

Heavy-light quark systems from the QCD instanton vacuum: $N_f = 1$ light flavor case

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We investigate heavy-light quark systems within the framework of the QCD instanton vacuum, focusing on the $N_f = 1$ light flavor case. We derive an effective heavy-light quark interaction from the low-energy QCD partition function and construct a heavy-meson effective Lagrangian. The physical residual mass of heavy mesons, Λ , is determined by employing compositeness and normalization conditions. We calculate the masses of D and B mesons and their weak decay constants to the leading order and next-to-leading order in the $1/m_Q$ expansion. The current results for f_D and f_B are in good agreement with recent lattice QCD data and Particle Data Group (PDG) average values.

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I. INTRODUCTION

A conventional heavy meson contains a heavy quark (antiquark) and a light antiquark (quark). In the limit of infinitely heavy quark mass, i.e., $m_Q \rightarrow \infty$, the spin of the heavy quark S_Q is conserved. Consequently, the spin of the light quark is also conserved: $S_L = S - S_Q$, where S is the spin of the heavy meson. This is known as heavy-quark spin symmetry [1–4], which makes the spin of the light quark a good quantum number. In this limit, a charm quark becomes indistinguishable from a bottom quark, leading to heavy-quark flavor symmetry. This implies that the infinitely heavy quark remains as a mere static color source in a heavy-light quark system, so the quark-gluon dynamics inside it is governed by the light quarks. Thus, heavy mesons are classified as the members of the flavor SU(3) representations: $(D, D_s) \in \mathbf{3}_0$ and $(D^*, D_s^*) \in \mathbf{3}_1$, where the subscripts denote the corresponding spin of the heavy meson. Heavy mesons with a light antiquark belong to the antitriplet ($\bar{\mathbf{3}}$). Since light quarks dictate the dynamics inside heavy mesons, chiral symmetry and its spontaneous breakdown come into crucial play in describing the heavy

mesons. Thus, as pointed out in Refs. [3,4], both heavy-quark spin-flavor symmetry and chiral symmetry must be considered when constructing an effective theory for the heavy-light quark system.

The instanton vacuum provides a theoretical framework for realizing the spontaneous breakdown of chiral symmetry (SB χ S) in quantum chromodynamics (QCD) [5–7] (see also the reviews [8,9]). In Ref. [6], it was quantitatively shown that the instanton medium, which consists of the superposition of the instantons (I 's) and anti-instantons ($\bar{I}$'s), was stabilized at $R/\rho \approx 3$, where R and ρ denote the average distance between I 's ($\bar{I}$'s) and the average size of the instanton. R and ρ were determined to be $R \simeq 1$ fm and $\rho \simeq 1/3$ fm [5–7,10]. Both R and ρ are related to Λ_{QCD} , allowing their determination once Λ_{QCD} is given. Lattice QCD results support these values [11–13]. These R and ρ values well explain the quark and gluon condensates, which characterize the QCD vacuum. The gluon condensate, proportional to the instanton density $N/V \equiv 1/R^4$ [9], agrees with the phenomenological estimate $\langle F_{\mu\nu} F_{\mu\nu} \rangle / 32\pi^2 \simeq (200 \text{ MeV})^4 > 0$ [14]. The quark condensate, the order parameter for SB χ S, is determined as [7]

$$\langle \bar{\psi}\psi \rangle = -\frac{1}{R^2\rho} \sqrt{\frac{N_c}{6.62}} \simeq -(253 \text{ MeV})^3, \quad (1)$$

at the scale of $\mu = 600 \text{ MeV}$, which is approximately identified by $\rho^{-1} \simeq 600 \text{ MeV}$ [9,15]. Here, N_c is the number of colors. The SB χ S mechanism involves quark propagation through the instanton medium. As quarks hop

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between random I 's and $\bar{I}$'s, their helicity changes by different handedness of I and $\bar{I}$ zero modes. This delocalization process realizes the spontaneous breakdown of chiral symmetry. This indicates that the quark acquires the dynamical quark mass, $M(k)$, which is momentum-dependent. $M(k)$ is related to the Fourier transform of the fermionic zero mode [7,9]. The value $M(0) \sim 350$ MeV is determined by the saddle-point equation. This framework also yielded $f_\pi \simeq 93$ MeV [7]. The effective low-energy QCD partition function derived from the instanton vacuum has been successfully applied to low-lying light hadrons and singly heavy baryons in the limit of the infinitely heavy-quark mass [16–27].

Instanton effects on quarkonia were also studied and found to be marginal. The instanton contribution to the heavy quark mass is $\Delta M_Q \simeq 70$ MeV [28–30], which is small compared to the charm quark mass. However, these effects allow for the values for parameters closed to physical ones in the heavy-quark potential, such as the strong coupling constant and heavy-quark mass. This approach has led to quantitative descriptions of charmonia spectra and decay rates [30–34]. The instanton vacuum thus provides a unified framework for investigating both light-quark and heavy-quark systems, making it crucial to explore effective interactions for heavy-light quark systems. Recently, effective heavy-light quark interactions from the instanton vacuum were sketched in Ref. [35]. In the present work, we aim to quantitatively complete the work of Ref. [35] by constructing a heavy-meson effective Lagrangian from the instanton vacuum. For simplicity, we focus on the case of the light-quark flavor number $N_f = 1$ and will present more complicated and general cases elsewhere.

This paper is organized as follows: in Sec. II, we show how to derive the heavy-light quark interaction from the instanton vacuum. In Sec. III we simplify this interaction and obtain the four-quark interaction for $N_f = 1$. In Sec. IV, we derive the heavy meson residual mass Λ , a key parameter predicted to be generated by nonperturbative corrections. We also calculate the heavy meson masses, M_D and M_B , and derive the weak decay constants f_D and f_B to the $1/m_Q$ leading order and next-to-leading order. The results are discussed in comparison with other works. The final section summarizes our findings and presents an outlook for future work.

II. HEAVY-LIGHT QUARK SYSTEMS FROM THE INSTANTON VACUUM

To derive the effective low-energy QCD partition function for the light quark degrees of freedom, we need to compute the quark determinant in the presence of the instantons and anti-instantons, and then average it over the collective coordinates of the instantons [7,9,36]. We will briefly recapitulate the derivation of the partition function

for the light quark systems and extend it to the heavy-light quark systems. We start from the quark determinant derived from the instanton vacuum [7,9,17,36,37]:

$$\text{Det}_N = \int D\psi D\psi^\dagger \exp \left(\sum_f \int d^4x \psi_f^\dagger (i\cancel{\partial} + im_f) \psi_f \right) \times \prod_f \left\{ \prod_+^{N_+} V_+[\psi_f^\dagger, \psi_f] \prod_-^{N_-} V_-[\psi_f^\dagger, \psi_f] \right\}, \quad (2)$$

where m_f denotes the current quark mass with flavor f . N_+ and N_- stand for the numbers of the instantons and anti-instantons, respectively. $V_\pm[\psi_f^\dagger, \psi_f]$ are the instanton-induced $2N_f$ quark-quark interactions written as follows [37]:

$$V_\pm[\psi_f^\dagger, \psi_f] = \int d^4x (\psi_f^\dagger(x)(i\cancel{\partial})\Phi_\pm(x - z_\pm, U_\pm)) \times \int d^4y (\Phi_\pm^\dagger(y - z_\pm, U_\pm)(i\cancel{\partial})\psi_f(y)). \quad (3)$$

$\Phi_\pm$ is the fermionic zero mode obtained in the presence of the (anti)instanton field. We need to average the quark determinant over the instanton zero modes $\xi_\pm$ that consist of its position, its size, and its orientation in color space. The instanton medium has low density since its packing fraction is very small ($\pi^2(\rho/R)^4 \approx 1$). This diluteness of the instanton medium allows us to perform the integration over the zero modes of the instantons independently. Before we proceed to derive the effective low-energy QCD partition function, we want to mention about the current quark mass m_f . In the current work, we focus on the $N_f = 1 + 1$ case and examine the heavy-quark spin-flavor symmetry and the effects of $1/m_Q$ corrections. Thus, effects of flavor SU(3) symmetry breaking will not be considered, so the chiral limit will be taken from now on.

After averaging the quark determinant over the zero modes, we obtain the effective low-energy QCD partition function [38,39] as follows:

$$Z_{\text{eff}} = \int D\psi D\psi^\dagger \exp \left(\int d^4x \sum_f^{N_f} \psi_f^\dagger i\cancel{\partial} \psi_f \right) Y_{N_f}^{N_+} Y_{N_f}^{N_-}, \quad (4)$$

where N_f denotes the number of light flavors, $Y_{N_f}^{N_\pm}$ represent the instanton-induced $2N_f$ quark-quark interactions:

$$Y_{N_f}^{N_\pm} = (i)^{N_f} \left(\frac{4\pi^2 \rho^2}{N_c} \right)^{N_f} \int \frac{d^4z}{V} \det_f iJ_\pm(z) \quad (5)$$

with

$$J_{\pm}(z)_{fg} = \int \frac{d^4 k d^4 l}{(2\pi)^8} \exp[-i(k-l)z] F(k^2) \psi_f^{\dagger}(k) \times \frac{1}{2} (1 \pm \gamma_5) \psi_g(l) F(l^2). \quad (6)$$

Here, $F(k)$ designates the form factor related to the zero-mode wave function in momentum space $\Phi_{\pm}(k; \xi_{\pm})$. It is expressed in terms of the modified Bessel functions I_i and K_i ,

$$F(k) = -k \frac{d}{dk} \left[I_0\left(\frac{k\rho}{2}\right) K_0\left(\frac{k\rho}{2}\right) - I_1\left(\frac{k\rho}{2}\right) K_1\left(\frac{k\rho}{2}\right) \right]. \quad (7)$$

The effective low-energy QCD partition function can further be bosonized. For details, we refer to Refs. [7,9].

We now turn our attention to the heavy-light quark system. Since the heavy quark mass is much larger than Λ_{QCD} , we consider the limit $m_Q \rightarrow \infty$. In this limit, the heavy quark and antiquark fields are separate, as $Q\bar{Q}$ pair creation does not occur, preventing overlap between the heavy quark and antiquark functional spaces. Furthermore, due to the infinite mass of the heavy quark, both the heavy quark and light quark spins are conserved. Consequently, the low-lying heavy mesons are classified as members belonging to either the flavor triplet (**3**) or antitriplet ($\bar{\mathbf{3}}$) representations.

In the limit of $m_Q \rightarrow \infty$, the heavy quark becomes a mere static color source. Thus, the heavy quark inside a heavy hadron is considered to be almost on shell, so that its momentum can be decomposed as $p_Q^{\mu} = m_Q v^{\mu} + k^{\mu}$, where v_{μ} is the heavy quark velocity and k_{μ} denotes the residual momentum. Note that the velocity of the heavy quark remains unchanged in the presences of the interactions due to $k/m_Q \ll 1$. This leads to the velocity superselection rule [2]. We introduce the covariant velocity-dependent projection operator $P_{\pm} = (1 \pm \not{v})/2$, which can be used to project out particle and antiparticle states, $Q_{\pm} = P_{\pm} Q$. In the rest frame, it becomes $(1 + \gamma_0)/2$, selecting the particle components from the Dirac spinor. We then redefine the heavy quark field by extracting the dominant part of the heavy-quark momentum:

$$Q(x) = e^{-im_Q v \cdot x} (h(x) + \mathcal{H}(x)), \quad (8)$$

where $h(x)$ is the large component of $Q(x)$,

$$h(x) = e^{im_Q v \cdot x} Q_+(x), \quad (9)$$

and $\mathcal{H}(x)$ is the small component given in the order of $1/m_Q$,

$$\mathcal{H}(x) = e^{im_Q v \cdot x} Q_-(x). \quad (10)$$

The h field contributes in the leading order, while the $\mathcal{H}$ field produces effects suppressed by factors of $1/m_Q$.

$\mathcal{H}$ can be effectively eliminated by integrating out the field using the Dirac equation. With the inverse heavy-quark mass $1/m_Q$ as an expansion parameter, we arrive at the heavy-quark effective Lagrangian¹:

$$\mathcal{L}_{\text{eff}} = -h^{\dagger} i(v \cdot D) h - \frac{i}{2m_Q} h^{\dagger} \left(D_{\perp}^2 - \frac{g}{2} \sigma_{\mu\nu} G^{\mu\nu} \right) h, \quad (11)$$

where $D_{\perp}$ is a component of the covariant derivative transversal to velocity v , defined as $D_{\perp\mu} = D_{\mu} - v_{\mu} v \cdot D$. The lowest-order Lagrangian, the first term of Eq. (11), reflects heavy-quark spin-flavor symmetry.

The heavy quark propagator in the instanton ensemble can be written in terms of the propagators in the single instanton background. Due to the small packing fraction, $\rho^4 N / (V N_c) \approx 0.004$, we can express the heavy quark propagator as $w = (i\partial_4 - i \sum_{\pm} A_{\pm,4})^{-1}$ [29,40] in the rest frame $v_{\mu} = (\vec{0}, 1)$. The inverse of the propagator is written as

$$\bar{w}^{-1} = i\partial_4 - \sum_{\pm} N_{\pm} \langle i\partial_4 (w_{\pm} - (i\partial_4)^{-1}) i\partial_4 \rangle, \quad (12)$$

where $\langle \hat{O} \rangle = \int d\xi_{\pm} \hat{O}$ denotes averaging over the single-instanton fields. $w_{\pm} = (i\partial_4 - iA_{\pm,4})^{-1}$ represents the heavy quark propagator in the single-instanton (anti-instanton) field. The heavy-quark propagator can be expressed as the Wilson line in large Euclidean time, allowing us to calculate the instanton contribution to the heavy quark mass $\Delta M_Q \simeq 70$ MeV. This Wilson line yields the instanton-induced heavy-quark potential [29]. Although the instanton effects on heavy quarkonium spectra are marginal [30,31,33,34], they enable one to use almost physical values for parameters such as the strong coupling constant and the charm quark mass. Note that these parameters are often treated as free variables to better describe quarkonium physics phenomenologically [30,31,33].

Combining Eqs. (4) and (11), we can derive the instanton-induced heavy-light quark interaction. This interaction depends on the total flavor number $N_f = N_{lf} + N_{hf}$, where N_{lf} and N_{hf} represent the numbers of light and heavy flavors, respectively. In the limit of $m_Q \rightarrow \infty$, the heavy quark becomes independent of its flavor, so we can express the total flavor number as $N_f = N_{lf} + 1$. In the following, we will derive the N_f instanton-induced heavy-light quark interaction.

We are now in a position to discuss both heavy and light quarks in the $N_{\pm}$ instanton ensemble. The corresponding fermionic determinant can be written as a product of light and heavy ones. We will replace the light-quark

¹Since the instantons are defined in Euclidean space, we also express the effective Lagrangian in Euclidean space.

determinant with an approximate one. The effective low-energy QCD partition function can then be written as

$$\begin{aligned} \mathcal{Z}_{\text{eff}} = & \left(\prod_{\pm}^{N_{\pm}} \int d\xi_{\pm} \right) \int D\psi D\psi^{\dagger} \prod_f^{N_{lf}} \exp \\ & \times \left(\int d^4x \psi_f^{\dagger} (i\partial) \psi_f \right) \prod_{\pm}^{N_{\pm}} (V_{\pm}[\psi_f^{\dagger}, \psi_f]) \\ & \times \int Dh Dh^{\dagger} \exp \left(\int d^4x h^{\dagger} \left(i\partial_4 - i \sum_{\pm} A_{\pm,4} \right) h \right), \end{aligned} \quad (13)$$

where $(\prod_{\pm}^{N_{\pm}} \int d\xi_{\pm})$ represents the average over the instanton collective coordinates. While one can calculate the heavy-quark propagator from Eq. (13), averaging over the instanton collective coordinates prevents immediate integration over light quark degrees of freedom. To overcome this, we introduce the identity operator,

$$\begin{aligned} \mathbf{1} = & \left(\prod_{\pm}^{N_{\pm}} \left\langle \prod_f V_{\pm}[\psi_f^{\dagger}, \psi_f] \right\rangle \right) \\ & \times \left(\prod_{\pm}^{N_{\pm}} \left\langle \prod_f V_{\pm}[\psi_f^{\dagger}, \psi_f] \right\rangle \right)^{-1}. \end{aligned} \quad (14)$$

We can group the first part of Eq. (14) together with the kinetic term for the light quarks. We can then add the light quark source fields, following the approach in Refs. [41,42]. This procedure allows us to integrate over the collective coordinates for the heavy-quark part separately:

$$\begin{aligned} w[\psi^{\dagger}, \psi] = & \left(\prod_{\pm}^{N_{\pm}} \left\langle \prod_f^{N_{lf}} V_{\pm}[\psi_f^{\dagger}, \psi_f] \right\rangle \right)^{-1} \left(\prod_{\pm}^{N_{\pm}} \int d\xi_{\pm} \right) \\ & \times \prod_f^{N_{lf}} V_{\pm}[\psi_f^{\dagger}, \psi_f] \left(i\partial_4 - i \sum_{\pm} A_{\pm,4} \right)^{-1}. \end{aligned} \quad (15)$$

We extend the method in Refs. [29,40] to the current case. Using the packing fraction $\rho^4 N / (V N_c) \ll 1$ as a

perturbative expansion parameter, we obtain an iterative solution up to linear order. This leads to an averaged inverse functional $w^{-1}[\psi^{\dagger}, \psi]$ in terms of a single-instanton average:

$$\begin{aligned} \bar{w}^{-1}[\psi^{\dagger}, \psi] = & i\partial_4 - \sum_{\pm} N_{\pm} \left\langle \prod_f^{N_f} V_{\pm}[\psi_f^{\dagger}, \psi_f] \right\rangle^{-1} \\ & \times \Delta_{H,\pm}[\psi^{\dagger}, \psi], \end{aligned} \quad (16)$$

where

$$\Delta_{H,\pm}[\psi^{\dagger}, \psi] = \left\langle \prod_f^{N_f} V_{\pm}[\psi_f^{\dagger}, \psi_f] i\partial_4 (w_{\pm} - (i\partial_4)^{-1}) i\partial_4 \right\rangle. \quad (17)$$

To derive the heavy-light quark interaction, we first combine Eq. (16) with the light quark part to obtain the heavy-quark propagator averaged over (anti)instanton collective coordinates:

$$\begin{aligned} \bar{w} = & \frac{1}{\mathcal{Z}[0]} \int D\psi^{\dagger} D\psi \prod_{\pm} \int d\lambda_{\pm} \exp \\ & \times \int d^4x \left\{ \psi_f^{\dagger} i\partial \psi_f + \lambda_{\pm} \left\langle \prod_f^{N_f} V_{\pm}[\psi^{\dagger}, \psi] \right\rangle \right\} \\ & \times \left(i\partial_4 - \sum_{\pm} \lambda_{\pm} \Delta_{H,\pm}[\psi^{\dagger}, \psi] \right)^{-1}. \end{aligned} \quad (18)$$

Here, the Lagrange multiplier $\lambda_{\pm}$ has to be integrated in the saddle point approximation. The function inside the exponent is responsible for the saddle point equation. Thus, we can safely say that the saddle point value $\lambda_{\pm} = \lambda_{\text{s.p.}}$ is defined in the light quark sector. According to Refs. [38,39] it defines the light quark dynamical mass $M(k) = \lambda_{\text{s.p.}} [2\pi\rho F(k)]^2 / N_c$. Next, we can rewrite the heavy-quark propagator in terms of the heavy-quark fields h and $h^{\dagger}$ in the form of the functional integration:

$$\begin{aligned} \bar{w} = & \frac{1}{\mathcal{Z}[0]} \prod_{\pm} \int d\lambda_{\pm} \int D\psi^{\dagger} D\psi \exp \int d^4x \left\{ \psi_f^{\dagger} i\partial \psi_f + \lambda_{\pm} \left\langle \prod_f^{N_f} V_{\pm}[\psi^{\dagger}, \psi] \right\rangle \right\} \\ & \times \int Dh^{\dagger} Dh h h^{\dagger} \exp \int d^4x \left\{ h^{\dagger} \left(i\partial_4 - \sum_{\pm} \lambda_{\pm} \Delta_{H,\pm} \right) h \right\}. \end{aligned} \quad (19)$$

From the last term of Eq. (19), we can identify the heavy-light quark interaction. Using definitions (3) and (17) one can express the interaction as

$$\begin{aligned}
S_{\text{int}} = & \sum_{\pm} \int d^4 z_{\pm} dU_{\pm} \frac{N_{\pm}}{V} \prod_{f=1}^{N_{lf}} \left\{ \frac{N_c V}{N_{\pm}} \int \frac{d^4 k_f d^4 l_f}{(2\pi)^8} e^{i(k_f - l_f) \cdot z_{\pm}} \sqrt{M(k_f) M(l_f)} \right. \\
& \times \psi_{\alpha_f i_f}^{\dagger}(k_f) (\gamma_{\mu} \gamma_{\nu} \gamma_{\pm})_{j_f}^{i_f} U_{\pm, \gamma_f}^{\alpha_f} (\tau_{\mu}^{\mp} \tau_{\nu}^{\pm})_{\delta_f}^{\gamma_f} U_{\beta_f}^{\dagger \delta_f} \psi^{\beta_f j_f}(l_f) \Big\} \\
& \times \int d^3 x dt dt' h_{\alpha_3}^{\dagger}(\vec{x}, t') U_{\pm, \gamma_3}^{\alpha_3} \langle t' | i \partial_4 (w_{\pm} - (i \partial_4)^{-1}) i \partial_4 | t \rangle_{\delta_3}^{\gamma_3} U_{\pm, \beta_3}^{\dagger \delta_3} h^{\beta_3}(\vec{x}, t). \quad (20)
\end{aligned}$$

To obtain the effective interaction with N_{lf} fixed, one must perform color integration and Fierz reordering. In the next section, we will discuss the instanton-induced $N_f = 1 + 1$ effective heavy-light interaction and its applications.

III. $N_f = 1 + 1$ HEAVY-LIGHT INTERACTION

The heavy quark fields can be represented in momentum space, resulting in integration over each quark momentum followed by a δ -function defining energy-momentum conservation. In this case, the color-orientation integral can be done straightforwardly as shown in the Appendix. To obtain color and spin contracted terms, we will use the notation $\psi_{ia}^{\dagger} \psi^{ia} = (\psi^{\dagger} \psi)$, where i and a represent spin and

color the indices, respectively. Then we carry out Fierz reordering. Furthermore, following Ref. [29], in $m_Q \rightarrow \infty$ limit, we can derive the instanton effect on the heavy-quark mass,

$$\begin{aligned}
& \frac{1}{N_c} \int d^3 x dt dt' e^{i \vec{p} \cdot \vec{x}} \text{tr}_c \langle -t' | i \partial_4 (w_{\pm} - (i \partial_4)^{-1}) i \partial_4 | t \rangle \\
& \rightarrow -\frac{V}{N} \Delta M_Q. \quad (21)
\end{aligned}$$

Finally, in the large N_c limit, we derive the instanton induced $N_f = 1 + 1$ effective heavy-light quark interaction given by

$$\begin{aligned}
S_{\text{int}} = & - \int \frac{d^4 k_1 d^4 k_2}{(2\pi)^8} \frac{d^4 p_1 d^4 p_2}{(2\pi)^8} (2\pi)^4 \delta^{(4)}(k_1 - k_2 + p_1 - p_2) F(k_1) F(k_2) M_q(0) \frac{\Delta M_Q}{N/V} \\
& \times \left[(\psi^{\dagger}(k_1) \psi(k_2)) (Q_+^{\dagger}(p_1) Q_+(p_2)) + \frac{1}{8} \sum_i (\psi^{\dagger}(k_1) \Gamma_i Q_+(p_1)) (Q_+^{\dagger}(p_2) \Gamma_i \psi(k_2)) \right], \quad (22)
\end{aligned}$$

where $\Gamma_i = (\mathbf{1}, \gamma_5, \gamma_{\mu}, i \gamma_{\mu} \gamma_5, \sigma_{\mu\nu} / \sqrt{2})$ denote the Dirac matrices. This interaction can be understood as follows: we began with heavy and light quarks in the background of the instanton medium. The dilute nature of this instanton medium allowed us to rewrite the QCD partition function as the partition function of the $N_{\pm}$ -instanton configuration multiplied by the instanton ensemble averaged action for heavy and light quarks. By performing a perturbative expansion, with respect to the instanton packing fraction, it enables us to simplify the ensemble average to the average over a single (anti-)instanton. This leads to an effective four-quark interaction between the heavy and light quarks, with a form factor $F(k)$ attached to each light quark leg. This interaction is characterized by a coupling $g^2 \sim M_q(0) \Delta M_Q R^4$, where $M(0)$ is the dynamical light quark mass, ΔM_Q is the instanton contribution to the heavy quark mass, and R is the average separation between instantons.

We now express the second term in the bracket of Eq. (22) in coordinate space, where the form factor F can be written as a function of the derivative,

$$\begin{aligned}
S_{\text{int}} = & -g^2 \int d^4 x (Q_+^{\dagger}(x) \Gamma_i F(\partial) \psi(x)) (F(\partial) \psi^{\dagger}(x) \Gamma_i Q_+(x)), \\
& g^2 = M_q(0) \Delta M_Q R^4 / 8, \quad (23)
\end{aligned}$$

where g^2 is given in the large N_c limit. To further develop this model, we can employ the bosonization. This involves replacing the heavy-light bilinears $(h^{\dagger} \Gamma_i F(\partial) \psi)$ with corresponding auxiliary boson fields Φ_i . This transformation yields the effective Lagrangian describing the interaction between heavy mesons and heavy-light quarks,

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & -\psi^{\dagger} (i \not{\partial} + i M_q(\partial)) \psi - h^{\dagger} i (v \cdot \partial) h + m_{\Phi}^2 \Phi_i^{\dagger} \Phi_i \\
& - G \Phi_i^{\dagger} F(\partial) \psi^{\dagger} \Gamma_i Q_+ - G Q_+^{\dagger} \Gamma_i F(\partial) \psi \Phi_i, \quad (24)
\end{aligned}$$

where $G = g m_{\Phi}$ and m_{Φ} stands for the mass of Φ . The attractive force between the heavy quark and the light antiquark is ensured by $G > 0$, guaranteeing the production of the heavy-light composite particle. Next, a renormalization process must be performed to identify the auxiliary

fields as corresponding physical meson fields. This crucial step will be discussed in detail in the following section.

IV. EQUIVALENT EFFECTIVE LAGRANGIAN

It is important to note that the auxiliary field Φ in Eq. (24) does not yet represent the physical heavy meson field, as the bosonization process does not inherently impose any physical constraints. Therefore, we need to ensure that this field is properly renormalized such that it acquires a physical mass. Once the compositeness condition $Z_\Phi = 0$ is considered, where Z_Φ denotes the wave function renormalization constant, we will show that Eq. (23) is equivalent to the following Lagrangian [43]:

$$\begin{aligned} \mathcal{L}'_{\text{eff}} = & -\psi^\dagger(i\not{\partial} + iM_q(\partial))\psi - h^\dagger i(v \cdot \partial)h + \partial_\mu \Phi_i^\dagger \partial_\mu \Phi_i \\ & + m_\Phi^2 \Phi_i^\dagger \Phi_i - G\Phi_i^\dagger F(\partial)\psi^\dagger \Gamma_i Q_+ - GQ_+^\dagger \Gamma_i F(\partial)\psi \Phi_i. \end{aligned} \quad (25)$$

The Φ field and its mass in Eq. (25) are required to be renormalized. They are related to the renormalized ones as follows:

$$\Phi = Z_\Phi^{1/2} \Phi_R, \quad Z_\Phi m_\Phi^2 = m_{\Phi_R}^2 + \delta m_\Phi^2. \quad (26)$$

We will briefly recapitulate how the compositeness condition can be proved [44,45]. In the limit $m_Q \rightarrow \infty$, the kinetic term and mass term for Φ can be expressed to zeroth-order of $1/m_Q$ as

$$\partial_\mu \Phi^\dagger \partial_\mu \Phi + m_\Phi^2 \Phi^\dagger \Phi = \phi_v^\dagger (-2v \cdot \partial - 2\Lambda) \phi_v + \mathcal{O}\left(\frac{1}{m_Q}\right), \quad (27)$$

where ϕ_v is related to $\Phi^\pm = \frac{1}{\sqrt{m_Q}} e^{\mp m_Q v \cdot x} \phi_v$ with $\Phi^+ = \Phi$ and $\Phi^- = \Phi^\dagger$ in Euclidean space. The heavy meson masses are defined as $m_\Phi = m_Q + \Lambda$ in the infinitely heavy quark mass limit. The term Λ is known as the residual mass of the heavy meson. It is related to the heavy meson momentum $P_\Phi = m_\Phi v = m_Q v + p_\Phi = m_Q v + \Lambda v$, which satisfies energy-momentum conservation: $P_\Phi = p_Q + p_q = m_Q v + k + p_q$, where p_Q denotes a heavy quark momentum, and

p_q designates a light quark momentum. Consequently, the residual momentum for the heavy meson $p_\Phi = \Lambda v$ satisfies the 4-momentum conservation $p_\Phi = p_q + k$.

Then, we can rewrite the Lagrangian of Eq. (25) as

$$\begin{aligned} \mathcal{L} = & -\psi^\dagger(i\not{\partial} + iM_q(\partial))\psi - h^\dagger i(v \cdot \partial)h \\ & - \phi_{vi}^\dagger (2v \cdot \partial + 2\Lambda) \phi_{vi} - G_0 \phi_{vi}^\dagger F(\partial) \psi^\dagger \Gamma_i h \\ & - G_0 h^\dagger \Gamma_i F(\partial) \psi \phi_{vi}, \end{aligned} \quad (28)$$

where G_0 is a rescaled coupling constant, which must be determined by the renormalization. For this Lagrangian to be the correct form of the effective Lagrangian, correlation functions derived from Eqs. (23) and (28) must be physically identical. We can compute the heavy-light quark correlation functions using Eq. (23). The momentum distribution of the quark inside a heavy meson is governed by the quark form factor $F(p)$. Since the average of $gF(p)$ in momentum space, which can be regarded as an effective coupling for the heavy-light quark vertex, is close to zero, i.e., $\langle gF(p_q) \rangle \approx 0$, the infinite quark-loop and meson-quark sums converge as shown in Figs. 1(a) and 1(b), respectively. Similar loop calculations can be found in Refs. [44,45]. The corresponding expressions for the correlation functions are given by

$$\begin{aligned} \mathcal{A}_{Qq}^a &= \frac{-F(p_q)F(p'_q)}{(v \cdot p_\Phi - i\Lambda_r)\Pi'(i\Lambda_r) + \Pi'(i\Lambda_r)}, \\ \mathcal{A}_{Qq}^b &= \frac{iG_0^2 F(p_q)F(p'_q)}{2(v \cdot p_\Phi - i\Lambda_r) - iG_0^2 \Pi'(i\Lambda_r)}, \end{aligned} \quad (29)$$

which determines the coupling constant $G_0^2 = 2i/\Pi'(i\Lambda_r)$. Therefore, we obtain the compositeness condition [43,44] as follows:

$$Z_\Phi = \left(1 + \frac{iG_0^2}{2} \Pi'(i\Lambda_r)\right) = 0, \quad (30)$$

where Λ_r is a physical residual mass. The fermion loop Π is given as

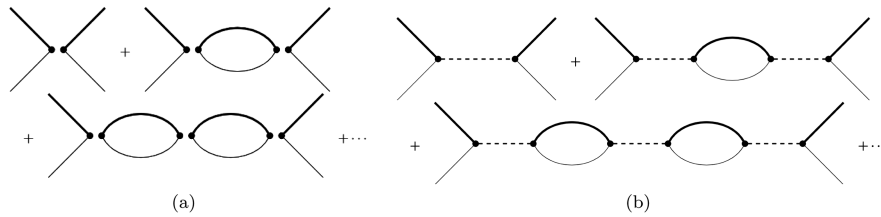


FIG. 1. In the left panel (a), the infinite sum for the quark-quark correlation function is drawn, whereas in the right panel (b) the meson-quark correlation function is depicted.

$$\Pi(v \cdot p_\Phi) = \int \frac{d^4 p_q}{(2\pi)^4} F^2(p_q) \text{tr}_D \left[\Gamma_P \frac{(1 + \not{p})}{2v \cdot (p - p_q) + i\epsilon} \right. \\ \left. \times \Gamma_P \frac{-(\not{p}_q - iM(p_q))}{p_q^2 + M^2(p_q)} \right]. \quad (31)$$

Note that $p = p_q + k$ corresponds to the definition of the heavy meson's residual momentum p_Φ . Equation (30) leads to the equivalence between the Lagrangians given in Eqs. (24) and (28). The coupling constant G_0 can be represented as $G_0 = g\mathcal{N}G_1$, where $\mathcal{N}$ is a normalization constant of heavy-meson state that will be introduced in the next section, and G_1 only stands for carrying the dimension of $\text{GeV}^{1/2}$.

V. NORMALIZATION AND RESIDUAL MASS OF HEAVY MESON

We are now in a position to compute the renormalized residual mass of the heavy meson. To obtain the physical mass, the auxiliary field ϕ_v satisfies the standard

normalization condition. The standard relativistic normalization is given as

$$\langle \Phi(\vec{p}') | \Phi(\vec{p}) \rangle = (2\pi)^3 2p_4 \delta^3(\vec{p}' - \vec{p}). \quad (32)$$

In $m_Q \rightarrow \infty$ limit, the heavy meson states can be expressed as $|\phi(v)\rangle = m_\Phi^{-1/2} |\Phi(\vec{p})\rangle$, then the nonrelativistic or heavy-quark mass independent normalization can be represented by [44,46]

$$\langle \phi(v') | \phi(v) \rangle = (2\pi)^3 2p_4 / m_\Phi \delta^3(\Lambda \vec{v}' - \Lambda \vec{v}) \\ = (2\pi)^3 2v_4 \delta^3(\Lambda \vec{v}' - \Lambda \vec{v}). \quad (33)$$

To confirm Eq. (33), let us define the wave packet of the heavy meson in momentum space. We find its form from the equation of motion from the effective Lagrangian (24), where the heavy-quark field is expressed as $\Phi_i \propto gF(\partial)\psi^\dagger \Gamma_i Q_+$. For convenience, we introduce the factors $\sqrt{2v_4}\mathcal{N}$, where $\mathcal{N}$ is a normalization constant. Then we can construct the wave packet Φ_v in momentum space as follows:

$$\Phi_{vi}(p_\Phi) = \langle p_{\Phi 4} | \phi_i(v) \rangle = \sqrt{2v_4} ig\mathcal{N} \int \frac{d^4 p_q d^4 k}{(2\pi)^8} (2\pi)^4 \delta^{(4)}(p_\Phi - k - p_q) F(p_q) \psi^\dagger(p_q) \Gamma_i h(k), \quad (34)$$

where the index i denotes a type of bilinears, i.e., scalar, vector, pseudoscalar, and so on. The one-particle state for a heavy meson is defined as $|\phi(v)\rangle \equiv \phi_v|0\rangle$. The form factor $F(p_q)$ encodes the quark-instanton dynamics in the

light-quark sector, which also stabilizes the wave packet. A diagrammatic form of the normalization is shown in Fig. 2.

It is straightforward to show that Eq. (34) satisfies the nonrelativistic normalization condition [3],

$$\langle \phi_i(v') | \phi_j(v) \rangle = \int \frac{dp_{\Phi 4}}{2\pi} \text{tr}_D \Phi_{vj}(p_\Phi) \Phi_{vi}^\dagger(p'_\Phi) \\ = 2v_4 g^2 \mathcal{N}^2 (2\pi)^3 \delta^3(\Lambda v' - \Lambda v) \int \frac{d^4 p_q}{(2\pi)^4} F^2(p_q) \text{tr}_D [S_q(p_q) \Gamma_i S_Q(p_\Phi - p_q) \Gamma_j] \\ = 2v_4 g^2 \mathcal{N}^2 \Pi(i\Lambda) (2\pi)^3 \delta^3(\Lambda v' - \Lambda v) \delta_{ij} = 2v_4 (2\pi)^3 \delta^3(\Lambda v' - \Lambda v) \delta_{ij}, \quad (35)$$

where we have replaced $\Lambda_r \rightarrow \Lambda$ for simplicity, and the functions S_q and S_Q denote the light and heavy quark propagators given in Eq. (31), as shown in Fig. 2. The normalization yields the normalization condition,

$g^2 \mathcal{N}^2 \Pi(i\Lambda) = 1$. The physical residual mass Λ is determined by the intersection of the compositeness and normalization conditions, as shown in Fig. 3. The result for Λ is $\Lambda \approx 262.8 \text{ MeV}$.

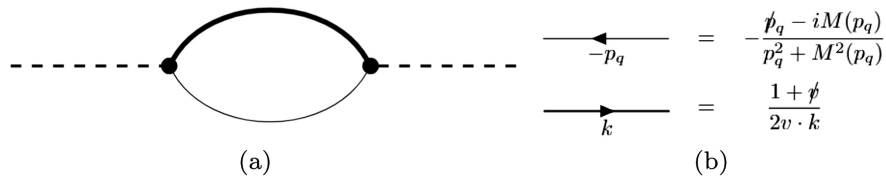


FIG. 2. (a) Diagrammatic form of the normalization for heavy meson states [47,48]. (b) Diagrammatic representations of the propagator of light and heavy quarks are $S_q(p_q)$ and $S_Q(k)$, respectively.

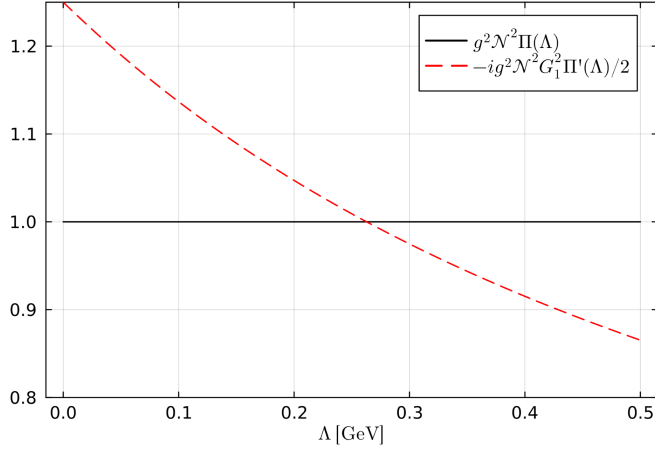


FIG. 3. The black solid line and red dashed curve draw the normalization condition and the renormalization condition with the rescaled function $\Pi(\Lambda)$ by the normalization constant $\mathcal{N}(\Lambda)$, respectively. The intersection point yields the physical residual mass, $\Lambda = 0.2628$ GeV.

Λ is typically considered to be on the order of $\Lambda \sim \Lambda_{\text{QCD}}$ [49], approximately 200–300 MeV, as it arises from heavy-light quark interactions in QCD. The current approach, based on the instanton vacuum, explicitly demonstrates this origin of Λ . This suggests that the residual mass quantifies the contribution of heavy-light quark interactions to a heavy meson mass, m_Φ . Instanton effects are known to generate dynamical quarks. We can decompose m_Φ as $m_\Phi = m_Q + \Delta M_Q + M_q + \Lambda$, where m_Q represents the charm (bottom) quark mass, ΔM_Q denotes instanton effects on the heavy-quark mass, and M_q signifies the dynamical quark mass emerging from $\text{SB}\chi\text{S}$. The final term, Λ , the physical residual mass, is evaluated through the heavy-light quark mass. Consequently, we determine the D and B meson masses to be $m_D = 1.942$, GeV, ($m_c = 1.27$, GeV) [50] and $m_B = 5.322$, GeV, (m_b (1S scheme) = 4.65, GeV) [50], respectively. These values approximate the average masses of pseudoscalar (D , B) and vector (D^* , B^*) mesons. In the $m_Q \rightarrow \infty$ limit, the masses of the pseudoscalar and vector heavy mesons are degenerate due to heavy quark-spin symmetry. This degeneracy is shifted by the $1/m_Q$ correction given in Eq. (11). Its mesonic matrix element can be expressed as

$$\frac{1}{4m_Q} \langle \phi_{\vec{J}} | h^\dagger \sigma_{\mu\nu} h G_{\mu\nu} | \phi_{\vec{J}} \rangle = \frac{1}{4m_Q} \langle h; \vec{S}_Q | h^\dagger \sigma_{\mu\nu} h | h; \vec{S}_Q \rangle$$

$$\langle q; \vec{S}_q | G_{\mu\nu} | q; \vec{S}_q \rangle = \frac{\lambda_2}{m_Q} \langle 2\vec{S}_Q \cdot \vec{S}_q \rangle, \quad (36)$$

where the states $|\phi_{\vec{J}}\rangle \equiv |\phi(v); \vec{J}; \vec{S}_Q; \vec{S}_q\rangle$ with the total spin $\vec{J} = \vec{S}_Q + \vec{S}_q$, and $|h(q); \vec{S}_{Q(q)}\rangle$ stand for each quark and its spin state. The eigenvalue of the heavy-light spin product is given as

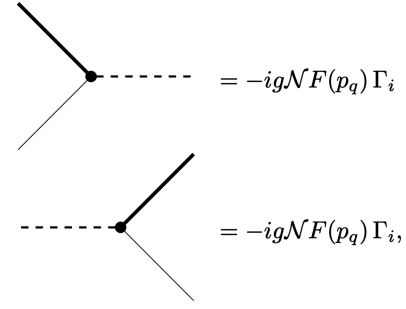


FIG. 4. Vertices modified by instanton effects. Γ_i are generically expressed as $\Gamma_V = -\gamma_\mu \epsilon_\mu$ and $\Gamma_P = i\gamma_5$. Here, ϵ_μ represents the polarization vector for the vector heavy meson.

$$s_{Qq} \equiv \langle 2\vec{S}_Q \cdot \vec{S}_q \rangle = J(J+1) - S_Q(S_Q+1) - S_q(S_q+1)$$

$$= \begin{cases} -3/2 & \text{pseudoscalar} \\ 1/2 & \text{vector} \end{cases}. \quad (37)$$

To obtain λ_2 , we have to compute the mesonic matrix element of the gluon operators from the instanton vacuum [54], which will be considered in future work.

VI. WEAK DECAY CONSTANTS OF D AND B MESONS

In the previous section, we presented the heavy-meson states and the residual mass from the instanton vacuum. Here, we will use the redefined vertex diagrams that incorporate the instanton effects, as illustrated in Fig. 4. These modified vertices encapsulate the nonperturbative effects of the instanton vacuum. In the limit of infinitely heavy quark mass, the weak decay constants of heavy mesons are defined in terms of axial-vector and vector currents:

$$\langle 0 | J_\mu^A | P \rangle = \langle 0 | q^\dagger \gamma_\mu \gamma_5 h | P \rangle = -f_P \sqrt{m_P} v_\mu,$$

$$\langle 0 | J_\mu^V | V \rangle = \langle 0 | q^\dagger \gamma_\mu h | V \rangle = if_V \sqrt{m_V} \epsilon_\mu, \quad (38)$$

where $|P\rangle$ and $|V\rangle$ denote the pseudoscalar and vector heavy mesons, respectively. To determine the $1/m_Q$ correction to the decay constants, we consider the heavy-quark field $Q(x)$ defined in Eq. (8). We can express $Q(x)$ as

$$Q(x) = e^{-im_Q v \cdot x} \left(1 - \frac{1}{2m_Q} \not{D}_\perp + \mathcal{O}(m_Q^{-2}) \right) h(x), \quad (39)$$

which leads to the following expressions for the vector (axial-vector) current:

$$J_\mu^{V(A)}(0) \simeq q^\dagger \gamma_\mu (\gamma_5) h - \frac{1}{2m_Q} q^\dagger \gamma_\mu (\gamma_5) \not{D}_\perp h + \mathcal{O}(m_Q^{-2}). \quad (40)$$

Considering that the matrix elements also depend on m_Q , we must account for the $1/m_Q$ corrections to the matrix elements given in Eq. (38) [46]:

$$\begin{aligned}
& \langle 0 | J_\mu^{V(A)} | V(P)(v) \rangle \\
&= \langle 0 | q^\dagger \gamma_\mu (\gamma_5) h | V(P)(v) \rangle \\
&\quad - \frac{1}{2m_Q} \langle 0 | q^\dagger \gamma_\mu (\gamma_5) \not{D}_\perp h | V(P)(v) \rangle \\
&\quad - \frac{1}{2m_Q} \int d^4 y \langle 0 | T \{ q^\dagger \gamma_\mu (\gamma_5) h(0), h^\dagger i D_\perp^2 h(y) \} | V(P)(v) \rangle \\
&\quad + \dots.
\end{aligned} \tag{41}$$

This expression is derived by modifying the heavy meson states as follows:

$$| \phi_v \rangle \rightarrow \left(1 - \frac{1}{2m_Q} \int d^4 x T \{ \mathcal{L}_Q^{1/m_Q}(x) \} + \dots \right) | \phi_v \rangle, \tag{42}$$

where $\mathcal{L}_Q^{1/m_Q}$ is the next-to-leading order (NLO) term of Eq. (11). The second term of $\mathcal{L}_Q^{1/m_Q}$ gives the spin-spin interaction term. After performing the instanton ensemble average, we obtain the numerical results for the matrix elements of gluon operators as

$$\begin{aligned}
\langle A_{I\mu} \rangle &= 0, \quad \langle G_{\mu\nu} \rangle = 0, \quad \langle A_I^2 \rangle = \frac{24\pi^2 \rho^2 n N_c^2}{N_c^2 - 1}, \\
\langle A_{I4}^2 \rangle &= \frac{1}{4} \langle A_I^2 \rangle,
\end{aligned} \tag{43}$$

where the instanton field $A_{I\mu}$ [29] and the strength tensor $G_{\mu\nu}$ are defined as

$$\begin{aligned}
A_{I\mu}(x) &= \eta_{\mu\nu a}^{(-)} \tau^a \frac{\rho^2 x_\nu}{x^2(x^2 + \rho^2)}, \\
G_{\mu\nu} &= \partial_\mu A_{I\nu} - \partial_\nu A_{I\mu} - i[A_\mu, A_\nu]
\end{aligned} \tag{44}$$

with the 't Hooft symbol $\eta_{\mu\nu a}^{(\pm)} = \epsilon_{a\mu\nu 4} \pm \delta_{a\mu} \delta_{4\nu} \mp \delta_{a\nu} \delta_{4\mu}$.

Since the heavy-light vector and axial-vector currents are not conserved, we need to renormalize them [46,49,51–53]:

$$\begin{aligned}
J_\mu^{V(A)} &= \sum_i C_i(\mu_0) \left(J_\mu^{V(A)(i)} - \frac{1}{2m_Q} \left(O_\mu^{V(A)(i)} + T_\mu^{V(A)(i)} \right) \right) \\
&\quad + \mathcal{O}\left(\frac{1}{m_Q^2}\right),
\end{aligned} \tag{45}$$

where $C_i(\mu_0)$ denote the Wilson coefficients, expressed as

$$C_1(\mu_0) = 1 + \frac{\alpha_s(\mu_0)}{\pi} \left(\ln \frac{\mu}{\mu_0} - \frac{4}{3} \right), \quad C_2(\mu_0) = -\frac{2\alpha_s(\mu_0)}{3\pi}. \tag{46}$$

Here, μ_0 is the normalization point from the current work. While, in the present approach, the value of the normalization point μ_0 is not uniquely determined from first principles, we can set it to be μ_0 approximately by $\mu_0 \approx \bar{\rho}^{-1}$. We can examine the plausibility of this value, following Refs. [15,54]. We define $\mu_0 = a/\bar{R}$ with $\bar{R}^{-1} \approx 200$ MeV. Here, a is a dimensionless parameter varied from 3 to 7 [54] in the variational estimate of bulk properties of the instanton medium. μ is the renormalization point at the matching scale. $J_\mu^{V(A)(i)}$ in Eq. (45) represent the leading-order currents defined as

$$J_\mu^{V(A)(1)} = q^\dagger \gamma_\mu (\gamma_5) h, \quad J_\mu^{V(A)(2)} = q^\dagger v_\mu (\gamma_5) h. \tag{47}$$

$O_\mu^{V(A)(i)}$ and $T_\mu^{V(A)(i)}$ stand for the currents arising from the NLO in the $1/m_Q$ expansion,

$$\begin{aligned}
O_\mu^{V(A)(1)} &= q^\dagger \gamma_\mu (\gamma_5) \not{D}_\perp h, \quad O_\mu^{V(A)(2)} = q^\dagger v^\mu (\gamma_5) \not{D}_\perp h, \\
T_\mu^{V(A)(1)} &= \int d^4 y \left\{ J_\mu^{V(A)(1)}(0), h^\dagger(y) i D_\perp^2 h(y) \right\}, \quad T_\mu^{V(A)(2)} = \int d^4 y \left\{ J_\mu^{V(A)(2)}(0), h^\dagger(y) i D_\perp^2 h(y) \right\}.
\end{aligned} \tag{48}$$

Note that

$$\langle 0 | J_\mu^{V(A)} | P(v) \rangle = -C_{12} F_{V(P)}^{(0)} v_\mu + \frac{C_{12}}{2m_Q} (F_{V(P),1}^{(1/m_Q)} + F_{V(P),2}^{(1/m_Q)}) v_\mu = -C_{12} F_P v_\mu, \tag{49}$$

where $C_{12} \equiv C_1(\mu_0) - C_2(\mu_0)$.

We first compute the decay constants for the pseudoscalar and vector heavy mesons from the leading-order currents. The results are obtained to be

$$\sum_i C_i(\mu_0) \langle 0 | J_\mu^{A(i)} | P \rangle = i C_{12} g \mathcal{N} \int \frac{d^4 p_q}{(2\pi)^4} F(p_q) \text{Tr}[\gamma_\mu \gamma_5 S_Q(i\Lambda - v \cdot p_q) \Gamma_P S_q(p_q)] = -C_{12} F_P^{(0)} v_\mu, \tag{50}$$

$$\sum_i C_i(\mu_0) \langle 0 | J_\mu^{V(i)} | V \rangle = i C_{12} g \mathcal{N} \int \frac{d^4 p_q}{(2\pi)^4} F(p_q) \text{Tr}[\gamma_\mu S_Q(i\Lambda - v \cdot p_q) \Gamma_V S_q(p_q)] = i C_{12} F_V^{(0)} \epsilon_\mu, \tag{51}$$

TABLE I. Total results for the heavy meson decay constants and masses of the D and B mesons in comparison with recent data from lattice QCD [55] and QCD sum rules [56]. In the last column, PDG average values [50] are given. The normalization point in the present work is given in the range of $\mu_0 = (0.6^{(+)} - 1.4^{(-)})$ GeV.

	This work	Lattice QCD [55]	QCD sum rule [56]	PDG average [50]
f_D [MeV]	$204.6^{+13.6}_{-4.5}$	209.0(2.4)	190(15)	203.8(4.7)(0.6)(1.4)
f_B [MeV]	$191.2^{+30.0}_{-8.4}$	192.0(4.3)	192^{+20}_{-15}	190.0(1.3)

where S_Q and S_q stand for the heavy-quark and light-quark propagator in Fig. 2, respectively. The explicit calculation yields the same results for $F_P^{(0)}$ and $F_V^{(0)}$,

$$F_P^{(0)} = F_V^{(0)} = \frac{4\pi^2 g \mathcal{N}}{(2\pi)^4} \int_0^\infty dp_q F(p_q) \frac{p_q [p_q^2 + 2(\Lambda + M_q(p_q))(\Lambda - \sqrt{p_q^2 + \Lambda^2})]}{p_q^2 + M_q^2(p_q)}. \quad (52)$$

As mentioned previously, the $N_f = 1 + 1$ heavy-light quark interactions do not remove the degeneracy due to the heavy-quark spin symmetry even with $1/m_Q$ corrections. The improvement can only be achieved by considering the $N_f = 2 + 1$ interactions. We will leave it as a future work.

We evaluate the decay constants for the D and B mesons to be in the range of

$$f_D = \frac{C_{12} F_D}{\sqrt{m_D}} = (295.0 - 322.6) \text{ MeV}, \quad f_B = \frac{C_{12} F_B}{\sqrt{m_B}} = (200.5 - 242.6) \text{ MeV}, \quad (53)$$

where we have used the value of μ_0 from 1.4 GeV to 0.6 GeV, respectively. If we use $\mu_0 = 1$ GeV, we obtain the decay constants as

$$f_D = 302.4 \text{ MeV}, \quad f_B = 209.8 \text{ MeV}. \quad (54)$$

We now compute the $1/m_Q$ corrections to the decay constants:

$$\begin{aligned}
F_{P,1}^{(1/m_Q)} &= g \mathcal{N} \int dp_q F(p_q) \left[\frac{p_q^3 (\Lambda - \sqrt{p_q^2 + \Lambda^2})}{2\pi^2 [p_q^2 + M_q^2(p_q)]} + \frac{p_q \Lambda \{p_q^2 + 2\Lambda(\Lambda - \sqrt{p_q^2 + \Lambda^2})\}}{4\pi^2 [p_q^2 + M_q^2(p_q)]} \right], \\
F_{P,2}^{(1/m_Q)} &= -g \mathcal{N} \int_0^\infty dp_q F(p_q) \frac{p_q}{4\pi^2 (p_q^2 + M_q(p_q)^2)} \\
&\quad \times \left[2 \left\{ p_q^2 (3\Lambda - \sqrt{p_q^2 + \Lambda^2}) + 4\Lambda^2 (\Lambda - \sqrt{p_q^2 + \Lambda^2}) \right\} + 3M_q(p_q) \left\{ p_q^2 + 2\Lambda (\Lambda - \sqrt{p_q^2 + \Lambda^2}) \right\} \right] \\
&\quad + 2g \mathcal{N} \frac{9\pi^2 p^2 N_c N}{(N_c^2 - 1)V} \int_0^\infty dp_q \frac{F(p_q) p_q (p_q^2 - (2\Lambda + M_q(p_q))(\sqrt{\Lambda^2 + p_q^2} - \Lambda))}{4\pi^2 (p_q^2 + M_q^2(p_q)) \sqrt{p_q^2 + \Lambda^2}}. \quad (55)
\end{aligned}$$

In Table I, we list the total results for the heavy meson decay constants, f_D and f_B , obtained at the normalization point given in the range of $\mu_0 = (0.6^{(+)} - 1.4^{(-)})$ GeV. The results are in good agreement with the Particle Data Group (PDG) average values [50]. We also compare the current results with those from lattice QCD [55] and QCD sum rules [56].

VII. SUMMARY AND OUTLOOK

In this work, we aimed at developing an approach to heavy-light quark systems based on the QCD instanton vacuum. We first derived an effective heavy-light quark interaction for the $N_f = 1 + 1$ case, providing a connection between light and heavy quark dynamics. By employing

the compositeness and normalization conditions, we determined the physical residual mass of heavy mesons to be $\Lambda \approx 262.8$ MeV. This allowed us to calculate the masses of D and B mesons, obtaining $m_D = 1.942$ GeV and $m_B = 5.322$ GeV, which approximate the average masses of pseudoscalar and vector heavy mesons. We then computed the weak decay constants for the D and B mesons to the leading order and next-to-leading order in the $1/m_Q$ expansion. Considering the normalization point of the current approach in the range of $\mu_0 = (0.6\text{--}1.4)$ GeV, we obtained $f_D = (204.6^{+13.6}_{-4.5})$ MeV and $f_B = (191.2^{+30.0}_{-8.4})$ MeV. The present results are in good agreement with recent lattice QCD results and PDG average values. The current study establishes a foundation for further investigations within the instanton vacuum framework. Future directions include extending the analysis to the $N_f = 2 + 1$ case to examine spin-spin interactions and their role in lifting the degeneracy between pseudoscalar and vector heavy mesons. The heavy-light quark interactions derived from the instanton vacuum may shed light on the nonperturbative structure of the heavy mesons and illustrate the interplay between heavy quark

symmetry and chiral dynamics in more complex heavy-light quark systems such as tetraquark hadrons.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: INTEGRATION IN COLOR SPACE

In this appendix, we compile the formulae for the integration over color orientation:

$$\begin{aligned} \int dU &= 1, \quad \int dU U_{\beta_1}^{\alpha_1} U_{\beta_2}^{\dagger \alpha_2} = \frac{1}{N_c} \delta_{\beta_2}^{\alpha_1} \delta_{\beta_1}^{\alpha_2}, \\ \int dU U_{\gamma_1}^{\alpha_1} U_{\beta_1}^{\dagger \delta_1} U_{\gamma_2}^{\alpha_2} U_{\beta_2}^{\dagger \delta_2} &= \frac{1}{N_c^2} \delta_{\beta_1}^{\alpha_1} \delta_{\delta_1}^{\alpha_2} \delta_{\beta_2}^{\delta_2} + \frac{1}{4(N_c^2 - 1)} [\lambda^c]_{\beta_1}^{\alpha_1} [\lambda^d]_{\delta_1}^{\alpha_2} [\lambda^c]_{\beta_2}^{\alpha_2} [\lambda^d]_{\gamma_2}^{\delta_2} \\ &= \frac{1}{N_c^2 - 1} \left[\delta_{\beta_1}^{\alpha_1} \delta_{\beta_2}^{\alpha_2} \left(\delta_{\gamma_1}^{\delta_1} \delta_{\gamma_2}^{\delta_2} - \frac{1}{N_c} \delta_{\gamma_2}^{\delta_1} \delta_{\gamma_1}^{\delta_2} \right) + \delta_{\beta_2}^{\alpha_1} \delta_{\beta_1}^{\alpha_2} \left(\delta_{\gamma_2}^{\delta_1} \delta_{\gamma_1}^{\delta_2} - \frac{1}{N_c} \delta_{\gamma_1}^{\delta_1} \delta_{\gamma_2}^{\delta_2} \right) \right], \end{aligned} \quad (\text{A1})$$

where Fierz identity for color matrices,

$$[\lambda^c]_{\beta_1}^{\alpha_1} [\lambda^c]_{\beta_2}^{\alpha_2} = 2 \left(\delta_{\beta_1}^{\alpha_1} \delta_{\beta_2}^{\alpha_2} - \frac{1}{N_c} \delta_{\beta_2}^{\alpha_1} \delta_{\beta_1}^{\alpha_2} \right), \quad (\text{A2})$$

are used.

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- [1] N. Isgur and M. B. Wise, *Phys. Lett. B* **232**, 113 (1989).
 - [2] H. Georgi, *Phys. Lett. B* **240**, 447 (1990).
 - [3] H. Georgi, in *Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 91): Perspectives in the Standard Model*, Boulder, CO (1991).
 - [4] M. B. Wise, in *Proceedings of the CCAST Symposium on Particle Physics at the Fermi Scale* (1993), pp. 71–114, [arXiv:hep-ph/9306277](https://arxiv.org/abs/hep-ph/9306277).
 - [5] E. V. Shuryak, *Nucl. Phys. B* **203**, 93 (1982).
 - [6] D. Diakonov and V. Y. Petrov, *Nucl. Phys. B* **245**, 259 (1984).
 - [7] D. Diakonov and V. Y. Petrov, *Nucl. Phys. B* **272**, 457 (1986).
 - [8] T. Schäfer and E. V. Shuryak, *Rev. Mod. Phys.* **70**, 323 (1998).
 - [9] D. Diakonov, *Prog. Part. Nucl. Phys.* **51**, 173 (2003).
 - [10] E. V. Shuryak, *Nucl. Phys. B* **203**, 140 (1982).
 - [11] M. C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, *Phys. Rev. D* **49**, 6039 (1994).
 - [12] P. de Forcrand, M. Garcia Perez, and I.-O. Stamatescu, *Nucl. Phys. B* **499**, 409 (1997).
 - [13] T. A. DeGrand, A. Hasenfratz, and T. G. Kovacs, *Nucl. Phys. B* **505**, 417 (1997).
 - [14] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys. B* **163**, 46 (1980).

- [15] H.-C. Kim, M. V. Polyakov, and K. Goeke, *Phys. Rev. D* **53**, R4715 (1996).
- [16] D. Diakonov, V. Y. Petrov, and P. V. Pobylitsa, *Nucl. Phys.* **B306**, 809 (1988).
- [17] M. M. Musakhanov and H.-C. Kim, *Phys. Lett. B* **572**, 181 (2003).
- [18] S.-i. Nam and H.-C. Kim, *Phys. Rev. D* **77**, 094014 (2008).
- [19] S.-i. Nam and H.-C. Kim, *Phys. Rev. D* **75**, 094011 (2007).
- [20] S.-i. Nam and H.-C. Kim, *Phys. Lett. B* **707**, 546 (2012).
- [21] H.-D. Son, S.-i. Nam, and H.-C. Kim, *Phys. Lett. B* **747**, 460 (2015).
- [22] S.-I. Shim and H.-C. Kim, *Phys. Lett. B* **772**, 687 (2017).
- [23] S.-I. Shim, A. Hosaka, and H.-C. Kim, *Phys. Lett. B* **795**, 438 (2019).
- [24] C. V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, *Prog. Part. Nucl. Phys.* **37**, 91 (1996).
- [25] D. Diakonov, in *Proceedings of Advanced Summer School on Nonperturbative Quantum Field Physics* (1997), pp. 1–55, [arXiv:hep-ph/9802298](#).
- [26] G.-S. Yang, H.-C. Kim, M. V. Polyakov, and M. Praszalowicz, *Phys. Rev. D* **94**, 071502 (2016).
- [27] J.-Y. Kim, H.-C. Kim, G.-S. Yang, and M. Oka, *Phys. Rev. D* **103**, 074025 (2021).
- [28] C. G. Callan, Jr., R. F. Dashen, D. J. Gross, F. Wilczek, and A. Zee, *Phys. Rev. D* **18**, 4684 (1978).
- [29] D. Diakonov, V. Y. Petrov, and P. V. Pobylitsa, *Phys. Lett. B* **226**, 372 (1989).
- [30] U. T. Yakhshiev, H.-C. Kim, M. M. Musakhanov, E. Hiyama, and B. Turimov, *Chin. Phys. C* **41**, 083102 (2017).
- [31] U. Yakhshiev, H.-C. Kim, and E. Hiyama, *Phys. Rev. D* **98**, 114036 (2018).
- [32] U. Yakhshiev, *J. Korean Phys. Soc.* **79**, 357 (2021).
- [33] K.-H. Hong, H.-C. Kim, and U. Yakhshiev, *Prog. Theor. Exp. Phys.* **2022**, 103D02 (2022).
- [34] M. Musakhanov, N. Rakhimov, and U. T. Yakhshiev, *Phys. Rev. D* **102**, 076022 (2020).
- [35] M. Musakhanov and U. Yakhshiev, *Int. J. Mod. Phys. E* **30**, 2141005 (2021).
- [36] M. Musakhanov, *Eur. Phys. J. C* **9**, 235 (1999).
- [37] K. Goeke, M. M. Musakhanov, and M. Siddikov, *Phys. Rev. D* **76**, 076007 (2007).
- [38] D. Diakonov and V. Petrov, in *Quark Cluster Dynamics* (Springer, Berlin, Heidelberg, 1993), pp. 288–297.
- [39] D. Diakonov, *Proc. Int. Sch. Phys. “Enrico Fermi”* **130**, 397 (1996).
- [40] P. V. Pobylitsa, *Phys. Lett. B* **226**, 387 (1989).
- [41] H.-C. Kim, M. Musakhanov, and M. Siddikov, *Phys. Lett. B* **608**, 95 (2005).
- [42] K. Goeke, H.-C. Kim, M. M. Musakhanov, and M. Siddikov, *Phys. Rev. D* **76**, 116007 (2007).
- [43] K.-i. Shizuya, *Phys. Rev. D* **21**, 2327 (1980).
- [44] H.-Y. Cheng, C.-Y. Cheung, and W.-M. Zhang, *Phys. Rev. D* **58**, 074003 (1998).
- [45] D. Lurié and A. J. Macfarlane, *Phys. Rev.* **136**, B816 (1964).
- [46] M. Neubert, *Phys. Rep.* **245**, 259 (1994).
- [47] H.-Y. Cheng, C.-Y. Cheung, C.-W. Hwang, and W.-M. Zhang, *Phys. Rev. D* **57**, 5598 (1998).
- [48] C.-W. Hwang, *Eur. Phys. J. C* **46**, 379 (2006).
- [49] A. F. Falk, M. Neubert, and M. E. Luke, *Nucl. Phys.* **B388**, 363 (1992).
- [50] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [51] M. Neubert, *Phys. Rev. D* **46**, 1076 (1992).
- [52] A. F. Falk and B. Grinstein, *Phys. Lett. B* **247**, 406 (1990).
- [53] A. V. Manohar and M. B. Wise, in *Heavy Quark Physics*, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology (Cambridge University Press, Cambridge, England, 2000).
- [54] D. Diakonov, M. V. Polyakov, and C. Weiss, *Nucl. Phys.* **B461**, 539 (1996).
- [55] Y. Aoki *et al.* (Flavour Lattice Averaging Group (FLAG)), *Eur. Phys. J. C* **82**, 869 (2022).
- [56] B. Pullin and R. Zwicky, *J. High Energy Phys.* **09** (2021) 023.