

Unified interpretation of the muon $g - 2$ anomaly, the 95 GeV diphoton, and $b\bar{b}$ excesses in the general next-to-minimal supersymmetric standard model

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We investigate three intriguing anomalies within the framework of the general next-to-minimal supersymmetric standard model. These anomalies include a significant deviation of the experimental results for the muon anomalous magnetic moment from its standard model prediction, with a confidence level of 5.1σ ; a joint observation by the CMS and ATLAS Collaborations of a diphoton excess with a local significance of 3.1σ in the invariant mass distribution around 95.4 GeV; and a reported excess in the $b\bar{b}$ production at the Large Electron-Positron Collider with a local significance of 2.3σ . Through analytical and numerical analyses, we provide unified interpretations across an extensive parameter space that remain consistent with current experimental restrictions from data on the Higgs boson at 125 GeV, B -physics measurements, and dark matter observables, as well as existing searches for supersymmetry and extra Higgs bosons. We attribute the muon anomaly to loops involving muon-smuon-neutralino and muon-sneutrino-chargino interactions, while attributing the diphoton and $b\bar{b}$ excesses to the resonant production of a singlet-dominated scalar. These proposed solutions are poised for experimental tests at the high-luminosity LHC and future linear colliders.

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I. INTRODUCTION

The muon anomalous magnetic moment $a_\mu \equiv (g - 2)/2$ stands as one of the most precisely measured quantities in particle physics. It holds a pivotal position as a crucial low-energy observable known for its sensitivity to potential new physics. The persistent deviation between the experimentally measured value and the prediction derived from the standard model (SM) has been regarded as a compelling indication of physics beyond the SM (BSM). Initial reports of this anomaly surfaced from the E821 experiment conducted at Brookhaven National Laboratory (BNL) [1] and have since been further substantiated by the run-1 data presented by E989 experiment at FermiLab in 2021 [2]. In a recent development, the FermiLab Muon $g - 2$ Experiment released a new measurement based on combined run-2 and run-3 data, demonstrating consistency with the earlier findings from BNL and run-1 data [3]. Upon comparing the updated world average $a_\mu^{\text{Exp}} = 116592059(22) \times 10^{-11}$

with the 2020 white paper prediction $a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11}$ [4], a notable discrepancy was revealed at a confidence level of 5.1σ ,

$$\Delta a_\mu \equiv a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (249 \pm 48) \times 10^{-11}. \quad (1.1)$$

While it is essential to acknowledge that uncertainties in the computation of leading-order hadronic contributions may contribute to the observed deviation [5,6],¹ there has been considerable interest in exploring the BSM origin of this anomaly [14,15].

As a highly promising BSM theory, the supersymmetric (SUSY) extension of the SM has garnered extensive attention for its exquisite structure and its ability to offer natural solutions to several long-standing puzzles within

¹The potential underestimation of theoretical uncertainties has been a subject of debate, thereby impeding the establishment of a conclusive comparison of the muon $g - 2$ measurement with theoretical predictions at present [7–9]. Notably, the BMW Collaboration conducted lattice calculations on the leading-order hadronic vacuum polarization contribution, achieving subpercent precision as detailed in [10]. This effort mitigated the observed discrepancy to a margin of 1.5σ [11]. Additionally, the determination of dispersive relations, utilizing the latest $e^+e^- \rightarrow \pi^+\pi^-$ cross section measurements by the CMD-3 experiment, demonstrates good alignment with the combined FermiLab-BNL result, despite exhibiting a discrepancy with all previous results [12,13].

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the SM, like the hierarchy problem, the unification of fundamental interactions, and the dark matter (DM) enigma [16–21]. Numerous studies on low-energy SUSY theories have consistently pointed toward quantum corrections from SUSY partners of SM particles (sparticles), such as smuon-neutralino or sneutrino-chargino loop effects, as potential contributors to the observed anomaly [22–30]. To explain Δa_μ within the SUSY framework, the associated sparticles must exhibit relatively low masses. However, such light sparticles have been subject to stringent constraints from the sparticle search experiments at the Large Hadron Collider (LHC) and dark matter detection experiments. In the minimal supersymmetric standard model (MSSM), which is the most economical realization of supersymmetry [17–19], the lightest neutralino, when serving as the lightest SUSY particle with a predominant bino field component, can fully account for the measured DM relic density [31]. Recent research on the phenomenology of the MSSM has revealed that, on the premise of explaining the muon $g - 2$ anomaly at a 2σ level, the bino DM achieves the correct relic density mainly through coannihilating with winolike electroweakinos and/or a muon-flavored slepton, and the Higgsino mass should be larger than about 500 GeV after considering the LHC experiments [32]. This implies a tuning of $\mathcal{O}(1\%)$ for predicting the Z -boson mass [33]. Furthermore, in the next-to-minimal supersymmetric standard model (NMSSM) with a $\mathbb{Z}_3$ symmetry ($\mathbb{Z}_3$) [34,35], another economical implementation of supersymmetry, the situation parallels that of the MSSM when the neutralino DM is bino dominated [36]. As for the singlino-like DM scenario, the constraints from dark matter detection experiments necessitate a relatively small value for λ . This requirement prefers the DM to coannihilate with the Higgsinos to achieve the correct relic density. Notably, this situation is associated with a small Bayesian evidence, indicating a certain level of fine-tuning within this framework [36–39]. Motivated by the mounting tension between natural interpretations of the $g - 2$ anomaly and the existing experimental constraints, we have explored the general form of the NMSSM, termed as “GNMSSM.” This exploration involves the deliberate omission of the *ad hoc* $\mathbb{Z}_3$ symmetry, a strategic choice driven by its advantageous role in circumventing the tadpole problem and resolving the cosmological domain-wall problem associated with the $\mathbb{Z}_3$ NMSSM [34]. Our findings indicate that the GNMSSM can establish a secluded dark matter sector capable of producing results consistent with the dark matter experimental observations. Additionally, it offers a robust explanation for the muon $g - 2$ anomaly across a broad parameter space that aligns well with various experimental results and naturally breaks the electroweak symmetry [40–42].

In March 2023, the CMS Collaboration released its latest analysis of searching for a light Higgs boson, confirming a previously reported excess at $m_{\gamma\gamma} = 95.4$ GeV with a local significance of 2.9σ by utilizing advanced analysis

techniques on data collected during the first, second, and third years of run 2 [43]. Additionally, the ATLAS Collaboration recently also published an analysis of the full run-2 LHC dataset (140 fb^{-1}) to search for diphoton signals in the mass range from 66 to 110 GeV [44]. This analysis revealed an excess at an invariant mass around 95 GeV with a local significance of 1.7σ . Notably, the renormalized diphoton production rates observed by both collaborations agree within uncertainties for the same mass value, suggesting the possibility that the excesses arose from the production of a single new particle. If confirmed, this could be the first sign of new physics in the Higgs boson sector [45]. After neglecting possible correlations, the combined signal strength corresponds to a 3.1σ local excess [45],

$$\mu_{\gamma\gamma}^{\text{exp}} \equiv \mu_{\gamma\gamma}^{\text{ATLAS+CMS}} = \frac{\sigma(pp \rightarrow \phi \rightarrow \gamma\gamma)}{\sigma_{\text{SM}}(pp \rightarrow H_{\text{SM}} \rightarrow \gamma\gamma)} = 0.24^{+0.09}_{-0.08}. \quad (1.2)$$

Here, ϕ is a postulated nonstandard scalar with $m_\phi = 95.4$ GeV responsible for the diphoton excess, and σ_{SM} denotes the cross section for a hypothetical SM Higgs boson H_{SM} at the same mass. Furthermore, the presence of a light ϕ could potentially be supported by the measurement of bottom quark pair events at the Large Electron-Positron (LEP) Collider in 2006. These events provided hints toward the production of ϕ through the process $e^+e^- \rightarrow Z\phi \rightarrow Z(b\bar{b})$ with a local significance of 2.3σ . They were characterized by an invariant mass of $b\bar{b}$ around 98 GeV and a signal strength of $\mu_{b\bar{b}}^{\text{exp}} = 0.117 \pm 0.057$ [46–48]. Considering the limited mass resolution for the dijets at LEP, there exists a possibility that the observed excess in $b\bar{b}$ events could be attributed to the same particle responsible for the diphoton excess.²

The intriguing speculation has sparked numerous investigations into the feasibility of accommodating both excesses in BSM theories [45,52–81]. Particularly, many of these inquiries involve explorations within the SUSY framework [48,51,64,80,82–99]. Our recent research has delved into the explanation of the observed excesses in the GNMSSM by attributing them to the resonant productions of a singlet-dominated CP -even Higgs boson [51]. We found that the observed excesses can be consistently explained in the GNMSSM across a broad parameter space without conflicting with existing experimental constraints. These constraints include data from the 125 GeV Higgs, dark matter physics, and collider searches for SUSY

²The purported 95 GeV scalar has also been suggested by the di- τ event surplus observed by the CMS Collaboration [49] and the WW excess revealed in Ref. [50]. Nevertheless, the reported signal strengths are characterized by substantial experimental uncertainties and implausibly high central values [51]. For the purpose of our analysis, we will disregard these anomalies.

particles and additional Higgs bosons. Considering the noteworthy implication of the anomalous magnetic moment and the merits of the GNMSSM Higgs sector, this research endeavors to present comprehensive explanations that jointly address both the muon $g - 2$ and the observed excesses while ensuring consistency with the various experimental constraints.

The structure of this paper is outlined as follows. In Sec. II, we provide a concise overview of the fundamental framework of the GNMSSM. This section also discusses the SUSY contributions to Δa_μ and analyzes the signal rates corresponding to $\gamma\gamma$ and $b\bar{b}$ events. Moving on to Sec. III, we conduct an extensive exploration of the model's parameter space and present the numerical outcomes through a combination of figures and tables. Furthermore, employing dedicated Monte Carlo simulations, we comprehensively analyze the constraints imposed by the searches for supersymmetry at the LHC. Finally, Sec. IV draws conclusions and offers insights based on the findings.

II. THEORETICAL PRELIMINARIES

A. The basics of GNMSSM

The GNMSSM incorporates the most general renormalizable couplings within its superpotential, as expressed by [34]

$$W_{\text{GNMSSM}} = W_{\text{Yukawa}} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{\kappa}{3} \hat{S}^3 + \mu \hat{H}_u \cdot \hat{H}_d + \frac{1}{2} \mu' \hat{S}^2 + \xi \hat{S}, \quad (2.1)$$

where W_{Yukawa} contains the quark and lepton Yukawa terms in the MSSM superpotential, $\hat{H}_u = (\hat{H}_u^+, \hat{H}_u^0)^T$ and $\hat{H}_d = (\hat{H}_d^0, \hat{H}_d^-)^T$ represent the up- and down-type $SU(2)_L$ doublet Higgs superfields, respectively, and $\hat{S}$ denotes the singlet Higgs superfield. The dimensionless coefficients λ and κ parametrize the interactions of the Higgs fields, similar to those in the $\mathbb{Z}_3$ NMSSM. The bilinear mass parameters μ and μ' and the singlet tadpole parameter ξ account for the $\mathbb{Z}_3$ -symmetry violating effects. They are advantageous for solving the tadpole problem [34,100] and the cosmological domain-wall problem of the $\mathbb{Z}_3$ NMSSM [101–103]. Note that one of these parameters can be eliminated by shifting the $\hat{S}$ field and redefining the other parameters [104]. Without loss of generality, we set ξ to be zero in this study. In such a scenario, the bilinear parameters may originate from an underlying discrete R symmetry, such as Z_4^R or Z_8^R , after the SUSY breaking and naturally reside at the electroweak scale [101,104–107]. These parameters can significantly influence the Higgs and DM physics of the $\mathbb{Z}_3$ NMSSM, which constitutes the focal point of this study.

The soft SUSY-breaking Lagrangian for the Higgs fields in the GNMSSM is given by

$$-\mathcal{L}_{\text{soft}} = \left[\lambda A_\lambda S H_u \cdot H_d + \frac{1}{3} \kappa A_\kappa S^3 + m_S^2 H_u \cdot H_d + \frac{1}{2} m_S'^2 S^2 + \xi' S + \text{H.c.} \right] + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2, \quad (2.2)$$

where H_u , H_d , and S denote the scalar components of the Higgs superfields, and $m_{H_u}^2$, $m_{H_d}^2$, and m_S^2 are their soft-breaking masses. After solving the conditional equations to minimize the scalar potential and expressing these masses in terms of the vacuum expectation values (VEVs) of the Higgs fields, namely, $\langle H_u^0 \rangle = v_u/\sqrt{2}$, $\langle H_d^0 \rangle = v_d/\sqrt{2}$, and $\langle S \rangle = v_s/\sqrt{2}$, with $v = \sqrt{v_u^2 + v_d^2} \simeq 246$ GeV, the Higgs sector is delineated by 11 independent parameters: $\tan\beta \equiv v_u/v_d$, v_s , the Yukawa couplings λ and κ , the soft-breaking trilinear coefficients A_λ and A_κ , the soft-breaking tadpole coefficient ξ' in dimension 3, the bilinear mass parameters μ and μ' , and their soft-breaking parameters m_S^2 and $m_S'^2$.

To clarify the properties of Higgs physics, it is customary to introduce the field combinations, specifically $H_{\text{SM}} \equiv \sin\beta \text{Re}(H_u^0) + \cos\beta \text{Re}(H_d^0)$, $H_{\text{NSM}} \equiv \cos\beta \text{Re}(H_u^0) - \sin\beta \text{Re}(H_d^0)$, and $A_{\text{NSM}} \equiv \cos\beta \text{Im}(H_u^0) - \sin\beta \text{Im}(H_d^0)$, where H_{SM} stands for the SM Higgs field, and H_{NSM} and A_{NSM} represent the additional doublet fields [108]. The elements of the CP -even Higgs boson mass matrix $\mathcal{M}_S^2$ in the bases $(H_{\text{NSM}}, H_{\text{SM}}, \text{Re}[S])$ are given by [34,109]

$$\begin{aligned} \mathcal{M}_{S,11}^2 &= \frac{\lambda v_s (\sqrt{2} A_\lambda + \kappa v_s + \sqrt{2} \mu') + 2m_S^2}{\sin 2\beta} + \frac{1}{2} (2m_Z^2 - \lambda^2 v^2) \sin^2 2\beta, \\ \mathcal{M}_{S,12}^2 &= -\frac{1}{4} (2m_Z^2 - \lambda^2 v^2) \sin 4\beta, \\ \mathcal{M}_{S,13}^2 &= -\frac{\lambda v}{\sqrt{2}} (A_\lambda + \sqrt{2} \kappa v_s + \mu') \cos 2\beta, \\ \mathcal{M}_{S,22}^2 &= m_Z^2 \cos^2 2\beta + \frac{1}{2} \lambda^2 v^2 \sin^2 2\beta, \\ \mathcal{M}_{S,23}^2 &= \frac{\lambda v}{\sqrt{2}} [(\sqrt{2} \lambda v_s + 2\mu) - (A_\lambda + \sqrt{2} \kappa v_s + \mu') \sin 2\beta], \\ \mathcal{M}_{S,33}^2 &= \frac{(A_\lambda + \mu') \sin 2\beta}{2\sqrt{2} v_s} \lambda v^2 + \frac{\kappa v_s}{\sqrt{2}} (A_\kappa + 2\sqrt{2} \kappa v_s + 3\mu') - \frac{\mu}{\sqrt{2} v_s} \lambda v^2 - \frac{\sqrt{2}}{v_s} \xi', \end{aligned} \quad (2.3)$$

while those for the CP -odd Higgs fields in the bases $(A_{\text{NSM}}, \text{Im}(S))$ take the following forms:

$$\begin{aligned}
\mathcal{M}_{P,11}^2 &= \frac{\lambda v_s(\sqrt{2}A_\lambda + \kappa v_s + \sqrt{2}\mu') + 2m_3^2}{\sin 2\beta}, \\
\mathcal{M}_{P,12}^2 &= \frac{\lambda v}{\sqrt{2}}(A_\lambda - \sqrt{2}\kappa v_s - \mu'), \\
\mathcal{M}_{P,22}^2 &= \frac{(A_\lambda + 2\sqrt{2}\kappa v_s + \mu') \sin 2\beta}{2\sqrt{2}v_s} \lambda v^2 - \frac{\kappa v_s}{\sqrt{2}}(3A_\kappa + \mu') \\
&\quad - \frac{\mu}{\sqrt{2}v_s} \lambda v^2 - 2m_S'^2 - \frac{\sqrt{2}}{v_s} \xi'.
\end{aligned} \tag{2.4}$$

After diagonalizing $\mathcal{M}_S^2$ and $\mathcal{M}_P^2$ with unitary matrices V and U , respectively, three CP -even and two CP -odd Higgs mass eigenstates, denoted as $h_i = \{h, H, h_s\}$ and $a_j = \{A_H, A_s\}$, respectively, are obtained,

$$\begin{aligned}
h_i &= V_{h_i}^{\text{NSM}} H_{\text{NSM}} + V_{h_i}^{\text{SM}} H_{\text{SM}} + V_{h_i}^S \text{Re}[S], \\
a_j &= U_{a_j}^{\text{NSM}} A_{\text{NSM}} + U_{a_j}^S \text{Im}[S],
\end{aligned} \tag{2.5}$$

where h represents the SM-like Higgs boson discovered at the LHC, H and A_H signify heavy doublet-dominated Higgs bosons, and h_s and A_s are singlet-dominated scalars. Additionally, the model predicts a pair of charged Higgs, $H^\pm = \cos \beta H_u^\pm + \sin \beta H_d^\pm$, with the squared mass given by

$$m_{H^\pm}^2 = \frac{\lambda v_s(\sqrt{2}A_\lambda + \kappa v_s + \sqrt{2}\mu') + 2m_3^2}{\sin 2\beta} + m_W^2 - \frac{1}{2} \lambda^2 v^2. \tag{2.6}$$

Under current experimental constraints, the Higgs sector exhibits the following characteristics. First, the H_{NSM} and $\text{Re}[S]$ components in the observed Higgs boson h are restricted to be less than about 10% [110–112], i.e., $\sqrt{(V_h^{\text{NSM}})^2 + (V_h^S)^2} \lesssim 0.1$ and $|V_h^{\text{SM}}| \sim 1$. In the limit of $\tan \beta \gg 1$, the composition of h primarily comes from the $\text{Re}[H_u^0]$ field, while H is dominated by the $\text{Re}[H_d^0]$ component. Second, the CP -even doublet scalar H exhibits a nearly degenerate mass with the CP -odd scalar A_H and the charged Higgs bosons $H^\pm$. The searches for additional Higgs bosons at the LHC, combined with indirect constraints from B physics, strongly support these bosons having significant masses, e.g., $m_H \gtrsim 0.5$ TeV [49,113]. Third, existing collider data allow the singlet-dominated scalar states to be moderately light and possess sizable doublet components [114]. This feature does not depend on the value of $\tan \beta$ and it serves as the foundational premise of this study.

The neutralino sector in the GNMSSM consists of the bino field $\tilde{B}$, the wino field $\tilde{W}$, the Higgsino fields $\tilde{H}_d^0$ and $\tilde{H}_u^0$, and the singlino field $\tilde{S}$. In the bases $\psi \equiv (\tilde{B}, \tilde{W}, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})$, the symmetric neutralino mass matrix is given by [34]

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta & 0 \\ & M_2 & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta & 0 \\ & & 0 & -\mu_{\text{tot}} & -\frac{1}{\sqrt{2}} \lambda v \sin \beta \\ & & & 0 & -\frac{1}{\sqrt{2}} \lambda v \cos \beta \\ & & & & m_N \end{pmatrix}, \tag{2.7}$$

where θ_W is the weak mixing angle, and M_1 and M_2 are the soft-breaking masses of the bino and wino fields, respectively. The Higgsino mass μ_{tot} and the singlino mass m_N are defined as $\mu_{\text{tot}} \equiv \lambda v_s / \sqrt{2} + \mu$ and $m_N \equiv \sqrt{2}\kappa v_s + \mu'$. Diagonalizing $\mathcal{M}$ by a rotation matrix N results in five mass eigenstates,

$$\tilde{\chi}_i^0 = N_{i1} \psi_1^0 + N_{i2} \psi_2^0 + N_{i3} \psi_3^0 + N_{i4} \psi_4^0 + N_{i5} \psi_5^0, \tag{2.8}$$

where $\tilde{\chi}_i^0$ ($i = 1, 2, 3, 4, 5$) are labeled in a mass-ascending order, and the matrix element N_{ij} parametrizes the component of the field ψ_j^0 in $\tilde{\chi}_i^0$. The lightest neutralino $\tilde{\chi}_1^0$ typically acts as a viable DM candidate, potentially being $\tilde{B}$ or $\tilde{S}$ dominated to acquire the measured DM relic density

[115]. A distinct feature of the GNMSSM is that the singletlike scalars may play crucial roles in DM physics when $\tilde{\chi}_1^0$ is $\tilde{S}$ dominated [115,116]. Specifically, due to the singlet-doublet coupling and the self-interaction term in the superpotential, these scalars may mediate the DM annihilation and the DM-nucleon scattering and may also present themselves as final states of the annihilation.

Recognizing that the parameters $\mu, \mu', m_3^2, m_S'^2$, and ξ' are not directly related to experimental measurements, we employ the masses of the heavy doublet Higgs fields, the CP -even and CP -odd singlet Higgs fields, and the Higgsino and singlino fields, denoted as $m_A \equiv \sqrt{\mathcal{M}_{P,11}^2}$, $m_B \equiv \sqrt{\mathcal{M}_{S,33}^2}$, $m_C \equiv \sqrt{\mathcal{M}_{P,22}^2}$, μ_{tot} , and m_N , respectively, as inputs. With this new set of parameters, the original

parameters are derived by the following identities:

$$\begin{aligned}
 \mu &= \mu_{\text{tot}} - \frac{\lambda}{\sqrt{2}} v_s, & \mu' &= m_N - \sqrt{2} \kappa v_s, \\
 m_3^2 &= \frac{m_A^2 \sin 2\beta}{2} - \lambda v_s \left(\frac{\kappa v_s}{2} + \frac{\mu'}{\sqrt{2}} + \frac{A_\lambda}{\sqrt{2}} \right), \\
 \xi' &= \frac{v_s}{\sqrt{2}} \left[\frac{(A_\lambda + \mu') \sin 2\beta}{2\sqrt{2} v_s} \lambda v^2 + \frac{\kappa v_s}{\sqrt{2}} (A_\kappa + 2\sqrt{2} \kappa v_s + 3\mu') \right. \\
 &\quad \left. - \frac{\mu}{\sqrt{2} v_s} \lambda v^2 - m_B^2 \right], \\
 m_S^2 &= \frac{1}{2} \left[m_B^2 - m_C^2 + \lambda \kappa \sin 2\beta v^2 - 2\sqrt{2} \kappa v_s \right. \\
 &\quad \left. \times \left(A_\kappa + \frac{\kappa}{\sqrt{2}} v_s + \mu' \right) \right]. \tag{2.9}
 \end{aligned}$$

Then Eqs. (2.3), (2.4), and (2.6) are rewritten as

$$\begin{aligned}
 \mathcal{M}_{S,11}^2 &= m_A^2 + \frac{1}{2} (2m_Z^2 - \lambda^2 v^2) \sin^2 2\beta, \\
 \mathcal{M}_{S,12}^2 &= -\frac{1}{4} (2m_Z^2 - \lambda^2 v^2) \sin 4\beta, \\
 \mathcal{M}_{S,13}^2 &= -\frac{\lambda v}{\sqrt{2}} (A_\lambda + m_N) \cos 2\beta, \\
 \mathcal{M}_{S,22}^2 &= m_Z^2 \cos^2 2\beta + \frac{1}{2} \lambda^2 v^2 \sin^2 2\beta, \\
 \mathcal{M}_{S,23}^2 &= \frac{\lambda v}{\sqrt{2}} [2\mu_{\text{tot}} - (A_\lambda + m_N) \sin 2\beta], \\
 \mathcal{M}_{S,33}^2 &= m_B^2, & \mathcal{M}_{P,11}^2 &= m_A^2, \\
 \mathcal{M}_{P,22}^2 &= m_C^2, & \mathcal{M}_{P,12}^2 &= \frac{\lambda v}{\sqrt{2}} (A_\lambda - m_N), \\
 m_{H^\pm}^2 &= m_A^2 + m_W^2 - \frac{1}{2} \lambda^2 v^2. \tag{2.10}
 \end{aligned}$$

In the case where λ equals zero, the singlet field corresponds to the physical particle state and its mass is interpreted as that of the particle. Although this scenario is not directly relevant to our study, we have confirmed that utilizing the new set of parameters as inputs can significantly expedite the process of identifying solutions to the excesses compared to using the old parameter set.

B. Muon $g - 2$

The SUSY contributions to the muon $g - 2$, denoted as a_μ^{SUSY} , mainly involve loops mediated by a smuon and a neutralino, as well as those containing a muon-flavored sneutrino and a chargino [22,23,26,117]. To highlight the essential characteristics of a_μ^{SUSY} , we present its expression in the mass insertion approximation instead of working in

the mass eigenstate basis.³ In the approximation at the lowest order, the contributions of a_μ^{SUSY} can be categorized into four types: WHL, BHL, BHR, and BLR, obtained from the Feynman diagrams involving $\tilde{W}^0 - \tilde{H}^0 - \tilde{\mu}_L$ (and $\tilde{W}^\pm - \tilde{H}^\pm - \tilde{\nu}_L$), $\tilde{B} - \tilde{H}^0 - \tilde{\mu}_L$, $\tilde{B} - \tilde{H}^0 - \tilde{\mu}_R$, and $\tilde{B} - \tilde{\mu}_L - \tilde{\mu}_R$ fields, respectively. Their contributions take the following forms [22,28,118]:

$$a_{\mu, \text{WHL}}^{\text{SUSY}} = \frac{\alpha_2 m_\mu^2 M_2 \mu \tan \beta}{8\pi m_{\tilde{\nu}_\mu}^4} \left\{ 2f_C \left(\frac{M_2^2}{m_{\tilde{\nu}_\mu}^2}, \frac{\mu^2}{m_{\tilde{\nu}_\mu}^2} \right) - \frac{m_{\tilde{\nu}_\mu}^4}{M_{\tilde{\mu}_L}^4} f_N \left(\frac{M_2^2}{M_{\tilde{\mu}_L}^2}, \frac{\mu^2}{M_{\tilde{\mu}_L}^2} \right) \right\}, \tag{2.11}$$

$$a_{\mu, \text{BHL}}^{\text{SUSY}} = \frac{\alpha_Y m_\mu^2 M_1 \mu \tan \beta}{8\pi M_{\tilde{\mu}_L}^4} f_N \left(\frac{M_1^2}{M_{\tilde{\mu}_L}^2}, \frac{\mu^2}{M_{\tilde{\mu}_L}^2} \right), \tag{2.12}$$

$$a_{\mu, \text{BHR}}^{\text{SUSY}} = -\frac{\alpha_Y m_\mu^2 M_1 \mu \tan \beta}{4\pi M_{\tilde{\mu}_R}^4} f_N \left(\frac{M_1^2}{M_{\tilde{\mu}_R}^2}, \frac{\mu^2}{M_{\tilde{\mu}_R}^2} \right), \tag{2.13}$$

$$a_{\mu, \text{BLR}}^{\text{SUSY}} = \frac{\alpha_Y m_\mu^2 M_1 \mu \tan \beta}{4\pi M_1^4} f_N \left(\frac{M_{\tilde{\mu}_L}^2}{M_1^2}, \frac{M_{\tilde{\mu}_R}^2}{M_1^2} \right), \tag{2.14}$$

where $M_{\tilde{\mu}_L}$ and $M_{\tilde{\mu}_R}$ represent the soft-breaking masses for left- and right-handed smuon fields, respectively, at the slepton mass scale, and they are approximately equal to smuon masses. The loop functions are given by

$$\begin{aligned}
 f_C(x, y) &= \frac{5 - 3(x + y) + xy}{(x - 1)^2(y - 1)^2} - \frac{2 \ln x}{(x - y)(x - 1)^3} \\
 &\quad + \frac{2 \ln y}{(x - y)(y - 1)^3}, \tag{2.15}
 \end{aligned}$$

$$\begin{aligned}
 f_N(x, y) &= \frac{-3 + x + y + xy}{(x - 1)^2(y - 1)^2} + \frac{2x \ln x}{(x - y)(x - 1)^3} \\
 &\quad - \frac{2y \ln y}{(x - y)(y - 1)^3}, \tag{2.16}
 \end{aligned}$$

satisfying $f_C(1, 1) = 1/2$ and $f_N(1, 1) = 1/6$.

The following points about a_μ^{SUSY} should be noted:

- (i) If all the dimensional SUSY parameters involved in a_μ^{SUSY} share a common value M_{SUSY} , a_μ^{SUSY} is proportional to $m_\mu^2 \tan \beta / M_{\text{SUSY}}^2$. This suggests that the muon $g - 2$ anomaly favors a large $\tan \beta$ and a moderately low SUSY scale.
- (ii) It was verified that the WHL contribution is usually much larger than the other contributions when μ_{tot} is less than 1 TeV and $\tilde{\mu}_L$ is not significantly heavier than $\tilde{\mu}_R$ [40].

³We verified that the difference between the a_μ^{SUSY} values calculated by the mass insertion approximation and the full expression is less than 3%.

- (iii) In contrast to the WHL, BHL, and BHR contributions that typically decrease monotonously with the magnitude of $|\mu_{\text{tot}}|$, the BLR contribution shows a linear dependence on μ_{tot} . Consequently, the scenario characterized by an exceptionally massive Higgsino (e.g., $\mu_{\text{tot}} \gtrsim 30$ TeV) is capable of achieving the central value of the muon $g-2$ anomaly, even when M_1 , $M_{\tilde{\mu}_L}$, and $M_{\tilde{\mu}_R}$ are at the TeV scale, as discussed in Ref. [119]. However, this situation necessitates significant fine-tuning to accurately predict m_Z , as noted in Ref. [33]. Additionally, it may conflict with vacuum stability or unitarity constraints due to the substantially enhanced coupling strength of the $\tilde{\mu}_L^* \tilde{\mu}_R h$ interaction [120].⁴ Therefore, in this study, we impose a limit of $|\mu_{\text{tot}}| \leq 1$ TeV to ensure that the theory complies with these constraints, resulting in a typically less significant BLR contribution.
- (iv) Two-loop (2L) contributions to a_μ , including 2L corrections to SM one-loop diagrams and those to

SUSY one-loop diagrams [25], are about -5% of the one-loop prediction [41]. Hence, these 2L contributions are neglected in this study.

- (v) When the smuon trilinear coefficient A_μ becomes comparable to or exceeds $\mu_{\text{tot}} \tan \beta$, it can significantly influence the $\tilde{\mu}_L - \tilde{\mu}_R$ transition, thereby substantially modifying the contribution $a_{\mu, \text{BHR}}^{\text{SUSY}}$. In deriving the aforementioned formulas for a_μ^{SUSY} , we have operated under the assumption that $A_\mu \ll \mu_{\text{tot}} \tan \beta$ and consequently neglected the effects of A_μ throughout the calculations.

C. $\gamma\gamma$ and $b\bar{b}$ signals

In the context of this study, the assumed origins of the excesses are the resonant productions of the singlet-dominated scalar h_s . The normalized diphoton signal strength, denoted as $\mu_{\gamma\gamma}$, in the narrow width approximation is expressed as follows:

$$\begin{aligned} \mu_{\gamma\gamma}|_{m_{h_s}=95.4 \text{ GeV}} &= \frac{\sigma_{\text{SUSY}}(pp \rightarrow h_s)}{\sigma_{\text{SM}}(pp \rightarrow h_s)} \times \frac{\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma\gamma)}{\text{Br}_{\text{SM}}(h_s \rightarrow \gamma\gamma)} \simeq \frac{\sigma_{\text{SUSY,ggF}}(pp \rightarrow h_s)}{\sigma_{\text{SM,ggF}}(pp \rightarrow h_s)} \times \frac{\Gamma_{\text{SUSY}}(h_s \rightarrow \gamma\gamma)}{\Gamma_{\text{SM}}(h_s \rightarrow \gamma\gamma)} \times \frac{\Gamma_{\text{SM}}^{\text{tot}}}{\Gamma_{\text{SUSY}}^{\text{tot}}} \\ &\simeq \frac{\Gamma_{\text{SUSY}}(h_s \rightarrow gg)}{\Gamma_{\text{SM}}(h_s \rightarrow gg)} \times \frac{\Gamma_{\text{SUSY}}(h_s \rightarrow \gamma\gamma)}{\Gamma_{\text{SM}}(h_s \rightarrow \gamma\gamma)} \times \frac{1}{R_{\text{Width}}}, \\ &\simeq |C_{h_s gg}|^2 \times |C_{h_s \gamma\gamma}|^2 \times \frac{1}{R_{\text{Width}}}, \end{aligned} \quad (2.17)$$

where the mass of h_s is fixed at 95.4 GeV. The production rate $\sigma(pp \rightarrow h_s)$, the decay branching ratio $\text{Br}(h_s \rightarrow \gamma\gamma)$, and width Γ , labeled with the subscript “SUSY,” refer to the predictions of the GNMSSM. The subscript “SM” denotes quantities assuming h_s to have SM couplings. Since the gluon fusion (ggF) process is the primary contribution to the Higgs production [43,44], we take $\sigma_{\text{SUSY}}(pp \rightarrow h_s)/\sigma_{\text{SM}}(pp \rightarrow h_s) \simeq \sigma_{\text{SUSY,ggF}}(pp \rightarrow h_s)/\sigma_{\text{SM,ggF}}(pp \rightarrow h_s) \simeq$

$\Gamma_{\text{SUSY}}(h_s \rightarrow gg)/\Gamma_{\text{SM}}(h_s \rightarrow gg) \simeq |C_{h_s gg}|^2$, where $C_{h_s gg}$ is the ratio of the h_s -gluon-gluon coupling strength to its SM prediction, denoted as $C_{h_s gg} \equiv |\mathcal{A}_{\text{SUSY}}^{h_s gg}/\mathcal{A}_{\text{SM}}^{h_s gg}|$. The normalized coupling strength of h_s to photons, $C_{h_s \gamma\gamma}$, is similarly defined. Additionally, the width $\Gamma_{\text{SUSY}}^{\text{tot}}$ and the ratio $R_{\text{Width}} \equiv \Gamma_{\text{SUSY}}^{\text{tot}}/\Gamma_{\text{SM}}^{\text{tot}}$ are acquired by

$$\begin{aligned} \Gamma_{\text{SUSY}}^{\text{tot}} &= \Gamma_{\text{SUSY}}(h_s \rightarrow b\bar{b}) + \Gamma_{\text{SUSY}}(h_s \rightarrow \tau\bar{\tau}) + \Gamma_{\text{SUSY}}(h_s \rightarrow c\bar{c}) + \Gamma_{\text{SUSY}}(h_s \rightarrow gg) + \cdots \\ &= |C_{h_s b\bar{b}}|^2 \times \Gamma_{\text{SM}}(h_s \rightarrow b\bar{b}) + |C_{h_s \tau\bar{\tau}}|^2 \times \Gamma_{\text{SM}}(h_s \rightarrow \tau\bar{\tau}) + |C_{h_s c\bar{c}}|^2 \times \Gamma_{\text{SM}}(h_s \rightarrow c\bar{c}) \\ &\quad + |C_{h_s g\bar{g}}|^2 \times \Gamma_{\text{SM}}(h_s \rightarrow g\bar{g}) + \cdots, \\ R_{\text{Width}} &= |C_{h_s b\bar{b}}|^2 \times \text{Br}_{\text{SM}}(h_s \rightarrow b\bar{b}) + |C_{h_s \tau\bar{\tau}}|^2 \times \text{Br}_{\text{SM}}(h_s \rightarrow \tau\bar{\tau}) + |C_{h_s c\bar{c}}|^2 \times \text{Br}_{\text{SM}}(h_s \rightarrow c\bar{c}) \\ &\quad + |C_{h_s g\bar{g}}|^2 \times \text{Br}_{\text{SM}}(h_s \rightarrow g\bar{g}) + \cdots, \\ &\simeq 0.801 \times |C_{h_s b\bar{b}}|^2 + 0.083 \times |C_{h_s \tau\bar{\tau}}|^2 + 0.041 \times |C_{h_s c\bar{c}}|^2 + 0.067 \times |C_{h_s g\bar{g}}|^2, \end{aligned} \quad (2.18)$$

⁴The vacuum stability constraints for the μ -term extended Z_3 NMSSM, which constitutes a subset of the GNMSSM parameter space, were recently analyzed in Ref. [40]. Additionally, an extensive discussion on unitarity constraints was presented in Ref. [121]. These studies, along with the cited references, are helpful to enhance researchers’ understanding of the constraints.

respectively, where $C_{h_s f \bar{f}}$ ($f = b, \tau, c$) are the normalized couplings of h_s to the fermion pair $f \bar{f}$ defined by $C_{h_s f \bar{f}} \equiv \mathcal{A}_{\text{SUSY}}^{h_s f \bar{f}} / \mathcal{A}_{\text{SM}}^{h_s f \bar{f}}$, and the branching ratios in the SM were obtained by the LHC Higgs Cross Section Working Group, which included all known higher-order QCD corrections [122].

Similarly, the normalized signal strength of the $b \bar{b}$ excess follows from

$$\begin{aligned} \mu_{b \bar{b}}|_{m_{h_s}=95.4 \text{ GeV}} &= \frac{\sigma_{\text{SUSY}}(e^+ e^- \rightarrow Z h_s)}{\sigma_{\text{SM}}(e^+ e^- \rightarrow Z h_s)} \times \frac{\text{Br}_{\text{SUSY}}(h_s \rightarrow b \bar{b})}{\text{Br}_{\text{SM}}(h_s \rightarrow b \bar{b})} \\ &= |C_{h_s V V}|^2 \times |C_{h_s b \bar{b}}|^2 \times \frac{1}{R_{\text{Width}}}. \end{aligned} \quad (2.19)$$

It is noteworthy that the produced Higgs boson must be CP -even to explain the $b \bar{b}$ excess, since a CP -odd one does not couple to the Z boson and thus gives no contributions. By contrast, the scalar may be either CP -even or CP -odd to account for the diphoton excess.

In the GNMSSM, the coupling $\mathcal{A}_{\text{SUSY}}^{h_s g g}$ is contributed by the loops mediated by quarks and squarks [123]. The code SPheno-4.0.5 [124,125] calculates it by the following formula:

$$\begin{aligned} \mathcal{A}_{\text{SUSY}}^{h_s g g} &= \sum_q \mathcal{A}_{\text{SUSY}}^{h_s g g, q} + \sum_{\tilde{q}} \mathcal{A}_{\text{SUSY}}^{h_s g g, \tilde{q}} \\ \mathcal{A}_{\text{SUSY}}^{h_s g g, q} &= \sum_q C_{h_s q \bar{q}}^{\text{tree}} \times \frac{m_q(Q)}{m_q(\text{pole})} \times \mathcal{A}_{\text{SM}}^{h_s g g, q} \end{aligned} \quad (2.20)$$

where $\mathcal{A}_{\text{SUSY}}^{h_s g g, q}$ and $\mathcal{A}_{\text{SUSY}}^{h_s g g, \tilde{q}}$ represent the quark and squark contributions, respectively, to the $h_s g g$ coupling in the GNMSSM [19], and $\mathcal{A}_{\text{SM}}^{h_s g g, q}$ denotes the quark loop contribution in the SM with its expression given in Ref. [126]. $C_{h_s q \bar{q}}^{\text{tree}}$ is the tree-level prediction of $C_{h_s q \bar{q}}$, and $m_q(Q)$ and $m_q(\text{pole})$ are the quark running mass at the scale $Q \simeq m_{h_s}$ and the pole mass, respectively. Additionally, the code has incorporated the higher-order QCD corrections to these quantities by the formulas in Refs. [127,128]. $\mathcal{A}_{\text{SUSY}}^{h_s \gamma \gamma}$ is similarly obtained, except that it receives additional contributions from the W boson, charged Higgs boson, and chargino-mediated loops [123]. We add that the supersymmetric contributions to $C_{h_s g g}$ and $C_{h_s \gamma \gamma}$ are not crucial in this study. Specifically, given the massiveness of the squarks, their contributions to the couplings are typically a few thousandths of the contribution from the SM particles. The charginos' contribution to $C_{h_s \gamma \gamma}$ only reaches 1% in an optimum case since λ is minor [129], and the charged Higgs contribution is at the level of 0.001%.

In this study, we obtained the normalized couplings of h_s to fermions, WW, and ZZ by their tree-level expressions.⁵

⁵The potentially large SUSY-QCD and SUSY-electroweak corrections to the bottom quark Yukawa coupling are minor in this study since gluino and squarks are very massive [130].

After neglecting the difference of the running mass and the pole mass and the supersymmetric contributions, we had the following relations [34]:

$$\begin{aligned} C_{h_s t \bar{t}} &= V_{h_s}^{\text{SM}} + V_{h_s}^{\text{NSM}} \cot \beta \simeq V_{h_s}^{\text{SM}}, \\ C_{h_s b \bar{b}} &= V_{h_s}^{\text{SM}} - V_{h_s}^{\text{NSM}} \tan \beta, \quad C_{h_s V V} = V_{h_s}^{\text{SM}}, \quad C_{h_s c \bar{c}} = C_{h_s t \bar{t}}, \\ C_{h_s \tau \bar{\tau}} &= C_{h_s b \bar{b}}, \quad C_{h_s g g} \simeq C_{h_s t \bar{t}}, \quad C_{h_s \gamma \gamma} \simeq V_{h_s}^{\text{SM}}, \end{aligned} \quad (2.21)$$

where the rotation matrix elements V_j^i were defined in Eq. (2.5). We concluded $V_{h_s}^{\text{SM}} \simeq 0.36$ and $(V_{h_s}^{\text{SM}} - V_{h_s}^{\text{NSM}} \tan \beta) \simeq 0.70 \times V_{h_s}^{\text{SM}} \simeq 0.25$ (or equivalently, $V_{h_s}^{\text{NSM}} \tan \beta \simeq 0.11$) to acquire the central values of $\mu_{\gamma \gamma}$ and $\mu_{b \bar{b}}$ and the preferred branching ratios to be $\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma \gamma) \simeq 1.86 \times \text{Br}_{\text{SM}}(h_s \rightarrow \gamma \gamma) \simeq 2.58 \times 10^{-3}$ and $\text{Br}_{\text{SUSY}}(h_s \rightarrow b \bar{b}) \simeq 0.90 \times \text{Br}_{\text{SM}}(h_s \rightarrow b \bar{b}) \simeq 72.6\%$. Alternatively, if we acquired $C_{h_s g g}$ and $C_{h_s \gamma \gamma}$ by the exact formulas, as we always did in this study, these couplings might deviate from $C_{h_s t \bar{t}}$ by 4% and 11%, respectively. In this case, we found the central values of $\mu_{\gamma \gamma}$ and $\mu_{b \bar{b}}$ corresponded to $V_{h_s}^{\text{SM}} \simeq 0.35$, $(V_{h_s}^{\text{SM}} - V_{h_s}^{\text{NSM}} \tan \beta) \simeq 0.81 \times V_{h_s}^{\text{SM}} \simeq 0.28$, or equivalently, $V_{h_s}^{\text{NSM}} \tan \beta \simeq 0.07$, $\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma \gamma) \simeq 1.77 \times \text{Br}_{\text{SM}}(h_s \rightarrow \gamma \gamma) \simeq 2.5 \times 10^{-3}$, and $\text{Br}_{\text{SUSY}}(h_s \rightarrow b \bar{b}) \simeq 0.95 \times \text{Br}_{\text{SM}}(h_s \rightarrow b \bar{b}) \simeq 76.1\%$. These results reveal that explaining the excesses requires an appropriate $C_{h_s t \bar{t}}$ and simultaneously a relatively suppressed $C_{h_s b \bar{b}}$, which are mainly decided by the Higgs mixings V_i^j . Particularly, the small deviations of $C_{h_s g g}$ and $C_{h_s \gamma \gamma}$ from $C_{h_s t \bar{t}}$ can significantly reduce $V_{h_s}^{\text{NSM}} \tan \beta$ needed to predict the central values of the excesses, but hardly change $V_{h_s}^{\text{SM}}$. The approximations also indicate that any reduction of $V_{h_s}^{\text{NSM}} \tan \beta$ can enhance $C_{h_s b \bar{b}}$, leading to the increase of $\mu_{b \bar{b}}$ and the decrease of $\mu_{\gamma \gamma}$ if $C_{h_s g g}$, $C_{h_s \gamma \gamma}$, and $C_{h_s V V}$ are fixed (note that $C_{h_s g g}$ and $C_{h_s \gamma \gamma}$ are insensitive to $C_{h_s b \bar{b}}$, and $C_{h_s V V}$ is independent of $C_{h_s b \bar{b}}$).

Furthermore, we point out that explaining the diphoton and $b \bar{b}$ excesses nontrivially relies on the parameters in the Higgs sector. Specifically, after neglecting the renormalization group running of the input parameters and the radiative corrections to $\mathcal{M}_S^2$ in Eq. (2.10), one can express the eigenstate equation of h_s as follows:

$$\sum_{j=\text{NSM,SM,S}} [\mathcal{M}_S^2]_j^i V_{h_s}^j = m_{h_s}^2 V_{h_s}^i. \quad (2.22)$$

Noting $m_{h_s}, m_h \ll m_A$, implying that the mixings of H_{NSM} with H_{SM} and $\text{Re}[S]$ are small, we acquire the following approximations:

$$\begin{aligned}
V_{h_s}^{\text{NSM}} &\simeq \frac{V_{h_s}^S}{\sqrt{2}} \times \frac{\lambda v \bar{A}_\lambda \cos 2\beta}{m_A^2}, \\
\lambda(\mu_{\text{tot}} - \bar{A}_\lambda \sin \beta \cos \beta) &\simeq \frac{V_{h_s}^{\text{SM}} V_{h_s}^S}{\sqrt{2}} \times \frac{m_{h_s}^2 - m_h^2}{v}, \\
m_B^2 &\simeq m_{h_s}^2 |V_{h_s}^S|^2 + m_h^2 |V_{h_s}^{\text{SM}}|^2, \\
\mathcal{M}_{S,22}^2 &\simeq m_{h_s}^2 |V_{h_s}^S|^2 + m_{h_s}^2 |V_{h_s}^{\text{SM}}|^2, \quad (2.23)
\end{aligned}$$

where $\bar{A}_\lambda$ is defined as $\bar{A}_\lambda \equiv A_\lambda + m_N$. These expressions indicate that the observed excesses have restricted m_B and $\mathcal{M}_{S,22}$ within narrow ranges. They also suggest

$$\lambda \simeq 0.06 \times \left(\frac{V_{h_s}^{\text{SM}}}{0.35} \right) \times \left(\frac{\mu_{\text{tot}} - \bar{A}_\lambda \sin \beta \cos \beta}{100 \text{ GeV}} \right)^{-1}, \quad (2.24)$$

and

$$\lambda \gtrsim 0.017 \times \frac{1}{|\cos 2\beta|} \times \left(\frac{\tan \beta}{50} \right)^{-1} \times \left(\frac{\bar{A}_\lambda}{2 \text{ TeV}} \right)^{-1} \times \left(\frac{m_A}{2 \text{ TeV}} \right)^2, \quad (2.25)$$

to predict $V_{h_s}^{\text{NSM}} \tan \beta \gtrsim 0.07$. Given that the LHC searches for electroweakinos have established $\mu_{\text{tot}} \gtrsim 200 \text{ GeV}$ [131], one can deduce that predicting the central values of the excesses favors $\lambda \lesssim 0.03$ under the condition $\bar{A}_\lambda \sin \beta \cos \beta \ll \mu_{\text{tot}}$. This in turn necessitates $\tan \beta / 50 \times \bar{A}_\lambda \gtrsim 1.8 \text{ TeV}$ when $m_A = 2 \text{ TeV}$. These requirements can be met when $\bar{A}_\lambda$ reaches several TeV and $\tan \beta$ takes sufficiently large values. On the other hand, for moderate values of $\tan \beta$ where $\mu_{\text{tot}} - \bar{A}_\lambda \sin \beta \cos \beta \simeq 10 \text{ GeV}$ is acquired, λ can be as large as approximately 0.5 to account for the excesses, as demonstrated in Ref. [98]. However, this scenario relies on a fine-tuned cancellation between two independent terms, μ_{tot} and $\bar{A}_\lambda \sin \beta \cos \beta$, and thus, its posterior probability density function (PDF) in Bayesian inference is suppressed.

III. EXPLANATIONS OF THE ANOMALIES

This section introduces the sampling strategy used in this work and provides detailed numerical results by which we

elucidate the characteristics of the GNMSSM in explaining the muon $g-2$ anomaly and the diphoton and $b\bar{b}$ excesses. The numerical calculation process begins with utilizing the package SARAH 4.15.3 [132–135] to build the model routines of the GNMSSM. Subsequently, the codes SPheno-4.0.5 [124,125] and FlavorKit [136] are employed to generate particle spectrum and compute low-energy flavor observables, respectively. We calculate the DM physics observables with the package MicrOMEGAs-5.0.4 [137–146]. Finally, we analyze the acquired samples by means of the posterior PDF in Bayesian inference and the profile likelihood (PL) in frequentist statistics [147]. We also discuss the impacts of the latest 125 GeV Higgs data on the explanations.

A. Research strategy

The discussion in Sec. II shows that the three anomalies depend on the parameters λ , $\tan \beta$, m_A , m_B , μ_{tot} , m_N , A_λ , M_1 , M_2 , $M_{\tilde{\mu}_L}$, and $M_{\tilde{\mu}_R}$ in the GNMSSM framework. Additionally, as demonstrated in Ref. [148], the DM physics in the GNMSSM is also influenced by parameters κ , A_κ , v_s , and m_C . Since our primary objective is to investigate whether the GNMSSM can account for these three anomalies under various experimental constraints, rather than performing a comprehensive global fitting analysis of the model, we fix the value of some unimportant parameters in this study, including $m_A = 2 \text{ TeV}$, $m_C = 800 \text{ GeV}$, and $v_s = 1 \text{ TeV}$. Furthermore, we determined A_κ through the condition $\xi' = 0$, which yields [148]

$$\begin{aligned}
\kappa A_\kappa &= \sqrt{2} m_B^2 / v_s + \lambda \mu v^2 / v_s^2 \\
&\quad - \lambda (A_\lambda + m_N - \sqrt{2} \sin 2\beta \kappa v_s) v^2 / (2 v_s^2) \\
&\quad + \sqrt{2} \kappa^2 v_s - 3 \kappa m_N,
\end{aligned}$$

while allowing the remaining parameters to vary.

We performed a sophisticated scan over the parameter space delineated in Table I, using the MultiNest algorithm with `nlive` = 12000 [149].⁶ We constructed the following likelihood function to guide the scan:

$$\mathcal{L} \equiv \mathcal{L}_{\Delta a_\mu} \times \mathcal{L}_{\gamma\gamma+b\bar{b}} \times \mathcal{L}_{\text{Res}} \quad (3.1)$$

$$\mathcal{L}_{\Delta a_\mu} = \exp \left[-\frac{\chi_{\Delta a_\mu}^2}{2} \right] = \exp \left[-\frac{1}{2} \left(\frac{a_\mu^{\text{SUSY}} - 2.49 \times 10^{-9}}{4.8 \times 10^{-10}} \right)^2 \right] \quad (3.2)$$

$$\mathcal{L}_{\gamma\gamma+b\bar{b}} = \exp \left[-\frac{\chi_{\gamma\gamma+b\bar{b}}^2}{2} \right] = \exp \left[-\frac{1}{2} \left(\frac{\mu_{\gamma\gamma} - 0.24}{0.08} \right)^2 - \frac{1}{2} \left(\frac{\mu_{b\bar{b}} - 0.117}{0.057} \right)^2 \right]_{m_{h_s} \simeq 95 \text{ GeV}}. \quad (3.3)$$

⁶`nlive` in the MultiNest algorithm signifies the number of active or live points to determine the iso-likelihood contour in each iteration [149,150]. The larger it is, the more detailed the scan process will be in surveying the parameter space.

TABLE I. The parameter space explored in this study, assuming all the inputs are flatly distributed in prior since they have clear physical meanings. Considering that the soft trilinear coefficients for the third-generation squarks, A_t and A_b , can significantly affect the SM-like Higgs boson mass by radiative corrections, we take $A_t = A_b$ and vary them. The unmentioned dimensional SUSY parameters are not crucial to this study, so we fix $M_3 = 3$ TeV, $m_A = 2$ TeV, $m_C = 800$ GeV, $v_s = 1$ TeV, and a shared value of 2 TeV for the others to be consistent with the LHC's search for new physics. We define all these parameters at the renormalization scale $Q_{\text{input}} = 1$ TeV and acquire the space by several trial scans over much broader parameter spaces.

Parameter	Prior	Range	Parameter	Prior	Range
λ	Flat	0.001–0.03	κ	Flat	−0.2–0.2
$\tan\beta$	Flat	5–60	m_B/GeV	Flat	90–120
$\mu_{\text{tot}}/\text{TeV}$	Flat	0.4–1.0	m_N/TeV	Flat	−1.0–1.0
A_t/TeV	Flat	1.0–3.0	A_λ/TeV	Flat	1.0–3.0
M_1/TeV	Flat	−1.0–−0.2	M_2/TeV	Flat	0.3–1.0
$M_{\tilde{\mu}_L}/\text{TeV}$	Flat	0.2–1.0	$M_{\tilde{\mu}_R}/\text{TeV}$	Flat	0.2–1.0

Here, $\mathcal{L}_{\text{Res}}$ represented the restrictions from relevant experiments on the theory: $\mathcal{L}_{\text{Res}} = 1$ by our definition if the limitations were satisfied; otherwise, $\mathcal{L}_{\text{Res}} = \exp[-100]$. These restrictions include:

- (i) Masses of the light Higgs bosons: Given the crucial importance of the radiative corrections to the SM-like Higgs boson mass in supersymmetric theories and the remarkable precision of the LHC in determining its value, the package SARAH 4.15.3 incorporates all one-loop and 2L corrections when calculating the Higgs mass spectrum [151,152]. Specifically, within the NMSSM framework, it has been recognized that the 2L correction to the mass beyond $\mathcal{O}(\alpha_s(\alpha_t + \alpha_b))$ —which constitutes the dominant 2L contribution in the MSSM—is also substantial, especially in scenarios where there is considerable mixing between the singlet and SM-doublet Higgs fields [153]. This correction alone can shift m_h by more than 2 GeV. Additionally, variations in the renormalization scale could modify the mass by over 2 GeV, even after incorporating the currently known corrections [153]. In contrast, the radiation corrections to m_{h_s} are less significant, as h_s lacks potentially intense interactions with other fields. Considering that the mass of h_s is estimated to be approximately 95.4 GeV to account for the observed excesses and that of h should be approximately 125 GeV to align with the LHC measurements of the SM-like Higgs boson mass [154], we set the theoretical and experimental uncertainties at 1 GeV for m_{h_s} and 3 GeV for m_h . Consequently, this study establishes the mass ranges of $94.4 \leq m_{h_s} \leq 96.4$ and $122 \leq m_h \leq 128$ GeV.
- (ii) Higgs data fit: Given that h corresponds to the discovered Higgs boson at the LHC, its properties should be consistent with corresponding measurements by the ATLAS and CMS Collaborations at the 95% confidence level. A p value larger than 0.05 was essential, tested by the code HiggsSignals-2.6.2 [155–158].

- (iii) Extra Higgs searches: This requirement is implemented through the code HiggsBounds-5.10.2 [159–163] and further via the code HiggsTools-1.2 [164].
- (iv) DM relic density: We take the central value of $\Omega h^2 = 0.120$ from the Planck-2018 data [31] and assume theoretical uncertainties of 20%, i.e., $0.096 \leq \Omega h^2 \leq 0.144$.
- (v) DM detections: The SI and SD DM-nucleon scattering cross sections are required to be lower than corresponding bounds from the LZ experiments [165]. The DM indirect searches from the observation of dwarf galaxies by the Fermi-LAT Collaboration are not considered as they impose no restrictions on the GNMSSM when $|m_{\tilde{\chi}_1^0}| \gtrsim 100$ GeV [166].
- (vi) B -physics observables: The branching ratios of $B_s \rightarrow \mu^+ \mu^-$ and $B \rightarrow X_s \gamma$ should be consistent with their experimental measurements at the 2σ level [167].
- (vii) Vacuum stability: The vacuum state of the scalar potential, composed of the Higgs fields and the last two-generation slepton fields,⁷ should be either stable or long-lived, as indicated in Ref. [169]. In contrast with the MSSM, the GNMSSM exhibits enhanced vacuum stability due to its incorporation of the singlet Higgs field as a dynamical degree of freedom, particularly in scenarios with small λ and large μ_{tot} values, as elaborated in Ref. [170].

⁷In traditional investigations of vacuum stability constraints, researchers typically scrutinize the scalar potential encompassing the Higgs fields and the fields of the third-generation squarks and sleptons. Within the parameter space delineated in Table I, the third-generation squark sector does not give rise to any charge and color-breaking (CCB) phenomena, mainly because A_t maintains a moderately small value. Specifically, it fulfills the CCB condition discussed in Ref. [168]: $|A_t| \lesssim \sqrt{3(M_{\tilde{t}_L}^2 + M_{\tilde{t}_R}^2 + \mu_{\text{tot}}^2)}$. Moreover, with $M_{\tilde{t}_L} = M_{\tilde{t}_R} = 2$ TeV, and $M_{\tilde{\mu}_L}$ and $M_{\tilde{\mu}_R}$ ranging within 1 TeV, the second-generation slepton sector might impose stricter constraints on the theory in terms of vacuum stability compared to the third-generation slepton sector, as analyzed in Ref. [40].

Nevertheless, excessively large soft-breaking trilinear coefficients may still lead to destabilization, as discussed in Ref. [171]. For our analysis, we utilized the package SARAH 4.15.3 to construct the model file for GNMSSM integration with the Vevacious code [172], allowing both the slepton fields and the neutral components of the Higgs fields to acquire potential VEVs. We evaluated the stability of this potential at zero temperature using the code VevaciousPlusPlus [173] (the C++ version of Vevacious).

- (viii) Unitarity constraint: This constraint imposes significant requirements on the partial wave decomposition of $2 \rightarrow 2$ scattering matrix elements, potentially limiting the maximum permissible values of the trilinear and quartic scalar couplings in the GNMSSM, as elaborated in Ref. [121]. We implement it by the package SARAH 4.15.3.

To get to know the impacts of the LHC's searches for supersymmetry on the scan results, the following processes

were studied by concrete Monte Carlo simulations:

$$\begin{aligned}
 pp &\rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^\pm, & i = 2, 3, 4, 5; & & j = 1, 2 \\
 pp &\rightarrow \tilde{\chi}_i^\pm \tilde{\chi}_j^\mp, & i, j = 1, 2; \\
 pp &\rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0, & i, j = 2, 3, 4, 5; \\
 pp &\rightarrow \tilde{\mu}_i \tilde{\mu}_j^*, & i, j = L, R; \\
 pp &\rightarrow \tilde{\nu}_\mu \tilde{\nu}_\mu^*.
 \end{aligned} \tag{3.4}$$

Specifically, in order to save computing time, program SModelS-3.0.0 [174], which contains the selection efficiencies of the experimental analyses in Table II, was first used to exclude the obtained samples. Given that this program's capability in implementing the LHC constraints was limited by its database and the strict prerequisites to use it, the rest samples were further surveyed by simulating the analyses listed in Tables III and IV. In this study, the cross sections for each process were calculated to next-to-leading order by

TABLE II. Experimental analyses included in the package SModelS-3.0.0.

Name	Scenario	Final state	Luminosity (fb ⁻¹)
ATLAS-1402-7029 [183]	$\tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow \tilde{\ell} \nu \tilde{\ell} \ell (\tilde{\nu} \nu), \ell \tilde{\nu} \tilde{\ell} \ell (\nu \nu) \rightarrow \ell \nu \tilde{\chi}_1^0 \ell \ell (\nu \nu) \tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow \tilde{\tau} \nu \tilde{\tau} \tau (\tilde{\nu} \nu), \tau \tilde{\nu} \tilde{\tau} \tau (\nu \nu) \rightarrow \tau \nu \tilde{\chi}_1^0 \tau \tau (\nu \nu) \tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow W^\pm \tilde{\chi}_1^0 Z \tilde{\chi}_1^0 \rightarrow \ell \nu \tilde{\chi}_1^0 \ell \ell \tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow W^\pm \tilde{\chi}_1^0 h \tilde{\chi}_1^0 \rightarrow \ell \nu \tilde{\chi}_1^0 \ell \ell \tilde{\chi}_1^0$	$3\ell + E_T^{\text{miss}}$	20.3
CMS-SUS-16-034 [184]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W \tilde{\chi}_1^0 Z (h) \tilde{\chi}_1^0$	$n\ell (n \geq 2) + n_j (n \geq 1) + E_T^{\text{miss}}$	35.9
CMS-SUS-16-039 [185]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow \ell \tilde{\nu} \ell \tilde{\ell}$ $\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow \tilde{\tau} \nu \tilde{\ell} \ell$ $\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow \tilde{\tau} \nu \tilde{\tau} \tau$ $\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W Z \tilde{\chi}_1^0 \tilde{\chi}_1^0$ $\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W H \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$n\ell (n > 0) (\tau) + E_T^{\text{miss}}$	35.9
CMS-SUS-16-045 [186]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W^\pm \tilde{\chi}_1^0 h \tilde{\chi}_1^0$	$1\ell 2b + E_T^{\text{miss}}$	35.9
CMS-SUS-16-048 [187]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow Z \tilde{\chi}_1^0 W \tilde{\chi}_1^0$ $\tilde{\chi}_2^0 \tilde{\chi}_1^0 \rightarrow Z \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$2\ell + E_T^{\text{miss}}$	35.9
CMS-SUS-17-004 [188]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W h (Z) \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$n\ell (n \geq 0) + n_j (n \geq 0) + E_T^{\text{miss}}$	35.9
CMS-SUS-17-009 [189]	$\tilde{\ell} \tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \ell \tilde{\chi}_1^0$	$2\ell + E_T^{\text{miss}}$	35.9
CMS-SUS-17-010 [190]	$\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow W^\pm \tilde{\chi}_1^0 W^\mp \tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow \nu \tilde{\ell} \ell \tilde{\nu}$	$2\ell + E_T^{\text{miss}}$	35.9
ATLAS-1803-02762 [191]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W Z \tilde{\chi}_1^0 \tilde{\chi}_1^0$ $\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow \nu \tilde{\ell} \ell \tilde{\ell}$ $\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow \nu \tilde{\ell} \nu \tilde{\ell}$ $\tilde{\ell} \tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \ell \tilde{\chi}_1^0$	$n\ell (n \geq 2) + E_T^{\text{miss}}$	36.1
ATLAS-1806-02293 [192]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W Z \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$n\ell (n \geq 2) + n_j (n \geq 0) + E_T^{\text{miss}}$	36.1
ATLAS-1812-09432 [193]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W h \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$n\ell (n \geq 0) + n_j (n \geq 0) + n_b (n \geq 0)$ $+ n\gamma (n \geq 0) + E_T^{\text{miss}}$	36.1
CMS-SUS-20-001 [194]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow Z \tilde{\chi}_1^0 W \tilde{\chi}_1^0$ $\tilde{\ell} \tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \ell \tilde{\chi}_1^0$	$2\ell + E_T^{\text{miss}}$	137

(Table continued)

TABLE II. (Continued)

Name	Scenario	Final state	Luminosity (fb ⁻¹)
CMS-SUS-20-004 [195]	$\tilde{\chi}_2^0 \tilde{\chi}_3^0 \rightarrow h \tilde{\chi}_1^0 h \tilde{\chi}_1^0$	$4b + E_T^{\text{miss}}$	137
CMS-SUS-21-002 [196]	$\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow W^\pm \tilde{\chi}_1^0 W^\mp \tilde{\chi}_1^0$ $\tilde{\chi}_{2/3}^0 \tilde{\chi}_1^\pm \rightarrow W^\pm \tilde{\chi}_1^0 Z(h) \tilde{\chi}_1^0$ $\tilde{\chi}_2^0 \tilde{\chi}_3^0 \rightarrow Z \tilde{\chi}_1^0 h \tilde{\chi}_1^0$	$4b + E_T^{\text{miss}}$	137
ATLAS-1908-08215 [197]	$\tilde{\ell} \tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \ell \tilde{\chi}_1^0 \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp$	$2\ell + E_T^{\text{miss}}$	139
ATLAS-1909-09226 [198]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow Wh \tilde{\chi}_1^0 \tilde{\chi}_1^\pm$	$1\ell + h(\rightarrow b\bar{b}) + E_T^{\text{miss}}$	139
ATLAS-1911-12606 [199]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow Z \tilde{\chi}_1^0 W \tilde{\chi}_1^0$ $\tilde{\chi}_2^0 \tilde{\chi}_1^0 \rightarrow Z \tilde{\chi}_1^0 \tilde{\chi}_1^0$ $\tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow W^+ \tilde{\chi}_1^0 W^- \tilde{\chi}_1^0$ $\tilde{\ell} \tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \ell \tilde{\chi}_1^0$	$2\ell + 2j + E_T^{\text{miss}}$	139
ATLAS-1912-08479 [200]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W(\rightarrow l\nu) \tilde{\chi}_1^0 Z(\rightarrow \ell\ell) \tilde{\chi}_1^0$	$3\ell + E_T^{\text{miss}}$	139
ATLAS-2106-01676 [131]	$\tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow W(\rightarrow \ell\nu) Z(\rightarrow \ell\ell) \tilde{\chi}_1^0 \tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow W(\rightarrow \ell\nu) h(\rightarrow \ell\ell) \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$3\ell + E_T^{\text{miss}}$	139
ATLAS-2108-07586 [201]	$\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm \rightarrow WW \tilde{\chi}_1^0 \tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow WZ(h) \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$4j + E_T^{\text{miss}}$	139
ATLAS-2204-13072 [202]	$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W(\rightarrow qq) \tilde{\chi}_1^0 Z(\rightarrow \ell\ell) \tilde{\chi}_1^0$	$2\ell + 2j + E_T^{\text{miss}}$	139
ATLAS-2209-13935 [203]	$\tilde{\ell} \tilde{\ell} \rightarrow \ell \ell \tilde{\chi}_1^0 \tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow W(\rightarrow \ell\nu) W(\rightarrow \ell\nu) \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$2\ell + E_T^{\text{miss}}$	139

TABLE III. Experimental analyses of the electroweakino production processes considered in this study, which are categorized by the topologies of the SUSY signals.

Scenario	Final state	Name
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow WZ \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$n\ell(n \geq 2) + nj(n \geq 0) + E_T^{\text{miss}}$	CMS-SUS-20-001(137 fb ⁻¹) [194]
		ATLAS-2106-01676(139 fb ⁻¹) [131]
		CMS-SUS-17-004(35.9 fb ⁻¹) [188]
		CMS-SUS-16-039(35.9 fb ⁻¹) [188]
		ATLAS-1803-02762(36.1 fb ⁻¹) [191]
		ATLAS-1806-02293(36.1 fb ⁻¹) [192]
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow \ell \tilde{\nu} \ell \tilde{\nu}$	$n\ell(n = 3) + E_T^{\text{miss}}$	CMS-SUS-16-039(35.9 fb ⁻¹) [185]
		ATLAS-1803-02762(36.1 fb ⁻¹) [191]
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow \tilde{\tau} \nu \ell \tilde{\ell}$	$2\ell + 1\tau + E_T^{\text{miss}}$	CMS-SUS-16-039(35.9 fb ⁻¹) [185]
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow \tilde{\tau} \nu \tilde{\tau} \tau$	$3\tau + E_T^{\text{miss}}$	CMS-SUS-16-039(35.9 fb ⁻¹) [185]
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow Wh \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$n\ell(n \geq 1) + nb(n \geq 0) + nj(n \geq 0) + E_T^{\text{miss}}$	ATLAS-1909-09226(139 fb ⁻¹) [204]
		CMS-SUS-17-004(35.9 fb ⁻¹) [188]
		CMS-SUS-16-039(35.9 fb ⁻¹) [185]
		ATLAS-1812-09432(36.1 fb ⁻¹) [193]
		CMS-SUS-16-034(35.9 fb ⁻¹) [184]
		CMS-SUS-16-045(35.9 fb ⁻¹) [186]
$\tilde{\chi}_1^\mp \tilde{\chi}_1^\pm \rightarrow WW \tilde{\chi}_1^0 \tilde{\chi}_1^0$	$2\ell + E_T^{\text{miss}}$	ATLAS-1908-08215(139 fb ⁻¹) [197]
		CMS-SUS-17-010(35.9 fb ⁻¹) [190]
$\tilde{\chi}_1^\mp \tilde{\chi}_1^\pm \rightarrow 2\tilde{\ell} \nu(\tilde{\nu} \ell)$	$2\ell + E_T^{\text{miss}}$	ATLAS-1908-08215(139 fb ⁻¹) [197]
		CMS-SUS-17-010(35.9 fb ⁻¹) [190]

(Table continued)

TABLE III. (Continued)

Scenario	Final state	Name
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow ZW\tilde{\chi}_1^0 \tilde{\chi}_1^\pm$	$2j(\text{large}) + E_T^{\text{miss}}$	ATLAS-2108-07586(139 fb ⁻¹) [201]
$\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow WW\tilde{\chi}_1^0 \tilde{\chi}_1^0$		
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow (h/Z)W\tilde{\chi}_1^0 \tilde{\chi}_1^\pm$	$j(\text{large}) + b(\text{large}) + E_T^{\text{miss}}$	ATLAS-2108-07586(139 fb ⁻¹) [201]
$\tilde{\chi}_2^0 \tilde{\chi}_3^0 \rightarrow (h/Z)Z\tilde{\chi}_1^0 \tilde{\chi}_1^0$		
$\tilde{\chi}_2^0 \tilde{\chi}_1^\mp \rightarrow h/ZW\tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow \gamma/Z\tilde{G}$	$2\gamma + n\ell(n \geq 0) + nb(n \geq 0) + nj(n \geq 0) + E_T^{\text{miss}}$	ATLAS-1802-03158(36.1 fb ⁻¹) [205]
$\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow WW\tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow \gamma/Z\tilde{G}$		
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow ZW\tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow h/Z\tilde{G}$	$n\ell(n \geq 4) + E_T^{\text{miss}}$	ATLAS-2103-11684(139 fb ⁻¹) [206]
$\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow WW\tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow h/Z\tilde{G}$		
$\tilde{\chi}_2^0 \tilde{\chi}_1^0 \rightarrow Z\tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow h/Z\tilde{G}$		
$\tilde{\chi}_1^\mp \tilde{\chi}_1^0 \rightarrow W\tilde{\chi}_1^0 \tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow h/Z\tilde{G}$		
$\tilde{\chi}_i^{0,\pm} \tilde{\chi}_j^{0,\mp} \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 + \chi_{\text{soft}} \rightarrow ZZ/H\tilde{G}\tilde{G}$	$n\ell(n \geq 2) + nb(n \geq 0) + nj(n \geq 0) + E_T^{\text{miss}}$	CMS-SUS-16-039(35.9 fb ⁻¹) [185]
		CMS-SUS-17-004(35.9 fb ⁻¹) [188]
		CMS-SUS-20-001(137 fb ⁻¹) [194]
$\tilde{\chi}_i^{0,\pm} \tilde{\chi}_j^{0,\mp} \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 + \chi_{\text{soft}} \rightarrow HH\tilde{G}\tilde{G}$	$n\ell(n \geq 2) + nb(n \geq 0) + nj(n \geq 0) + E_T^{\text{miss}}$	CMS-SUS-16-039(35.9 fb ⁻¹) [185]
		CMS-SUS-17-004(35.9 fb ⁻¹) [188]
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow W^*Z^*\tilde{\chi}_1^0 \tilde{\chi}_1^0$	$3\ell + E_T^{\text{miss}}$	ATLAS-2106-01676(139 fb ⁻¹) [131]
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow Z^*W^*\tilde{\chi}_1^0 \tilde{\chi}_1^0$	$2\ell + nj(n \geq 0) + E_T^{\text{miss}}$	ATLAS-1911-12606(139 fb ⁻¹) [199]
		ATLAS-1712-08119(36.1 fb ⁻¹) [207]
		CMS-SUS-16-048(35.9 fb ⁻¹) [187]
$\tilde{\chi}_2^0 \tilde{\chi}_1^\pm + \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp + \tilde{\chi}_1^\pm \tilde{\chi}_1^0$	$2\ell + nj(n \geq 0) + E_T^{\text{miss}}$	ATLAS-1911-12606(139 fb ⁻¹) [199]
		ATLAS-1712-08119(36.1 fb ⁻¹) [207]
		CMS-SUS-16-048(35.9 fb ⁻¹) [187]

program Prospino2 [175]. 50000 events were generated for both electroweakino and slepton production processes, by package MadGraph_aMC@NLO [176,177], and their parton shower and hadronization were finished by program PYTHIA8 [178]. Detector simulations were implemented with program Delphes [179]. Finally, the event files were put into the package CheckMATE-2.0.26 [180–182] to calculate the R value defined by $R \equiv \max\{S_i/S_{i,\text{obs}}^{95}\}$ for all the involved analyses, where S_i represents the simulated event number of the i th signal region (SR), and $S_{i,\text{obs}}^{95}$ is the corresponding 95% confidence level upper limit. Evidently, $R > 1$ indicates that the sample is experimentally excluded if the

involved uncertainties are neglected [40], while $R < 1$ means that it is consistent with the experimental analyses.

B. Numerical results

The scan process yields more than 92000 samples that are consistent with the experimental restrictions, among which about 50000 samples can simultaneously explain the muon $(g-2)$ anomaly and the diphoton and $b\bar{b}$ excesses at a level of 2σ . According to their predictions on the major component of DM, the obtained samples can be classified into the bino-dominated case and the singlino-dominated case. They achieve the measured relic abundance primarily

TABLE IV. Same as Table III, but for the slepton production processes.

Scenario	Final state	Name
$\tilde{\ell} \tilde{\ell} \rightarrow \ell\ell\tilde{\chi}_1^0 \tilde{\chi}_1^0$	$2\ell + E_T^{\text{miss}}$	ATLAS-1911-12606(139 fb ⁻¹) [199]
		ATLAS-1712-08119(36.1 fb ⁻¹) [207]
		ATLAS-1908-08215(139 fb ⁻¹) [197]
		CMS-SUS-20-001(137 fb ⁻¹) [194]
		ATLAS-1803-02762(36.1 fb ⁻¹) [191]
		CMS-SUS-17-009(35.9 fb ⁻¹) [189]

by coannihilating with the wino- and Higgsino-like electro-weakinos, respectively, and account for 26% and 74% of the total Bayesian evidence.

For both cases, we initially present in Fig. 1 the statistical distributions of the 12 input parameters listed in Table I, including one-dimensional profile likelihoods (delineated as green lines) and marginal posterior probability density functions (plotted as black lines).⁸ The distributions of the bino-DM and singlino-DM cases are shown in upper and lower parts in each panel, respectively. The violet and orange bands represent the 1σ and 2σ confidence intervals, and the green dots mark the best points corresponding to $\chi^2_{\gamma\gamma+b\bar{b}} + \chi^2_{\Delta a_\mu} = 0.20$ for the bino-DM case and $\chi^2_{\gamma\gamma+b\bar{b}} + \chi^2_{\Delta a_\mu} = 0.04$ for the singlino-DM case. The yellow and golden bands denote the 1σ and 2σ credible regions and the black dots denote the modes of the posterior probability density function. The PL distributions indicate that the GNMSSM can account for the muon ($g - 2$) anomaly and the diphoton and $b\bar{b}$ excesses at a level of 2σ in broad parameter space except that m_B is restricted within a narrow range. In fact, as noted in the approximations in Eqs. (2.23), m_B is fixed by the relation $m_B^2 \simeq m_{h_s}^2 |V_{h_s}^S|^2 + m_h^2 |V_{h_s}^{SM}|^2$, where $V_{h_s}^S$ and $V_{h_s}^{SM}$ are determined by the signal rates of the excesses.

The distributions in Fig. 1 reflect the impacts of the inputs on the observables involved in this study. They include the following:

- (i) μ_{tot} . In the bino-DM case, the SI DM-nucleon scattering cross section is inversely proportional to μ_{tot} , with the LZ experiment alone stipulating a minimal μ_{tot} of approximately 380 GeV [208]. When taking into account constraints from both the muon $g - 2$ anomaly and results from the LHC, this minimum threshold increases to around 500 GeV [32]. Additionally, only explaining the diphoton and $b\bar{b}$ excesses at the 2σ level has set a lower bound on μ_{tot}

⁸The profile likelihood is the most significant likelihood value in a specific parameter space [147]. Given a set of input parameters $\Theta \equiv (\Theta_1, \Theta_2, \dots)$, one can acquire the one-dimensional PL by changing the other parameters to maximize the likelihood function, i.e.,

$$\mathcal{L}(\Theta_A) = \max_{\Theta_1, \dots, \Theta_{A-1}, \Theta_{A+1}, \dots} \mathcal{L}(\Theta).$$

The PL reflects the preference of a theory on the parameter space. For a given point Θ_A , it represents the capability of the point in the theory to account for experimental data. By contrast, the one-dimensional marginal posterior PDFs of parameter Θ_A is obtained by integrating the posterior PDF from the Bayesian theorem $P(\Theta)$ over the rest of the model inputs,

$$P(\Theta_A) = \int P(\Theta) d\Theta_1 d\Theta_2 \dots d\Theta_{A-1} d\Theta_{A+1} \dots$$

It reflects the preference for the samples acquired in the scan.

of approximately 540 GeV [51]. Contrastingly, in the singlino-DM scenario, the scattering rate notably diverges from the bino-DM case as delineated by Eq. (2.30) in Ref. [116], permitting a significantly lower μ_{tot} . Specifically, as demonstrated in Ref. [41], a μ_{tot} value as low as approximately 200 GeV is still viable even after incorporating the muon $g - 2$ anomaly and LHC constraints. Such a μ_{tot} in the singlino-DM case is also capable of explaining the diphoton and $b\bar{b}$ excesses [51].

Furthermore, the WHL contribution predominantly influences a_μ^{SUSY} , manifesting a declining trend as μ_{tot} enhances. This aspect elucidates the descending tendencies of the PL lines for μ_{tot} beyond approximately 750 GeV in bino-DM and 425 GeV in singlino-DM scenarios. Correspondingly, the posterior PDF curves of μ_{tot} peak around these values, highlighting the critical interplay between this parameter and experimental constraints.

After considering these factors, the comprehensive interpretation of the muon $g - 2$ anomaly, as well as the diphoton and $b\bar{b}$ excesses, suggests a 2σ confidence interval for the μ_{tot} parameter within the scanning range presented in Table I: (580, 1000) GeV for the bino-DM case and (400, 610) GeV for the singlino-DM case.

- (ii) λ , A_λ , and m_N . Given the characteristics of μ_{tot} in both cases and the relation in Eq. (2.24), the bino-DM case demonstrates a preference for a significantly smaller value of λ compared to the singlino-DM case when an appropriate $V_{h_s}^{SM}$ is required to account for the observed excesses. Moreover, Eq. (2.25) suggests that the sum of A_λ and m_N should exhibit a much larger magnitude in the bino-DM scenario than in the singlino-DM scenario. To achieve this outcome, it is strongly favored for m_N in the bino-DM case to maintain a positive sign similar to A_λ and vary within a wide range from 300 to 1000 GeV, where the lower bound arises from requiring $m_N > |M_1|$ to predict a bino-DM, while the upper bound is defined by the constraints of the scanning area. By contrast, within the singlino-DM framework, $|m_N| \simeq |m_{\tilde{\chi}_1^0}| \simeq \mu_{\text{tot}}$ is predicted since the DM primarily coannihilates with the Higgsinos to obtain its measured abundance. Considering that explaining the muon $g - 2$ anomaly at a 2σ level requires $m_{\tilde{\chi}_1^0} \lesssim 600$ GeV [41] and μ_{tot} values listed in Table I are greater than 400 GeV, one can deduce that $400 \lesssim |m_N| \lesssim 600$ GeV. In addition, due to the cancellation between m_N and A_λ in Eq. (2.25), it becomes challenging for negative values of m_N within the singlino-DM case to account for the diphoton and $b\bar{b}$ excesses at a 1σ level. This feature leads to the suppression of the posterior PDF for a negative m_N , which is explicitly shown in Fig. 1.

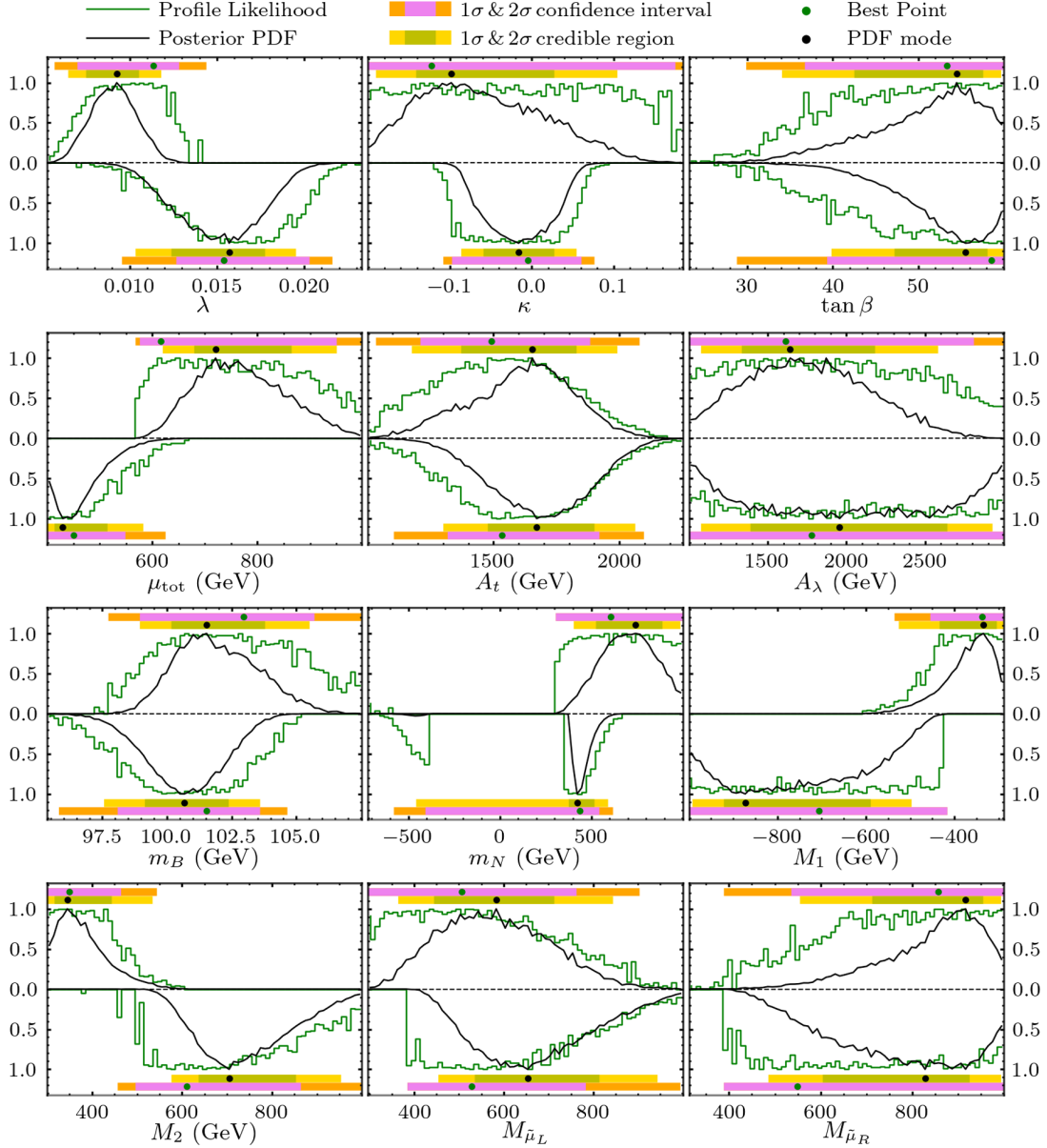


FIG. 1. One-dimensional profile likelihoods (green line) and posterior probability density functions (black line) of the input parameters λ , κ , $\tan\beta$, μ_{tot} , A_t , A_λ , m_B , m_N , M_1 , M_2 , $M_{\tilde{\mu}_L}$, and $M_{\tilde{\mu}_R}$ before implementing the restrictions from the LHC's search for supersymmetry. The upper (lower) part in each panel is obtained by the samples predicting the bino-dominated (singlino-dominated) DM. The violet and orange bands show the 1σ and 2σ confidence intervals, respectively, and the green dots mark the best point, corresponding to $\chi^2_{\gamma\gamma+b\bar{b}} + \chi^2_{\Delta a_\mu} = 0.16$ for the bino-DM case and $\chi^2_{\gamma\gamma+b\bar{b}} + \chi^2_{\Delta a_\mu} = 0.02$ for the singlino-DM case. The yellow and golden bands denote the 1σ and 2σ credible regions and the black dots are the modes of the posterior probability density function. All the statistical measures have been briefly introduced in Ref. [147].

It is noteworthy that in the bino-DM scenario, the contribution of charginos to $C_{h_s\gamma\gamma}$ is consistently less than 1% of the SM contribution, attributed to the small values of λ and the massiveness of the Higgsinos [129]. In contrast, as highlighted in our previous research [51], this contribution can approach 1% within the singlino-DM framework. Additionally, the presence of a relatively substantial λ value within the singlino-DM framework proves

advantageous for achieving the central values of $\mu_{\gamma\gamma}$ and $\mu_{b\bar{b}}$ [51]. Collectively, these characteristics render the singlino-DM scenario more suitable for explaining these excesses.

- (iii) M_1 and M_2 . In the GNMSSM, the impact of parameters M_1 and M_2 on the diphoton and $b\bar{b}$ signals is negligible. Therefore, their distributions are primarily determined by DM physics and the muon $g - 2$ anomaly. In the case of bino-DM, it is

expected that the relation $|M_1| \simeq |m_{\tilde{\chi}_1^0}| \simeq M_2$ holds since the DM mainly coannihilates with winolike electroweakinos to achieve the observed abundance. The permissible range for $|M_1|$ is approximately 210–620 GeV, where the lower threshold arises from constraints imposed by LHC in searching for sparticles, while the upper bound is derived from addressing the muon $g - 2$ anomaly [32]. As $|M_1|$ exceeds around 450 GeV, it becomes challenging to effectively account for the muon $g - 2$ anomaly at a 1σ level, resulting in diminished PL and posterior PDF distributions. It is remarkable that a negative value of M_1 is preferred in this scenario as it can suppress SI DM-nucleon scattering through cancellation of different contributions [208]. By contrast, the singlino-DM scenario reveals distinct characteristics: $|M_1|$ only needs to surpass $|m_N|$ to predict appropriate dark matter properties, given that its influence on $\delta a_\mu^{\text{SUSY}}$ is minor, and an effective explanation of muon $g - 2$ anomaly requires $M_2 \lesssim 1.5$ TeV [41]. Furthermore, when M_2 increases beyond 700 GeV, this scenario becomes challenging in explaining the muon $g - 2$ anomaly at a 1σ level as reflected by diminishing PL and posterior PDF distributions of M_2 .

- (iv) $\tan\beta$. As indicated by the condition to obtain a substantial $V_{h_s}^{\text{NSM}} \tan\beta$ in Eq. (2.25) and the formulas of $\delta a_\mu^{\text{SUSY}}$ in Eqs. (2.11)–(2.14), a significant $\tan\beta$ is preferred for explaining both the muon $g - 2$ anomaly and the diphoton/ $b\bar{b}$ excesses. This characteristic is reflected by corresponding PL and PDF distributions in Fig. 1.
- (v) $M_{\tilde{\mu}_L}$ and $M_{\tilde{\mu}_R}$. In both scenarios, the impact of these two parameters is confined solely to $\delta a_\mu^{\text{SUSY}}$. Notably, $\delta a_\mu^{\text{SUSY}}$ exhibits a greater sensitivity to $M_{\tilde{\mu}_L}$ in comparison to $M_{\tilde{\mu}_R}$, leading to a preference of a moderately light $M_{\tilde{\mu}_L}$ when addressing the muon $g - 2$ anomaly. In addition to the PL and PDF plots of these parameters shown in Fig. 1, we will delve deeper into the smuon spectrum through scattering plots in subsequent discussions.
- (vi) A_t . The trilinear coefficient A_t significantly influences the SM-like Higgs boson mass m_h , the mixing $V_{h_s}^{\text{SM}}$, and the $h_s gg$ and $h_s \gamma\gamma$ coupling strengths via top-squark-mediated loops. Given that we have fixed the soft-breaking masses of squarks at 2 TeV, A_t s around 1.7 TeV are preferred to make the theory accessible to explain the excesses.
- (vii) κ . Regarding the bino-DM case, Eqs. (2.9) and (2.10) indicate that κ does not directly impact the observables involved in this study, except for its influence on the scalar potential of the singlet field and consequently affecting the electroweak symmetry breaking. Therefore, the naturalness in realizing the symmetry breaking determines its posterior

PDF [209]. In the framework of the singlino-DM scenario, however, there is an additional effect related to κ : since the amplitude of the t -channel h_s -mediated contribution to the SI DM-nucleon scattering is proportional to $\kappa V_{h_s}^{\text{SM}}$ in the leading-order approximation, and considering this contribution's potential significance, the LZ results favor a small value of κ [116].

We note that the PLs and posterior PDFs depicted in Fig. 1 can significantly differ from those presented in Ref. [51], which were derived by a similar methodology but neglected the muon $g - 2$ anomaly in the likelihood function.

Second, we generate a bilateral diagram in which we project the samples of the bino-DM case and those of the singlino-DM case onto the $\mu_{\text{tot}} - \tan\beta$, $\mu_{\text{tot}} - M_{\tilde{\mu}_L}$, $\mu_{\text{tot}} - m_{\tilde{\chi}_1^0}$, and $m_{\tilde{\chi}_1^0} - M_2$ planes in the left and right sections of Fig. 2, respectively. The different colors distinguish four combinations of the constraints from the latest DM detections, the $\gamma\gamma$ and $b\bar{b}$ signal rates with $\chi^2_{\gamma\gamma+b\bar{b}} \leq 2.3$, the muon $g - 2$ measurement with $\chi^2_{\Delta a_\mu} \leq 4.0$, and the LHC's search for sparticles. As a useful supplement to Fig. 2, we also list in Table V the numbers of these colored samples. From these scattering plots and Table V, one can infer the following facts:

- (i) The $m_{\tilde{\chi}_1^0} - M_2$ panel for the bino-DM case demonstrates that the DM achieves the measured relic abundance through coannihilating with the winolike electroweakinos. We verified that the dominant contribution to this abundance arises from the channels $\tilde{\chi}_2^0 \tilde{\chi}_1^- \rightarrow d_i \bar{u}_i$, where $i = 1, 2$ represents the first two generations of quarks. Additionally, our analysis reveals that the scenarios where binolike DM coannihilates with winolike electroweakinos along with either muon-flavored sneutrino or smuons contribute less than 1% of total Bayesian evidence.
- (ii) The $m_{\tilde{\chi}_1^0} - \mu_{\text{tot}}$ panel for the singlino-DM case indicates that the DM predominantly coannihilated with Higgsino-like electroweakinos to attain the measured relic abundance. Our analysis suggests that there is also a possibility of simultaneous coannihilation of the DM with Higgsinos and muon-flavored sleptons; however, this scenario only contributes to less than 0.2% of the total Bayesian evidence. Moreover, we have identified a few samples featuring singlino-wino coannihilation. Nevertheless, these samples pose challenges in explaining the 95 GeV excesses at a 1σ level due to the reason listed in the following item.
- (iii) The condition stated in Eq. (2.24) is crucial for the GNMSSM to address the diphoton and $b\bar{b}$ excesses, while its impact on forecasting the DM observables is negligible. This elucidates why some samples capable of interpreting the DM experiments fail to explain the excesses, as shown in Fig. 2 and Table V.

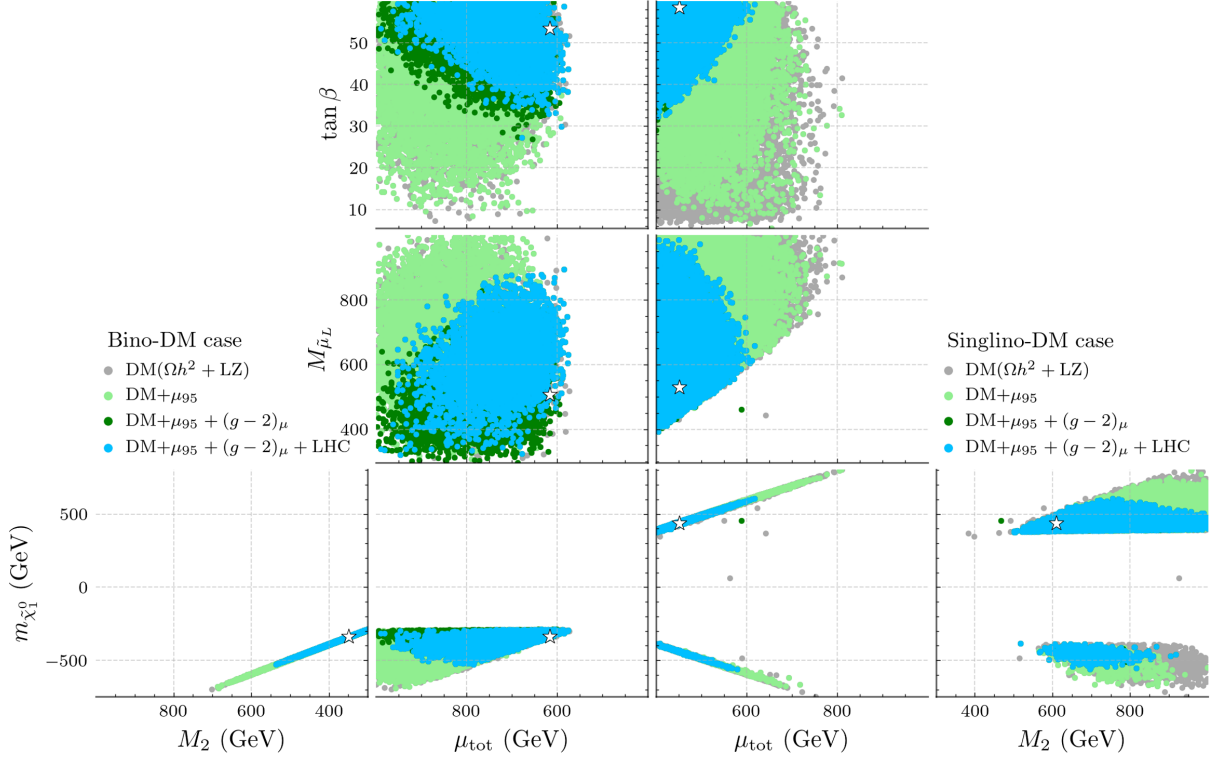


FIG. 2. The samples for the bino-DM case obtained by the scan in Sec. III A and those for the singlino-DM case, which are projected onto the $\mu_{\text{tot}} - \tan \beta$, $\mu_{\text{tot}} - M_{\tilde{\mu}_L}$, $\mu_{\text{tot}} - m_{\tilde{\chi}_1^0}$, and $m_{\tilde{\chi}_1^0} - M_2$ planes in the left and right sections, respectively. The colors distinguish four combinations of constraints from the latest DM detections, the $\gamma\gamma$ and $b\bar{b}$ signal rates with $\chi^2_{\gamma\gamma+b\bar{b}} \leq 2.3$ (1σ level for 2 degrees of freedom), the muon $g-2$ measurement with $\chi^2_{\Delta a_\mu} \leq 4.0$ (2σ level), and the LHC's search for sparticles.

- (iv) The impacts of $\tan \beta$, M_1 , M_2 , μ_{tot} , $M_{\tilde{\mu}_L}$, and $M_{\tilde{\mu}_R}$ on $\delta a_\mu^{\text{SUSY}}$ are delineated in Eqs. (2.11)–(2.14). Among these parameters, only μ_{tot} plays a crucial role in explaining the excesses at 95 GeV. This characteristic accounts for the distinct regions in each panel of Fig. 2 favored by the excesses and the muon $g-2$ anomaly.
- (v) The LHC search for sparticles has long been acknowledged as a stringent constraint on the SUSY explanations of the muon $g-2$ anomaly (see, for example, the analysis presented in Ref. [32]). In relation to this study, the manifestation of this phenomenon is clearly observed in the bino-DM scenario, while it remains ambiguous in the singlino-DM scenario (refer to Table V). This difference is

attributed to the substantial mass of the DM particle in the latter case (specifically, $|m_{\tilde{\chi}_1^0}| \gtrsim 400$ GeV in the singlino-DM case versus $|m_{\tilde{\chi}_1^0}| \gtrsim 300$ GeV in the bino-DM case), which in turn diminishes the restrictions imposed by the LHC.

Additionally, our analysis reveals that the maximal prediction of $\delta a_\mu^{\text{SUSY}}$ diminishes as $|m_{\tilde{\chi}_1^0}|$ increases, once the constraints from the LHC are accounted for. In the bino-DM scenario, $\delta a_\mu^{\text{SUSY}}$ can reach a value of 2.7×10^{-9} when $|m_{\tilde{\chi}_1^0}|$ is approximately 300 GeV. By contrast, in the singlino-DM scenario, it can achieve a value of 3.0×10^{-9} for $|m_{\tilde{\chi}_1^0}|$ around 400 GeV. The ability to predict a higher $\delta a_\mu^{\text{SUSY}}$ at the same $|m_{\tilde{\chi}_1^0}|$ in the

TABLE V. Numbers of the samples in the bino- and singlino-DM scenarios, respectively, classified by the experimental constraints they satisfy. In Fig. 2, these samples are labeled with gray, green, dark green, and blue colors, respectively.

Scenario	Constraints			
	No.			
	DM($\Omega h^2 + \text{LZ}$) Gray	DM + μ_{95} Green	DM + $\mu_{95} + (g-2)_\mu$ Dark green	DM + $\mu_{95} + (g-2)_\mu + \text{LHC}$ Blue
Bino-DM	17368	14416	8408	4770
Singlino-DM	59038	38731	20851	20539

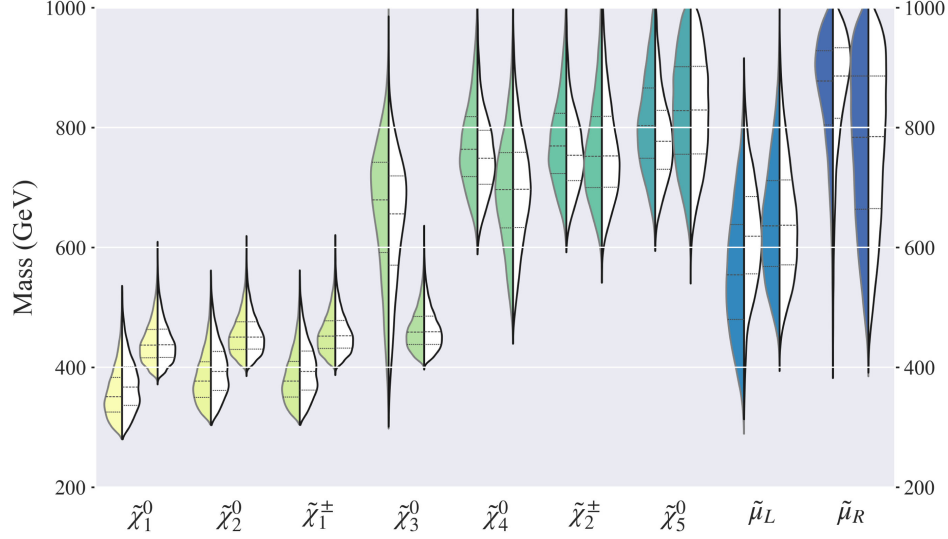


FIG. 3. Mass distribution for the involved sparticles, which are scaled by counting sample numbers. The left and right violins for each sparticle represent the bino- and singlino-DM cases, respectively. The colored half in each violin is plotted with the dark green samples in Fig. 2, while the white is drawn with the blue samples. The dashed line labels the quartile ranges.

latter scenario arises from the more complex decay chains of the involved sparticles, leading to less stringent LHC constraints. This aspect was emphasized in our prior work [41].

- (vi) The unified explanation and the LHC restrictions set bounds on $\tan\beta$ and the masses of the involved sparticles, i.e., $\tan\beta \gtrsim 26$, $580 \lesssim \mu_{\text{tot}} \lesssim 1000$, $300 \lesssim |m_{\tilde{\chi}_1^0}|$, $M_2 \lesssim 550$, and $300 \lesssim M_{\tilde{\mu}_L} \lesssim 900$ GeV for the bino-DM case and $\tan\beta \gtrsim 32$, $400 \lesssim |m_{\tilde{\chi}_1^0}|$,

$\mu_{\text{tot}} \lesssim 600$, $500 \lesssim M_2 \lesssim 1000$ and $400 \lesssim M_{\tilde{\mu}_L} \lesssim 1000$ GeV for the singlino-DM case. To display the bounds more clearly, we plot the mass distributions of the involved sparticles by “violin” diagram in Fig. 3.

Third, we study the properties of the Higgs bosons. We project the samples in Fig. 2 onto $|C_{h_s b\bar{b}}| - |C_{h_s t\bar{t}}|$, $|C_{h_s gg}| - |C_{h_s t\bar{t}}|$, and $|C_{h_s \gamma\gamma}| - |C_{h_s t\bar{t}}|$ planes to obtain Fig. 4 and onto $\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma\gamma) - \text{Br}_{\text{SUSY}}(h_s \rightarrow b\bar{b})$

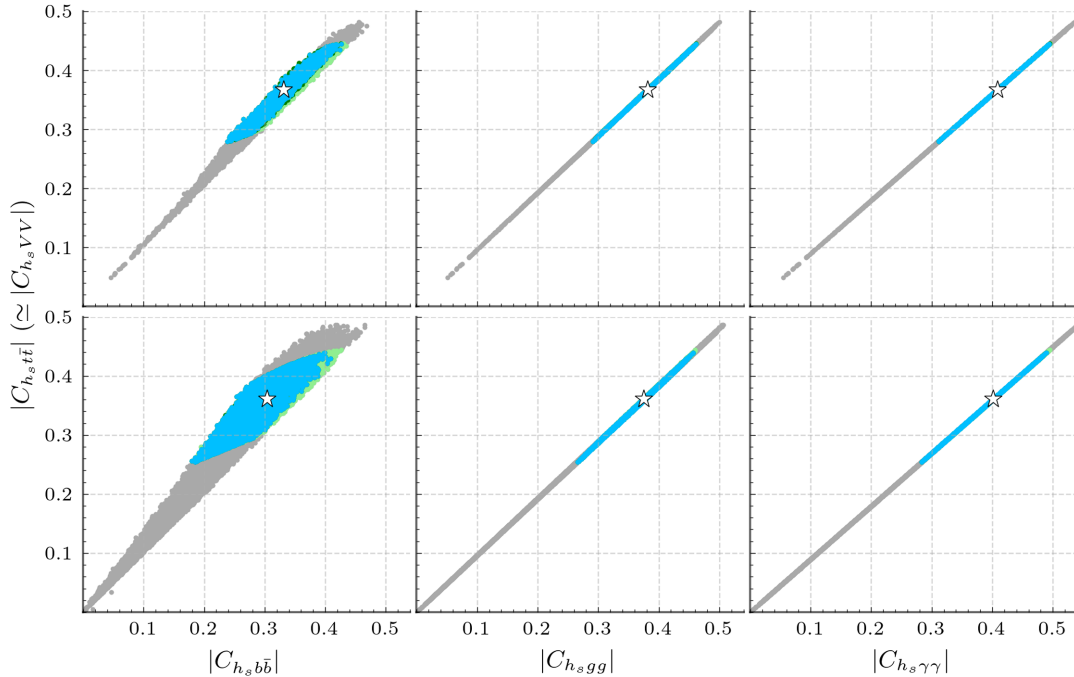


FIG. 4. Projection of the samples in Fig. 2 onto $|C_{h_s b\bar{b}}| - |C_{h_s t\bar{t}}|$, $|C_{h_s gg}| - |C_{h_s t\bar{t}}|$, and $|C_{h_s \gamma\gamma}| - |C_{h_s t\bar{t}}|$ planes. The upper and lower panels correspond to the bino- and singlino-DM cases, respectively.

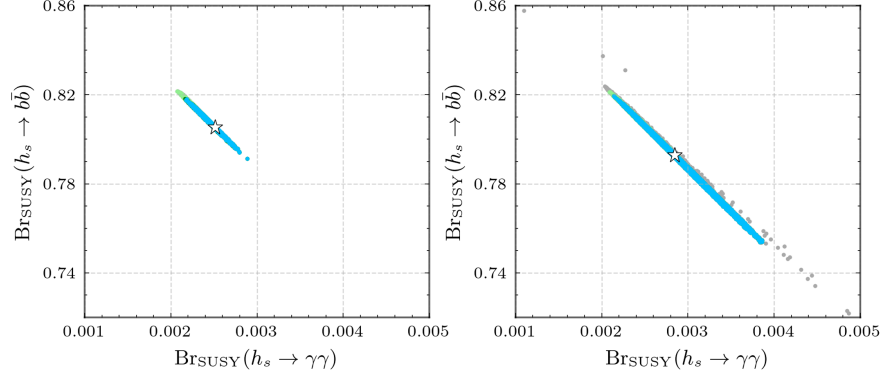


FIG. 5. Projection of the samples in Fig. 2 onto $\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma\gamma) - \text{Br}_{\text{SUSY}}(h_s \rightarrow b\bar{b})$ planes. The left and right panels correspond to the bino- and singlino-DM cases, respectively.

planes to yield Fig. 5. These figures reveal that both $|C_{h_s gg}|$ and $|C_{h_s \gamma\gamma}|$ are slightly higher than $|C_{h_s t\bar{t}}|$, while $|C_{h_s b\bar{b}}|$ is relatively suppressed. Additionally, there is a strong correlation between $\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma\gamma)$ and $\text{Br}_{\text{SUSY}}(h_s \rightarrow b\bar{b})$ in explaining the excesses at 95 GeV, whereas $C_{h_s b\bar{b}}$ loosely correlates with $C_{h_s t\bar{t}}$. The best point for the bino-DM is located at $|C_{h_s t\bar{t}}| \simeq |C_{h_s VV}| \simeq 0.37$, $|C_{h_s b\bar{b}}| \simeq 0.33$, $|C_{h_s gg}| \simeq 0.38$, and $|C_{h_s \gamma\gamma}| \simeq 0.41$, leading to $\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma\gamma) \simeq 0.25\%$, $\text{Br}_{\text{SUSY}}(h_s \rightarrow b\bar{b}) \simeq 81\%$, $\mu_{\gamma\gamma} = 0.22$, $\mu_{b\bar{b}} = 0.13$, and $\chi^2_{\gamma\gamma+b\bar{b}} = 0.12$. In contrast, the optimal configuration for the singlino-DM case corresponds to $|C_{h_s t\bar{t}}| \simeq |C_{h_s VV}| \simeq 0.35$, $|C_{h_s b\bar{b}}| \simeq 0.29$, $|C_{h_s gg}| \simeq 0.37$, and $|C_{h_s \gamma\gamma}| \simeq 0.39$, predicting $\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma\gamma) \simeq 0.28\%$, $\text{Br}_{\text{SUSY}}(h_s \rightarrow b\bar{b}) \simeq 79\%$, $\mu_{\gamma\gamma} = 0.24$, $\mu_{b\bar{b}} = 0.12$, and $\chi^2_{\gamma\gamma+b\bar{b}} \simeq 0$. It is

noteworthy that the singlino-DM case allows significantly wider ranges of the couplings to account for the excesses at the 1σ level when compared to the bino-DM case, revealing its greater adaptability in forecasting Higgs observables. The underlying reasons for these phenomena were discussed in Sec. II C.

We also examine the characteristics of the SM-like Higgs boson in a similar manner and present the results in Figs. 6 and 7. These figures indicate that the couplings of h are in agreement with those of the SM Higgs boson, with uncertainties of up to 10%, and that h exhibits properties similar to those of h_s . However, there are two notable distinctions. First, both $C_{hb\bar{b}}$ and $C_{h\gamma\gamma}$ take higher values compared to $C_{ht\bar{t}}$, with the latter surpassing $C_{hV\bar{V}}$ by approximately 10%. Second, $\text{Br}_{\text{SUSY}}(h_s \rightarrow \gamma\gamma)$ and

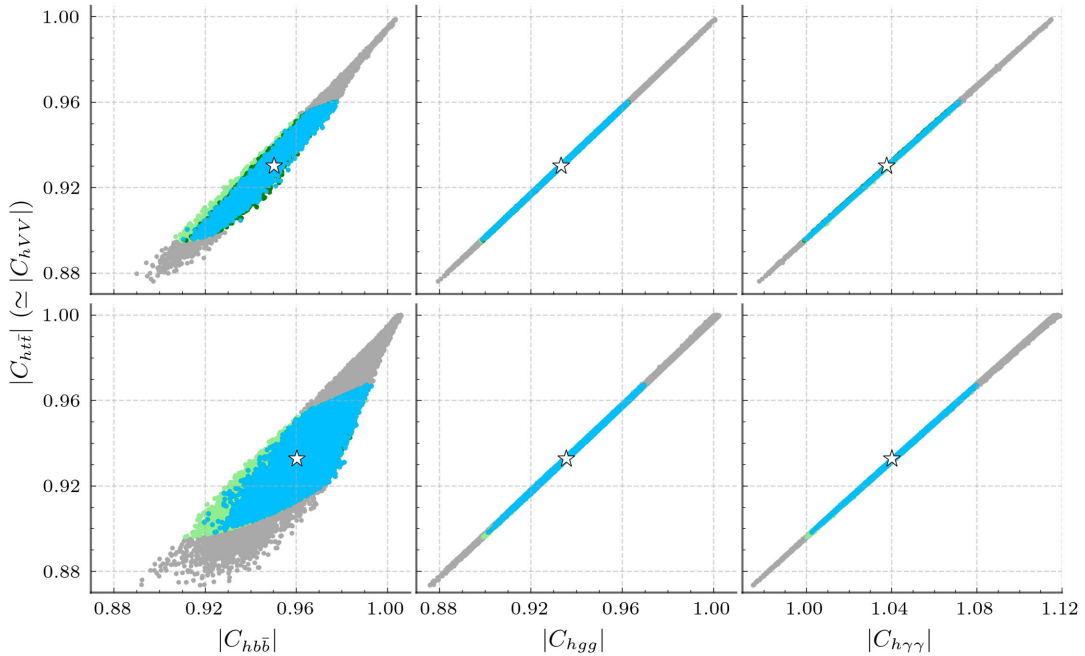


FIG. 6. The same as Fig. 4, except that the couplings of the SM-like Higgs boson are displayed.

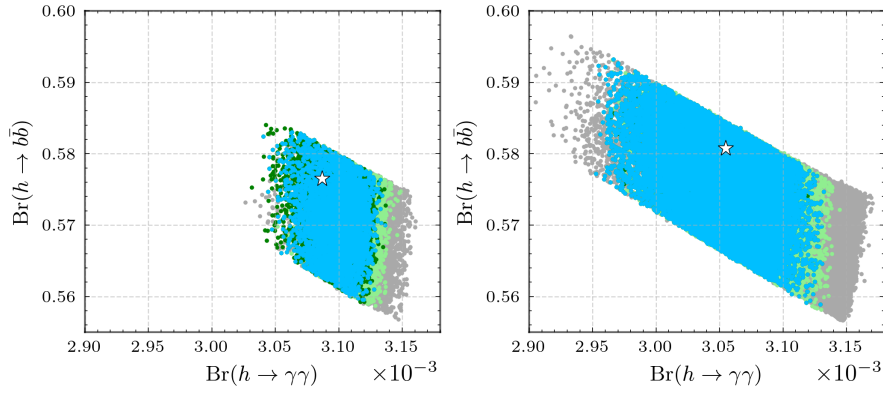


FIG. 7. The same as Fig. 5, except that the branching ratios of the SM-like Higgs boson are shown.

$\text{Br}_{\text{SUSY}}(h_s \rightarrow b\bar{b})$ display a loose correlation and both vary within relatively broad ranges. Specifically, $\text{Br}(h \rightarrow b\bar{b})$ varies from 55.8% to 59.5%, which is consistent with its SM prediction of $(57.7 \pm 1.8)\%$ [122], while $\text{Br}(h \rightarrow \gamma\gamma)$ ranges from 2.95×10^{-3} to 3.13×10^{-3} , significantly exceeding the SM prediction of $(2.28 \pm 0.11) \times 10^{-3}$ [122]. We add that considering $C_{hgg} < 1$, the diphoton signal of h remains consistent with its SM prediction.

Finally, we elucidate the physics underlying the observed excesses by providing detailed information on two benchmark points in Table VI. These points represent the most favored solutions for the anomalies within the two types of DM scenarios, as indicated by stars in the earlier diagrams. Given that numerous characteristics of these points have been previously discussed, our focus now lies

on highlighting the most significant signal at the LHC. Specifically, we concentrate on Higgsino-like and winolike electroweakino pair productions for points P1 and P2, respectively, leading to rich productions of $V_1 V_2 V_3^* V_4^*$ state from the sparticle decays, where V_i with $i = 1, 2, 3, 4$ symbolizes either a W or Z boson. As suggested by the SRs A14 and A44 in Ref. [185], an optimal detection strategy for the production involves searching for events with three electrons or muons that include at least one opposite-sign same-flavor pair. The two SRs are distinct by the requirement for the invariant mass of the lepton pair to be less than 75 GeV and greater than 105 GeV, respectively. Both points anticipate an R value less than 1, implying their compatibility with restrictions from the LHC's search for sparticles [40]. Additionally, we make several observations regarding these two benchmark points:

TABLE VI. Details of the best points for the bino- and singlino-dominated DM cases. Both of them can explain the $(g - 2)_\mu$ anomaly and the diphoton and $b\bar{b}$ excesses at a 1σ level simultaneously, aligned with the LHC restrictions. They achieve the measured abundance primarily by coannihilating with the wino- and Higgsino-dominated electroweakinos, respectively. d_i , u_i , and ℓ_i appearing in the annihilation final state denote the i th generation of down-type quarks, up-type quarks, and leptons, respectively.

Benchmark point P1				Benchmark point P2			
λ	0.011	A_t	1491 GeV	λ	0.015	A_t	1534 GeV
κ	-0.122	A_λ	1614 GeV	κ	-0.005	A_λ	1780 GeV
$\tan\beta$	53.37	M_1	-338.0 GeV	$\tan\beta$	58.54	M_1	-706.3 GeV
μ_{tot}	616.5 GeV	M_2	350.0 GeV	μ_{tot}	451.0 GeV	M_2	610.0 GeV
m_B	103.0 GeV	$M_{\tilde{\mu}_L}$	506.3 GeV	m_B	101.5 GeV	$M_{\tilde{\mu}_L}$	528.8 GeV
m_N	604.3 GeV	$M_{\tilde{\mu}_R}$	856.8 GeV	m_N	437.1 GeV	$M_{\tilde{\mu}_R}$	549.2 GeV
m_{h_s}	95.92 GeV	$m_{\tilde{\chi}_1^0}$	-337.2 GeV	m_{h_s}	96.1 GeV	$m_{\tilde{\chi}_1^0}$	436.8 GeV
m_h	124.9 GeV	$m_{\tilde{\chi}_2^0}$	-361.8 GeV	m_h	124.8 GeV	$m_{\tilde{\chi}_2^0}$	448.2 GeV
m_H	2412 GeV	$m_{\tilde{\chi}_3^0}$	603.8 GeV	m_H	2377 GeV	$m_{\tilde{\chi}_3^0}$	-462.3 GeV
m_{A_s}	797.7 GeV	$m_{\tilde{\chi}_4^0}$	-635.9 GeV	m_{A_s}	799.9 GeV	$m_{\tilde{\chi}_4^0}$	655.2 GeV
m_{A_H}	2412 GeV	$m_{\tilde{\chi}_5^0}$	642.9 GeV	m_{A_H}	2377 GeV	$m_{\tilde{\chi}_5^0}$	-712.0 GeV
$m_{H^\pm}$	2444 GeV	$m_{\tilde{\mu}_L}$	512.5 GeV	$m_{H^\pm}$	2404 GeV	$m_{\tilde{\mu}_L}$	539.0 GeV
$m_{\tilde{\chi}_1^\pm}$	362.1 GeV	$m_{\tilde{\mu}_R}$	857.9 GeV	$m_{\tilde{\chi}_1^\pm}$	449.9 GeV	$m_{\tilde{\mu}_R}$	554.1 GeV
$m_{\tilde{\chi}_2^\pm}$	645.1 GeV	$m_{\tilde{\nu}_L}$	506.0 GeV	$m_{\tilde{\chi}_2^\pm}$	655.5 GeV	$m_{\tilde{\nu}_L}$	533.4 GeV
Ωh^2	0.11			Ωh^2	0.10		

(Table continued)

TABLE VI. (Continued)

Benchmark point P1		Benchmark point P2	
$\sigma_p^{SI}, \sigma_n^{SD}$	$8.6 \times 10^{-47} \text{ cm}^2, 1.3 \times 10^{-42} \text{ cm}^2$	$\sigma_p^{SI}, \sigma_n^{SD}$	$1.0 \times 10^{-47} \text{ cm}^2, 5.8 \times 10^{-46} \text{ cm}^2$
$a_\mu^{\text{SUSY}}, \chi_{\Delta_{a_\mu}}^2$	$2.59 \times 10^{-9}, 0.04$	$a_\mu^{\text{SUSY}}, \chi_{\Delta_{a_\mu}}^2$	$2.56 \times 10^{-9}, 0.02$
$a_\mu^{\text{WHL}}, a_\mu^{\text{BHL}}$	$2.93 \times 10^{-9}, -1.68 \times 10^{-10}$	$a_\mu^{\text{WHL}}, a_\mu^{\text{BHL}}$	$2.71 \times 10^{-9}, -1.64 \times 10^{-10}$
$a_\mu^{\text{BHR}}, a_\mu^{\text{BLR}}$	$1.20 \times 10^{-10}, -2.24 \times 10^{-10}$	$a_\mu^{\text{BHR}}, a_\mu^{\text{BLR}}$	$3.09 \times 10^{-10}, -2.32 \times 10^{-10}$
$\mu_{\gamma\gamma}, \mu_{b\bar{b}}, \Delta_{\gamma\gamma+b\bar{b}}$	0.22, 0.13, 0.12	$\mu_{\gamma\gamma}, \mu_{b\bar{b}}, \Delta_{\gamma\gamma+b\bar{b}}$	0.24, 0.12, 0.0
$V_{h_s}^S, V_{h_s}^{\text{SM}}, V_{h_s}^{\text{NSM}}$	-0.930, 0.367, 6.76×10^{-4}	$V_{h_s}^S, V_{h_s}^{\text{SM}}, V_{h_s}^{\text{NSM}}$	-0.933, 0.361, 9.66×10^{-4}
$C_{h_s gg}, C_{h_s VV}, C_{h_s \gamma\gamma}, C_{h_s i\bar{i}}, C_{h_s b\bar{b}}$	0.382, 0.367, 0.409, 0.367, 0.331	$C_{h_s gg}, C_{h_s VV}, C_{h_s \gamma\gamma}, C_{h_s i\bar{i}}, C_{h_s b\bar{b}}$	0.366, 0.351, 0.391, 0.351, 0.291
$C_{h\gamma\gamma}, C_{hVV}, C_{h\gamma\gamma}, C_{h i\bar{i}}, C_{h b\bar{b}}$	0.376, 0.361, 0.402, 0.361, 0.304	$C_{h\gamma\gamma}, C_{hVV}, C_{h\gamma\gamma}, C_{h i\bar{i}}, C_{h b\bar{b}}$	0.936, 0.933, 1.040, 0.933, 0.960
$N_{11}, N_{12}, N_{13}, N_{14}, N_{15}$	-0.994, -0.005, -0.096, -0.050, -0.000	$N_{11}, N_{12}, N_{13}, N_{14}, N_{15}$	0.004, 0.044, -0.111, 0.110, -0.987
$N_{21}, N_{22}, N_{23}, N_{24}, N_{25}$	0.007, 0.978, -0.182, 0.105, -0.001	$N_{21}, N_{22}, N_{23}, N_{24}, N_{25}$	-0.025, -0.280, 0.680, -0.657, -0.162
$N_{31}, N_{32}, N_{33}, N_{34}, N_{35}$	-0.001, -0.009, 0.024, -0.026, 0.999	$N_{31}, N_{32}, N_{33}, N_{34}, N_{35}$	-0.122, -0.050, -0.701, -0.702, -0.002
$N_{41}, N_{42}, N_{43}, N_{44}, N_{45}$	-0.103, 0.055, 0.699, 0.706, 0.001	$N_{41}, N_{42}, N_{43}, N_{44}, N_{45}$	0.008, -0.958, -0.167, 0.234, 0.002
$N_{51}, N_{52}, N_{53}, N_{54}, N_{55}$	0.032, -0.203, -0.685, 0.698, 0.036	$N_{51}, N_{52}, N_{53}, N_{54}, N_{55}$	0.992, -0.006, -0.068, -0.106, -0.000
Coannihilations	Fractions (%)	Coannihilations	Fractions (%)
$\tilde{\chi}_2^0 \tilde{\chi}_1^- \rightarrow d_i \bar{u}_i / \nu_{\ell_i} \ell_i^- / ZW^- / \gamma W^-$	29.3/7.75/6.35/1.69	$\tilde{\chi}_2^0 \tilde{\chi}_1^- \rightarrow d_i \bar{u}_i / \nu_{\ell_i} \ell_i^- / hW^- / ZW^-$	27.3/9.33/3.25/2.70
$\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow q_i \bar{q}_i / ZZ / W^- W^+ / \gamma Z$	13.7/4.50/3.90/2.55	$\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp \rightarrow q_i \bar{q}_i / W^- W^+ / hZ / \gamma Z$	12.9/3.44/1.82/1.58
$\tilde{\chi}_2^0 \tilde{\chi}_2^0 \rightarrow W^- W^+$	7.68	$\tilde{\chi}_3^0 \tilde{\chi}_1^- \rightarrow d_i \bar{u}_i / ZW^-$	6.63/1.10
Decays	Branching ratios (%)	Decays	Branching ratios (%)
$\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^*$	100	$\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^*$	100
$\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^\pm W^\mp / \tilde{\chi}_2^0 h / \tilde{\chi}_1^0 Z / \tilde{\chi}_2^0 h_s$	64.7/24.4/4.81/3.64	$\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^\pm W^\mp / \tilde{\chi}_2^0 h / \tilde{\chi}_1^0 Z^*$	48.5/47.1
$\tilde{\chi}_4^0 \rightarrow \tilde{\chi}_1^\pm W^\mp / \tilde{\chi}_2^0 Z / \tilde{\chi}_1^0 h / \tilde{\chi}_2^0 h$	61.5/27.0/7.29/1.41	$\tilde{\chi}_4^0 \rightarrow \tilde{\chi}_1^\pm W^\mp / \tilde{\chi}_3^0 Z / \tilde{\chi}_2^0 h / \nu_\mu \tilde{\nu}_\mu / \mu \tilde{\mu}_L$	49.3/18.1/13.5/8.10/6.98
$\tilde{\chi}_5^0 \rightarrow \tilde{\chi}_1^\pm W^\mp / \tilde{\chi}_2^0 h / \tilde{\chi}_1^0 Z / \tilde{\chi}_2^0 h_s$	62.3/21.4/8.49/3.68	$\tilde{\chi}_5^0 \rightarrow \tilde{\chi}_1^\pm W^\mp / \mu \tilde{\mu}_R / \tilde{\chi}_3^0 h / \mu \tilde{\mu}_L / \nu_\mu \tilde{\nu}_\mu$	29.7/25.2/11.7/8.35/8.23
$\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 W^{*\pm}$	100	$\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 W^{*\pm}$	100
$\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_2^0 W^\pm / \tilde{\chi}_1^\pm Z / \tilde{\chi}_1^0 h / \tilde{\chi}_1^0 W^\pm$	31.6/30.1/23.0/10.4	$\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_2^0 W^\pm / \tilde{\chi}_1^\pm Z / \tilde{\chi}_1^0 h / \nu_\mu \tilde{\nu}_\mu / \mu \tilde{\mu}_L$	45.1/21.9/14.8/7.81/7.27
$\tilde{\mu}_L \rightarrow \nu_\mu \tilde{\chi}_1^\pm / \mu \tilde{\chi}_2^0 / \mu \tilde{\chi}_1^0$	57.5/29.7/12.7	$\tilde{\mu}_L \rightarrow \nu_\mu \tilde{\chi}_1^\pm / \mu \tilde{\chi}_2^0 / \mu \tilde{\chi}_3^0 / \mu \tilde{\chi}_1^0$	51.5/41.2/6.07/1.28
$\tilde{\mu}_R \rightarrow \mu \tilde{\chi}_1^0 / \mu \tilde{\chi}_2^0 / \nu_\mu W^\pm$	99.0/0.37/0.22	$\tilde{\mu}_R \rightarrow \mu \tilde{\chi}_3^0 / \mu \tilde{\chi}_2^0 / \nu_\mu \tilde{\chi}_1^\pm$	89.8/8.35/1.52
$\tilde{\nu}_\mu \rightarrow \mu \tilde{\chi}_1^\pm / \nu_\mu \tilde{\chi}_2^0 / \nu_\mu \tilde{\chi}_1^0$	59.1/28.7/12.1	$\tilde{\nu}_\mu \rightarrow \mu \tilde{\chi}_1^\pm / \nu_\mu \tilde{\chi}_2^0 / \nu_\mu \tilde{\chi}_1^0$	75.2/23.9/0.74
$h_s \rightarrow b\bar{b} / \tau^+ \tau^- / gg / c\bar{c} / W^+ W^- / \gamma\gamma$	80.5/8.93/5.07/4.69/0.46/0.25	$h_s \rightarrow b\bar{b} / \tau^+ \tau^- / gg / c\bar{c} / W^+ W^- / \gamma\gamma$	79.3/8.79/5.75/5.28/0.54/0.29
$h \rightarrow b\bar{b} / W^+ W^- / \tau^+ \tau^- / gg / \gamma\gamma$	57.6/25.8/6.69/4.61/0.31	$h \rightarrow b\bar{b} / W^+ W^- / \tau^+ \tau^- / gg / \gamma\gamma$	58.1/25.4/67.4/4.57/0.30
$H \rightarrow b\bar{b} / \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp / \tau^+ \tau^- / \tilde{\chi}_2^0 \tilde{\chi}_4^0$	56.7/16.1/14.3/7.21	$H \rightarrow b\bar{b} / \tau^+ \tau^- / \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp / \tilde{\chi}_2^0 \tilde{\chi}_4^0$	61.9/14.7/12.2/5.89
$A_s \rightarrow b\bar{b} / \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp / \tau^+ \tau^- / \tilde{\chi}_2^0 \tilde{\chi}_2^0$	69.0/12.8/10.5/6.60	$A_s \rightarrow b\bar{b} / \tau^+ \tau^-$	86.6/13.2
$A_H \rightarrow b\bar{b} / \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp / \tau^+ \tau^- / \tilde{\chi}_2^0 \tilde{\chi}_4^0$	56.7/15.8/14.3/7.21	$A_H \rightarrow b\bar{b} / \tau^+ \tau^- / \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp / \tilde{\chi}_2^0 \tilde{\chi}_4^0$	61.9/14.7/11.6/5.71
$H^+ \rightarrow t\bar{b} / \tau^+ \nu_\tau / \tilde{\chi}_2^0 \tilde{\chi}_2^+ / \tilde{\chi}_3^0 \tilde{\chi}_1^+ / \tilde{\chi}_4^0 \tilde{\chi}_1^+$	55.7/15.6/8.84/8.32/7.2	$H^+ \rightarrow t\bar{b} / \tau^+ \nu_\tau / \tilde{\chi}_2^0 \tilde{\chi}_2^+ / \tilde{\chi}_3^0 \tilde{\chi}_1^+ / \tilde{\chi}_3^0 \tilde{\chi}_2^+$	60.9/16.1/7.00/6.69/6.04
R value: 0.47	Signal region: SR-A14 in Ref. [185]	R value: 0.50	Signal region: SR-A44 in Ref. [185]
$\chi_{\text{tot}}^2 \equiv \chi_{\Delta_{a_\mu}}^2 + \chi_{\gamma\gamma+b\bar{b}}^2$	0.16	$\chi_{\text{tot}}^2 \equiv \chi_{\Delta_{a_\mu}}^2 + \chi_{\gamma\gamma+b\bar{b}}^2$	0.02

- (i) Considering that the analyses in Ref. [185] utilized data from the initial year of LHC run 2, accumulating 35.9 fb^{-1} , reanalyzing the complete run-2 dataset using the same strategy is anticipated to significantly enhance the LHC's sensitivity to these signals.
- (ii) For point P1's winolike and point P2's Higgsino-like electroweakino pair productions at the LHC, their ability to provide substantial evidence for supersymmetry is hindered by the large DM mass. Specifically, these processes are effective in probing supersymmetry only when the DM is lighter than

approximately 220 and 190 GeV, respectively, as demonstrated in Ref. [131] through examinations of compressed sparticle mass spectrum cases.

- (iii) The slepton pair productions at both points prove ineffective for probing supersymmetry due to the complex decay chains involved with the sleptons, compounded by the substantial masses of the DM and the sleptons, as indicated in Refs. [194,197].
- (iv) As suggested in Refs. [210–213], the electroweakino and slepton sectors of these points could be exhaustively explored at both high-luminosity LHC and a high-energy e^+e^- collider with $\sqrt{s} > 1 \text{ TeV}$.

- (v) Research expounded in Refs. [61,214–216] underscores that future Higgs factories stand to meticulously scrutinize the properties of a Higgs boson with a mass of approximate 95 GeV.

C. Impacts of the recent 125 GeV Higgs data

The ongoing operation of the LHC has led to a substantial accumulation of 125 GeV Higgs boson data, enabling increasingly precise measurements of its properties [110–112]. These enhanced precision measurements provide an opportunity to establish more stringent constraints on theories beyond the SM. To leverage these new measurements, we refined the Higgs data fitting in Sec. III A using HiggsTools-1.2, the latest version of HiggsTools [164], to reassess previously scanned parameter points. This updated version marks a significant advancement over the HiggsSignals-2.6.2 by incorporating 22 additional physical observables, thereby increasing the total number of observables available for fitting Higgs properties to 129 [164].

In our study, we initially specified particular parameter values: $\tan\beta = 20$ and $\lambda = \kappa = 10^{-4}$. Additionally, we set all mass-dimensional parameters in the Higgs, sfermion, and gaugino sectors to 2 TeV, with A_t being the sole exception. Through meticulous adjustment of A_t , we successfully reproduced a Higgs particle with a mass of approximately 125.1 GeV. This particle shares identical features with the SM Higgs boson, with the crucial difference that its mass is derived from sophisticated theoretical calculations rather than being an input parameter. Upon incorporating a mass uncertainty of 3 GeV into our analysis, the Higgs data fitting process yielded a χ^2 value of approximately 176.3, denoted as $\chi^2_{\text{SM},125} \simeq 176.3$.

To evaluate how recent Higgs data influence the theoretical interpretation of the observed excesses, we mapped the parameter points from Fig. 2 onto the $\mu_{\gamma\gamma} - \mu_{b\bar{b}}$ plane in Fig. 8, where the color bar indicates the difference between their χ^2 values (denoted as χ^2_{125}) in the 125 GeV Higgs data fit using HiggsTools-1.2 and $\chi^2_{\text{SM},125}$. Our analysis demonstrates strong compatibility between these parameter points and the current Higgs data, as evidenced by the following observations:

- (i) The benchmark points P1 and P2 presented in Table VI exhibit χ^2_{125} values of 164.7 and 165.9 respectively, both falling below $\chi^2_{\text{SM},125}$.
- (ii) Within the experimentally acceptable range defined by $\chi^2_{125} \lesssim \chi^2_{\text{SM},125} + 6.18$ [217], the signal strengths $\mu_{\gamma\gamma}$ and $\mu_{b\bar{b}}$ can achieve maximum values of 0.37 and 0.21, respectively.

It is noteworthy that $\chi^2_{\text{SM},125}/\nu \simeq 1.38$ significantly exceeds $1 + \sqrt{2/\nu} = 1.125$, where $\nu \equiv n_{\text{obs}} - n_{\text{param}}$ denotes the degrees of freedom with $n_{\text{obs}} = 129$ representing the number of physical observables in HiggsTools and $n_{\text{param}} = 1$ corresponding to the Higgs boson mass in the SM [218]. As emphasized in Ref. [164], this discrepancy indicates

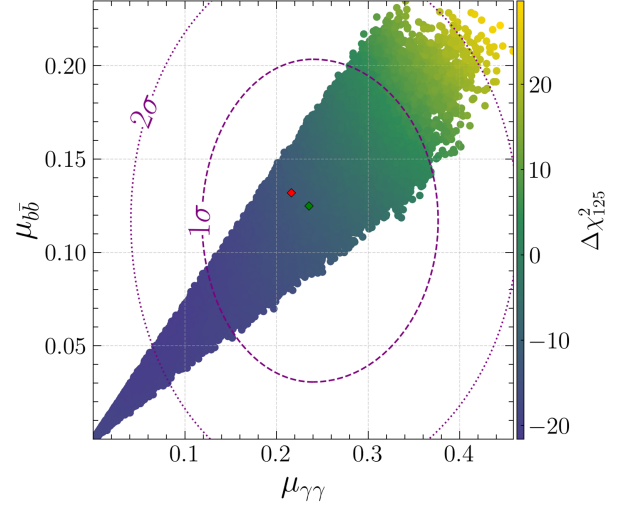


FIG. 8. Projection of the parameter points from Fig. 2 onto the $\mu_{\gamma\gamma} - \mu_{b\bar{b}}$ plane. The color bar represents $\Delta\chi^2_{125}$, defined as the difference between χ^2_{125} and $\chi^2_{\text{SM},125}$: $\Delta\chi^2_{125} \equiv \chi^2_{125} - \chi^2_{\text{SM},125}$. All fits for the 125 GeV Higgs data were performed using HiggsTools-1.2, yielding $\chi^2_{\text{SM},125} = 176.3$ for a fully SM-like Higgs boson with an assumed mass uncertainty of 3 GeV (see text for details). The benchmark points P1 and P2 from Table VI are indicated by red and green dots, respectively.

substantial systematic uncertainties in the data incorporated within the HiggsTools.

IV. CONCLUSION

Recently, FermiLab has updated its measurement of the muon anomalous magnetic moment based on run-2 and run-3 data, revealing a significant deviation from the current SM prediction at a confidence level of 5.1σ . Despite uncertainties in calculating leading-order hadronic contributions, this observation likely indicates the presence of new physics. Moreover, both the CMS and ATLAS Collaborations have recently reported an excess in the diphoton invariant mass distribution at $m_{\gamma\gamma} \simeq 95.4$ GeV with compatible signal strengths. By combining these novel findings with a similar excess previously reported by CMS, the local significance has increased to 3.1σ . Additionally, it is noteworthy that the invariant mass of the diphoton signal coincides with that of the observed $b\bar{b}$ excess at the LEP, suggesting they may originate from a same CP -even Higgs boson with its mass around 95.4 GeV.

The GNMSSM stands out for its theoretical advantages and a versatile parameter space, which enables a comprehensive exploration of Higgs and DM phenomenology. In this study, we apply the GNMSSM to simultaneously explain the muon $g - 2$ anomaly, as well as the diphoton and $b\bar{b}$ excesses. We demonstrate that the anomaly can be entirely attributed to the loops involving muon-smuon-neutralino and muon-sneutrino-chargino interactions. By employing the mass insertion approximation of a_μ^{SUSY} , we

find a preference for a large $\tan\beta$ value and a sub-TeV SUSY scale. Additionally, we address the excesses by considering resonant productions of the singlet-dominated scalar h_s . Our derivation of approximate formulas for signal strengths highlights how crucial input parameters such as λ , $\tan\beta$, μ_{tot} , A_λ , and m_N interact with each other. Our conclusion reveals that the unified explanations favor a moderately large SM Higgs field component in h_s , with a suppressed $h_s b\bar{b}$ coupling relative to $h_s t\bar{t}$ coupling. Small deviations in $C_{h_s gg}$ and $C_{h_s \gamma\gamma}$ from $C_{h_s t\bar{t}}$ enhance the theory's ability to explain the excesses.

To comprehensively investigate the impact of experimental restrictions on the unified explanation, encompassing those from Higgs physics, B physics, DM physics, and collider searches for sparticles, we undertake an exploration across the GNMSSM parameter space. Our analysis yields the following key findings:

- (i) The GNMSSM can simultaneously account for the muon ($g-2$) anomaly and the diphoton and $b\bar{b}$ excesses at a 2σ level within a wide range of parameters without conflicting with constraints from the SM-like Higgs data fit, the B -physics measurements, the Planck and LZ experiments, and the vacuum stability considerations, as well as the LHC's searches for sparticles and extra Higgs bosons.
- (ii) DM physics significantly influences the unified explanation through the crucial parameter μ_{tot} . Specifically, the preferred samples are classified into either the bino-DM scenario or singlino-DM scenario based on their dominant DM component. They achieve the observed relic abundance primarily by coannihilating with wino- and Higgsino-like electroweakinos, respectively, contributing to 26% and 74% of the total Bayesian evidence. The bino-DM scenario favors $\mu_{\text{tot}} \gtrsim 600$ GeV mainly due to restrictions imposed by the LZ experiment, whereas in the singlino-DM scenario $\mu_{\text{tot}} \simeq |m_{\tilde{\chi}_1^0}|$ can be as low as 400 GeV. As indicated in this study, the difference has profound implications in $\delta a_\mu^{\text{SUSY}}$, $\mu_{\gamma\gamma}$, and $\mu_{b\bar{b}}$.
- (iii) The WHL contribution dominates a_μ^{SUSY} calculation. It sets upper bounds on μ_{tot} , M_2 , and $M_{\tilde{\mu}_L}$ in explaining the muon $g-2$ anomaly. Additionally, the BHL and BLR contributions consistently counteract other terms when considering negative M_1 values, which are preferred in the bino-DM case to suppress DM-nucleon scattering.
- (iv) Constraints arising from the LHC's searches for sparticles slightly narrow down the parameter regions in the bino-DM case but have minimal impacts on the unified explanation in the singlino-DM case due to our *ad hoc* assumption regarding the range of μ_{tot} , i.e., $\mu_{\text{tot}} \geq 400$ GeV. Incorporating the LHC restrictions sets lower bounds on masses of the sparticles involved in the explanation, such as neutralinos, charginos, and muon-flavored sleptons.

Before concluding our study, we believe it is important to share the following remarks about our findings:

- (i) In formulating the likelihood function, we have adopted overly idealistic assumptions regarding the prolific generation of diphoton and $b\bar{b}$ events at colliders. Furthermore, considering that increasingly comprehensive analyses of the experimental data are being carried out, it is likely that we might have underestimated the impact of the LHC's search for SUSY in excluding the parameter space of the GNMSSM. Consequently, although the results presented here provide valuable insights into the fundamental physics of the observed excesses, they do not possess strict statistical significance. For instance, $\chi_{\text{tot}}^2 = 0.02$ for the optimal point P2 in Table VI merely indicates that the GNMSSM can predict the central values of the excesses, and it outperforms the SM, which yields $\chi_{\text{tot}}^2 = 40.1$ in accounting for the observed excesses. It should not be overinterpreted.

We remind the reader that a more realistic methodology for constructing the likelihood function can be found in Ref. [219], which addresses the global fitting of grand unified theory scale SUSY models to experimental data.

- (ii) As demonstrated by this study, addressing the diphoton excess, the $b\bar{b}$ excess, and the muon $g-2$ anomaly demands a considerable $V_{h_s}^{\text{SM}}$, an appropriate cancellation between $V_{h_s}^{\text{SM}}$ and $V_{h_s}^{\text{NSM}} \tan\beta$, and, depending on the value of $\tan\beta$, the presence of moderately light electroweakinos and muon-type sleptons, respectively. This unified interpretation integrates these distinct requirements to delineate specific GNMSSM parameter configurations. Some of these constraints would be alleviated if any excess diminishes, thereby expanding the permissible parameter space. For example, if future theoretical and experimental advancements reveal that Δa_μ is significantly smaller than its current estimation, the results could be accommodated simply by increasing the sparticle mass values and, in this case, moderately light sparticles are no longer necessary. As shown in this study, such adjustments in the GNMSSM's parameter space would not impact the explanations for the other anomalies and the constraints from the LHC's search for SUSY.
- (iii) The muon $g-2$ anomaly has been comprehensively investigated within the h_1 and h_2 scenarios of the GNMSSM [41,42], where the SM-like Higgs boson h corresponds to the lightest and next-to-lightest CP -even Higgs bosons, respectively. It has been observed that the h_1 scenario provides a much broader parameter space compared to those of the MSSM and the Z_3 NMSSM for explaining the anomaly [41]. This is attributed to more relaxed

constraints from DM experiments and the LHC's search for SUSY. In contrast, the h_2 scenario has difficulty accounting for the anomaly, as its parameter space is characterized by $\mu_{\text{tot}} \lesssim 500$ GeV and $\tan\beta \lesssim 30$, with fixed $A_\lambda = 2$ TeV and $\mu' = 0$ [42]. This parameter configuration tends to result in a relatively low $\mathcal{M}_{S,23}^2$ in Eq. (2.3), thereby avoiding overly large singlet-doublet Higgs mixing, which is more in line with the observed Higgs data. However, as indicated by the study in Ref. [42], it faces strict limitations from the LHC's searches for SUSY. Our study improves upon Ref. [42] by choosing the field masses m_A , m_B , and m_C (instead of the soft-breaking masses in the original Lagrangian adopted in Ref. [42]) as theoretical inputs and simultaneously allowing A_λ to vary within reasonable ranges during the parameter scan of the GNMSSM, as detailed in Table I. We draw conclusions that significantly differ from those of Ref. [42]: the GNMSSM can still explain the muon $g - 2$ anomaly across a vast parameter space that complies with experimental constraints. The key difference stems from the fact that our refined scanning strategy affects the posterior PDF of the samples, particularly showing that a considerable proportion of samples suggest $\tan\beta > 30$ and $\mu_{\text{tot}} > 300$ GeV. This case demands a relatively small λ and a moderate cancellation between different contributions to $\mathcal{M}_{S,23}^2$, effectively reducing its size by typically 10%. Such a case was neglected in the study of Ref. [42]. Indeed, it is the limitations in the analysis of Ref. [42] that motivated us to extend our previous study of the diphoton and $b\bar{b}$ excesses in the GNMSSM, as elaborated in Ref. [51], to also incorporate the muon $g - 2$ anomaly.

- (iv) We note that the three anomalies were also investigated in Ref. [98] within the framework of the Z_3 NMSSM, a special implementation of the GNMSSM [34]. That study was conducted later than ours and it was found that these anomalies could be accounted for in a parameter space characterized by $\lambda \gtrsim 0.2$, $\tan\beta \lesssim 20$, and the presence of at least one electroweakino or slepton lighter than approximately 200 GeV. These findings are significantly different from our current results.

The disparity in the two findings can be mainly attributed to our distinct research objectives. As discussed in Sec. III A, this study specifically

focused on examining whether the GNMSSM could explain these three anomalies under various experimental constraints, rather than carrying out a comprehensive global fitting analysis. We deliberately excluded the parameter regions discussed in Ref. [98] based on the following two crucial considerations:

- (a) As elaborated below Eq. (2.25), the condition $\mu_{\text{tot}} - \bar{A}_\lambda \sin\beta \cos\beta \simeq 10$ GeV is necessary for the substantial λ case to account for the $\gamma\gamma$ and $b\bar{b}$ excesses. This condition demands a strong cancellation between two independent terms μ_{tot} and $\bar{A}_\lambda \sin\beta \cos\beta$, typically resulting in a net magnitude reduction by one order. Such fine-tuning can significantly suppress the posterior PDF considered in this study, rendering this explanation statistically less probable.
- (b) In scenarios where $\tan\beta \lesssim 20$, moderately light sparticles are needed to explain the muon $g - 2$ anomaly. However, as demonstrated in Ref. [42], this scenario is subject to strict limitations from LHC supersymmetry searches, making viable solutions extremely rare. To appropriately account for LHC constraints in these cases, dedicated Monte Carlo simulations of the electroweakino and slepton production processes using CheckMATE are essential [41]. This requirement stems from the fact that the capabilities of the code SModelS in this domain are significantly restricted when analyzing numerous realistic SUSY scenarios, especially in cases involving long decay chains of heavy sparticles, due to both its database limitations and strict usage prerequisites. While many previous studies, including that in Ref. [98], overlooked this conclusion, our earlier research demonstrated that the SModelS could effectively exclude parameter points in only a small fraction of cases for both the Z_3 NMSSM [36] and the GNMSSM [41,42].

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