensures that Eq. (A5) manifests a $\text{SU}(3)_f \otimes \text{SU}(2)_s$ symmetry. Then the LECs $\tilde{g}_s$ and $\tilde{g}_a$ determined from the $I = \frac{1}{2}$ $P_\psi^N(4440)$ and $P_\psi^N(4457)$ states can also be applied to the $\Sigma_c \bar{D}^*$ system with $I = \frac{3}{2}$.

By including the matrix elements of $\langle \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2 \rangle$ and $\langle \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \rangle$, the explicit expressions of the effective potentials for $[\Sigma_c \bar{D}]_{\frac{1}{2}}^{\frac{1}{2}}$ and $[\Sigma_c \bar{D}]_{\frac{3}{2}}^{\frac{1}{2}}$ states in our quark-level description are [78]

$$V_{[\Sigma_c \bar{D}]_{\frac{1}{2}}^{\frac{1}{2}}} = -\frac{10}{3}\tilde{g}_s + \frac{40}{9}\tilde{g}_a, \quad (\text{A6})$$

$$V_{[\Sigma_c \bar{D}]_{\frac{3}{2}}^{\frac{1}{2}}} = -\frac{10}{3}\tilde{g}_s - \frac{20}{9}\tilde{g}_a. \quad (\text{A7})$$

The $\tilde{g}_s$ and $\tilde{g}_a$ in the scenario 1 are also given in Eq. (25), if we compare the Eqs. (A3) and (A4) with Eqs. (A6) and (A7), we can easily find that the signs of the two LECs in the hadron-level description are consistent with that of the quark-level description, i.e., the $\mathbb{D}_1$ and $\tilde{g}_s$ have the same signs, while the $\mathbb{D}_2$ and $\tilde{g}_a$ have opposite signs. We also checked that all the contact potentials of the dihadron systems involved in this work can be consistently described in our quark-level description or in the hadron-level description.

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