

Roles of the scalar $f_0(500)$ and $f_0(980)$ in the process $D^0 \rightarrow \pi^0 \pi^0 \bar{K}^0$

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Motivated by the near-threshold enhancement and the dip structure around 1 GeV in the $\pi^0 \pi^0$ invariant mass distribution of the process $D^0 \rightarrow \pi^0 \pi^0 \bar{K}^0$ observed by the CLEO Collaboration, we have investigated this process by taking into account the contribution from the S -wave pseudoscalar meson–pseudoscalar meson interactions within the chiral unitary approach, and also the one from the intermediate resonance $\bar{K}^*(892)$. Our results are in good agreement with the CLEO measurements, which implies that the near-threshold enhancement near the $\pi^0 \pi^0$ threshold is mainly due to the contributions from the scalar meson $f_0(500)$ and the intermediate $\bar{K}^*$, and the cusp structure around 1 GeV in the $\pi^0 \pi^0$ invariant mass distribution should be associated with the scalar meson $f_0(980)$.

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I. INTRODUCTION

In the classical quark model, hadrons are categorized into mesons and baryons, where mesons are composed of a quark-antiquark pair, and baryons are composed of three quarks [1,2]. Although most of the mesons could be well described within the classical quark models, some mesons have exotic properties that are difficult to be explained. For instance, the light scalar mesons $a_0(980)$, $f_0(980)$, and $f_0(500)$ should have more complicated components and attract many theoretical studies [3]. However, there are still different explanations for their structure, such as a compact tetraquark, the molecular state, and the mixing of different components [4–12].

The hadronic decays of the charmed hadrons could provide valuable insights for understanding the short-distance weak interaction and the long-distance quantum chromodynamics (QCD) interaction [13–17]. Especially, the final state interactions are the key ingredients in the production of the light scalar mesons, i.e., $f_0(500)$, $f_0(980)$, and $a_0(980)$, which are of particular interest for exploring the nature of the light scalar mesons [18–24].

Recently, BESIII, BABAR, and Belle/Belle II have accumulated a lot of experimental data about the multibody decays of the charmed hadrons, which provide an important platform to investigate these light scalar mesons [25–31]. For instance,

the BESIII and BABAR Collaborations have performed the amplitude analysis of the process $D_s^+ \rightarrow K^+ K^- \pi^+$ and observed the near-threshold enhancement in the $K^+ K^-$ invariant mass distribution [32,33]. In Ref. [21], the authors have investigated this process by taking into account the S -wave pseudoscalar-pseudoscalar interactions within the chiral unitary approach, and concluded that scalar meson $f_0(980)$ gives the dominant contribution for the near-threshold enhancement in the $K^+ K^-$ invariant mass distribution. In addition, the authors of Ref. [34] have studied the decay $D_s^+ \rightarrow \pi^+ \pi^0 \pi^0$ and concluded that the cusp structure around 1 GeV in the $\pi^0 \pi^0$ invariant mass distribution could be associated with the scalar meson $f_0(980)$, which favors the $no-q\bar{q}$ nature of $f_0(980)$.

In 2011, the CLEO Collaboration has conducted a Dalitz plot analysis on $D^0 \rightarrow K_s^0 \pi^0 \pi^0$ using the dataset of 818 pb⁻¹ of $e^+ e^-$ collisions accumulated at $\sqrt{s} = 3.77$ GeV [35] and concluded that the combined $\pi^0 \pi^0$ S -wave contributes $(28.9 \pm 6.3 \pm 3.1)\%$ of the total decay rate, while the $D^0 \rightarrow \bar{K}^{*0} \pi^0$ contributes $(65.6 \pm 5.3 \pm 2.5)\%$. In addition, the decay branching fraction of the $D^0 \rightarrow \bar{K}^{*0} \pi^0 \pi^0$ was also measured to be $(1.058 \pm 0.038 \pm 0.073)\%$ [35]. In the $\pi^0 \pi^0$ invariant mass distribution, a cusp structure followed by a dip appears around 1 GeV, which may be associated with the scalar meson $f_0(980)$, and an enhancement was also found near the $\pi^0 \pi^0$ threshold. Meanwhile, one could find a clear peak of $\bar{K}^*(892)$ in the $\pi^0 \bar{K}$ invariant mass distribution. Because of the conservation of the charge parity, there is no contribution from the intermediate vector meson ρ in the process $D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0$, and thus this mode could provide a clearer way to investigate the S -wave $\pi\pi$ substructure than the process $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ [36,37].

In this work, we will investigate the process $D^0 \rightarrow \pi^0 \pi^0 \bar{K}^0$ by considering the scalar $f_0(980)$ generated

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from the S -wave pseudoscalar–pseudoscalar meson interaction within the chiral unitary approach. In addition, we also consider the contribution from the intermediate vector meson $\bar{K}^*(892)$.

This paper is organized as follows. In Sec. II, we will present the mechanism for the process $D^0 \rightarrow \pi^0 \pi^0 \bar{K}^0$, and our results and discussions will be shown in Sec. III, followed by a short summary in the last section.

II. FORMALISM

In Sec. II A, we first present the mechanism of the Cabibbo-favored process $D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0$ via the S -wave pseudoscalar meson–pseudoscalar meson interactions, which could dynamically generate the scalar mesons $f_0(500)$ and $f_0(980)$. Meanwhile, this process could also happen via the intermediate vector meson $\bar{K}^*(892)$, and the corresponding formalism and the double differential decay width are given in Sec. II B.

A. S -wave pseudoscalar–pseudoscalar interactions

Analogous to Refs. [38–40], the Cabibbo-favored process $D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0$ could be divided into three steps: the weak decay, the hadronization, and the final state interactions. For the first step, the c quark of the initial state D^0 meson weakly decays into an s quark and a W^+ boson, followed by the W^+ boson decaying into $\bar{d}u$ quarks. Next, as depicted in Fig. 1, the s quark from the c decay and the $\bar{d}$ quark from the W^+ decay hadronize into the $\bar{K}^0$ meson, while the u quark from the W^+ decay and the $\bar{u}$ of the initial D^0 , together with the antiquark–quark pair $q\bar{q}$ ($= \bar{u}u + \bar{d}d + \bar{s}s$) created from the vacuum with the quantum numbers $J^{PC} = 0^{++}$, hadronize into two mesons as follows:

$$u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{u} = \sum_{i=1}^3 M_{1i} M_{i1} = (M^2)_{11}, \quad (1)$$

where $i = 1, 2, 3$ represent u, d, s quarks. The M is the $q\bar{q}$ matrix

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}, \quad (2)$$

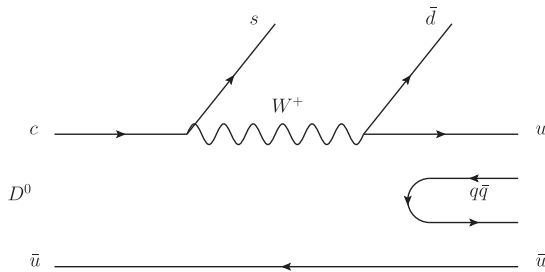


FIG. 1. Feynman diagram of $D^0 \rightarrow \bar{K}^0 u \bar{u}$ and hadronization of the $u\bar{u}$ through $q\bar{q}$ with vacuum quantum numbers.

which could also be expressed with the pseudoscalar meson basis as follows [41,42]:

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \end{pmatrix}, \quad (3)$$

where we adopt the approximate $\eta - \eta'$ mixing [43,44].¹ Then all the possible meson-meson components of Eq. (1) are

$$(M^2)_{11} = \frac{1}{2} \pi^0 \pi^0 + \frac{1}{3} \eta \eta + \frac{2}{\sqrt{6}} \pi^0 \eta + \pi^+ \pi^- + K^+ K^-, \quad (4)$$

where we have omitted possible components involving the η' because it has a large mass and does not play a role in the generation of $f_0(980)$. Meanwhile, since the $\pi^0 \eta$ system has the isospin $I = 1$, and does not couple to $f_0(500)$ and $f_0(980)$, we do not consider its contribution.

As we discussed above, the Cabibbo-favored process $D^0 \rightarrow \pi^0 \pi^0 \bar{K}^0$ could happen via the tree diagram of Fig. 2(a), and the final state interaction of the S -wave pseudoscalar meson–pseudoscalar meson of Fig. 2(b). Then, the amplitude for Figs. 2(a) and 2(b) could be expressed as

$$\begin{aligned} \mathcal{T}^S = V_p & \left(h_{\pi^0 \pi^0} + h_{\pi^0 \pi^0} \times \frac{1}{2} G_{\pi^0 \pi^0} t_{\pi^0 \pi^0 \rightarrow \pi^0 \pi^0} \right. \\ & + h_{\eta \eta} \times \frac{1}{2} \times G_{\eta \eta} t_{\eta \eta \rightarrow \pi^0 \pi^0} \\ & + h_{\pi^+ \pi^-} \times G_{\pi^+ \pi^-} t_{\pi^+ \pi^- \rightarrow \pi^0 \pi^0} \\ & \left. + h_{K^+ K^-} \times G_{K^+ K^-} t_{K^+ K^- \rightarrow \pi^0 \pi^0} \right), \end{aligned} \quad (5)$$

where the V_p stands for the strength of the weak production vertex and could be treated as a constant, and the coefficients h_i represent the production weights of the corresponding components in the preliminary decay, which could be obtained from Eq. (4),

¹According to *Review of Particle Physics* (PRR) [45], the mixing angle is between -10° and -20° , and recent lattice calculations support the value $\theta_P = -14.1^\circ \pm 2.8^\circ$ by reproducing the masses of the η and η' [46]. In this work, we adopt the mixing from Ref. [44] as follows:

$$\begin{aligned} \eta & \sim \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} - s\bar{s}), \\ \eta' & \sim \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} + 2s\bar{s}). \end{aligned}$$

$$h_{\pi^0\pi^0} = \frac{1}{2}, \quad h_{\eta\eta} = \frac{1}{3}, \quad h_{\pi^+\pi^-} = 1, \quad h_{K^+K^-} = 1. \quad (6)$$

Here we introduce the extra factor $1/2$ for the loop systems of $\pi^0\pi^0$ and $\eta\eta$ with identical particles, as done in Ref. [47]. The G_l is the loop function of the two-meson propagator, and the $t_{i \rightarrow j}$ is the transition amplitude between the coupled channels [7]. The t -matrix can be obtained by solving the Bethe-Salpeter equation [7]

$$T = [1 - VG]^{-1}V, \quad (7)$$

where V is a 5×5 matrix of the interaction kernel for five channels $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^- , $K^0\bar{K}^0$, and $\eta\eta$ with isospin $I = 0$, and the explicit expressions of the 5×5 matrix elements are given by [48,49]

$$\begin{aligned} V_{11} &= -\frac{1}{2f^2}s, & V_{12} &= -\frac{1}{\sqrt{2}f^2}(s - m_\pi^2), \\ V_{13} &= -\frac{1}{4f^2}s, & V_{14} &= -\frac{1}{4f^2}s, \\ V_{15} &= -\frac{1}{3\sqrt{2}f^2}m_\pi^2, & V_{22} &= -\frac{1}{2f^2}m_\pi^2, \\ V_{23} &= -\frac{1}{4\sqrt{2}f^2}s, & V_{24} &= -\frac{1}{4\sqrt{2}f^2}s, \\ V_{25} &= -\frac{1}{6f^2}m_\pi^2, & V_{33} &= -\frac{1}{2f^2}s, & V_{34} &= -\frac{1}{4f^2}s, \\ V_{35} &= -\frac{1}{12\sqrt{2}f^2}(9s - 6m_\eta^2 - 2m_\pi^2), \\ V_{44} &= -\frac{1}{2f^2}s, & V_{45} &= -\frac{1}{12\sqrt{2}f^2}(9s - 6m_\eta^2 - 2m_\pi^2), \\ V_{55} &= -\frac{1}{18f^2}(16m_K^2 - 7m^2 - \pi), \end{aligned} \quad (8)$$

with the pion decay constant $f = 93$ MeV and the invariant mass square s of the meson-meson system. The masses of the mesons are taken from RPP [45]. We have taken the unitary normalization $|\eta\eta\rangle \rightarrow \frac{1}{\sqrt{2}}|\eta\eta\rangle$, $|\pi^0\pi^0\rangle \rightarrow \frac{1}{\sqrt{2}}|\pi^0\pi^0\rangle$ to account for the identity of particles when using the G function without an extra factor in Eq. (7) [48]. The loop function for a two-meson system is

$$G_l = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P - q)^2 - m_1^2 + i\epsilon} \frac{1}{q^2 - m_2^2 + i\epsilon}, \quad (9)$$

where m_1 and m_2 are the masses of mesons in the l -channel, q is the four-momentum of one meson, and P is the total four-momentum, with $s = P^2$. Since the loop function G in Eq. (9) is logarithmically divergent [50], we have two methods to solve this singular integral, using either the three-momentum cutoff method or the dimensional regularization method, by introducing free parameters, which are determined by giving rise to the pole position of the

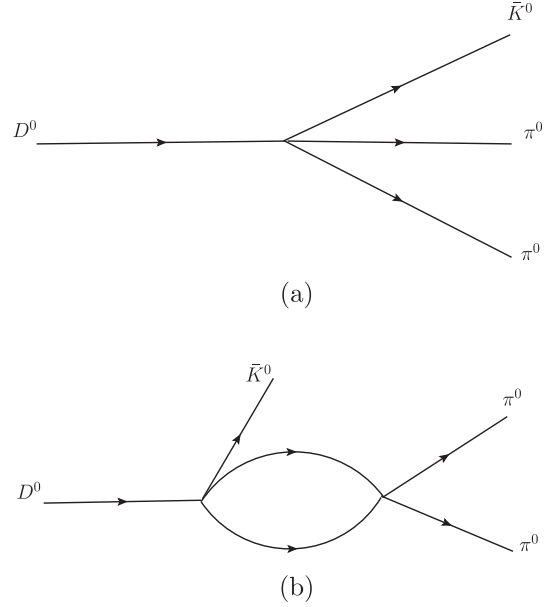


FIG. 2. The tree diagram (a) and the S -wave pseudoscalar meson-pseudoscalar meson interactions (b) for the process $D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0$.

resonance. In this work, the integral of the loop function is performed integrating exactly for q^0 and implementing a cutoff $q_{\max}$ for the three-momentum [7],

$$G_l = \int_0^{q_{\max}} \frac{|\vec{q}|^2 d|\vec{q}|}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2 [s - (\omega_1 + \omega_2)^2 + i\epsilon]}, \quad (10)$$

where the meson energies are $\omega_i = \sqrt{\vec{q}^2 + m_i^2}$ with the subindex i standing for the two intermediate mesons in the l -channel. It should be stressed that, in Ref. [7], only the $\pi\pi$ and KK channels were considered, while in this work the channel $\eta\eta$ is also considered, since the threshold of this channel is not so far away from the mass of $f_0(980)$. The results obtained with or without the $\eta\eta$ channel are very similar but the cutoff needed with two channels is $q_{\max} = 903$ MeV, while it is 600 MeV for three channels [48]. The effective inclusion of new channels by means of a change in the cutoff is common in many works [21,48].

In addition, the most reliable prediction range of the S -wave pseudoscalar meson-pseudoscalar meson interaction within the chiral unitary approach is up to 1100–1200 MeV, and one cannot use this model for higher invariant masses. To present a reasonable $\pi^0\pi^0$ invariant mass distribution, we evaluate the Gt combinations of Eq. (5) up to $M_{\text{cut}} = 1.1$ GeV, and from there on we multiply Gt by a smooth factor to make it gradually decreasing at large $M_{\pi^0\pi^0}$, as done in Ref. [18],

$$Gt(M_{\pi^0\pi^0}) = Gt(M_{\text{cut}}) e^{-\alpha(M_{\pi^0\pi^0} - M_{\text{cut}})}, \quad M_{\pi^0\pi^0} > M_{\text{cut}}, \quad (11)$$

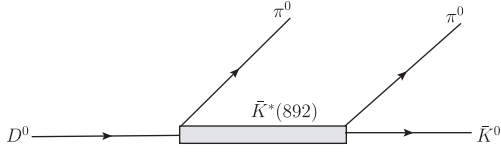


FIG. 3. The decay $D^0 \rightarrow \pi^0 \bar{K}^*(892)^0 \rightarrow \pi^0 \pi^0 \bar{K}^0$ via the intermediate vector $\bar{K}^*(892)^0$.

Here, we consider $\alpha = 0.0054$ MeV, which is the smoothing factor [18].

B. The contribution from $\bar{K}^*(892)$

In addition to the contribution from the S wave pseudoscalar meson–pseudoscalar meson interactions, we have also considered the contribution from intermediate vector meson $\bar{K}^*(892)$, as depicted in Fig. 3.

Since the vector meson $\bar{K}^*(892)^0$ decays into $\pi^0 \bar{K}^0$ in the P -wave, the corresponding Lagrangian for the vertex of VPP is given as

$$\mathcal{L} = -ig\langle [P, \partial_\mu P] V^\mu \rangle, \quad (12)$$

where $g = M_V/2f$ is the effective coupling, and P and V represent the matrix of the pseudoscalar meson and vector meson, respectively. The matrix of pseudoscalar mesons is

given by Eq. (2), and the matrix of the vector mesons V is written as [51]

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^* \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \quad (13)$$

Thus, the potential of $\bar{K}^*(892)^0 \rightarrow \pi^0 \bar{K}^0$ (we denote this π as the first π_1) is

$$V_{\bar{K}^{*0} \rightarrow \pi^0 \bar{K}^0} = \langle i\mathcal{L} \rangle = \frac{g}{\sqrt{2}} (p_{\bar{K}^0} - p_{\pi_1})_\mu \epsilon_{\bar{K}^{*0}}^\mu. \quad (14)$$

For the vertex of $D^0 \rightarrow \pi^0 \bar{K}^{*0}$ (we denote this π as the second π_2), we could have the potential with the same structure,

$$V_{D^0 \rightarrow \pi^0 \bar{K}^*} = g_{D^0 \pi^0 \bar{K}^{*0}} \epsilon_{\bar{K}^{*0}}^\nu (p_{D^0} + p_{\pi_2})_\nu, \quad (15)$$

where p_{D^0} and p_{π_2} stand for D^0 and π^0 four-momenta, respectively, and $g_{D^0 \pi^0 \bar{K}^{*0}}$ is an effective coupling for the vertex of $D^0 \rightarrow \pi^0 \bar{K}^{*0}$. Then the amplitude for $D^0 \rightarrow \pi^0 \bar{K}^*(892) \rightarrow \pi^0 \pi^0 \bar{K}^0$ could be written as [51,52]

$$\begin{aligned} \mathcal{T}_{\bar{K}^*} = & \frac{g_{D^0 \pi^0 \bar{K}^{*0}} g}{\sqrt{2}} \frac{1}{M_{\pi^0 \bar{K}^0}^2 - m_{\bar{K}^*}^2 + iM_{\bar{K}^*} \Gamma_{\bar{K}^*}} \left[-(m_{\bar{K}^0}^2 - m_{\pi_1}^2) \left(1 - \frac{M_{\pi_1 \bar{K}^0}^2}{m_{\bar{K}^*}^2} \right) + 2p_{\pi_1} p_{\pi_2} \frac{m_{\bar{K}^*}^2 + m_{\bar{K}^0}^2 - m_{\pi_1}^2}{m_{\bar{K}^*}^2} \right. \\ & \left. + 2p_{\bar{K}^0} p_{\pi_2} \frac{m_{\bar{K}^0}^2 - m_{\bar{K}^*}^2 - m_{\pi_1}^2}{m_{\bar{K}^*}^2} \right] + \text{exchange}(\pi_1^0 \leftrightarrow \pi_2^0), \end{aligned} \quad (16)$$

with

$$\begin{aligned} M_{\pi_1 \bar{K}^0}^2 &= (p_{\bar{K}^0} + p_{\pi_1})^2, \\ p_{\pi_1^0} \cdot p_{\pi_2^0} &= \frac{M_{\pi_1^0 \pi_2^0}^2 - 2m_{\pi^0}^2}{2}, \\ p_{\bar{K}^0} \cdot p_{\pi_2^0} &= \frac{m_{D^0}^2 + m_{\pi_1^0}^2 - M_{\pi_1^0 \pi_2^0}^2 - M_{\pi_1^0 \bar{K}^0}^2}{2}, \end{aligned} \quad (17)$$

$$(18)$$

where the exchange ($\pi_1^0 \leftrightarrow \pi_2^0$) stands for the replacement particle π_1^0 of the term with particle π_2^0 . With the above formalism, the double differential decay width can be expressed as

$$\frac{d^2\Gamma}{dM_{\pi_1^0 \pi_2^0} dM_{\pi_1^0 \bar{K}^0}} = \frac{M_{\pi_1^0 \pi_2^0} M_{\pi_1^0 \bar{K}^0}}{128\pi^3 M_{D^0}^3} (|T^S|^2 + |\mathcal{M}^{\bar{K}^*}|^2). \quad (19)$$

One could obtain the invariant mass distributions $d\Gamma/dM_{\pi_1^0 \pi_2^0}$ and $d\Gamma/dM_{\pi_1^0 \bar{K}^0}$ by integrating Eq. (19) over

each of the invariant mass variables. For a given value of M_{12} , the range of M_{23} is determined by [45]

$$\begin{aligned} (m_{23}^2)_{\min} &= (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2} \right)^2, \\ (m_{23}^2)_{\max} &= (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2} \right)^2, \end{aligned} \quad (20)$$

where E_2^* and E_3^* are the energies of particles 2 and 3 in the M_{12} rest frame, which are written as

$$\begin{aligned} E_2^* &= \frac{M_{12}^2 - m_1^2 + m_2^2}{2M_{12}}, \\ E_3^* &= \frac{M_{D^0}^2 - M_{12}^2 - m_3^2}{2M_{12}}, \end{aligned} \quad (21)$$

where m_1 , m_2 , and m_3 are the masses of involved particles 1, 2, and 3, respectively. All the masses and widths of the particles involved in this work are taken from the RPP [45].

III. RESULTS AND DISCUSSIONS

In our formalism discussed above, there are two parameters: the normalization factor V_p of Eq. (5), and the effective couplings $g_{D^0\pi^0\bar{K}^*}$. Since the experimental measurements of CLEO are the $\pi^0\pi^0$ and $\pi^0K_S^0$ invariant mass square distributions, we could introduce a parameter $\beta = g_{D^0\pi^0\bar{K}^*}g/V_p$ to account for the relative weight of the contribution from $\bar{K}^{*0}$ with respect to the one from the S -wave pseudoscalar meson–pseudoscalar meson interactions. In addition, the bin sizes of the $\pi^0\pi^0$ and $\pi^0K_S^0$ invariant mass square distributions are 0.038 GeV^2 and 0.055 GeV^2 , respectively. Thus, to account for the different bin sizes of the $\pi^0\pi^0$ and $\pi^0K_S^0$ invariant mass square distributions simultaneously, we need to adopt two different normalization factors V_p and V'_p for the $\pi^0K_S^0$ and $\pi^0\pi^0$ invariant mass square distributions, which have the relationship $(V'_p)^2 = \frac{1}{2} \times \frac{38}{55} (V_p)^2$, where we introduce the factor $\frac{1}{2}$ because there are two π^0 in the final states.

First, we have performed a fit to the experimental data of the $\pi^0\pi^0$ and π^0K^0 invariant mass distributions [35]. Then we obtain $\chi^2/\text{d.o.f.}$ is $237.8/(47 + 48 - 2) = 2.56$, and the fit parameters $(V'_p)^2 = (6.275 \pm 0.2472) \times 10^9$ and $\beta = (9.5195 \pm 0.313) \times 10^{-2}$.

Next, we have calculated the $\pi^0\pi^0$ and $\pi^0\bar{K}^0$ invariant mass distribution with the fitted parameters in Fig. 4. The black points with error bars labeled as “CLEO data” are the CLEO-c data taken from Ref. [35]. One can find that, in the $\pi^0\pi^0$ invariant mass distribution, our results could well reproduce the structure of the cusp followed by a dip, which could be associated with the scalar meson $f_0(980)$, dynamically generated from the S -wave pseudoscalar meson–pseudoscalar meson interactions. Meanwhile, our

results show that the peak around $M_{\pi^0\pi^0}^2 = 0.2 \text{ GeV}^2$ is mainly due to the scalar meson $f_0(500)$ and the reflection of the vector meson $\bar{K}^{*0}$. Within the coupled-channel Bethe-Salpeter approach with the meson-meson potentials provided by the lowest order chiral Lagrangian [7], one can simultaneously find a broad resonance $f_0(500)$, which couples strongly to the $\pi\pi$ channel, and the one $f_0(980)$, which couples strongly to the $K\bar{K}$ channel, which implies that these two states have a different dynamically internal structure.

In addition, our results for the $\pi\bar{K}^0$ invariant mass distribution agree well with the CLEO measurements. The narrow peak structure around $M_{\pi^0\bar{K}^0}^2 = 0.7 \text{ GeV}^2$ should be associated with the vector meson $K^*(892)$, while the contribution from the S -wave pseudoscalar meson–pseudoscalar meson interaction serves as a smooth background in the $\pi\bar{K}^0$ invariant mass distribution.

We also present the Dalitz plot of “ $M_{\pi^0\bar{K}^0}^2$ ” and “ $M_{\pi^0\pi^0}^2$ ” in Fig. 5, which clearly shows the structures of $f_0(980)$ and $\bar{K}^*(892)$. One can also find that the peak near the $\pi^0\pi^0$ threshold is mainly due to the scalar meson $f_0(500)$ and the reflection of the vector meson $\bar{K}^{*0}$.

Finally, with the fitted parameters, we could estimate that the S -wave $\pi^0\pi^0$ interaction contributes about 47.7% of the total decay rate, larger than the experimental data $(28.9 \pm 6.3 \pm 3.1)\%$, while the intermediate resonance $K^*(892)$ contributes about 52.3% of the total decay rate, consistent with the experimental data $(65.6 \pm 5.3 \pm 2.5)\%$ within the uncertainties [35]. It should be pointed out that the contribution from the scalar $f_0(500)$ was not taken into account by CLEO [35], which plays an important role in the near-threshold enhancement of the $\pi^0\pi^0$ invariant mass distribution.

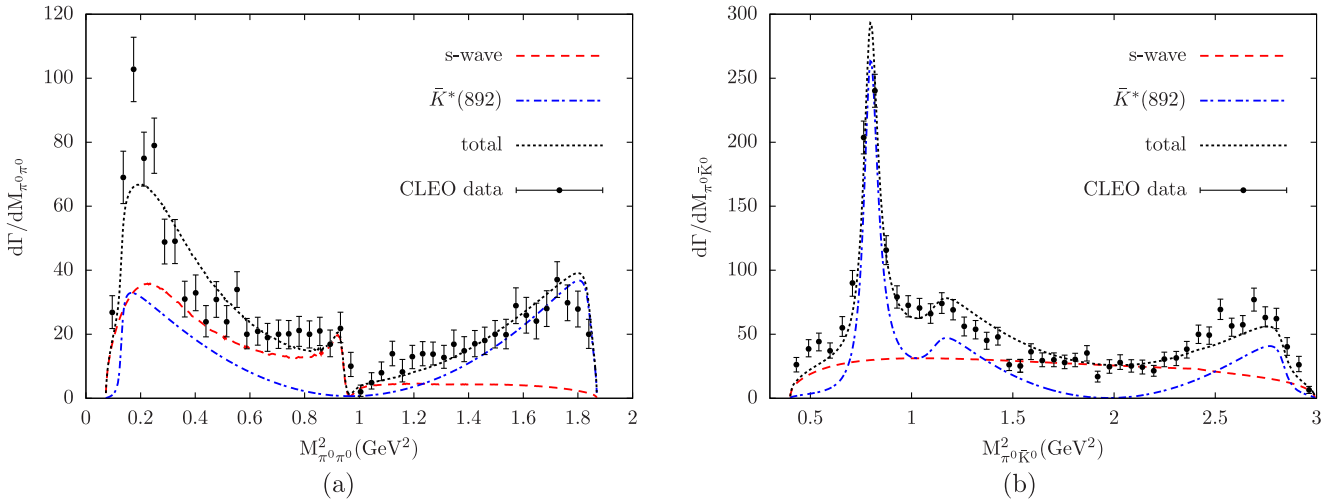


FIG. 4. (a) $\pi^0\pi^0$ and (b) $\pi^0\bar{K}^0$ invariant mass distributions for the process $D^0 \rightarrow \pi^0\pi^0\bar{K}^0$. The blue dashed curves and red dotted curves correspond to the contributions from the S -wave pseudoscalar meson–pseudoscalar interactions and the vector meson $\bar{K}^{*0}$, respectively; the solid black curves stand for the results of the full model. In addition, The experimental data of CLEO are represented by the points with error bars [35].

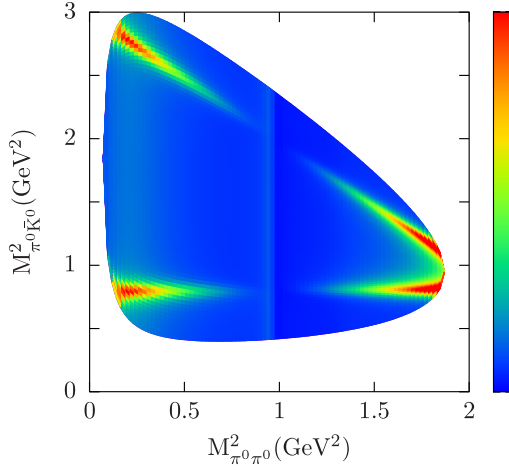


FIG. 5. The Dalitz plot of $M^2_{\pi^0\pi^0}$ vs $M^2_{\pi^0\bar{K}^0}$ for the process $D^0 \rightarrow \pi^0\pi^0\bar{K}^0$.

IV. SUMMARY AND CONCLUSION

Motivated by the CLEO measurements on the process $D^0 \rightarrow \pi^0\pi^0\bar{K}^0$ [35], we have investigated this process by taking into account the contribution from the S -wave pseudoscalar meson–pseudoscalar meson interaction within the chiral unitary method, which could dynamically generate intermediate resonance states $f_0(980)$ and $f_0(500)$, and also the contribution from the intermediate vector meson $\bar{K}^*(892)$.

First, we have performed a fit to the experimental data of the $\pi^0\pi^0$ and $\pi^0 K_S^0$ invariant mass distributions. With the fitted parameters, our results could well reproduce the cusp structure following a dip around 1 GeV in the $\pi^0\pi^0$

invariant mass distribution, which is mainly due to the contribution of scalar $f_0(980)$. Our results indicate that the peak near the $\pi^0\pi^0$ threshold is due to the scalar meson $f_0(500)$ and the reflection of $\bar{K}^*$. Our results of the $\pi^0\bar{K}^0$ invariant mass distribution are in good agreement with the CLEO data, which support that the peak around 892 MeV in the $\pi^0\bar{K}^0$ invariant mass distribution should be associated with $\bar{K}^*(892)$.

As we discussed in the Introduction, there are different explanations for the nature of the scalar mesons $f_0(500)$ and $f_0(980)$. Our work, taking into account S -wave pseudoscalar meson–pseudoscalar meson interaction within the chiral unitary method, could well describe the CLEO measurements of the process $D^0 \rightarrow \pi^0\pi^0\bar{K}^0$, especially the cusp structure following a dip around 1 GeV in the $\pi^0\pi^0$ invariant mass distribution, which favors the molecular explanation of the scalar mesons $f_0(500)$ and $f_0(980)$.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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