

W mass and muon $g - 2$ in an inert 2HDM extended by a singlet complex scalar

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The deviations of the recent measurements of the muon magnetic moment and the W -boson mass from their SM predictions hint to new physics beyond the SM. In this article, we address the observed discrepancies in the W -boson mass and muon anomalous magnetic moment in the Inert Two Higgs Doublet Model extended by a complex scalar field singlet under the SM gauge group. The model is constrained from the existing Large Electron Positron Collider (LEP) data and the measurements of partial decay widths to gauge bosons at LHC. It is shown that a large subset of the constrained parameter space of the model can accommodate both the experimentally measured as well as SM global fit value of W -boson mass while simultaneously explaining the observed muon $g - 2$ anomaly.

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I. INTRODUCTION

The departures of low-energy observables from their Standard Model (SM) predictions can provide indirect clues for physics beyond the SM. The disappearing observed W boson mass anomaly and the prevailing discrepancy in anomalous magnetic moment of muon provide a stringent test of the SM [1] and should be explained by any proposed model beyond SM.

Until its recent measurement by the CMS collaboration [2], the most precise known value of the mass of W boson m_W was

$$m_W^{\text{CDF}} = (80.4335 \pm 0.0094) \text{ GeV}, \quad (1)$$

a measurement done by the CDF Collaboration [3] on their full Run-2 dataset of 8.8 fb^{-1} . This value deviates from the global average of the other experiments [4,5]

$$m_W^{\text{PDG}} = (80.377 \pm 0.012)(80.3692 \pm 0.0133) \text{ GeV}. \quad (2)$$

A global fit to electroweak data, used to predict m_W in the standard model, yields the value [6]

$$m_W^{\text{SM}} = (80.3499 \pm 0.0094) \text{ GeV}, \quad (3)$$

which is about 7σ below the value reported by CDF. Such a significant discrepancy calls for a thorough investigation of physics beyond the Standard Model [7].

However, recently, the CMS Collaboration has reported their first measurement of W mass [2]

$$m_W^{\text{CMS}} = (80.3602 \pm 0.0099) \text{ GeV}, \quad (4)$$

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with a precision very similar to that of the recent CDF measurement [3] and better than that of all previous results. This value of W mass not is consistent with the expectation from the SM electroweak fit within experimental uncertainties as well as the present world average (excluding CDF). However, the CDF measurement is way above this value.

Another long-standing discrepancy is in the muon anomalous magnetic moment where the direct measurements of muon ($g-2$) are precisely made and have been confirmed in several experiments [8]. The most recent measurement of the anomalous muon magnetic moment by the Fermilab Muon $g-2$ Experiment [9] using data collected in 2019 and 2020 gives

$$a_\mu = \frac{(g-2)_\mu}{2} = 116592057(25) \times 10^{-11} (0.21 \text{ ppm}) \quad (5)$$

resulting in the new world average

$$a_\mu^{\text{exp}} = 116592059(22) \times 10^{-11} (0.19 \text{ ppm}). \quad (6)$$

The SM prediction is given by [10]

$$a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11} \quad (7)$$

amounting to about 5σ discrepancy

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.49 \pm 0.48) \times 10^{-9}. \quad (8)$$

This SM prediction uses the conservative leading order data-driven computation of hadronic vacuum polarization (HVP) [11] based on the available data sets for the $e^+e^- \rightarrow$ hadrons cross section and the techniques applied for the evaluation of the HVP dispersive integral. There is however, tension between the results of hadron vacuum polarization from Lattice simulations of QCD [12,13]. This and the recent measurements of e^+e^- going to hadrons by CMD3 Collaboration will make SM predictions closer to the experimental measurements. The prospects for improvements of the uncertainties in the SM prediction [10] may make it closer to the experimental measurements [14,15]. However, how this discrepancy would play out by future analysis is not yet settled.

The additional quantum corrections induced by new particles in a model beyond SM might account for the observed anomaly in the W -boson mass as well as the muon magnetic moment. These twin problems have been addressed recently (either individually or simultaneously) in many models beyond the SM [16] with varying degrees of success.

In an earlier work [17], the authors have addressed the observed discrepancies in the anomalous magnetic moment of muons and electrons by I2HDM to include a complex scalar field and a charged singlet vector-like lepton. In this spirit we revisit our earlier model albeit without the introduction of a charged vector-like lepton and discuss the constraints on the model parameters from the Large Electron Positron Collider (LEP) data and recent Higgs decay data. Using this constrained model, we attempt to address the possibility of explaining the observed upward pull for m_W and muon $g-2$.

The rest of this article is organized as follows: Sec. II briefly reviews our model. In Sec. III, we discuss the constraints on model parameters coming from the Higgs decay and the LEP data. The additional contributions to muon anomalous magnetic moment and W -mass in our model are discussed in Sec. IV. The corresponding numerical results of the regions in parameter space that simultaneously satisfy the experimental results of both observables, namely the W mass and the muon $g-2$, are given in Sec. V. In the end, we summarize our results in the Sec. VI.

II. THE MODEL

The I2HDM consists of two $SU(2)_L$ doublets of complex scalar fields: SM-like doublet Φ_1 and another doublet Φ_2 (the inert doublet) possessing the same quantum numbers as Φ_1 but with no direct coupling to fermions. We consider a model with the scalar sector of I2HDM extended by a neutral complex gauge singlet scalar field Φ_3 . After electroweak symmetry breaking, Φ_1 as well as Φ_3 acquire nonzero real vacuum expectation values, v_{SM} and v_s respectively. We invoke a Z_2 symmetry under which all SM fields and Φ_1 are even. The inert doublet fields Φ_2 and the singlet scalar Φ_3 are odd under this Z_2 symmetry. Due to this symmetry the scalar fields in Φ_2 do not mix with the SM-like field from Φ_1 . The Z_2 symmetry also ensures that the SM gauge bosons and fermions are forbidden to have direct interaction with the inert doublet and additional complex scalar singlet. We, however, allow an explicit breaking of Z_2 symmetry in the Yukawa Lagrangian $\mathcal{L}_Y$ in order to facilitate coupling of SM leptons with CP odd pseudoscalars.

The part of the Lagrangian different from the SM Lagrangian is written as

$$\mathcal{L} \supset \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}}, \quad (9a)$$

$$\begin{aligned} \mathcal{L}_{\text{scalar}} = & (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D_\mu \Phi_2) \\ & + (D_\mu \Phi_3)^* (D_\mu \Phi_3) - V_{\text{scalar}}, \end{aligned} \quad (9b)$$

$$\begin{aligned}
V_{\text{scalar}} &= V_{2\text{HDM}}(\Phi_1, \Phi_2) + V_{\text{Singlet}}(\Phi_3) + V_{\text{Mix}}(\Phi_1, \Phi_2, \Phi_3) \\
&= -\frac{1}{2}m_{11}^2(\Phi_1^\dagger\Phi_1) - \frac{1}{2}m_{22}^2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\
&\quad + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{H.c.}] \\
&\quad - \frac{1}{2}m_{33}^2\Phi_3^*\Phi_3 + \frac{\lambda_8}{2}(\Phi_3^*\Phi_3)^2 + \lambda_{11}|\Phi_1|^2\Phi_3^*\Phi_3 + \lambda_{13}|\Phi_2|^2\Phi_3^*\Phi_3 - i\kappa[(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)(\Phi_3 - \Phi_3^*)], \quad (9c)
\end{aligned}$$

where

$$\Phi_1 \equiv \begin{bmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_{\text{SM}} + \phi_1^0 + i\eta_1^0) \end{bmatrix}; \quad \Phi_2 \equiv \begin{bmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(\phi_2^0 + i\eta_2^0) \end{bmatrix} \quad \text{and} \quad \Phi_3 \equiv \frac{1}{\sqrt{2}}(v_s + \phi_3^0 + i\eta_3^0) \quad (9d)$$

and $D_\mu\Phi_i (i = 1, 2, 3)$ is the covariant derivative for the field Φ_i , where all couplings in the scalar potential and Yukawa sector are real in order to preserve the CP invariance. Here, we have invoked an additional global $U(1)$ symmetry under $\Phi_3 \rightarrow e^{i\alpha}\Phi_3$ to reduce the number of free parameters in the scalar potential, which is however allowed to be softly broken by the κ term. Further, the Yukawa terms are given by

$$\begin{aligned}
-\mathcal{L}_{\text{Yukawa}} &= y_u \overline{Q_L} \widetilde{\Phi_1} u_R + y_d \overline{Q_L} \Phi_1 d_R + y_l \overline{L_L} \Phi_1 e_R \\
&\quad + y_1 \overline{L_L} \Phi_2 e_R + \text{H.c.} \quad (10)
\end{aligned}$$

The stability of the scalar potential given in (9c) has been discussed in the article [17] and the reader may refer to it for the copositivity conditions on the scalar potential and its minimization.

The absence of mixing among the imaginary component of the inert doublet with the real component of either the first SM-like doublet or the singlet results in the decoupling of the mass matrices for neutral scalars and pseudoscalars. The 2×2 CP -even neutral scalar mass matrix arises due to the mixing of the real components of SM like first doublet Φ_1 and the singlet Φ_3 . Diagonalization of this CP -even mass matrix by orthogonal rotation matrix parametrized in terms of the mixing angle θ_{13} results in two mass eigenstates h_1 and h_3 with masses given by

$$m_{h_1}^2 = \cos^2\theta_{13}\lambda_1 v_{\text{SM}}^2 + \sin(2\theta_{13})v_s\lambda_{11}v_{\text{SM}} + \sin^2\theta_{13}v_s^2\lambda_8, \quad (11a)$$

$$m_{h_3}^2 = \sin^2\theta_{13}\lambda_1 v_{\text{SM}}^2 - \sin(2\theta_{13})v_s\lambda_{11}v_{\text{SM}} + \cos^2\theta_{13}v_s^2\lambda_8, \quad (11b)$$

with

$$\tan 2\theta_{13} = \frac{\lambda_{11}v_{\text{SM}}v_s}{\lambda_1 v_{\text{SM}}^2 - \lambda_8 v_s^2}. \quad (11c)$$

Similarly, the diagonalization of mass matrix for CP -odd scalars η_2^0 and η_3^0 gives the pseudoscalar mass eigenstates A^0 and P^0 with masses given by

$$\begin{aligned}
m_{A^0}^2 &= \frac{1}{2}(\bar{\lambda}_{345}v_{\text{SM}}^2 - m_{22}^2 + \lambda_{13}v_s^2)\cos^2\theta_{23} \\
&\quad - \sqrt{2}\kappa v_{\text{SM}}\sin 2\theta_{23}, \quad (12a)
\end{aligned}$$

$$\begin{aligned}
m_{P^0}^2 &= \frac{1}{2}(\bar{\lambda}_{345}v_{\text{SM}}^2 - m_{22}^2 + \lambda_{13}v_s^2)\sin^2\theta_{23} \\
&\quad + \sqrt{2}\kappa v_{\text{SM}}\sin 2\theta_{23} \quad (12b)
\end{aligned}$$

$$\kappa = -\frac{1}{2\sqrt{2}v_{\text{SM}}}(m_{P^0}^2 + m_{A^0}^2)\tan(2\theta_{23}). \quad (12c)$$

where $\bar{\lambda}_{345} = \lambda_3 + \lambda_4 - \lambda_5$ and the mixing angle θ_{23} is given by Out of the remaining neutral and charged scalar mass eigenstates, η_1^0 and $\phi_1^\pm$ are the massless Nambu-Goldstone bosons and the masses of ϕ_2^0 and $\phi_2^\pm$, which are renamed as h_2 and $H^\pm$, respectively, are given by

$$m_{h_2}^2 = \frac{1}{2}[-m_{22}^2 + (\lambda_3 + \lambda_4 + \lambda_5)v_{\text{SM}}^2 + \lambda_{13}v_s^2], \quad (13a)$$

$$m_{H^\pm}^2 = -m_{22}^2 + \lambda_3 v_{\text{SM}}^2 + \lambda_{13} v_s^2. \quad (13b)$$

It should be noted that the parameter λ_2 appears only in the quartic interaction of Z_2 -odd particles coming from the inert doublet Φ_2 and does not contribute to the mass spectrum. It is therefore not constrained by our analysis.

The remaining 11 parameters in the scalar potential (9c), namely, m_{11} , m_{22} , m_{33} , $\lambda_{i=1,3,4,5,8,11,13}$, and κ can now be expressed in terms of the Vacuum Expectation Values (VEVs), masses, and mixing angles

TABLE I. Yukawa couplings.

y_{ffh_1}	$(\sqrt{2}m_f/v_{\text{SM}})\cos\theta_{13}$	y_{llh_2}	y_1
y_{ffh_3}	$-(\sqrt{2}m_f/v_{\text{SM}})\sin\theta_{13}$	y_{llP^0}	$-iy_1\sin\theta_{23}$
y_{lvH^-}	y_1	y_{llA^0}	$iy_1\cos\theta_{23}$

$$v_{\text{SM}}, v_s, m_{22}^2, m_{h_1}^2, m_{h_2}^2, m_{h_3}^2, m_{H^\pm}^2, m_{A^0}^2, m_{P^0}^2, \theta_{13}, \theta_{23}. \quad (14)$$

For the relations among the mass parameters and scalar couplings of the Lagrangian, the reader is referred to the Appendix A. Further, the dimension-full scalar triple couplings of the charged Higgs bosons with neutral scalars are expressed as $g_{h_1H^+H^-} = (v_{\text{SM}}\lambda_{h_1H^+H^-})$, where

$$\lambda_{h_1H^+H^-} = \lambda_3 \cos\theta_{13} + \frac{v_s}{v_{\text{SM}}}\lambda_{13} \sin\theta_{13}, \quad (15a)$$

$$\lambda_{h_3H^+H^-} = \frac{v_s}{v_{\text{SM}}}\lambda_{13} \cos\theta_{13} - \lambda_3 \sin\theta_{13} \quad (15b)$$

are the dimensionless couplings.

The Yukawa interactions given in (10) can be rewritten in terms of mass eigenstates as

$$\begin{aligned} -\mathcal{L}_{\text{SM Fermions}}^{\text{Yukawa}} = & \sum_{s_i \equiv h_1, h_3} \frac{y_{ffs_i}}{\sqrt{2}} (v_{\text{SM}}\delta_{s_i, h_1} + s_i) \bar{f}f \\ & + \frac{y_{llh_2}}{\sqrt{2}} (h_2 \bar{l}l^-) + \sum_{s_i \equiv P^0, A^0} \frac{y_{lls_i}}{\sqrt{2}} (s_i \bar{l}l^- \gamma_5 l^-) \\ & + [y_{lvH^-} (\bar{\nu}_l P_R l^- H^+) + \text{H.c.}], \end{aligned} \quad (16)$$

where f and l^- represent SM fermions and SM charged

leptons respectively. The Yukawa couplings with scalar/pseudoscalar mass eigenstates are listed in Table I.

III. CONSTRAINTS ON PARAMETER SPACE

The theoretical constraints and existing experimental observations restrict the parameter space of any model beyond the SM. The following physical parameters of the model affect the observables considered by us in this article

Masses: $m_{h_1}, m_{h_2}, m_{h_3}, m_{H^\pm}, m_{A^0}, m_{P^0}$

Mixing angles: θ_{13}, θ_{23}

Couplings: $y_1, \lambda_{h_1H^+H^-}, \lambda_{h_3H^+H^-}$ (17)

We discuss below various constraints imposed on these parameters.

A. Theoretical constraints

Let us first consider theoretical limitations on the scalar potential of our model. The scalar potential given in (9c) should satisfy the stability and copositivity conditions listed in Ref. [17]. Further, tree level perturbative unitarity requires that

$$|\lambda_i| \leq 4\pi, \quad \text{and} \quad |y_1| < \sqrt{4\pi}, \quad (18)$$

where λ_i are all the quartic scalar couplings and y_1 is the Yukawa coupling.

The relations among mass parameters and scalar couplings of the Lagrangian, along with the copositivity conditions, result in the following two mutually exclusive allowed regions of parameter space:

$$\Theta(|\lambda_5| - \lambda_4) = \begin{cases} \Theta[m_{H^\pm}^2 - (m_{A^0}^2 + m_{P^0}^2)] & \text{for } m_{h_2}^2 > m_{A^0}^2 + m_{P^0}^2 : \text{region I} \\ \Theta[m_{h_2}^2 - m_{H^\pm}^2] & \text{for } m_{h_2}^2 < m_{A^0}^2 + m_{P^0}^2 : \text{region II.} \end{cases} \quad (19)$$

These two regions I and II correspond to $\lambda_5 > 0$ and $\lambda_5 < 0$, respectively, [as per Eq. (A5) in Appendix A]. In this article we explore the phenomenology rich region I given by

$$m_{h_2}^2 > m_{A^0}^2 + m_{P^0}^2 \quad \text{and} \quad m_{H^\pm}^2 > m_{A^0}^2 + m_{P^0}^2. \quad (20)$$

Given the aforementioned mass hierarchy, no viable scalar dark matter exists in this region. Also, the nonvanishing Yukawa coupling y_1 in the Lagrangian (10) prevents the lightest pseudoscalar from being a dark matter candidate by permitting the pseudoscalar to decay to leptons.

Now we consider the constraints from some experimental observations in the next section. In all these calculations,

the values of parameters α , the Fermi constant G_F , and Z boson mass m_Z are taken to be the measured values [5].

B. Constraints from Higgs decay

Since LHC data favors a scalar eigenstate H with mass ~ 125 GeV [5], we identify CP even lightest neutral scalar h_1 , coming predominantly from the doublet Φ_1 [Eq. (11)] with the observed scalar H and take $m_{h_1} = 125$ GeV. Further, the couplings of h_1 with a pair of fermions and gauge bosons are the corresponding SM Higgs couplings but suppressed by $\cos\theta_{13}$ due to $\Phi_1 - \Phi_3$ mixing.

We now compare the total Higgs decay width in SM [18,19]

$$\Gamma(h^{\text{SM}} \rightarrow \text{all}) \sim 4.07 \text{ MeV} \quad (21)$$

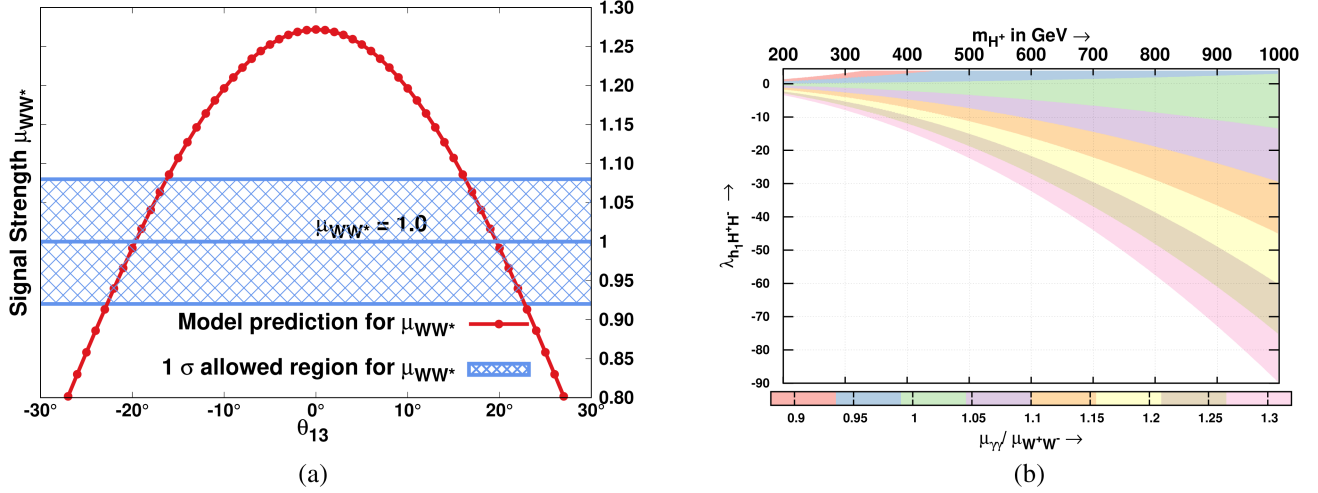


FIG. 1. Constraints on parameters θ_{13} , $g_{h_1 H^+ H^-}$ and $m_{H^\pm}$ from the measurements of the partial Higgs decay widths to the gauge bosons at the LHC [5]. (a) The solid red curve shows the variation of μ_{WW^*} computed in our model with the CP-even mixing angle θ_{13} . The shaded blue region depicts the allowed one sigma region for the measured $\mu_{WW^*} = 1.00 \pm 0.08$ [5]. (b) Color density map for the constraint $\mu_{\gamma\gamma}/\mu_{WW^*} = 1.1 \pm 0.11$ at 2σ level [5] corresponding to the $\theta_{13} = 20^\circ$ in $\lambda_{h_1 H^+ H^-} - m_{H^\pm}$ plane.

with the recently measured total Higgs decay width at the Large Hadron Collider [5]

$$\Gamma(H \rightarrow \text{all})_{\text{LHC}} = 3.2_{-1.7}^{+2.4} \text{ MeV}. \quad (22)$$

We examine the bounds on partial decay widths of 125 GeV h_1 at LHC and determine the constrained parameter space by demanding that, in our model, h_1 decays can account for the measured value of the total Higgs decay width. To this end, we define the signal strength μ_{XY} w.r.t. h_1 production via dominant gluon fusion in $p - p$ collision, followed by its decay to XY pairs in the narrow width approximation as

$$\begin{aligned} \mu_{XY} &= \frac{\sigma(pp \rightarrow h_1 \rightarrow XY)}{\sigma(pp \rightarrow h \rightarrow XY)^{\text{SM}}} \\ &= \frac{\Gamma(h_1 \rightarrow gg)}{\Gamma(h^{\text{SM}} \rightarrow gg)} \frac{\text{BR}(h_1 \rightarrow XY)}{\text{BR}(h^{\text{SM}} \rightarrow XY)} \\ &= \cos^2 \theta_{13} \frac{\text{BR}(h_1 \rightarrow XY)}{\text{BR}(h^{\text{SM}} \rightarrow XY)}. \end{aligned} \quad (23)$$

The partial decay width of $h_1 \rightarrow WW^*$ channel is related to the corresponding value in SM as

$$\Gamma(h_1 \rightarrow WW^*) = \cos^2 \theta_{13} \Gamma(h^{\text{SM}} \rightarrow WW^*) \quad (24)$$

giving the signal strength

$$\mu_{WW^*} = \cos^4 \theta_{13} \frac{\Gamma(h^{\text{SM}} \rightarrow \text{all})}{\Gamma(H \rightarrow \text{all})_{\text{LHC}}} \simeq 1.27 \cos^4 \theta_{13}. \quad (25)$$

Thus, the signal strength μ_{WW^*} depends on only one parameter of the model, namely, θ_{13} , which can be strongly

constrained by the observed value, $\mu_{WW^*} = 1.00 \pm 0.08$ [5]. The one sigma band around the central value of the observed μ_{WW^*} is shown in the Fig. 1(a), which restricts the value of θ_{13} to $19.7^\circ \lesssim |\theta_{13}| \lesssim 22.8^\circ$. Throughout this work, we take $\theta_{13} = 20^\circ$.

We now calculate the partial decay width of $h_1 \rightarrow \gamma\gamma$ channel at one loop in our model that may be parametrized as

$$\Gamma(h_1 \rightarrow \gamma\gamma) = \cos^2 \theta_{13} |1 + \zeta_{\gamma\gamma}|^2 \Gamma(h^{\text{SM}} \rightarrow \gamma\gamma), \quad (26)$$

where the SM Higgs partial decay width in $\gamma\gamma$ channel and the dimensionless parameter $\zeta_{\gamma\gamma}$ are given by [17,20]

$$\Gamma(h^{\text{SM}} \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 m_h^3}{128 \sqrt{2} \pi^3} \left| \frac{4}{3} \mathcal{M}_{1/2}^{\gamma\gamma} \left(\frac{4m_t^2}{m_h^2} \right) + \mathcal{M}_1^{\gamma\gamma} \left(\frac{4m_W^2}{m_h^2} \right) \right|^2 \quad (27a)$$

$$\zeta_{\gamma\gamma} = \frac{v_{\text{SM}}}{\cos \theta_{13}} \left[\frac{\frac{g_{h_1 H^+ H^-}}{2m_{H^\pm}^2} \mathcal{M}_0^{\gamma\gamma} \left(\frac{4m_{H^\pm}^2}{m_{h_1}^2} \right)}{\mathcal{M}_1^{\gamma\gamma} \left(\frac{4m_W^2}{m_{h_1}^2} \right) + \frac{4}{3} \mathcal{M}_{1/2}^{\gamma\gamma} \left(\frac{4m_t^2}{m_{h_1}^2} \right)} \right]. \quad (27b)$$

The loop form factors $\mathcal{M}_{0,1/2,1}^{\gamma\gamma}$ in the above equations are defined in the Appendix B. Using the relations (24) and (26), the ratio of signal strengths becomes

$$\frac{\mu_{\gamma\gamma}}{\mu_{WW^*}} = \frac{\Gamma(h_1 \rightarrow \gamma\gamma)}{\Gamma(h_1 \rightarrow WW^*)} \times \frac{\Gamma(h^{\text{SM}} \rightarrow WW^*)}{\Gamma(h^{\text{SM}} \rightarrow \gamma\gamma)} = |1 + \zeta_{\gamma\gamma}|^2. \quad (28)$$

The average experimental values of signal strengths $\mu_{\gamma\gamma} = 1.10 \pm 0.07$ and $\mu_{WW^*} = 1.00 \pm 0.08$ [5] give $\mu_{\gamma\gamma}/\mu_{WW^*} = 1.1 \pm 0.11$. The value for this ratio in our model depends only upon the parameters θ_{13} , $m_{H^\pm}$ and $\lambda_{h_1 H^+ H^-}$. Varying the $m_{H^\pm}$ between 210 GeV and 1 TeV and fixing $\theta_{13} = 20^\circ$, we find that the one sigma and two sigma constraints on $\mu_{\gamma\gamma}/\mu_{WW^*}$ restricts the charged Higgs couplings to the lightest CP even scalar within a allowed range. This allowed range depends upon the value of $m_{H^\pm}$. For example, for $m_{H^\pm} = 1$ TeV, the range allowed by $\mu_{\gamma\gamma}/\mu_{WW^*}$ is

$$\begin{aligned} -60 < \lambda_{h_1 H^+ H^-} < 3 & \text{ at } 1\sigma \\ -90 < \lambda_{h_1 H^+ H^-} < 4 & \text{ at } 2\sigma \end{aligned} \quad (29)$$

In Fig. 1(b), we exhibit the contours satisfying $\mu_{\gamma\gamma}/\mu_{WW^*}$ at 2σ level for $\theta_{13} = 20^\circ$ in the $\lambda_{h_1 H^+ H^-} - m_{H^\pm}$ plane.

Since the experimental uncertainty for $\mu_{Z\gamma}$ [5] is large, we do not expect any more constraints on the model parameters from $h_1 \rightarrow Z\gamma$ decay channel [17].

C. Constraints from LEP II data

The scalar and pseudoscalar masses along with the Yukawa coupling y_1 in our model can be constrained from the existing LEP II data either (a) by investigating the direct

pair production of scalars and pseudoscalars or (b) by production of pair of fermions mediated by these additional physical scalars or pseudoscalars. The direct neutral scalar and pseudoscalar pair production channels

$$e^+ e^- \rightarrow Z^* \rightarrow A^0/P^0 + h_i \quad (30)$$

constrain the sum of neutral Higgs masses ($\sum_{i=1}^3 m_{h_i} + m_{A_0} + m_{P^0}$) to be $\gtrsim 200$ GeV [21]. To be consistent with these bounds from LEP, we perform our analysis for all scalar and pseudoscalar masses above 210 GeV.

The production cross section of fermion pairs gets a contribution from additional scalars and pseudoscalars in the model through new leptonic Yukawa coupling y_1 . This additional contribution should be in agreement with the electroweak precision measurements conducted by LEP experiments. The combined analysis of DELPHI and L3 at LEP II at $\sqrt{s} = 200$ GeV estimate the cross section of muon pair production to [21]

$$\sigma(e^+ e^- \rightarrow \mu^+ \mu^-) = 3.072 \pm 0.108 \pm 0.018 \text{ pb.} \quad (31)$$

The excess contribution to this cross section in our model over the SM one can be written as

$$\begin{aligned} \sigma_{\mu^+ \mu^-}^{\text{Excess}} = & \frac{s}{64\pi} \sqrt{\frac{s - 4m_\mu^2}{s - 4m_e^2}} \left[y_1^2 \left(-\frac{\cos^2 \theta_{23}}{s - m_{A^0}^2} - \frac{\sin^2 \theta_{23}}{s - m_{P^0}^2} + \frac{1}{s - m_{h_2}^2} \right) + \frac{2m_e m_\mu}{v_{\text{SM}}^2} \left(\frac{\cos^2 \theta_{13}}{s - m_{h_1}^2} + \frac{\sin^2 \theta_{13}}{s - m_{h_3}^2} \right) \right]^2 \\ & - \left[\frac{2m_e m_\mu}{v_{\text{SM}}^2} \left(\frac{1}{s - m_{h_{\text{SM}}}^2} \right) \right]^2. \end{aligned} \quad (32)$$

We compute this contribution to μ -pair production cross section given by Eq. (32) and put constraints on the model parameters by accommodating this excess contribution within the 1σ uncertainty (0.1095 pb) in the cross section

$\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)$ given by (31). Figure 2 depicts the density maps for $|y_1|$ in the $m_{h_2} - \theta_{23}$ plane corresponding to various values of m_{A^0} and pseudoscalar mass ratio $R_P = m_{P^0}/m_{A^0} = 0.5, 1, 2$. The value of m_{h_3} is taken to

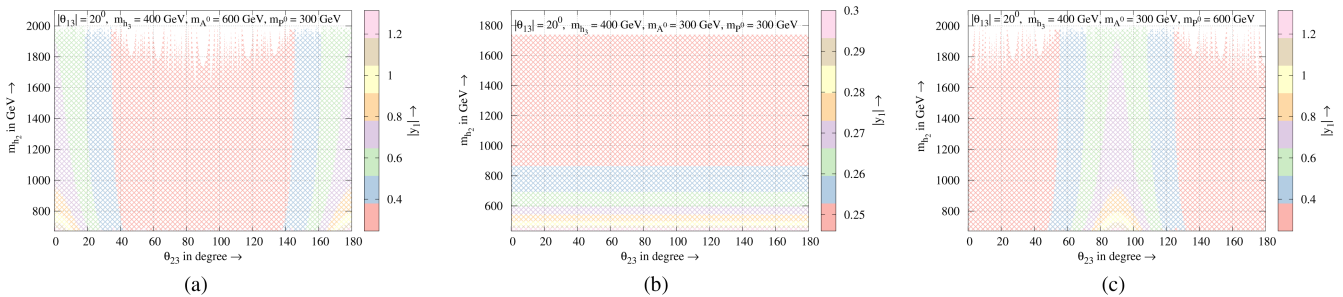


FIG. 2. Yukawa coupling $|y_1|$ color density maps for $\theta_{13} = 20^\circ$ and $m_{h_3} = 400$ GeV in the $m_{h_2} - \theta_{23}$ plane satisfying the constraints from combined analysis of DELPHI and L3, namely, $\sigma(e^+ e^- \rightarrow \mu^+ \mu^-) = 3.072 \pm 0.108 \pm 0.018$ pb at $\sqrt{s} = 200$ GeV [21] corresponding to the three parameter sets (a) $m_{A^0} = 600$ GeV, $R_P (= m_{P^0}/m_{A^0}) = 0.5$. (b) $m_{A^0} = 300$ GeV, $R_P = 1$. (c) $m_{A^0} = 300$ GeV, $R_P = 2$.

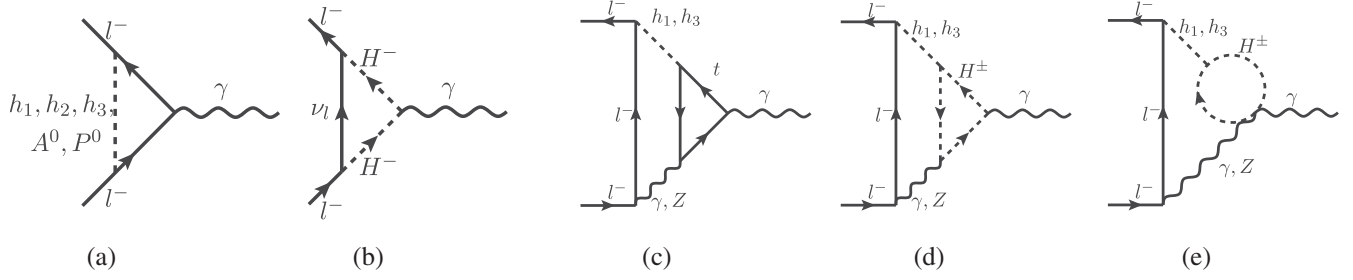


FIG. 3. One-loop and two-loop dominant diagrams contributing to $g - 2$ of charged lepton l . (a) Leptons one-loop. (b) Charged scalars one-loop. (c) Barr-Zee top triangle. (d) Barr-Zee $H^\pm$ triangle. (e) Barr-Zee $H^\pm$ bubble.

be 400 GeV in these plots. For a given m_{A^0} and m_{P^0} , the value of m_{h_2} has a lower limit determined by (20).

The following observations may be noted from Eq. (32):

- (1) The cross section is found to be less sensitive to the variation of m_{h_3} since, for $\theta_{13} \approx 20^\circ$, the term proportional to $(m_e m_\mu)/v_{\text{SM}}^2$ is negligibly tiny. This enables the LEP data to tightly constrain the magnitude of the $|y_1|$ and $|\theta_{23}|$ for the varying scalar and pseudoscalar masses up to a TeV scale.
- (2) The permitted range of $|y_1|$ is primarily governed by the choice of θ_{23} and the pseudoscalar mass ratio $R_P = m_{P^0}/m_{A^0}$. With the exception of $R_P = 1$, we note that the allowed values of $|y_1|$ are not very sensitive to m_{h_2} . This is because the matrix element squared $|\mathcal{M}_{\mu^+\mu^-}^{\text{NP}}|^2$ in Eq. (32) becomes independent of θ_{23} for $m_{A^0} = m_{P^0}$, and hence the allowed values of $|y_1|$ are dictated by values of m_{h_2} and m_{A^0} . This is evident from the $|y_1|$ color density map given in Fig. 2(b).
- (3) The color density maps in Figs. 2(a) and 2(c) show the concave and convex profiles of $|y_1|$ w.r.t. θ_{23} for $R_P < 1$ and $R_P > 1$, respectively, due to the presence of $\cos^2(\theta_{23})$ and $\sin^2(\theta_{23})$ with the respective propagators for pseudoscalars A^0 and P^0 . The convexity/concavity profile is more pronounced for lower scalar and pseudoscalar masses.

The dominant direct charged Higgs pair production channels at the e^+e^- collider

$$e^+e^- \rightarrow \gamma^*/Z^* \rightarrow H^+ + H^- \quad (33)$$

limits the charged Higgs mass between 80 and 100 GeV [22]. Assuming the branching ratio for the model predicted dominant decay channel of charged Higgs $\text{Br}(H^+ \rightarrow \tau + \nu_\tau)$ to be unity, the ALEPH collaboration at LEP [22] gives the combined 95% CL lower bound of 94 GeV on the mass of the charged Higgs boson. The LEP constraints on the masses of pseudoscalars and the model restriction $\Theta[m_{H^\pm}^2 - (m_{A^0}^2 + m_{P^0}^2)]$ as given in Eq. (20) ensures that the probed charged Higgs mass is significantly higher than the lower bound obtained from LEP.

We now proceed to look for viable regions of the parameter space already constrained in this section that account for the observed measurements of the anomalous magnetic dipole moment for muon and the W -boson mass in the next two sections.

IV. CALCULATION OF $(g-2)_\mu$ AND W -BOSON MASS

We explore in this and the following section how our model can account for the positive pull in the observed muon anomalous magnetic moment and the W -boson mass. We discuss the formalism for computing both quantities in this section, while the next section provides multivariate numerical analysis.

A. Muon anomalous magnetic moment

Now, we compute the dominant one- and two-loop contributions to the anomalous magnetic moment of a charged lepton (l) in our model and then subtract the SM contributions from the same. This difference in the anomalous magnetic moment Δa_l arises due to the exchange of the additional spectrum of charged and neutral scalars and pseudoscalars in the I2HDM at the one- and two-loop level of the perturbative calculations. Based on the Lagrangian given in Eqs. (9c) and (10), the dominant Feynman diagrams at one loop and two loop Barr-Zee diagrams are given in Fig. 3. Note that the Barr-Zee diagrams involving W -bosons are not allowed by Z_2 symmetry.

The excess contribution to lepton Δa_l at the one-loop level is given by

$$\begin{aligned} \delta a_l^{1\text{loop}} = & \frac{1}{16\pi^2} \left[2 \frac{m_l^4}{v_{\text{SM}}^2} \left(\frac{\cos^2 \theta_{13}}{m_{h_1}^2} + \frac{\sin^2 \theta_{13}}{m_{h_3}^2} - \frac{1}{m_{h_{\text{SM}}}^2} \right) \mathcal{I}_1 \right. \\ & + m_l^2 \left(\frac{\cos^2 \theta_{23}}{m_{A^0}^2} + \frac{\sin^2 \theta_{23}}{m_{P^0}^2} \right) y_1^2 \mathcal{I}_2 \\ & \left. + \frac{m_l^2}{m_{h_2}^2} y_1^2 \mathcal{I}_1 + |y_1|^2 \frac{m_l^2}{m_{H^\pm}^2} \mathcal{I}_3 \right] \quad (34) \end{aligned}$$

where the one loop integral functions $\mathcal{I}_1$, $\mathcal{I}_2$, and $\mathcal{I}_3$ are defined in Appendix C in Eqs. (C1a)–(C1c), respectively.

We observe that the one-loop amplitudes in Fig. 3(a) are negative and positive, corresponding to mediating pseudo-scalars and scalars, respectively, while the contribution from the charged Higgs loop in Fig. 3(b) is negative and competitively much smaller in magnitude. It is to be noted that, for $m_{A^0} = m_{P^0}$, the one-loop contribution becomes independent of the mixing angle θ_{23} .

The contributions of two loop diagrams, some of which may dominate in spite of an additional loop suppression

$$\delta a_l^{2\text{loop}} = \frac{\alpha_{\text{em}}}{4\pi^3} \left[\frac{m_l}{v_{\text{SM}}} \frac{m_t}{v_{\text{SM}}} \left\{ \sin^2 \theta_{13} f\left(\frac{m_t^2}{m_{h_3}^2}\right) - \cos^2 \theta_{13} f\left(\frac{m_t^2}{m_{h_1}^2}\right) + f\left(\frac{m_t^2}{m_{h_{\text{SM}}}^2}\right) \right\} \right. \\ \left. - \frac{m_l^2}{4} \frac{m_l}{v_{\text{SM}}^2} \left\{ \frac{\cos \theta_{13}}{m_{h_1}^2} g_{h_1 H^+ H^-} \tilde{f}\left(\frac{m_{H^\pm}^2}{m_{h_1}^2}\right) - \frac{\sin \theta_{13}}{m_{h_3}^2} g_{h_3 H^+ H^-} \tilde{f}\left(\frac{m_{H^\pm}^2}{m_{h_3}^2}\right) \right\} \right], \quad (35)$$

where the two loop integral functions f and $\tilde{f}$ are given by the Eqs. (C2a) and (C2b), respectively, in Appendix C.

It should be noted that the Barr-Zee diagrams of the type shown in Fig. 3(d) with W -boson and charged Higgs $H^\pm$ replacing scalars (h_1/h_3) and W boson replacing γ/Z that are usually present in a Two Higgs Doublet Model

factor, play a crucial role in the estimation of anomalous Muon Dipole Moment (MDM). It is shown in the literature that the dominant two-loop Barr-Zee diagrams mediated by neutral scalars and pseudoscalars can become relevant for certain mass scales so that their contribution to the muon anomalous MDM is of the same order to that of one loop diagrams [23]. The additional contributions to the lepton Δa_l at the two-loop level are given by

(THDM) do not contribute in this model because such a diagram will involve $WW h_2$ coupling which is forbidden in our model by the imposed Z_2 symmetry.

With $m_{h_1} \approx m_{h_{\text{SM}}} = 125$ GeV and using the dimensionless couplings defined in Eqs. (15a) and (15b), the two-loop Bar-Zee contribution in Eq. (36) can be simplified to

$$\delta a_l^{2\text{loop}} = \frac{\alpha_{\text{em}}}{4\pi^3} \left[\frac{m_l}{v_{\text{SM}}} \frac{m_t}{v_{\text{SM}}} \sin^2 \theta_{13} \left\{ f\left(\frac{m_t^2}{m_{h_3}^2}\right) - f\left(\frac{m_t^2}{m_{h_1}^2}\right) \right\} \right. \\ \left. - \frac{1}{4} \frac{m_l}{v_{\text{SM}}} \left\{ (\lambda_{h_1 H^+ H^-}) \frac{m_l^2}{m_{h_1}^2} \cos \theta_{13} \tilde{f}\left(\frac{m_{H^\pm}^2}{m_{h_1}^2}\right) - \lambda_{h_3 H^+ H^-} \frac{m_l^2}{m_{h_3}^2} \sin \theta_{13} \tilde{f}\left(\frac{m_{H^\pm}^2}{m_{h_3}^2}\right) \right\} \right]. \quad (36)$$

Keeping the dimensionless parameters $\lambda_{h_i H^+ H^-}$ ($i = 1, 3$) at a reasonable value (say $\lesssim 10$) and with the range of masses considered by us (i.e. $m_{h_3}, m_{A^0}, m_{P^0} > 200$ GeV and $m_{H^\pm} > \sqrt{m_{A^0}^2 + m_{P^0}^2}$), the suppression due to factor $m_l^2/m_{h_i}^2$ in the second term (lower line in above equation) compared to $(m_l m_t/v_{\text{SM}}^2)$ makes the contribution of charged Higgs to two-loop muon magnetic moment in Figs. 3(d) and 3(e) negligibly small even for $\lambda_{h_i H^+ H^-}$ as large as $\sim 10^3$. However, the dimensionless parameter $\lambda_{h_1 H^+ H^-}$ is restricted from the observation of signal strength ratio $\mu_{\gamma\gamma}/\mu_{WW^*}$ as discussed in Sec. III B while $\lambda_{h_3 H^+ H^-}$ is not restricted.

The Barr-Zee contributions are thus found to dominantly depend on the mixing angle θ_{13} and the scalar masses m_{h_1} and m_{h_3} .

On analyzing the combined contribution from the one- and two-loop diagrams for the constrained parameter space obtained in the preceding section, we find that, depending on the mass range of the scalars and pseudoscalars, both the one- and two-loop Barr-Zee contributions can be

significant. In fact, by demanding the total anomalous magnetic dipole moment, to agree within one sigma of the measured value as stated in Eq. (8), we can further limit the parameter space.

B. W mass computation

In this subsection, we compute the W -boson mass in our extended inert two Higgs doublet models. The mass of the W -boson can be predicted from muon decay in terms of three precisely measured quantities, namely, the Fermi constant, G_μ ; the fine structure constant, α_{em} ; and the mass of the Z -boson, m_Z via

$$m_W^2 \left(1 - \frac{m_W^2}{m_Z^2} \right) = \frac{\pi \alpha_{\text{em}}}{\sqrt{2} G_\mu (1 - \Delta r)}, \quad (37)$$

where Δr represents the quantum corrections to the relation and is a function of the scalar and pseudoscalar masses and the gauge couplings. This relationship is usually employed for predicting the W -boson mass m_W by an iterative procedure since Δr is itself a function of m_W . The SM

contribution to Δr at the full two-loop level, augmented by all the known three-loop contributions and the four-loop strong corrections, has been computed [24]. The discrepancy between the measured and the SM value may be resolved via quantum corrections that modify Δr . Defining $(\Delta r)' = \Delta r|_{\text{NP}} - \Delta r|_{\text{SM}}$ and using measured values of G_μ , α_{em} , and m_Z as input to the $SU(2) \times U(1)$ gauge theory, the relation

$$m_W^2 = (m_W^{\text{SM}})^2 \left(1 + \frac{s_w^2}{c_w^2 - s_w^2} (\Delta r)' \right) \quad (38)$$

gives the prediction of W -boson mass [25]. Here $s_w = \sin \theta_w$ and $c_w = \cos \theta_w$, θ_w being the weak angle and $(\Delta r)'$ represents the measure of deviations of the quantum corrections in a new physics model from those in SM. It is possible to parametrize $(\Delta r)'$ in terms of the oblique parameters S , T , and U as

$$\Delta r' = \frac{\alpha}{s_w^2} \left(-\frac{1}{2} \Delta S + c_w^2 \Delta T + \frac{c_w^2 - s_w^2}{4s_w^2} \Delta U \right), \quad (39)$$

where ΔS , ΔT , and ΔU are the deviations from their corresponding SM values in the estimation of the oblique parameters in any new physics models [26]. These deviations are caused by additional radiative corrections resulting from the additional scalars and pseudoscalars in the computation of self-energy amplitudes of the SM gauge bosons. The electroweak precision measurements estimate the deviations in the precision observables as [5]

$$\begin{aligned} \Delta S &= -0.02 \pm 0.10, & \Delta T &= 0.03 \pm 0.12, \\ \Delta U &= 0.01 \pm 0.11. \end{aligned} \quad (40)$$

Defining $\Delta m_W = m_W - m_W^{\text{SM}}$ and approximating $\Delta m_W^2 \simeq 2m_W^{\text{SM}} \Delta m_W$, the discrepancy between the SM prediction and experimental value of W mass can be computed using the relation

$$\Delta m_W = \frac{\alpha m_W^{\text{SM}}}{2(c_w^2 - s_w^2)} \left(-\frac{1}{2} \Delta S + c_w^2 \Delta T + \frac{c_w^2 - s_w^2}{4s_w^2} \Delta U \right). \quad (41)$$

Since the contribution from ΔU is small, henceforth we consider only the corrections from ΔS and ΔT to Δm_W .

We compute the deviations ΔS and ΔT in our model at one loop level coming from scalars and pseudoscalars h_i , P^0 , and A^0 . The explicit expressions for the same are given in the Appendix D. The equation (41) can then be solved iteratively to determine the prediction of m_W in our model.

V. THE ANOMALIES

In this section we demonstrate the simultaneous explanation of the twin anomalies while satisfying all the

constraints discussed so far. Our numerical analysis algorithm is designed as follows:

- (i) Based on the LHC constraint on the partial decay width of Higgs to WW^* and identifying the lightest scalar in the spectrum to be $m_{h_1} = 125$ GeV, we fix the CP -even mixing angle $|\theta_{13}| = 20^\circ$.
- (ii) As discussed in Sec. III, we divide the parameter space into three regions of pseudoscalar mass ratios: $R_P = m_{P^0}/m_{A^0} \equiv 0.5, 1$, and 2 . Each such region is further investigated for three choices of CP -odd mixing angle $\theta_{23} \equiv 30^\circ, 45^\circ$, and 60° .
- (iii) In general all scalar and pseudoscalar masses are varied between 200 GeV and 1 TeV. However, in accordance with Eq. (20), m_{h_2} and $m_{H^\pm}$ are varied in range $\sqrt{m_{A^0}^2 + m_{P^0}^2} < m_{h_2}, m_{H^\pm} \leq 1$ TeV. For the purpose of demonstration and paucity of space, we choose specific mass combinations for (m_{A^0}, m_{P^0}) : (600, 300) GeV, (300, 300) GeV, and (300, 600) GeV corresponding to $R_P = m_{P^0}/m_{A^0} \equiv 0.5, 1$, and 2 , respectively.
- (iv) The magnitude of the Yukawa coupling is kept below the perturbative limit, $|y_1| \leq \sqrt{4\pi}$, and is strongly constrained from the LEP data. The triple scalar coupling $\lambda_{h_1 H^+ H^-}$ is varied in the allowed range for a given value of $m_{H^\pm}$ while $|\lambda_{h_3 H^+ H^-}|$ is probed in the range 0 to 10^3 .
- (v) Next, we scan the constrained parameter hyperspace to search for a simultaneous solution for W mass lying in the range [80.3395 : 80.5275] and the anomalous magnetic moment of muon lying within one sigma band $\Delta a_\mu \in [2.01 : 2.97] \times 10^{-9}$ [9] given in Eq. (8). The specified range of m_W is chosen in order to include m_W^{CMS} [2], m_W^{SM} [6], as well as m_W^{CDF} [3].

Following the analysis, Fig. 4 illustrates this allowed parameter space in the $m_{h_2} - m_{H^\pm}$ plane for various combinations of θ_{23} and R_P that satisfy the one sigma permissible range for Δa_μ . The lower limits of m_{h_2} and $m_{H^\pm}$ in these plots are set by the constraint (20). The contour satisfying a specific value of $m_W = m_W^{\text{CDF}} + n\sigma^{\text{CDF}}$ with $n \in [10, 10]$ is represented by the loci of points in a given color. The lowest (uppermost) contour corresponds to $n = -10$ ($n = 10$). The loci of red, green, and black points correspond to the central values of m_W^{SM} , m_W^{CMS} , and m_W^{CDF} , respectively.¹ The choice of $|y_1|$ for a given set of scalar and pseudoscalar masses is essentially dictated by the LEP constraint and hence varies within a narrow range as mentioned in the legend. We make some important

¹While this work was under review, the new result of W -mass measurement by CMS collaboration was announced [2]. Although we have centered the contours around the CDF value of m_W , it may be noted that there is enough parameter space that favors the CMS central value as well as SM global fit values of m_W as shown in Fig. 4.

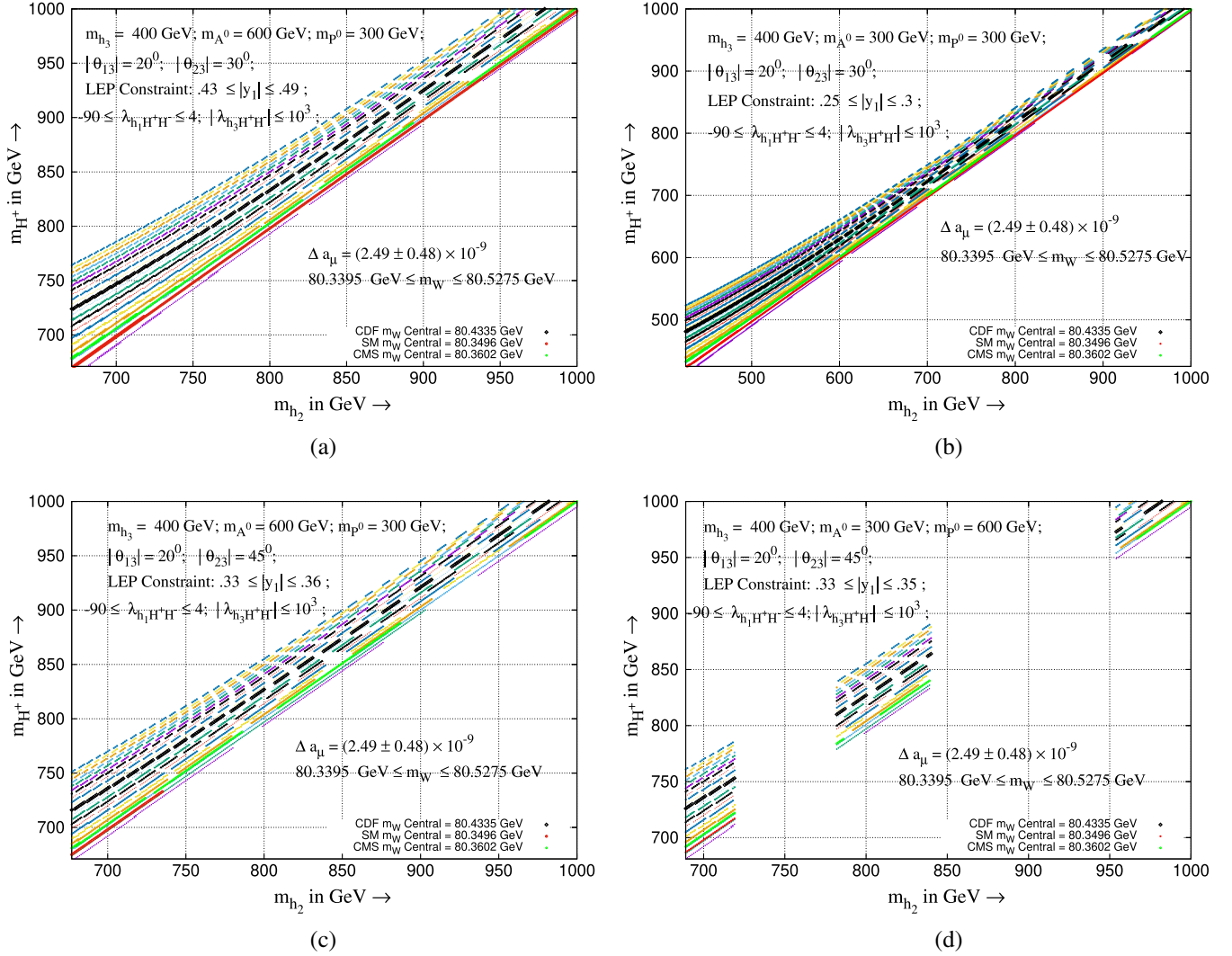


FIG. 4. This figure exhibits the allowed parameter space in the $m_{h_2} - m_{H^\pm}$ plane for the parameter sets (a) $\theta_{23} = 30^\circ$, $R_P = \frac{m_{P^0}}{m_{A^0}} = 0.5$, (b) $\theta_{23} = 30^\circ$, $R_P = \frac{m_{P^0}}{m_{A^0}} = 1$, (c) $\theta_{23} = 45^\circ$, $R_P = \frac{m_{P^0}}{m_{A^0}} = 0.5$ and (d) $\theta_{23} = 45^\circ$, $R_P = \frac{m_{P^0}}{m_{A^0}} = 2$. In each panel, the loci of points in a given color depict a contour satisfying simultaneously (i) LEP and partial Higgs decay width constraints from LHC, (ii) a specific value of $m_W = m_W^{\text{CDF}} + n\sigma^{\text{CDF}}$, with $n \in [-10, 10]$, and (iii) muon anomalous magnetic moment in the range $[2.01 : 2.97] \times 10^{-9}$ (1σ band of Δa_μ). The lowest (uppermost) contour corresponds to $n = -10$ ($n = 10$). The loci of red, green and black points correspond to the central values of m_W^{SM} , m_W^{CMS} and m_W^{CDF} , respectively.

observations on the contour plots based on the model analysis:

- (i) Since the Yukawa coupling of leptons with A^0 and P^0 is proportional to $\cos \theta_{23}$ and $\sin \theta_{23}$ respectively, the behavior of the contour plots for $\theta_{23} = 30^\circ$ and $R_P = 0.5$ is very similar to the case with $\theta_{23} = 60^\circ$ and $R_P = 2$. Hence we show a plot for only one of them in Fig. 4(a).

On a similar note, for cases where the mass ratio R_P is unity, the LEP constraints and the value of Δa_μ become independent of the mixing angle θ_{23} , and hence, similar patterns are obtained in the contour plots for all θ_{23} . We have therefore depicted only one of them for $\theta_{23} = 30^\circ$ in Fig. 4(b).

- (ii) No viable solution for m_W in the required range is found for $R_P = 2(0.5)$ at fixed $m_{h_3} = 400$ GeV, keeping all other parameters constant, for $\theta_{23} = 30^\circ(60^\circ)$. However, given a lower (or higher) value of m_{h_3} , the solution does exist. This is also evident from the m_{h_3} color density plot in Figs. 5(b) and 5(e).
- (iii) The long discontinuities of loci of points in the contour plots of Fig. 4 indicate the noncompliance of the model parameters to accommodate measured values of both observables simultaneously in the required range.

Finally, we exhibit the sensitivity of model parameter space through the m_{h_3} color density maps in the $\Delta a_\mu - m_W$ plane in Fig. 5 for different combinations of R_P and θ_{23} .

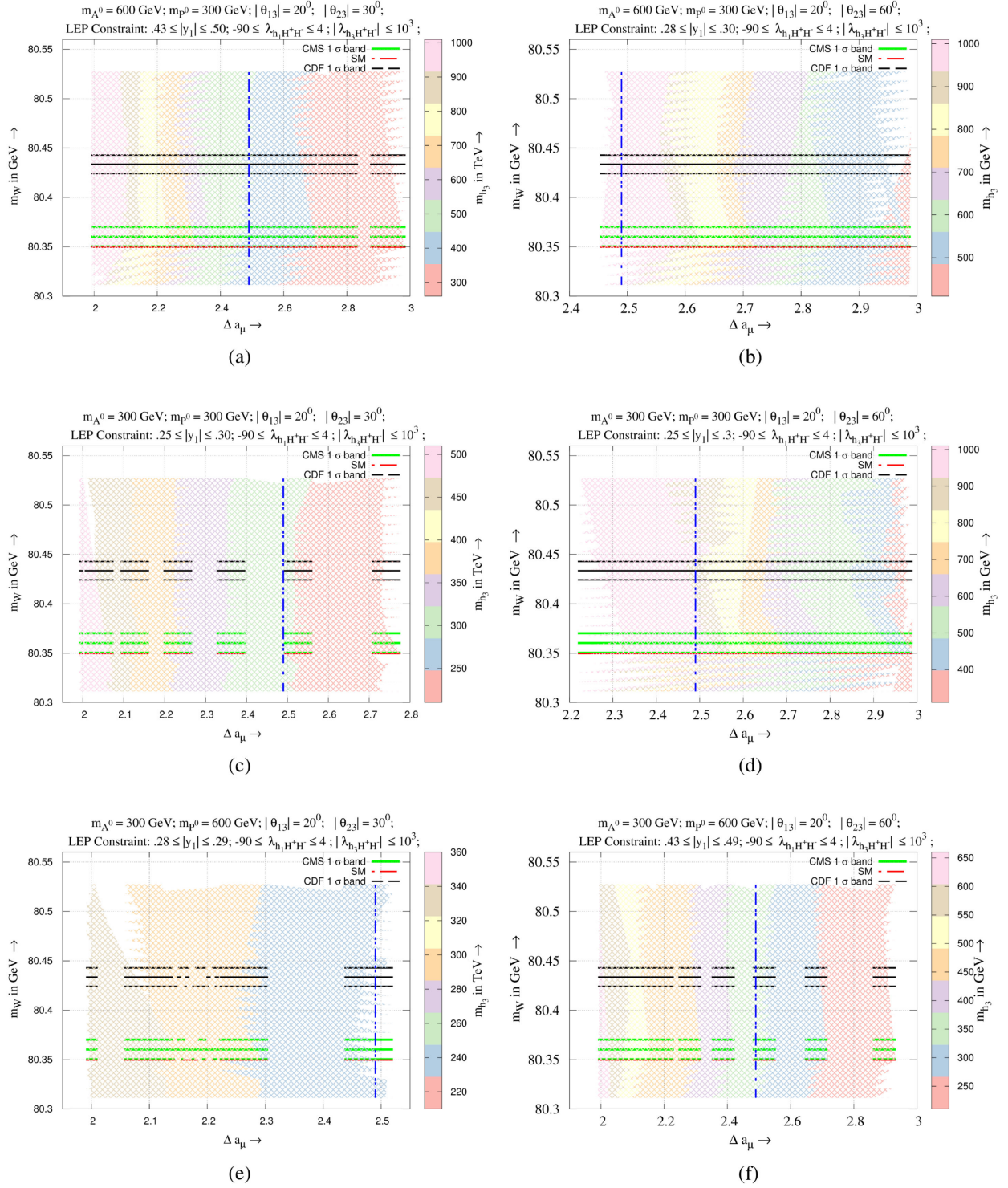


FIG. 5. Model prediction for twin anomalies through the m_{h_3} color density maps in the $\Delta a_\mu - m_W$ plane corresponding to (a) $\theta_{23} = 30^\circ$, $R_P = 0.5$, $m_{A^0} = 600$ GeV, (b) $\theta_{23} = 60^\circ$, $R_P = 0.5$, $m_{A^0} = 600$ GeV, (c) $\theta_{23} = 30^\circ$, $R_P = 1$, $m_{A^0} = 300$ GeV, (d) $\theta_{23} = 60^\circ$, $R_P = 1$, $m_{A^0} = 300$ GeV, (e) $\theta_{23} = 30^\circ$, $R_P = 2$, $m_{A^0} = 300$ GeV, and (f) $\theta_{23} = 60^\circ$, $R_P = 2$, $m_{A^0} = 300$ GeV. The density plot satisfy LEP limits and partial Higgs decay width constraint from LHC. The black horizontal dashed lines correspond to m_W^{CDF} 1σ band given by Eq. (1) while the red horizontal dashed line corresponds to m_W^{SM} predicted value Eq. (3). The recently announced value of m_W by CMS Eq. (4) is shown by green horizontal line. The vertical blue line corresponds to the central value of Δa_μ given by Eq. (8). The values of Δa_μ are shown only in the 1σ band, i.e. in the range $[2.01:2.97] \times 10^9$.

The black horizontal lines correspond to 1σ band of m_W^{CDF} given by (1) while the red horizontal line corresponds to the central m_W^{SM} value (3). We also depict the recently announced value of m_W by CMS (4) by green horizontal line. Similarly, the vertical blue line corresponds to the central value of Δa_μ given by (8). A couple of observations from Fig. 5 are given below:

- (i) For a given R_P , lower values of m_{h_3} are favored for lower values of mixing angle θ_{23} . Similarly, for a given value of θ_{23} , lower values of m_{h_3} are favored for higher R_P .
- (ii) For $\theta_{23} = 30^\circ$ and $R_P = 2$, $m_{A^0} = 300$ GeV, the common parameter space allowed by m_W value favors Δa_μ in the lower half of 1σ band while for $\theta_{23} = 60^\circ$ and $R_P = 0.5$, $m_{A^0} = 600$ GeV the parameter space allowed by m_W value favors Δa_μ in the upper half of 1σ band. This can be inferred from Figs. 5(b) and 5(e).

Thus, we see that fairly large mutually exclusive regions in the parameter space of the model are available that accommodate the CDF, SM, and CMS values of W -boson mass while simultaneously solving the anomaly of Δa_μ .

VI. SUMMARY

In this article, we have considered a minimal extension of the inert 2HDM with the inclusion of a Z_2 odd $SU(2)$ complex scalar singlet to explain the deviations of the recent measurements of the muon anomalous magnetic moment and the W -boson mass from their SM predictions. Implementing the stability and minimization conditions on the scalar potential, we have parametrized the model in terms of three neutral CP -even and two CP -odd scalar masses, one charged Higgs mass, one mixing angle each for the CP -even and CP -odd pair of scalars, the Yukawa coupling, and the scalar triple couplings of charged scalar.

We identify the lightest scalar h_1 of the spectrum of this extended model with SM-like Higgs ($m_{h_1} = 125$ GeV) observed at LHC. The model is then constrained by the recent measurements of the partial decay widths of Higgs to gauge bosons at the LHC that fix the CP -even mixing angle $\theta_{13} \approx 20^\circ$. The average experimental values of signal strengths $\mu_{\gamma\gamma} = 1.10 \pm 0.07$ and $\mu_{WW^*} = 1.00 \pm 0.08$ [5] provide the allowed range for neutral scalar triple coupling $\lambda_{h_1 H^+ H^-}$ with the charged Higgs. Further, the existing LEP data for $\sigma(e^+e^- \rightarrow \mu^+\mu^-) = 3.072 \pm 0.108 \pm 0.018$ pb [21] is used to constrain the relation between the Yukawa couplings and the masses of the scalar and pseudoscalars as stated in Eq. (32).

We then compute the contribution of the model to the anomalous magnetic moment of the charged lepton, Δa_μ , at the one loop level arising from the Feynman diagrams due to the exchange of the neutral scalar, pseudoscalar, and charged scalars as given in (34). The contribution of the

dominant Bar-Zee diagrams at the two-loop level presented in Eq. (36) is also included in the computation of Δa_μ .

Next, we calculate the contribution to the precision observables ΔS and ΔT at the one loop level from the scalars and pseudoscalars in the extended I2HDM as given in Appendix D. This deviation of the precision variables from the SM prediction is fed into the nonlinear relation for the W -boson mass in Eq. (41). We then solve this nonlinear equation iteratively by varying the model's parameters to compute the contribution to W -boson mass in the model.

The constrained model is systematically scanned and analyzed to accommodate both experimental observations simultaneously. For simplicity and brevity, the analysis is reported for three pseudoscalar mass combinations (m_{A^0}, m_{P^0}) $\equiv$ (300, 300) GeV, (300, 600) GeV, and (600, 300) GeV and three choices of the pseudoscalar mixing angle $\theta_{23} \equiv 30^\circ, 45^\circ$, and 60° . The m_{h_2} , m_{h_3} , and m_{H^+} are varied up to 1 TeV, while the lower limits for m_{h_2} and m_{H^+} are fixed by Eqs. (13a) and (13b), respectively. Maintaining the unitarity of Yukawa couplings, the coupling $|y_1|$ is varied in range $|y_1| < \sqrt{4\pi}$. The allowed values of $|y_1|$ are fixed from the LEP data and the one sigma range for $\Delta a_\mu = (249 \pm 48) \times 10^{-11}$ [9].

Our analysis can be summarized from the four panels in Fig. 4, where each panel consists of 22 contour plots in the $m_{h_2} - m_{H^\pm}$ plane for various combinations of θ_{23} and ratio $R_P = m_{P^0}/m_{A^0}$. The contours correspond to various m_W values in the range [80.3395:80.5275] including the three central values, namely, m_W^{CDF} , m_W^{CMS} , and m_W^{SM} , and simultaneously satisfy the one sigma permissible range for Δa_μ at fixed $m_{h_3} = 400$ GeV. The observations are further reinforced by depicting the allowed common parameter space in the color density plots for m_{h_3} in the $\Delta a_\mu - m_W$ plane in Fig. 5 for different combinations of R_P and θ_{23} .

Thus, the LEP and LHC data-constrained parameter hyperspaces of the said model accommodate recent observations of both Δa_μ and m_W . Although we have worked with a restricted parameter space, the simultaneous solution space of the parameters is, however, fairly large and also spans over other choices of the mass combinations for pseudoscalars with the mixing angle $20^\circ \leq |\theta_{23}| \leq 80^\circ$.

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APPENDIX A: SCALAR COUPLINGS IN TERMS OF MASS PARAMETERS

The minimization of the potential leads to THE following relations:

$$m_{11}^2 = \lambda_1 v_{\text{SM}}^2 + \lambda_{11} v_s^2, \quad (\text{A1})$$

$$m_{33}^2 = \lambda_8 v_s^2 + \lambda_{11} v_{\text{SM}}^2. \quad (\text{A2})$$

Further, the mass relations given by the equations can be combined to give the following equations relating the couplings appearing in the scalar potential (9c) with the physical mass parameters:

$$\lambda_3 = \frac{1}{v_{\text{SM}}^2} [2m_{H^\pm}^2 + m_{22}^2 - \lambda_{13} v_s^2], \quad (\text{A3})$$

$$\lambda_4 = \frac{1}{v_{\text{SM}}^2} [m_{h_2}^2 + m_{A^0}^2 + m_{P^0}^2 - 2m_{H^\pm}^2], \quad (\text{A4})$$

$$\lambda_5 = \frac{1}{v_{\text{SM}}^2} [m_{h_2}^2 - m_{A^0}^2 - m_{P^0}^2], \quad (\text{A5})$$

$$\lambda_8 = \frac{1}{v_s^2} [m_{h_1}^2 + m_{h_3}^2 - \lambda_1 v_{\text{SM}}^2], \quad (\text{A6})$$

$$\lambda_{11} = \frac{1}{v_{\text{SM}} v_s} (\lambda_1 v_{\text{SM}}^2 - \lambda_8 v_s^2) \tan(2\theta_{13}), \quad (\text{A7})$$

$$\kappa = -\frac{1}{2\sqrt{2}v_{\text{SM}}} (m_{P^0}^2 + m_{A^0}^2) \tan(2\theta_{23}). \quad (\text{A8})$$

Thus, considering VEVs v_{SM} and v_s ; mixing angles θ_{13} and θ_{23} ; coupling λ_{13} ; and masses $m_{22}^2, m_{h_1}^2, m_{h_2}^2, m_{h_3}^2, m_{H^\pm}^2, m_{A^0}^2$, and $m_{P^0}^2$ to be the free parameters, we can express $m_{11}^2, m_{33}^2, \lambda_3, \lambda_4, \lambda_5, \lambda_8, \lambda_{11}$ and κ in terms of the above free parameters.

APPENDIX B: DEFINITION OF LOOP FORM FACTORS

The loop amplitudes used in Eqs. (27a) and (27b) are expressed in terms of dimensionless parameter τ , which is essentially a function of the ratios of the mass squared of physical scalars, pseudoscalars, gauge bosons, and fermions.

$$\mathcal{M}_0^{\gamma\gamma}(\tau) = -\tau[1 - \tau f(\tau)], \quad (\text{B1a})$$

$$\mathcal{M}_{1/2}^{\gamma\gamma}(\tau) = 2\tau[1 + (1 - \tau)f(\tau)], \quad (\text{B1b})$$

$$\mathcal{M}_1^{\gamma\gamma}(\tau) = -[2 + 3\tau + 3\tau(2 - \tau)f(\tau)] \quad (\text{B1c})$$

where

$$f(\tau) = \begin{cases} \arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) & \text{for } \tau \geq 1, \\ -\frac{1}{4} \left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right) - i\pi \right]^2 & \text{for } \tau < 1. \end{cases} \quad (\text{B2})$$

APPENDIX C: ONE LOOP AND TWO LOOP FUNCTIONS FOR MDM

The integrals required to compute the one loop contribution to the muon magnetic moment of leptons (34) are given by

$$\mathcal{I}_1(r^2) = \int_0^1 dx \frac{(1+x)(1-x)^2}{(1-x)^2 r^2 + x}, \quad (\text{C1a})$$

$$\mathcal{I}_2(r^2) = \int_0^1 dx \frac{-(1-x)^3}{(1-x)^2 r^2 + x}, \quad (\text{C1b})$$

$$\mathcal{I}_3(r^2) = \int_0^1 dx \frac{-x(1-x)}{1 - (1-x)r^2} \quad (\text{C1c})$$

with $r = \frac{m_l}{m_{s_i}}$ and $s_i = h_1, h_2, h_3, A^0, P^0$.

The integrals contributing to the muon magnetic moment of leptons at the two loop level given in Eq. (36) are defined as

$$f(r^2) = \frac{r^2}{2} \int_0^1 dx \frac{1-2x(1-x)}{x(1-x)-r^2} \ln \left[\frac{x(1-x)}{r^2} \right], \quad (\text{C2a})$$

$$\tilde{f}(r^2) = \int_0^1 dx \frac{x(1-x)}{r^2 - x(1-x)} \ln \left[\frac{x(1-x)}{r^2} \right]. \quad (\text{C2b})$$

APPENDIX D: THE OBLIQUE PARAMETERS

The precision observables derived from the radiative corrections of the gauge Boson propagator are essentially the two point vacuum polarization tensor functions of $\Pi_{ij}^{\mu\nu}(q^2)$; q^2 is the four-momentum of the vector boson ($V = W, Z$ or γ). Following the prescription of the Ref. [27], the vacuum polarization tensor functions corresponding to pair of gauge Bosons V_i, V_j can be written as

$$i\Pi_{ij}^{\mu\nu}(q) = ig^{\mu\nu}A_{ij}(q^2) + iq^\mu q^\nu B_{ij}(q^2); \quad (\text{D1a})$$

$$A_{ij}(q^2) = A_{ij}(0) + q^2 F_{ij}(q^2). \quad (\text{D1a})$$

The oblique parameters are defined as

$$S \equiv \frac{1}{g^2} (16\pi \cos^2 \theta_W^2) \left[F_{ZZ}(m_Z^2) - F_{\gamma\gamma}(m_Z^2) \right. \\ \left. + \left(\frac{2 \sin \theta_W^2 - 1}{\sin \theta_W \cos \theta_W} \right) F_{Z\gamma}(m_Z^2) \right], \quad (\text{D2a})$$

$$T \equiv \frac{1}{\alpha_{em}} \left[\frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2} \right], \quad (D2b)$$

$$U \equiv \frac{1}{g^2} (16\pi) \left[F_{WW}(m_W^2) - F_{\gamma\gamma}(m_W^2) - \frac{\cos \theta_W}{\sin \theta_W} F_{Z\gamma}(m_W^2) \right] - S, \quad (D2c)$$

α_{em} being the fine structure constant. It is worthwhile to mention that although $A_{ij}(0)$ and F_{ij} are divergent by themselves the total divergence associated with each precision parameter in Eqs. (D2a)–(D2c) vanish on taking into account a gauge invariant set of one loop diagrams contributing for a given pair of gauge Bosons. The additional contribution to the oblique parameters (apart from SM) in our model can be computed to give

$$\Delta S = \frac{G_F \alpha_{em}^{-1}}{2\sqrt{2}\pi^2} \sin^2(2\theta_W) [\sin^2\theta_{13} \{m_Z^2(\mathcal{B}_0(m_Z^2; m_Z^2, m_{h_1}^2) - \mathcal{B}_0(m_Z^2; m_Z^2, m_{h_3}^2)) + \mathcal{B}_{22}(m_Z^2; m_Z^2, m_{h_3}^2) - \mathcal{B}_{22}(m_Z^2; m_Z^2, m_{h_1}^2)\} + \cos^2\theta_{23} \mathcal{B}_{22}(m_Z^2; m_{h_2}^2, m_{A^0}^2) + \sin^2\theta_{23} \mathcal{B}_{22}(m_Z^2; m_{h_2}^2, m_{P^0}^2) - \mathcal{B}_{22}(m_Z^2; m_{H^\pm}^2, m_{H^\pm}^2)] \quad (D3a)$$

where

$$\mathcal{B}_{22}(q^2; m_1^2, m_2^2) = B_{22}(q^2; m_1^2, m_2^2) - B_{22}(0; m_1^2, m_2^2), \quad (D3b)$$

$$\mathcal{B}_0(q^2; m_1^2, m_2^2) = B_0(q^2; m_1^2, m_2^2) - B_0(0; m_1^2, m_2^2), \quad (D3c)$$

$$\begin{aligned} \Delta T = \frac{G_F \alpha_{em}^{-1}}{2\sqrt{2}\pi^2} & [\sin^2\theta_{13} \{m_W^2(B_0(0; m_W^2, m_{h_1}^2) - B_0(0; m_W^2, m_{h_3}^2)) - m_Z^2(B_0(0; m_Z^2, m_{h_1}^2) - B_0(0; m_Z^2, m_{h_3}^2)) \\ & + B_{22}(0; m_W^2, m_{h_3}^2) - B_{22}(0; m_W^2, m_{h_1}^2) + B_{22}(0; m_Z^2, m_{h_1}^2) - B_{22}(0; m_Z^2, m_{h_3}^2)\} \\ & - \frac{1}{2} A_0(m_{H^\pm}^2) + B_{22}(0; m_{H^\pm}^2, m_{h_2}^2) + \cos^2\theta_{23} (B_{22}(0; m_{H^\pm}^2, m_{A^0}^2) - B_{22}(0; m_{h_2}^2, m_{A^0}^2)) \\ & + \sin^2\theta_{23} (B_{22}(0; m_{H^\pm}^2, m_{P^0}^2) - B_{22}(0; m_{h_2}^2, m_{P^0}^2))]. \end{aligned} \quad (D4)$$

The Veltman Passarino Loop Integrals A_0 , B_0 , B_{22} in the above expressions are defined as

$$A_0(m^2) = m^2(\Delta + 1 - \ln m^2), \quad (D5a)$$

$$B_0(q^2; m_1^2, m_2^2) = \Delta - \int_0^1 dx \ln(X - i\epsilon), \quad (D5b)$$

$$B_{22}(q^2; m_1^2, m_2^2) = \frac{1}{4}(\Delta + 1) \left[m_1^2 + m_2^2 - \frac{1}{3}q^2 \right] - \frac{1}{2} \int_0^1 dx X \ln(X - i\epsilon) \quad (D5c)$$

where $X \equiv m_1^2 x + m_2^2(1-x) - q^2 x(1-x)$ and $\Delta \equiv \frac{2}{4-d} + \ln(4\pi) + \gamma_E$ in d spacetime dimensions. For the Feynman rules and Feynman diagrams involved in the computation of vacuum polarization functions for ΔS and ΔT , one is referred to Ref. [17].

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