

Floquet prethermalization of Z_2 lattice gauge theory on superconducting qubits

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Simulating nonequilibrium dynamics of a quantum many-body system is one of the promising applications of quantum computing. We simulate the time evolution of one-dimensional Z_2 lattice gauge theory on IBM's superconducting 156-qubit device, `ibm_fez`. We consider the Floquet circuit made of the Trotter decomposition of Hamiltonian evolution and focus on its dynamics toward thermalization. Quantum simulation with the help of error mitigation is successful in running the Floquet circuit made of 38 and 116 qubits up to 10 Trotter steps in the best case. This is enough to reach the early stage of prethermalization. Our work would be a benchmark for the potential power of quantum computing for high-energy physics problems.

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I. INTRODUCTION

Simulating nonequilibrium quantum systems is one of the promising targets of quantum computers. Nonequilibrium dynamics of quantum field theory will be able to be studied by quantum simulation, but we may need to await the realization of fault-tolerant quantum computers. At present, the depth of executable circuits is severely limited due to harmful device noise. This implies that the number of Trotter steps is severely limited, and we can do simulations only for very short times to get Trotter errors under control. The practical implementation of continuous time evolution, say, Hamiltonian simulation, with a classically intractable system size may be unrealistic for near-term quantum devices. Discrete time evolution, such as Floquet circuits [1–4], is a first step toward practical application.

By making the time step very large in the Lie-Suzuki-Trotter decomposition of Hamiltonian evolution, we obtain a quantum circuit feasible for simulation with the near-term quantum devices. Although we cannot faithfully simulate Hamiltonian evolution, such a quantum circuit exhibits

interesting nonequilibrium dynamics, as in the Floquet system. It is known that the Floquet system eventually heats up to infinite temperature by acquiring energy continuously from the driving force, but the heating time can be exponentially long with driving frequency, i.e., inverse of time step [5–10]. Indeed, the generic Floquet system prethermalizes before reaching the infinite temperature state as the generic Hamiltonian system thermalizes [5–10]. We refer to this phenomenon as Floquet prethermalization. Regarding the Trotter evolution with a large time step, we expect that Floquet prethermalization occurs for generic Hamiltonians as long as the Trotter step is smaller than the threshold [11], which is able to be simulated with a shallower circuit and feasible for near-term quantum devices. Indeed, it was proposed to study the prethermal state in a classically intractable system using near-term quantum devices [2].

In this paper, we carry out the digital quantum simulations of Floquet circuits in lattice gauge theory. We consider the simplest model of lattice gauge theory—the Z_2 lattice gauge theory with fermions on a one-dimensional chain with open boundaries. In this geometry, all the gauge fields can be eliminated by solving the Gauss law constraints, and the local gauge-fermion interaction is replaced by the nonlocal fermion-fermion interaction, i.e., the long-range Coulomb interaction. While such a reduction is used in many benchmark studies of quantum simulations [12–19], it cannot be generalized to higher-dimensional gauge theories. The simulation in the presence of fermions and gauge fields is a good exercise toward the future [20–23]. In this study,

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we do not remove fermions or gauge fields but simulate both of them. From another perspective, all-to-all interactions use $\mathcal{O}(N^2)$ two-qubit gates in one Trotter step (or more if we have only local connectivity) with the system size N , which is too many, and must be approximated on near-term quantum devices [19].

We use IBM's latest device "ibm_fez," which consists of 156 superconducting qubits, for quantum simulations. Error mitigation is necessary for interpreting the data by noisy simulation. We examine three mitigation schemes. The first one is zero-noise extrapolation (ZNE), available from Qiskit's estimator [24]. The second and third ones involve mitigation of amplitude damping by division. In the second method, the damping rate is estimated by measuring the Gauss law operator. In the third method, the damping rate is estimated by simulating a Floquet circuit with special parameters, such that the circuit becomes trivial if there exist no noises [3].

The rest of this paper is organized as follows. We introduce the $\mathbf{Z}_2$ lattice gauge theory and the Floquet circuit in Sec. II. We first show the classical simulations with the matrix product state (MPS) ansatz in Sec. III. The parameter dependencies of Floquet prethermalization are checked by the classical MPS simulations. After that, we show the main results of the quantum simulations in Sec. IV. We explain the circuit details, show the results with the three mitigation methods, and discuss the interpretation of the results based on the effective fidelity and circuit volume of the simulated circuits. Finally, we give some comments on future applications of quantum computers to high-energy physics in Sec. V.

II. METHODS

A. $\mathbf{Z}_2$ lattice gauge theory

Here, we overview the Hamiltonian formulation of the $(1+1)$ -dimensional $\mathbf{Z}_2$ lattice gauge theory with the Wilson fermion. We considered a one-dimensional lattice with open boundary conditions, which has N sites and $(N-1)$ links. The total state vector can be written as

$$|\Psi\rangle = \prod_{x=1}^{N-1} |g(x, x+1)\rangle \prod_{x=1}^N |\psi_1(x)\rangle |\psi_2(x)\rangle, \quad (1)$$

where x labels the site position. The gauge fields are defined based on links, and each field $|g(x, x+1)\rangle$ at the link in between x and $x+1$ is two-dimensional. The link operator $Z_g(x, x+1)$ and the conjugate operator $X_g(x, x+1)$ act on these fields. Fermions are two-component spinors, and each component $|\psi_i(x)\rangle$ is two-dimensional. The creation and annihilation operators satisfy the anticommutation relation $\{\psi_i(x), \psi_j^\dagger(y)\} = \delta_{xy} \delta_{ij}$. The Hamiltonian is given by the gauge part

$$H_g = -K \sum_{x=1}^{N-1} X_g(x, x+1), \quad (2)$$

the fermion part

$$H_f = \sum_{x=1}^N (1+m) \psi^\dagger(x) \gamma^0 \psi(x), \quad (3)$$

and the gauge-fermion coupling part

$$H_{gf} = -\frac{1}{2} \sum_{x=1}^{N-1} Z_g(x, x+1) \{ \psi^\dagger(x) \gamma^0 (1 - \gamma^1) \psi(x+1) + \psi^\dagger(x+1) \gamma^0 (1 + \gamma^1) \psi(x) \}, \quad (4)$$

where K and m are the gauge parameter and fermion mass, respectively. We used the lattice unit and eliminated the lattice spacing. Taking $\gamma^0 = \sigma_1$, $\gamma^1 = \sigma_3$, and performing the Jordan-Wigner transformation with open boundary conditions, we can rewrite the Hamiltonian terms as

$$H_f = \frac{1}{2} \sum_{x=1}^N (1+m) \{ X_1(x) X_2(x) + Y_1(x) Y_2(x) \} \quad (5)$$

and

$$H_{gf} = -\frac{1}{2} \sum_{x=1}^{N-1} Z_g(x, x+1) \{ X_1(x) X_2(x+1) + Y_1(x) Y_2(x+1) \}. \quad (6)$$

The theory can be mapped to $(3N-1)$ -qubit systems with linear connectivity [25]. [If we employ the staggered fermion, we need $4N-1$ qubits to simulate the same physical volume because of the gauge interaction between $\psi_1(x)$ and $\psi_2(x)$.]

The theory has local $\mathbf{Z}_2$ gauge symmetry. We considered the theory in the presence of external static charges, such as infinitely heavy electrons. The Gauss law operator is defined by

$$G(x) = X_g(x-1, x) e^{-i\pi(\rho(x) + \rho_s(x))} X_g(x, x+1) \\ = -X_g(x-1, x) Z_1(x) Z_2(x) e^{-i\pi\rho_s(x)} X_g(x, x+1), \quad (7)$$

with the dynamical charge density

$$\rho(x) = \psi_1^\dagger(x) \psi_1(x) + \psi_2^\dagger(x) \psi_2(x) - 1 \\ = \frac{1}{2} \{ Z_1(x) + Z_2(x) \}, \quad (8)$$

and the external charge density $\rho_s(x)$. We take

$$\rho_s = \begin{cases} 1, & x = 1, N, \\ 0, & \text{otherwise,} \end{cases} \quad (9)$$

i.e., we put charges at the boundaries of the system as external probes. The physical state satisfies the Gauss law constraint

$$G(x)|\Psi\rangle = |\Psi\rangle \quad (10)$$

for all x . Here, the operators outside the boundaries are understood as $X_g(-1, 0) = X_g(N, N+1) = 1$.

B. Floquet circuit

We simulated the Floquet circuit obtained from the first-order Lie-Suzuki-Trotter decomposition of Hamiltonian evolution. The Floquet evolution operator over a single period reads

$$U_F = e^{-iH_{gf}dt} e^{-i(H_f + H_g)dt}, \quad (11)$$

where dt is the step size. The time evolution with U_F can be understood as a periodically driven quantum system described by the time-dependent Hamiltonian

$$H(t) = \begin{cases} H_f + H_g, & t \in [0, T/2), \\ H_{gf}, & t \in [T/2, T) \end{cases} \quad (12)$$

with the driving period $T = 2dt$. The evolution of the state vector over N_t Trotter steps is given by

$$|\Psi(N_t)\rangle = (U_F)^{N_t} |\Psi(0)\rangle. \quad (13)$$

We chose the ground state of $H_f + H_g$ in the presence of external charges (9) as the initial state $|\Psi(0)\rangle$, which was easily prepared on circuits since $[H_f, H_g] = 0$. The fermion part of $|\Psi(0)\rangle$ is given by a product of Bell pairs, while the gauge field part of $|\Psi(0)\rangle$ is given by a product of eigenstates of operators $X_g(x, x+1)$. The evolution conserves the Gauss law constraint because $[U_F, G(x)] = 0$. To study thermalization in the Floquet circuit, we computed the expectation value of the local fermion Hamiltonian

$$\mathcal{H}(x_c) = \frac{1}{2}(1+m)\{X_1(x_c)X_2(x_c) + Y_1(x_c)Y_2(x_c)\} \quad (14)$$

and the Gauss law operator

$$G(x_c) = -X_g(x_c - 1, x_c)Z_1(x_c)Z_2(x_c)X_g(x_c, x_c + 1) \quad (15)$$

at the center $x_c = (N+1)/2$. We fixed $K = m = 1.0$ for the simulations, for which the Hamiltonian is nonintegrable.

III. CLASSICAL MPS SIMULATION

Before discussing the quantum simulation results using IBM's superconducting quantum devices, let us first show the classical simulation results based on MPSs. MPSs are tensor networks that are particularly useful for describing a one-dimensional gapped quantum system, including our Z_2 lattice gauge theory. The purpose of the classical simulations is to estimate the system size and step size dependence of Floquet circuits. We used iTensor [26] for the classical MPS simulations. For another application of iTensor to lattice gauge theories, see, e.g., Ref. [27].

First, we fixed the system size with $N = 13$ (as seen below, $N = 13$ is large enough to discuss the large volume limit). We show the dependence on the Trotter step size dt in Fig. 1. As a representative of small dt , which is necessary for Hamiltonian evolution, we show the Trotter evolution with $dt = 0.5$. Thermalization was reached after many Trotter steps (>25 –30 steps). It is too hard of a task for the present noisy quantum devices. To find more rapid thermalization that could be simulated even in the present noisy devices, we made the step size very large. We saw “prethermalization,” i.e., plateau behavior before true thermalization to infinite temperature states, without oscillation for $dt = 1.0$, and with oscillation for $dt = 1.4$. There were prethermalization plateaus during 8–16 Trotter steps for $dt = 1.0$ and 8–25 Trotter steps for $dt = 1.4$. Those Floquet circuits may reach the infinite temperature state after the prethermalization plateau, as we can see in the case of $dt = 1.0$ (see Fig. 1), but this needs many more Trotter steps. We employed $dt = 1.0$ and $dt = 1.4$ and focused on

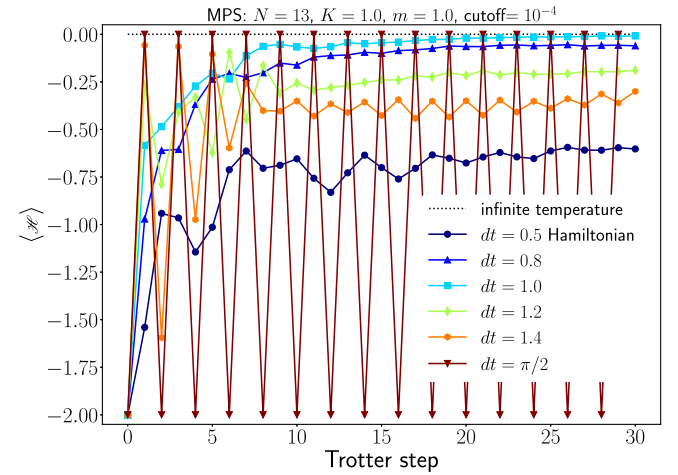


FIG. 1. Classical simulation results of Trotter time step dependence of Floquet prethermalization. The system size is $N = 13$, the gauge parameter is $K = 1.0$, and the fermion mass is $m = 1.0$. The cutoff for tensor contraction is 10^{-4} , which is small enough for the convergence of the results. Lines are just for eye guides. $dt = 0.5$ is close to Hamiltonian dynamics and the other dt are Floquet dynamics, respectively. Rapid prethermalization is observed in $dt = 1.0$ and 1.4 .

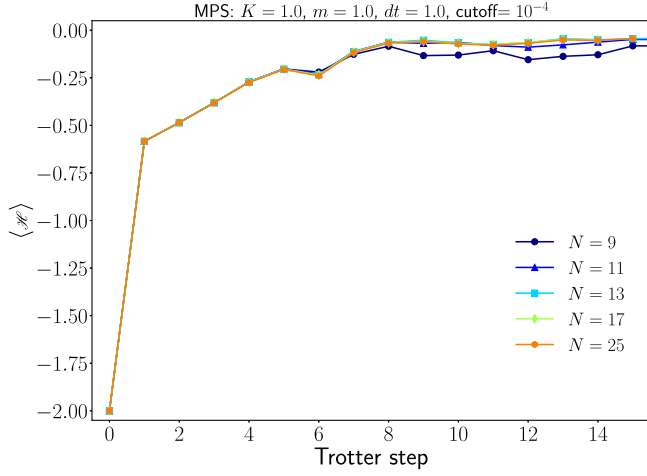


FIG. 2. Classical simulation results of the system size dependence of Floquet prethermalization. The step size, gauge parameter, and fermion mass are $dt = 1.0$, $K = 1.0$, and $m = 1.0$, respectively. The cutoff for tensor contraction is 10^{-4} , which is small enough for the convergence of results. Lines are just for eye guides.

the prethermal plateau regimes in the following quantum simulation with real devices.

We show the system size dependence in Figs. 2 and 3. In both cases, $N = 13$ is large enough to simulate the prethermalization dynamics, namely, Floquet circuits with up to 15 or 20 Trotter steps. Importantly, this is much smaller than the number of qubits inside causal cones at 15 or 20 Trotter steps in a large system. We note that the state at late times with $dt = 1.0$ is closer to the infinite temperature state than $dt = 1.4$, and it is computationally demanding in MPS calculations. This is the reason why we simulated only up to 15 Trotter steps for $dt = 1.0$ with large system sizes. We note also that true thermalization to infinite temperature states showed more severe system size

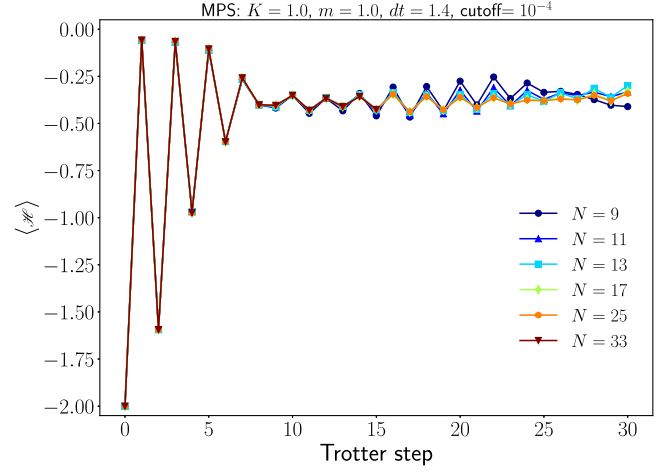


FIG. 3. Classical simulation results of the system size dependence of Floquet prethermalization. The step size, gauge parameter, and fermion mass are $dt = 1.4$, $K = 1.0$, and $m = 1.0$, respectively. The cutoff for tensor contraction is 10^{-4} , which is small enough for the convergence of results. Lines are just for eye guides.

dependence, as seen in Fig. 3, although it may not be distinguishable in noisy quantum simulations.

IV. QUANTUM SIMULATION

A. Qiskit compiling details

We performed quantum simulation using IBM's latest superconducting quantum device, named `ibm_fez`. The device consists of 156 qubits with heavy-hex connectivity. Mitigation of errors is inevitable for the present noisy quantum devices. We tested three mitigation methods using $N = 13$ (38 qubits), and then applied the best one for simulating a larger system, $N = 39$ (116 qubits), to see a potential power of the present quantum devices.

As for gate counting of the Floquet circuit, since $e^{-i\frac{\theta}{2}(XX+YY)}$ uses two CZ gates [28,29],

$$e^{-i\frac{\theta}{2}(XX+YY)} = \begin{array}{c} |\psi_1(x)\rangle \text{---} R_X(\frac{\pi}{2}) \text{---} \bullet \text{---} R_X(\theta) \text{---} \bullet \text{---} R_X(-\frac{\pi}{2}) \text{---} \\ |\psi_2(x)\rangle \text{---} R_X(\frac{\pi}{2}) \text{---} \oplus \text{---} R_Z(\theta) \text{---} \oplus \text{---} R_X(-\frac{\pi}{2}) \text{---} \end{array}, \quad (16)$$

$e^{-i(H_f+H_g)dt}$ uses $2N$ CZ gates for each Trotter step. We followed Ref. [22] for the circuit implementation of $e^{-iH_{gf}dt}$. We need six CZ gates to implement $e^{i\theta(XZX+YZY)}$,

$$e^{i\theta(XZX+YZY)} = \begin{array}{c} |\psi_2(x)\rangle \text{---} \oplus \text{---} H \text{---} \oplus \text{---} H \text{---} S \text{---} \oplus \text{---} S^\dagger \text{---} \\ |g(x, x+1)\rangle \text{---} H \text{---} \bullet \text{---} R_X(\theta) \text{---} \bullet \text{---} Z \text{---} R_X(\theta) \text{---} \bullet \text{---} H \text{---} \\ |\psi_1(x+1)\rangle \text{---} \oplus \text{---} H \text{---} \oplus \text{---} H \text{---} S \text{---} \oplus \text{---} S^\dagger \text{---} \end{array}, \quad (17)$$

so that $e^{-iH_{gf}dt}$ uses $6(N-1)$ CZ gates for each Trotter step. Furthermore, the initial state is prepared on the circuit as

$$|\Psi(0)\rangle = \begin{aligned} &|0_1(1)\rangle \text{---} [H] \text{---} [Z] \text{---} \bullet \\ &|0_2(1)\rangle \text{---} [X] \text{---} \oplus \\ &|0_g(1,2)\rangle \text{---} [H] \text{---} [Z] \text{---} \text{---} \\ &\vdots \\ &|0_g(N-1,N)\rangle \text{---} [H] \text{---} [Z] \text{---} \text{---} \\ &|0_1(N)\rangle \text{---} [H] \text{---} [Z] \text{---} \bullet \\ &|0_2(N)\rangle \text{---} [X] \text{---} \oplus \end{aligned} \quad (18)$$

In short, the total number of CZ gates used for N_t Trotter steps of the N -site system is thus $(8N-6)N_t + N$.

We used Qiskit's estimator to compute the expectation values of $\mathcal{H}(x_c)$ (14) and $G(x_c)$ (15). The total number of shots used to compute each expectation value was 100,000. We enabled readout error mitigation based on the Pauli twirling, named as twirled readout error eXtinction (TREX), in all the experimental data. The dynamical decoupling was disabled in all the simulations. We enabled Pauli twirling of two-qubit gates. The number of twirled circuits (num_randomizations in Qiskit's twirling options) was set to 16 in the simulations for $N = 13$ with ZNE, 64 in the simulations for $N = 13$ with other mitigation, and 32 in the simulations for $N = 39$. In the ZNE mitigation, circuits equivalent to the original one but with amplified noise rates were executed, and the associated results were extrapolated to the zero-noise limit. Noise amplification was performed by the two-qubit gate folding scheme, which replaces each two-qubit gate with a sequence of the gate and its inverse, whose length is specified by a parameter $\mathcal{N}$, called the noise factor. Specifically, each native two-qubit gate U is replaced with $U(U^\dagger U)^{\frac{\mathcal{N}-1}{2}}$, and $\mathcal{N} = 1$ corresponds to the original circuit without noise amplification. We used noise factors of $\mathcal{N} = 1, 3$, and 5 , and extrapolated the results to the limit of $\mathcal{N} = 0$ with the exponential fitting. Changing num_randomizations to 16, 32, 64, and 96 for $N = 13$, we found that 32 or 64 was optimal for our circuit. However, we could not use the optimal value with ZNE due to the limitations of the computational resources.

In addition, it may be optional since the growth of the effective causal cone is much slower than that of the causal cone, but we removed unnecessary gates outside the causal

cone using pytket [30]. We first wrote the Qiskit code of a circuit and translated it to tket. Then, we removed unnecessary gates using tket's compilation pass RemoveDiscarded, and we retranslated the reduced circuit to Qiskit. Finally, we executed the built Qiskit code on the real device.

B. Zero-noise extrapolation

The simulation results of ZNE are shown in Fig. 4 (see also the raw data in Figs. 5 and 6). The ZNE results

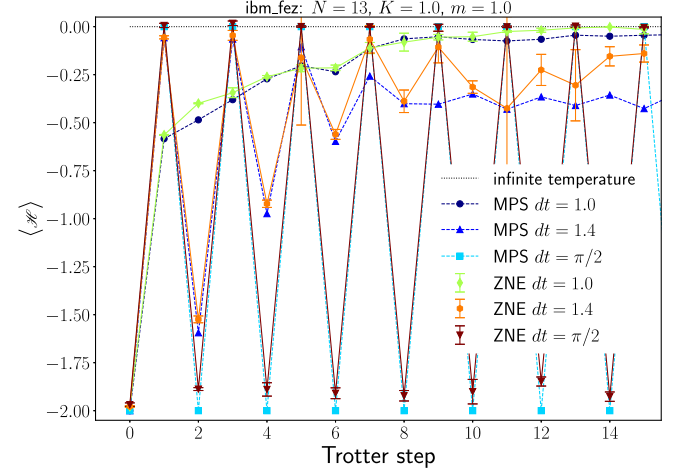


FIG. 4. Quantum simulation results of Trotter time step dependence of Floquet prethermalization using `ibm_fez`. The system size is $N = 13$ (38 qubits on the real device). The gauge parameter is $K = 1.0$, and the fermion mass is $m = 1.0$. “MPS” shows the results of the classical MPS calculations, and “ZNE” shows the mitigated results by ZNE. Lines are just for eye guides.

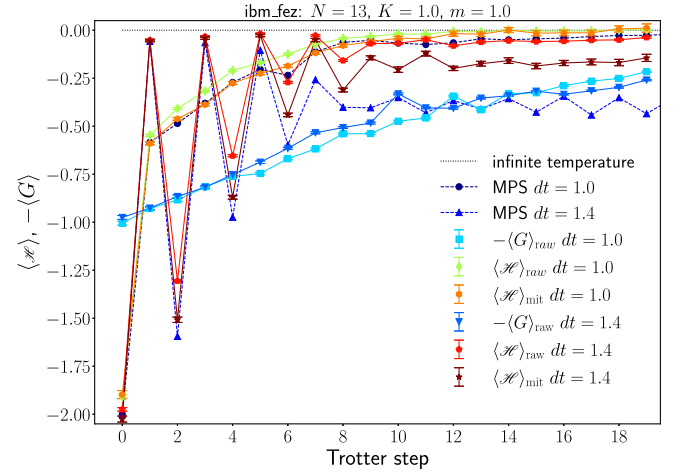


FIG. 5. Quantum simulation results of Trotter time step dependence of Floquet prethermalization using `ibm_fez`. The system size is $N = 13$ (38 qubits on the real device). The gauge parameter is $K = 1.0$, and the fermion mass is $m = 1.0$. “MPS” shows the results of the classical MPS calculations. $\langle G \rangle_{\text{raw}}$ is the measured value of the Gauss law operator, $\langle \mathcal{H} \rangle_{\text{raw}}$ is the measured value of the local Hamiltonian, and $\langle \mathcal{H} \rangle_{\text{mit}}$ is the mitigated result by a division by $\langle G \rangle_{\text{raw}}$. Lines are just for eye guides.

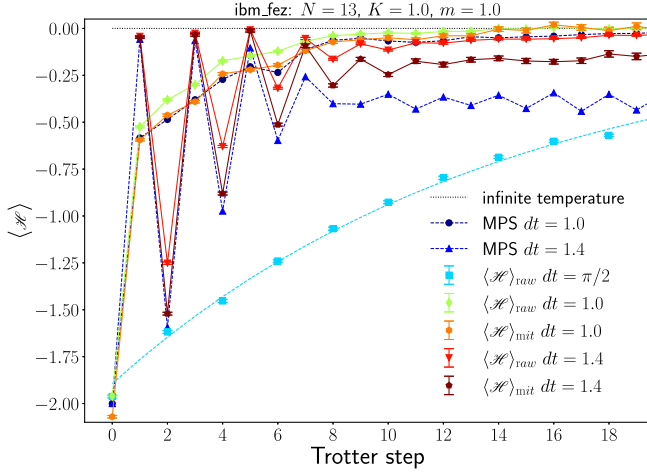


FIG. 6. Quantum simulation results of Trotter time step dependence of Floquet prethermalization using `ibm_fez`. The system size is $N = 13$ (38 qubits on the real device). The gauge parameter is $K = 1.0$, and the fermion mass is $m = 1.0$. “MPS” shows the results of the classical MPS calculations. $\langle \mathcal{H} \rangle_{\text{raw}}$ is the measured value and $\langle \mathcal{H} \rangle_{\text{mit}}$ is the mitigated result by a division by the fitting result of $dt = \pi/2$. Lines are just for eye guides, while the curve is the result of exponential fitting of $\pi/2$ that is used in error mitigation of $dt = 1.0$ and 1.4 .

quantitatively agreed with the MPS results in small Trotter steps. In particular, quantum simulations can nicely reproduce the MPS results up to 10 Trotter steps for $dt = 1.0$. This demonstrates that the current quantum device has the power to simulate physics-motivated problems, that is, thermalization dynamics and the very early stage of the prethermal plateau. On the other hand, the results of $dt = 1.4$ were not as good as those of $dt = 1.0$, particularly in 7–10 Trotter steps. The origin of this difference is discussed in detail in Sec. IV E.

ZNE seems to work remarkably well. Nevertheless, we cannot say that ZNE is the best one. The computational costs of ZNE are very large. In fact, we could simulate only up to 15 Trotter steps if we enabled both the Pauli twirling and ZNE. In the following subsections, we employed two other methods that are less computationally demanding and may be useful for simulating systems with a large number of qubits.

C. Mitigation by division: The Gauss law operator

To correct amplitude decay due to decoherence, we need to estimate a rescaling factor. Lattice gauge theories have a very nice property useful for estimating the factor. The Gauss law constraint is an exact relationship in gauge theories, and its expectation value is protected by local gauge symmetry. The mitigation by it is universally applicable to any gauge theory.

If the initial state satisfies the Gauss law constraint $\langle G(x) \rangle = 1$, it must not be violated during time evolution, but it is violated in noisy simulation. As shown in Fig. 5,

the expectation value of the Gauss law operator decreased as the Trotter step increased. We identified the expectation value of the Gauss law operator as a damping factor and performed error mitigation as follows:

$$\langle \mathcal{H}(x_c) \rangle_{\text{mit}} = \frac{\langle \mathcal{H}(x_c) \rangle_{\text{raw}}}{\langle G(x_c) \rangle_{\text{raw}}}. \quad (19)$$

Here, the data obtained by enabling only TREX and Pauli twirling of two-qubit gates are referred to as raw data. The raw data and the mitigated results are shown in Fig. 5. The error mitigation works and quantum simulations agreed with the MPS results for small Trotter steps. However, we can see that mitigation by the Gauss law cannot capture the dip at 6 Trotter steps for $dt = 1.0$ that ZNE can. Again, although quantum simulations quantitatively agreed with the MPS results up to 10 Trotter steps for $dt = 1.0$, the results of $dt = 1.4$ were not as good as those of $dt = 1.0$ in 6–10 Trotter steps.

While the local Hamiltonian $\mathcal{H}(x_c)$ acts on two qubits (two fermions), the Gauss law operator $G(x_c)$ acts on four qubits (two fermions and two gauge links). We expected that the difference in operator sizes may not have an effect after many gates were applied, but the results could not reproduce the amplitudes in the prethermal plateau. This might imply that it is important to make operator sizes the same in the numerator and denominator of Eq. (19).

D. Mitigation by division: $dt = \frac{\pi}{2}$ circuit

To estimate a rescaling factor using the same operator that we will measure, we employed the error mitigation used in Ref. [3]. The error mitigation is based on a global depolarizing noise model: at each Trotter step, we assume that

$$\rho_{\text{noisy}} = f\rho_{\text{ideal}} + (1-f)\frac{I}{2^{3N-1}}, \quad (20)$$

where ρ_{noisy} is the noisy quantum state, ρ_{ideal} is the quantum state that could be obtained through noiseless time evolution, $I/2^{3N-1}$ is the maximally mixed state, and f is the depolarization parameter to be estimated. Since $\mathcal{H}$ is traceless, noisy and ideal expectation values of $\mathcal{H}$ are related as $\langle \mathcal{H} \rangle_{\text{raw}} = f\langle \mathcal{H} \rangle_{\text{ideal}}$. We estimated f by simulating the same circuit for which $\mathcal{H}$ was to be measured with special parameters, such that the circuit is exactly solvable if there is no error. In our case, if we set $dt = \frac{\pi}{2}$, the expectation value of the local Hamiltonian takes $-2 \rightarrow 0 \rightarrow -2 \rightarrow 0 \dots$. Thus, at even Trotter steps, the depolarization parameter can be estimated as $f = -\frac{1}{2}\langle \mathcal{H}_{\frac{\pi}{2}}(x_c) \rangle_{\text{raw}}$ by assuming that f is independent of parameters of the circuit (dt in our case). In Fig. 6, the actual data exponentially decreased as the Trotter step N_t increased. Using even N_t , we fit the results of the noisy simulations by assuming a form of

$$\langle \mathcal{H}_{\frac{\pi}{2}}(x_c) \rangle_{\text{raw}} = -Ae^{-\lambda N_t}, \quad (21)$$

where A and λ are the fitting parameters. Using the fitting results, we performed error mitigation of $dt \neq \frac{\pi}{2}$ for all the Trotter steps as

$$\langle \mathcal{H}(x_c) \rangle_{\text{mit}} = \frac{\langle \mathcal{H}(x_c) \rangle_{\text{raw}}}{f} = \frac{2}{A} e^{\lambda N_t} \langle \mathcal{H}(x_c) \rangle_{\text{raw}}. \quad (22)$$

Here, the data obtained by enabling only TREX and Pauli twirling of two-qubit gates are referred to as raw data. The results of error mitigation are shown with the raw data in Fig. 6. In fact, the results of error mitigation were very similar to those based on the Gauss law operator. Although the operator size is the same in this error mitigation method, the effective circuit volume may largely differ in thermalized and trivial circuits. This may be the reason why we could not reproduce the prethermal plateau in this error mitigation method. We may need a solvable circuit, such that the growth of operator sizes is the same as the circuit that we would like to simulate, but it is, in general, hard to prepare such a circuit. This strongly restricts the number of Trotter steps that we can simulate correctly, and we can rely on ZNE if the available Trotter step is short.

Finally, we performed a large-scale simulation that used more than 100 qubits. The system size was $N = 39$, for which we used 116 of the 156 qubits of `ibm_fez`. We show the results of the quantum simulations in Fig. 7. We used the error mitigation based on division. This may not be surprising since $N = 13$ is large enough to compute the observable in a large system, as seen in MPS simulations,

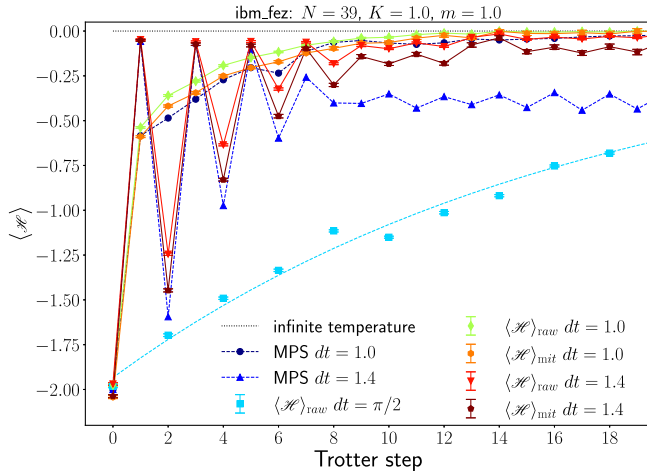


FIG. 7. Quantum simulation results of Trotter time step dependence of Floquet prethermalization using `ibm_fez`. The system size is $N = 39$ (116 qubits on the real device). The gauge parameter is $K = 1.0$, and the fermion mass is $m = 1.0$. “MPS” shows the results of the classical MPS calculations. $\langle \mathcal{H} \rangle_{\text{raw}}$ is the measured value and $\langle \mathcal{H} \rangle_{\text{mit}}$ is the mitigated result by a division by the fitting result of $dt = \pi/2$. Lines are just for eye guides, while the curve is the result of exponential fitting of $\pi/2$ that is used in error mitigation of $dt = 1.0$ and 1.4 .

but we can see that the simulations that use more than 100 qubits even quantitatively agree with the MPS results. This demonstrates the capability of IBM’s superconducting quantum devices for studying quantum many-body dynamics in the era of the quantum utility scale.

E. Discussion

Assuming the global depolarizing channel, the experimental results before error mitigation $\langle \hat{O} \rangle_{\text{raw}}$ are related to ideal results without noises $\langle \hat{O} \rangle_{\text{ideal}}$ as [31]

$$\langle \hat{O} \rangle_{\text{raw}} = F_{\text{eff}} \langle \hat{O} \rangle_{\text{ideal}} + \frac{1 - F_{\text{eff}}}{2^{3N-1}} \text{Tr} \hat{O}, \quad (23)$$

with $F_{\text{eff}} = (1 - p)^{V_{\text{eff}}}$, where F_{eff} , V_{eff} , and p are the effective fidelity, circuit volume, and typical value of gate errors, respectively. Here, we estimated F_{eff} and V_{eff} to deepen our understanding of the quantum simulation results. We used the raw data $\langle \mathcal{H}(x_c) \rangle_{\text{raw}}$ and the MPS results $\langle \mathcal{H}(x_c) \rangle_{\text{ideal}}$. We set $p = 2.75 \times 10^{-3}$, which is the median CZ error of `ibm_fez` in executing the circuits. F_{eff} is related to the experimentally achievable signal-to-noise ratio of the observable. If we set $F_{\text{eff}} = 0.1$ as a typical value for the fidelity threshold, this is equal to $V_{\text{eff}} = 836$ in terms of the CZ gates.

We show the estimated values of the effective fidelity F_{eff} and circuit volume V_{eff} in Fig. 8. For comparison, we show the results of $N = 13$ and $N = 39$. In both system sizes, F_{eff} of $dt = 1.0$ was close to that of $dt = \frac{\pi}{2}$ up to 10 Trotter steps. This fact supports our observation that the mitigation by the $dt = \frac{\pi}{2}$ circuit worked well for $dt = 1.0$. On the other hand, F_{eff} of $dt = 1.4$ decayed more rapidly, particularly in the 7–10 Trotter steps. The high effective fidelity of $dt = 1.0$ (that is, the smallness of the effective causal cone seen in V_{eff}) is considered to be the reason why we could simulate larger Trotter steps for $dt = 1.0$ using ZNE.

The effective circuit volume V_{eff} is interpreted as the effective number of entangling gates contributing to expectation values. The estimated values (except for the peak at 5 Trotter steps for $dt = 1.4$) were much below the threshold value $V_{\text{eff}} = 836$ up to 10 Trotter steps. This implies that the simulation output contained reliable signals in this range. V_{eff} of $dt = 1.0$ increased linearly with up to 10 Trotter steps. V_{eff} of $dt = 1.4$ exhibited similar linearity in a broader range, although it had a more complex structure, as expected from the oscillatory behavior of the observable. Physically, such a time interval corresponds to thermalization dynamics until the onset of a prethermal plateau. Importantly for us, V_{eff} was much smaller than the number of two-qubit gates inside the causal cones. This implies that, at least in our system of the Z_2 lattice gauge theory, thermalization dynamics can be simulated more easily than expected from the two-qubit gate counting. (Unfortunately, this may imply that it can be simulated easily in classical simulations as well.)

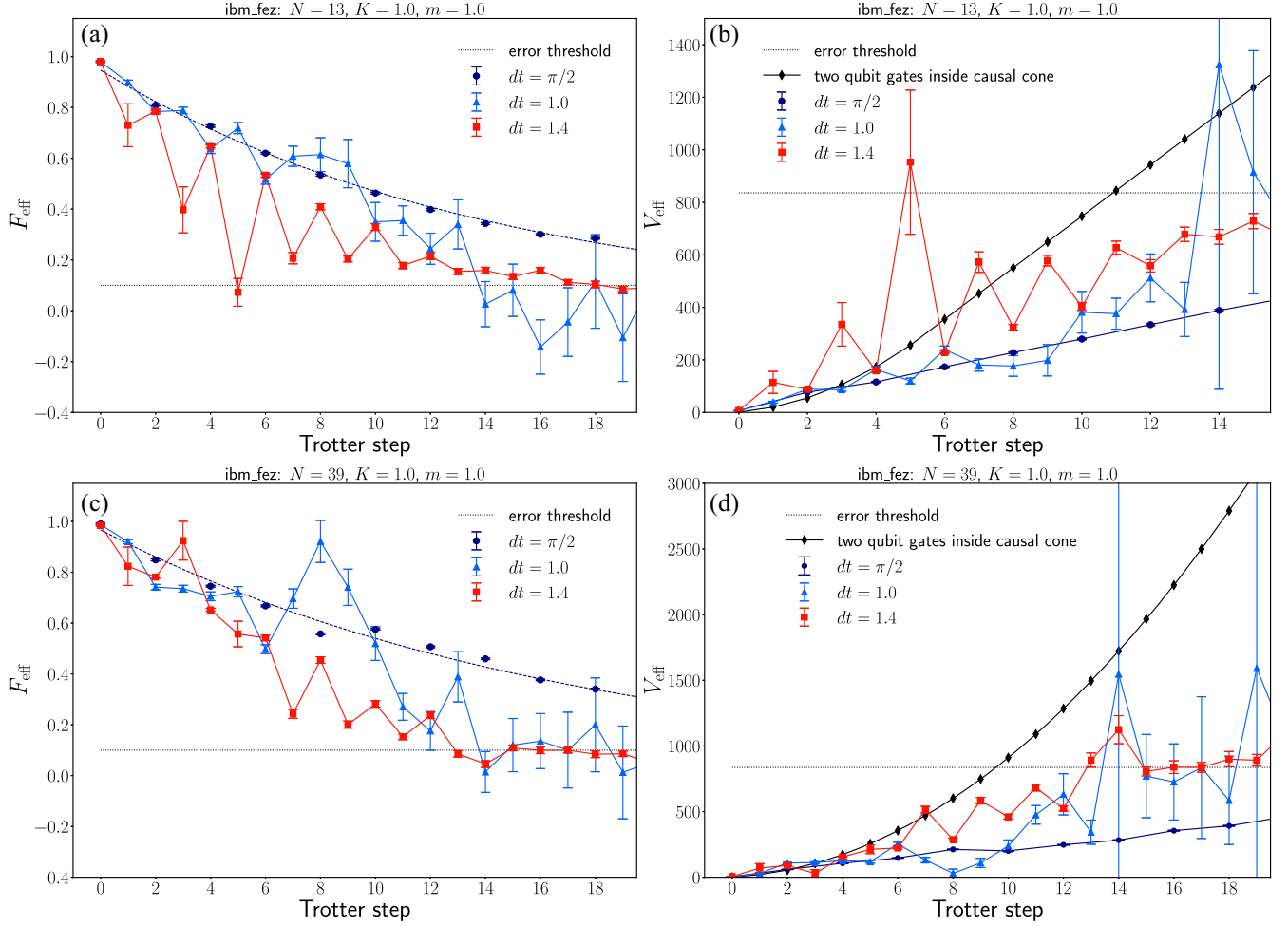


FIG. 8. Effective fidelity F_{eff} of quantum circuits simulated using `ibm_fez` for $N = 13$ (a) and for $N = 39$ (c), and corresponding circuit volume V_{eff} for $N = 13$ (b) and for $N = 39$ (d). The gauge parameter is $K = 1.0$, and the fermion mass is $m = 1.0$. We estimated F_{eff} by dividing $\langle \mathcal{H} \rangle_{\text{raw}}$ by the corresponding “MPS” results in Fig. 6. Lines are just for eye guides, while the curve is the result of exponential fitting of $\pi/2$. V_{eff} is related to F_{eff} as $F_{\text{eff}} = (1 - p)^{V_{\text{eff}}}$, and we used $p = 2.75 \times 10^{-3}$, which is the median CZ error of `ibm_fez`, when we executed the circuits. A typical value of the “error threshold” $F_{\text{eff}} = 0.1$ in (a) and (c) corresponds to $V_{\text{eff}} = 836$ in (b) and (d). We show only up to 15 Trotter steps in (b), since the fidelity in (a) becomes negative due to large noises above 15 Trotter steps. For comparison, we show the number of two-qubit gates inside causal cones in (b) and (d).

V. SUMMARY AND OUTLOOK

We have simulated the Floquet time evolution of the one-dimensional $\mathbf{Z}_2$ lattice gauge theory on a superconducting quantum computer. Although current quantum computers have a strong limitation on the circuit depth, it can be used for simulating short-time physics, such as the emergence of the Floquet prethermal plateau, by implementing error mitigation. We have tested three methods for the error mitigation. The ZNE was the most reliable but computationally demanding among the three. The mitigation methods of amplitude damping are more economical because the damping rates can be estimated by the circuits with the same depth as the original circuit. The estimates by the Gauss law operator and the solvable circuit were almost equally good, but the latter seemed optimal in our case.

This work demonstrated that current quantum computing has potential for application to high-energy physics problems. We should challenge quantum simulations of more general cases, for example, two- or three-dimensional lattice gauge theories [32–38]. Simulating Floquet prethermalization in higher-dimensional systems would be one of the promising directions to show the quantum advantage. Also, once thermalized states are made by the simulation, it would be interesting to use them to study the dynamical properties of finite-temperature systems. We will leave the detailed studies of those problems for the future.

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- [1] T. Eckstein, R. Mansuroglu, P. Czarnik, J.-X. Zhu, M. J. Hartmann, L. Cincio, A. T. Sornborger, and Z. Holmes, *npj Quantum Inf.* **10**, 84 (2024).
 - [2] Y. Yang, A. Christianen, S. Coll-Vinent, V. Smelyanskiy, M. C. Bañuls, T. E. O'Brien, D. S. Wild, and J. I. Cirac, *PRX Quantum* **4**, 030320 (2023).
 - [3] K. Shinjo, K. Seki, T. Shirakawa, R.-Y. Sun, and S. Yunoki, *arXiv:2403.16718*.
 - [4] K. Seki, Y. Kikuchi, T. Hayata, and S. Yunoki, *arXiv:2405.07613*.
 - [5] A. Lazarides, A. Das, and R. Moessner, *Phys. Rev. E* **90**, 012110 (2014).
 - [6] L. D'Alessio and M. Rigol, *Phys. Rev. X* **4**, 041048 (2014).
 - [7] D. A. Abanin, W. De Roeck, and F. m. c. Huveneers, *Phys. Rev. Lett.* **115**, 256803 (2015).
 - [8] T. Mori, T. Kuwahara, and K. Saito, *Phys. Rev. Lett.* **116**, 120401 (2016).
 - [9] T. Kuwahara, T. Mori, and K. Saito, *Ann. Phys. (Amsterdam)* **367**, 96 (2016).
 - [10] T. Mori, T. N. Ikeda, E. Kaminishi, and M. Ueda, *J. Phys. B* **51**, 112001 (2018).
 - [11] M. Heyl, P. Hauke, and P. Zoller, *Sci. Adv.* **5**, eaau8342 (2019).
 - [12] N. Klcio, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage, *Phys. Rev. A* **98**, 032331 (2018).
 - [13] W. A. de Jong, K. Lee, J. Mulligan, M. Płoskoń, F. Ringer, and X. Yao, *Phys. Rev. D* **106**, 054508 (2022).
 - [14] N. H. Nguyen, M. C. Tran, Y. Zhu, A. M. Green, C. H. Alderete, Z. Davoudi, and N. M. Linke, *PRX Quantum* **3**, 020324 (2022).
 - [15] Y. Y. Atas, J. F. Haase, J. Zhang, V. Wei, S. M. L. Pfaendler, R. Lewis, and C. A. Muschik, *Phys. Rev. Res.* **5**, 033184 (2023).
 - [16] S. Schuster, S. Kühn, L. Funcke, T. Hartung, M.-O. Pleinert, J. von Zanthier, and K. Jansen, *Phys. Rev. D* **109**, 114508 (2024).
 - [17] T. Angelides, P. Naredi, A. Crippa, K. Jansen, S. Kühn, I. Tavernelli, and D. S. Wang, *arXiv:2312.12831*.
 - [18] R. C. Farrell, M. Illa, A. N. Ciavarella, and M. J. Savage, *PRX Quantum* **5**, 020315 (2024).
 - [19] R. C. Farrell, M. Illa, A. N. Ciavarella, and M. J. Savage, *Phys. Rev. D* **109**, 114510 (2024).
 - [20] J. Mildenberger, W. Mruczkiewicz, J. C. Halimeh, Z. Jiang, and P. Hauke, *arXiv:2203.08905*.
 - [21] D. Pomarico, L. Cosmai, P. Facchi, C. Lupo, S. Pascazio, and F. V. Pepe, *Entropy* **25**, 608 (2023).
 - [22] C. Charles, E. J. Gustafson, E. Hardt, F. Herren, N. Hogan, H. Lamm, S. Starecheski, R. S. Van de Water, and M. L. Wagman, *Phys. Rev. E* **109**, 015307 (2024).
 - [23] T. Hayata, K. Nakayama, and A. Yamamoto, *Phys. Rev. D* **109**, 034501 (2024).
 - [24] A. Javadi-Abhari, M. Treinish, K. Krsulich, C. J. Wood, J. Lishman, J. Gacon, S. Martiel, P. D. Nation, L. S. Bishop, A. W. Cross, B. R. Johnson, and J. M. Gambetta, *arXiv:2405.08810*.
 - [25] T. V. Zache, F. Hebenstreit, F. Jendrzejewski, M. K. Oberthaler, J. Berges, and P. Hauke, *Quantum Sci. Technol.* **3**, 034010 (2018).
 - [26] M. Fishman, S. R. White, and E. M. Stoudenmire, *SciPost Phys. Codebases* **4** (2022).10.21468/SciPostPhysCodeb.4
 - [27] T. Hayata, Y. Hidaka, and K. Nishimura, *J. High Energy Phys.* **07** (2024) 106.
 - [28] G. Vidal and C. M. Dawson, *Phys. Rev. A* **69**, 010301 (2004).
 - [29] M. Foss-Feig, D. Hayes, J. M. Dreiling, C. Figgatt, J. P. Gaebler, S. A. Moses, J. M. Pino, and A. C. Potter, *Phys. Rev. Res.* **3**, 033002 (2021).
 - [30] S. Sivarajah, S. Dilkes, A. Cowtan, W. Simmons, A. Edgington, and R. Duncan, *Quantum Sci. Technol.* **6**, 014003 (2020).
 - [31] K. Kechedzhi, S. V. Isakov, S. Mandrà, B. Villalonga, X. Mi, S. Boixo, and V. Smelyanskiy, *Future Gener. Comput. Syst.* **153**, 431 (2024).
 - [32] A. Yamamoto, *Prog. Theor. Exp. Phys.* **2021**, 013B06 (2021).
 - [33] E. J. Gustafson and H. Lamm, *Phys. Rev. D* **103**, 054507 (2021).
 - [34] E. Zohar, *Phil. Trans. R. Soc. A* **380**, 20210069 (2021).
 - [35] L. Lumia, P. Torta, G. B. Mbeng, G. E. Santoro, E. Ercolessi, M. Burrello, and M. M. Wauters, *PRX Quantum* **3**, 020320 (2022).
 - [36] T. V. Zache, D. González-Cuadra, and P. Zoller, *Phys. Rev. Lett.* **131**, 171902 (2023).
 - [37] T. Hayata and Y. Hidaka, *J. High Energy Phys.* **09** (2023) 126.
 - [38] T. Hayata and Y. Hidaka, *J. High Energy Phys.* **09** (2023) 123.