

Capped black hole in five dimensions

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We present the first non-BPS (Bogomol'nyi-Prasad-Sommerfield) exact solution of an asymptotically flat, stationary spherical black hole having domain of outer communication with nontrivial topology in five-dimensional minimal supergravity. It describes a charged rotating black hole capped by a disk-shaped bubble. The existence of the “capped black hole” shows the nonuniqueness of spherical black holes.

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According to the uniqueness theorem for charged rotating black holes in the bosonic sector of five-dimensional minimal supergravity [1], under the assumptions of the existence of two commuting axial isometries and spherical topology of horizon cross sections, an asymptotically flat, stationary charged rotating black hole with a nonextremal horizon is uniquely characterized by the mass, charge, and two independent angular momenta and therefore is described by the five-dimensional Cvetič-Youm solution [2]. Therefore, it seems that there is no room for another spherical black hole in a class of asymptotically flat, regular solutions with the same symmetry. However, the topological censorship theorem [3] gives us the possibility of another black hole with spherical topology, since in the uniqueness theorem [1], the intersection of the exterior region of a black hole and the time slice Σ is assumed to have the trivial topology $\mathbb{R}^4 \setminus \mathbb{B}^4$.

The topological censorship theorem says that the domain of outer communication (DOC) must be simply connected. This implies that, for four dimensions, the topology of the black hole exterior region admits a trivial topology of $\text{DOC} \cap \Sigma = \mathbb{R}^3 \setminus \mathbb{B}^3$, whereas for higher dimensions, the DOC can have nontrivial topologies in the sense that the time slice can have homology groups of a higher rank. Even in higher dimensions, for static asymptotically flat spacetimes, a black hole solution with such nontrivial DOC does not exist since the uniqueness theorems [4,5] say that the higher-dimensional Schwarzschild and

Reissner-Nordström solutions [6] are the only vacuum black hole solution and the charged black hole solution, respectively. Therefore, if such a solution exists, it must not lie in a class of static but stationary solutions. In fact, Kunduri and Lucietti [7] have constructed a four parameter family of a supersymmetric black hole solution such that it has a spherical horizon topology and two bubbles of a two-cycle in the DOC in the five-dimensional minimal supergravity.

In this Letter, we show the existence of an exact solution of an asymptotically flat, stationary, nonsupersymmetric black hole with the horizon cross section of trivial topology S^3 and DOC of nontrivial topology in the bosonic sector of the five-dimensional minimal supergravity. As shown below, the horizon looks like it is being capped by a disk-shaped bubble [8], and so we call the black hole a capped black hole. Our solution is regular in the sense that it does not have any one of the curvature singularities, conical singularities, Dirac-Misner string singularities, or orbifold singularities on and outside the horizon and, in addition, is free from closed timelike curves (CTCs). This is the first example of such a black hole as a non-Bogomol'nyi-Prasad-Sommerfield (BPS) solution.

The action of the bosonic sector in the five-dimensional minimal supergravity is given by

$$S = \frac{1}{16\pi G_5} \int \left[R \star 1 - \frac{1}{2} F \wedge \star F - \frac{1}{3\sqrt{3}} F \wedge F \wedge A \right], \quad (1)$$

where $F := dA$. Applying the Harrison transformation of Eq. (119) in Ref. [9] to the vacuum rotating black lens constructed in Ref. [10] (see also [11]), we can obtain the metric and gauge potential for a capped black hole, respectively, as (see Supplemental Material [12])

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$$\begin{aligned}
ds^2 = & -D^{-2}H(y, x)H^{-1}(x, y)(dt + \Omega'_\psi d\psi + \Omega'_\phi d\phi)^2 \\
& + DH^{-1}(y, x)[F(y, x)d\psi^2 - 2J(x, y)d\psi d\phi \\
& - F(x, y)d\phi^2] \\
& + \frac{\ell^2 DH(x, y)}{4(1-\gamma)^3(1-\nu)^2(1-a^2)(x-y)^2} \left[\frac{dx^2}{G(x)} - \frac{dy^2}{G(y)} \right], \quad (2)
\end{aligned}$$

where $(c, s) := (\cosh \alpha, \sinh \alpha)$ with the parameter α corresponding to the electric charge. The functions D and $(\Omega'_\psi, \Omega'_\phi)$ are given by

$$D = H^{-1}(x, y)(c^2 H(x, y) - s^2 H(y, x)), \quad (4)$$

$$\Omega'_\psi = c^3 \Omega_\psi(x, y) - s^3 \Omega_\phi(y, x), \quad (5)$$

$$\Omega'_\phi = c^3 \Omega_\phi(x, y) - s^3 \Omega_\psi(y, x). \quad (6)$$

$$\begin{aligned}
A = & \sqrt{3}csD^{-1}H^{-1}(x, y)[\{H(x, y) - H(y, x)\}dt \\
& - \{cH(y, x)\Omega_\psi(x, y) - sH(x, y)\Omega_\phi(y, x)\}d\psi \\
& - \{cH(y, x)\Omega_\phi(x, y) - sH(x, y)\Omega_\psi(y, x)\}d\phi], \quad (3)
\end{aligned}$$

Remaining functions $(G, H, F, J, \Omega_\phi, \Omega_\psi)$ are written as

$$G(\xi) = (1 - \xi^2)(1 + \nu\xi), \quad (7)$$

and

$$\begin{aligned}
H(x, y) = & [\nu b^2 c_1^2 (1 + \nu)^2 (\gamma - \nu)(1 - \gamma)(1 - x^2) + \nu(1 - \gamma)(\gamma - \nu)(b(1 - \nu)^2(1 - x) - 2c_1(1 + \nu x))^2 \\
& + b^2 c_1^2 (1 + \nu)^3 (x + 1)(\gamma - \nu)^2](1 + y)^2 - [d_5(1 - x)^2 + d_6(1 - x^2) + d_7(1 + x)(1 + \nu x)](1 + y) \\
& + 4(1 - a^2)(1 - \gamma)^3(1 - \nu)^4(1 - x) + 2c_2^2(1 - \gamma)(1 - \nu)^2(1 - x^2) - 4d_1(1 - \gamma)(1 - \nu)^2(1 + \nu)(1 + x), \quad (8)
\end{aligned}$$

$$\begin{aligned}
F(x, y) = & 2\ell^2(1 - a^2)^{-1}(x - y)^{-2}[4\{(1 - a^2)^2(y - 1)(1 - \gamma)^3(1 - \nu)^3 - (1 + y)d_1^2\}(1 + y\nu)G(x) \\
& + \nu^{-1}(1 - \gamma)^{-1}G(x)G(y)\{(1 - a^2)(1 - \gamma)d_4 - \nu c_2^2(a - b)^2(1 - \gamma)^2(\gamma - \nu)y + \nu x(\gamma - \nu)(c_1 c_3 - b d_1)^2\} \\
& + 4\{(1 - \nu)c_2 - (1 - ab)(\gamma - \nu)(1 + \nu)c_1\}^2(1 + x\nu)(1 + x)G(y) \\
& + \nu^{-1}(1 - \nu)^3(\gamma - \nu)\{d_3^2(1 - x^2)G(y) - c_3^2(1 - y^2)G(x)\}], \quad (9)
\end{aligned}$$

$$\begin{aligned}
J(x, y) = & 2\ell^2(1 + x)(1 + y)(1 - a^2)^{-1}(x - y)^{-1}[4d_1\{(a - b)(1 - \gamma)(\gamma - \nu)(1 + \nu) - a d_2\}(1 + \nu x)(1 + \nu y) \\
& - d_3 c_3(1 - \nu)^3(\gamma - \nu)(1 - x)(1 - y) - c_2(a - b)(\gamma - \nu)(c_1 c_3 - b d_1)(1 - x)(1 - y)(1 + \nu x)(1 + \nu y)], \quad (10)
\end{aligned}$$

$$\begin{aligned}
\Omega_\psi(x, y) = & v_0 \ell (1 + y)(1 - \nu) \nu^{-1} H^{-1}(y, x)[c_2(c_1 c_3 - b d_1)(1 - x)(1 + x\nu)(1 + y\nu) \\
& - d_2 c_3(1 - \nu)^2(1 - x) + d_1(1 + x\nu)\{2\nu(1 - ab)(1 - \gamma)(1 + \nu)(1 + x) + c_3(1 - 3\nu - x(1 + \nu))\}], \quad (11)
\end{aligned}$$

$$\begin{aligned}
\Omega_\phi(x, y) = & v_0 \ell (1 + x) H^{-1}(y, x)[d_1 b(1 + x)\{d_2(1 + y)(1 + y\nu) + \nu c_3(1 - y^2)(1 - \nu)\} \\
& + 2(a - b)(1 + \nu)^{-1}(1 - \gamma)^2\{2d_1(1 + x\nu)(1 + y\nu)^2 - \nu c_3(1 - \nu)^2(1 - y)(x + y + \nu + xy\nu)\}], \quad (12)
\end{aligned}$$

where the constants $v_0, c_p, d_q (p = 1, 2, 3, q = 1, \dots, 7)$ are given by

$$\begin{aligned}
v_0 &:= \sqrt{2(\gamma^2 - \nu^2)(1 - a^2)^{-1}(1 - \gamma)^{-1}}, \quad c_1 := a(1 - \gamma) + b(\gamma - \nu), \quad c_2 := 2a\nu(1 - \gamma) + b(\gamma - \nu)(1 + \nu), \\
c_3 &:= 2\nu(1 - \gamma) + b^2(\gamma - \nu)(1 + \nu), \quad d_1 := c_1^2(\nu + 1) - (1 - \gamma)(1 - \nu)^2, \\
d_2 &:= bc_1(\nu + 1)(\gamma - \nu) + 2\nu(1 - \gamma)(1 - \nu), \quad d_3 := b(1 - a^2)(1 - \gamma)(\nu + 1) - ac_3, \\
d_4 &:= b^2(\gamma - \nu)[c_1^2(\nu + 1)^2(1 - \nu^2 - 3(1 - \gamma)\nu) - (1 - \gamma)(1 - \nu)^4(2\nu + 1)] + (1 - \gamma)[(c_2(1 - \nu) - 2\nu^2 c_1)^2 \\
& - 4\nu^2 c_1^2(3\nu^2 + 1 - \gamma(\nu + 2))], \\
d_5 &:= (1 - \gamma)(1 - \nu)^3[(\gamma - 3\nu)(b^2(\nu - 1)(\gamma - \nu) - c_1^2) - 2bc_1(3\nu - 1)(\gamma - \nu)], \\
d_6/(1 - \nu) &:= d_7/2 := c_1(1 - \gamma)(1 - \nu^2)(c_2 - (1 - \gamma)(a - b)(\gamma - \nu)). \quad (13)
\end{aligned}$$

The coordinates (x, y, t, ψ, ϕ) have the ranges $-1 \leq x \leq 1, -1/\nu \leq y \leq -1, -\infty < t < \infty, 0 \leq \psi, \phi \leq 2\pi$ with the asymptotic infinity at $x \rightarrow y \rightarrow -1$ and the event horizon at $y = -1/\nu$. Moreover, $x = -1$ and $y = -1$ correspond to the rotation axes around the ϕ direction and the ψ direction, respectively, which extend to the asymptotic infinity, and $x = 1$ corresponds to the disk-shaped bubble touching the horizon at $(x, y) = (1, -1/\nu)$. The parameters $(\ell, \nu, \gamma, a, b, \alpha)$ have the ranges $\ell > 0, 0 < \nu < \gamma < 1, -\infty < \alpha < \infty$ and

$$\begin{cases} \frac{a(a-2)(1-a+a^2)}{(1-2a)(1+2a-2a^2)} < b < 0 & (0 < a < a_*) \\ -1 < b < 0 & (a_* \leq a < 1) \end{cases}, \quad (14)$$

with the constraints

$$\tanh^3 \alpha = (a - b)/(1 - ab), \quad (15)$$

$$\begin{aligned} \nu &= 1 - \frac{2b(1-a^2)^2}{b(2a^2(a-1)^2 + 1) + a((a-1)^3 - 1)}, \\ \gamma &= \nu + \frac{(1-\nu)(1-a+a^2)}{1 - (1+2b)a + (1+b)a^2}, \end{aligned} \quad (16)$$

where $a_* = 0.347\dots$ is a root of $a^3 - 3a + 1 = 0$. Thus, the conditions (15) and (16) reduce the number of the independent parameters from six to three. As seen below, ℓ denotes the scale merely, and ν controls the size of the horizon or the smallness of the bubble. α is related to an electric charge via below Eq. (23). Figure 1 illustrates the parameter region in the (a, b) plane.

The asymptotic infinity lies at $x \rightarrow y \rightarrow -1$ since in terms of the standard spherical coordinates (r, θ) defined as

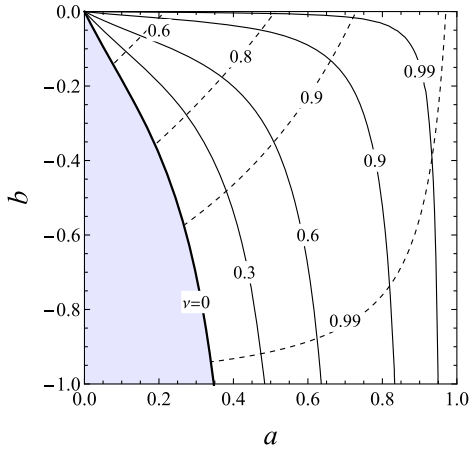


FIG. 1. The allowed region for (a, b) in Eq. (14). The thick and dashed curves correspond to $\nu = \text{constant}$ and $\tanh \alpha = \text{constant}$, which are given by Eqs. (15) and (16), respectively.

$(x+1, y+1) = \frac{4(1-\nu)\ell^2}{r^2}(\cos^2 \theta, \sin^2 \theta)$, the metric asymptotes to the five-dimensional Minkowski metric at $r \rightarrow \infty$ ($x \rightarrow y \rightarrow -1$) as $ds^2 \simeq -dt^2 + dr^2 + r^2(d\theta^2 + \cos^2 \theta d\phi^2 + \sin^2 \theta d\psi^2)$. The metric has the apparent singularities at $x = \pm 1, y = -1$, but these are merely coordinate singularities. In particular, the Killing vectors ∂_ϕ and ∂_ψ vanish at $x = -1$ and $y = -1$, respectively, which correspond to the rotation axes around the ϕ direction and the ψ direction, respectively, and extend to asymptotic infinity. The choice of the 2π periodicities for ϕ, ψ guarantees the absence of conical singularities on these rotation axes.

Under the conditions (15) and (16), the Killing vector $\partial_{\phi'} := \partial_\phi + \partial_\psi$ vanishes at $x = 1$ ($-1/\nu \leq y \leq -1$), and hence there do not exist Dirac-Misner string singularities. Furthermore, from Eq. (16), we can find $\Delta\phi' = 2\pi$, which means the absence of conical singularities at $x = 1$. At $x = 1$, another Killing vector ∂_ψ vanishes at $(x, y) = (1, -1)$ but does not vanish for $-1/\nu \leq y < -1$ including $(x, y) = (1, -1/\nu)$, and then since $x = 1$ ($-1/\nu \leq y \leq -1$) where the $U(1)$ Killing vector $\partial_{\phi'}$ vanishes is topologically D^2 , this denotes a two-dimensional disk-shaped bubble.

The Killing vector $v_H := \partial_t + \omega_\psi \partial_\psi + \omega_\phi \partial_\phi$ becomes null at $y = -1/\nu$ ($-1 \leq x \leq 1$), and so $y = -1/\nu$ ($-1 \leq x \leq 1$) corresponds to a Killing horizon with the surface gravity

$$\kappa = \frac{(1-a^2)^{3/2}(1-\gamma)^2 \sqrt{\nu(\nu+1)(\gamma+\nu)^{-1}}}{c_3 \ell (c^3(1-\nu-ac_1) + s^3(b-a)(\gamma-\nu))}, \quad (17)$$

where the horizon angular velocities ω_ψ, ω_ϕ are given by

$$\omega_\psi = -\frac{c_3 \omega_\phi}{d_3} = \frac{v_0 c_3 \kappa}{2(1-\gamma) \sqrt{\nu(\nu+1)(\gamma+\nu)(1-a^2)}}. \quad (18)$$

The two Killing vectors, $v_\phi := \partial_\phi = {}^t(0, 1)$ and $v_\psi := \partial_\psi = \partial_\psi + \partial_\phi = {}^t(1, 1)$, vanish on $x = -1$ and $x = 1$, respectively, following $\det(v_\phi, v_\psi) = -1$. Hence the theorem in Ref. [13] shows that the spacial cross section of the Killing horizon at $y = -1/\nu$ is topologically S^3 .

Absence of curvature singularities and CTCs. The metric and its inverse seem divergent at the surfaces $H(x, y) = 0$ and $D = 0$, where the curvature singularities may appear, e.g., the Kretschmann scalar can be written as $R^{\mu\nu\rho\lambda} R_{\mu\nu\rho\lambda} \propto H^{-6}(x, y) D^{-6}$. It is easy to check that the five terms in Eq. (8) are non-negative within the range specified in Eq. (14) for Eq. (16) and $-1 \leq x \leq 1, -1/\nu \leq y \leq -1$, hence $H(x, y) \geq 0$. Therefore, $H(x, y) = 0$ if and only if all of five terms vanish at the same time; however, this does not occur. Thus, it turns out that $H(x, y) > 0$ everywhere outside and on the horizon. The positivity of $H(x, y)$ directly implies $D > 0$ because $H(x, y)D > c^2[H(x, y) - H(y, x)] > 0$, where $H(x, y) - H(y, x)$ can be written as the

sum of non-negative terms that do not vanish at the same time.

Having shown the positivity of $H(x, y)$ and D , one can also show the absence of curvature singularities at the boundaries of the C-metric coordinates (x, y) . (i) One can see that $y = -1/\nu$ is a regular horizon by the transformation $dx^i \rightarrow dx^i - v_H^i(1 - \nu^2)dy/(2\kappa\nu G(y))$ and the gauge transformation $A \rightarrow A - d[(1 - \nu^2)(A_i|_H v_H^i) \int^y dy/(2\kappa\nu G(y))]$ ($i = t, \psi, \phi$), where $A_i|_H := A_i(y = -1/\nu)$. (ii) Near $x = \pm 1$, in terms of $x = \pm 1 \mp C_\pm r^2$ with positive constants $C_\pm$, the metric behaves as $ds^2 \simeq \alpha_{00}dt^2 + 2\alpha_{01}dtd\phi_1 + \alpha_{11}d\phi_1^2 + \beta_1 dy^2 + \beta_2(dr^2 + r^2 d\phi_2^2)$, where $\alpha_{ij}(y)$ ($i, j = 0, 1$) is a certain regular, nondegenerate matrix and $\beta_i(y)$ ($i = 1, 2$) are some positive function. We have set $(\phi_1, \phi_2) = (\psi, \phi)$ at $x = -1$ and $(\phi_1, \phi_2) = (\psi - \phi, \phi)$ at $x = 1$. (iii) It can be also shown that near $y = -1$, in terms of $y = -1 - C_0 r^2$ with a positive constant C_0 and $(\phi_1, \phi_2) = (\phi, \psi)$, the metric takes a similar regular form, where the role of y is replaced with x . (iv) Finally, one can show that, at the point $(x, y) = (1, -1)$, the metric is regular because, by introducing the new coordinates $(t', \psi', \phi', x, y) := (C_1 t, \psi - \phi, \phi, 1 - C_2(1 + \nu)r^2 \sin^2 \theta, -1 - C_2(1 - \nu)r^2 \cos^2 \theta)$ with appropriate constants, C_1, C_2 , the metric behaves as the Minkowski metric $ds^2 \simeq -dt'^2 + dr^2 + r^2(\cos^2 \theta d\psi'^2 + \sin^2 \theta d\phi'^2) d\theta^2$. Therefore, the spacetime does not have any curvature singularities outside and on the horizon. Moreover, the absence of CTCs requires that the two-dimensional angular part $g_{IJ}(I, J = \phi, \psi)$ of the metric is non-negative on and outside the horizon (which is determined from the non-negativity of the trace and determinant for $-1 \leq x \leq 1$ and $-1/\nu \leq y \leq -1$). We can show the non-negativity numerically and hence the absence of CTCs in the DOC.

Physical quantities. For the ADM (Arnowitt-Deser-Misner) mass, two angular momenta are computed as

$$M = \frac{3\pi\ell^2(1 + 2s^2)(\gamma + \nu)(c_3(1 - \nu) - d_1)}{4G_5(1 - a^2)(1 - \gamma)^2(1 + \nu)}, \quad (19)$$

$$J_\psi = c^3 J_1 + s^3 J_2, \quad J_\phi = c^3 J_2 + s^3 J_1, \quad (20)$$

where J_1 and J_2 correspond to the angular momenta in the neutral case,

$$J_1 = \frac{\pi\ell^3 v_0(c_3 d_2 - c_1 c_2 c_3 - (c_3 - b c_2) d_1)}{4G_5 \nu(1 - a^2)(1 - \gamma)^3}, \quad (21)$$

$$J_2 = \frac{\pi\ell^3 v_0(a - b)(2c_3 \nu + d_1)}{2G_5(1 - a^2)(1 - \gamma)(1 + \nu)}. \quad (22)$$

The electric charge over a three-dimensional closed surface S surrounding a horizon and a bubble is defined as

$$Q := \frac{1}{8\pi G_5} \int_S \left(\star F + \frac{1}{\sqrt{3}} F \wedge A \right) = -\frac{2M \tanh(2\alpha)}{\sqrt{3}}, \quad (23)$$

which obviously follows the BPS bound $M \geq \frac{\sqrt{3}}{2}|Q|$.

In addition to these conserved charges, the existence of the disk-shaped bubble D at $x = 1$ enables one to define the magnetic flux (this is not a conserved charge) as

$$q := \frac{1}{4\pi} \int_D F = \frac{\sqrt{3}\ell s c d_1 v_0(b d_2 s(\gamma - \nu) - 2c\nu \tilde{d}_2)}{2(c^2 \tilde{d}_2^2(\gamma - \nu) + 2\nu s^2 \tilde{d}_2^2)}, \quad (24)$$

where $\tilde{d}_2 := d_2 - (\gamma - 1)(\nu - 1)(\gamma + \nu)$. The area of a constant time slice through the horizon is written as

$$A_H = 8\ell^3 \pi^2 c_3 \sqrt{\nu(\gamma + \nu)(1 + \nu)^{-1}(1 - a^2)^{-3}(1 - \gamma)^{-2}} \times (c^3(1 - \nu - ac_1) - s^3(a - b)(\gamma - \nu)). \quad (25)$$

Phase space. Now, to consider the phase diagram, we introduce the angular momenta and horizon area normalized by the mass scale $r_M := \sqrt{8G_5 M/3\pi}$ as $j_\psi := 4G_5 J_\psi/\pi r_M^3$, $j_\phi := 4G_5 J_\phi/\pi r_M^3$, and $a_H := \sqrt{2}A_H/\pi^2 r_M^3$. Similarly, we introduce the normalized angular momenta $j_\psi^{\text{CY}}, j_\phi^{\text{CY}}$ and the normalized horizon area a_H^{CY} for the Cvetič-Youm solution [14]. Figure 2 shows the allowed region of regular black holes without CTCs in the $j_\psi - j_\phi$ plane. The angular momenta have the ranges $0 < j_\phi < 1/(2\sqrt{2}) = 0.354, \dots, 0.347 \dots < j_\psi < 1$.

The solid curves correspond to various $Q = \text{const}$ (various values of $\tanh \alpha$), each of which has end points at $\nu = 0$ and $\nu = 1$. The region is bounded by the dashed line $j_\psi = j_\phi$, which can only be reached at the BPS limit $M = \sqrt{3}|Q|/2(\alpha \rightarrow \infty)$. Since the metric is no longer asymptotically flat at the BPS limit, the capped black hole does not admit equal angular momenta $j_\psi = j_\phi$. Moreover, since each $Q = \text{const}$ curve, as seen in the middle panel of Fig. 2, is not closed, the capped black hole is uniquely specified by the conserved charges of its mass, two angular momenta, and electric charge. As in the right panel of Fig. 3, each $Q = \text{const}$ curve can be decomposed into three parts, (i) $0 < \nu < \nu_{\text{ext}}$, (ii) $\nu_{\text{ext}} < \nu < \nu_{\text{crit}}$, and (iii) $\nu_{\text{crit}} < \nu < 1$, where at $\nu = \nu_{\text{ext}}$, the Cvetič-Youm black hole with the same conserved charges becomes extremal, and at $\nu = \nu_{\text{crit}}$ has the same entropy as the capped black hole. Hence, for (i), the capped black hole has larger angular momenta beyond the extremal Cvetič-Youm black hole, whereas for (ii) and (iii), the capped black hole and the Cvetič-Youm black hole have the same mass, two angular momenta, and electric charge, which obviously shows the nonuniqueness property of black holes even

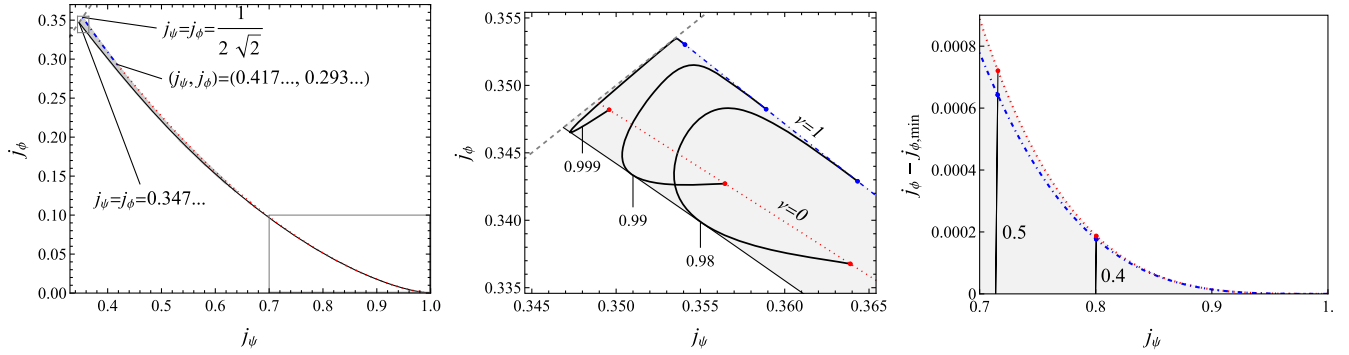


FIG. 2. The allowed region of two angular momenta and the constant curves for an electric charge ($\tanh \alpha$). Left: the whole region is shown by the shaded region. Middle/right: close-ups of $0.348... \leq j_\psi \leq 0.365$ and $0.7 \leq j_\psi \leq 1$, respectively, in the left panel. The red dashed and blue dot-dashed lines represent $\nu = 0$ and $\nu = 1$, respectively, which have an intersection at the point $(j_\psi, j_\phi) = (0.417..., 0.293...)$. Middle: some $\tanh \alpha = \text{const}$ curves are plotted near the BPS limit ($\tanh \alpha \rightarrow \infty$), where the curve approaches the black dashed line $j_\psi = j_\phi$. As seen on the right, for $j_\psi \rightarrow 1$ ($\alpha \rightarrow 0$), the region gets narrowed toward a point $(j_\psi, j_\phi) = (1, 0)$, where j_ψ and the difference $j_\phi - j_{\phi, \min}$ from the lower bound of j_ϕ are plotted due to lacking in resolution.

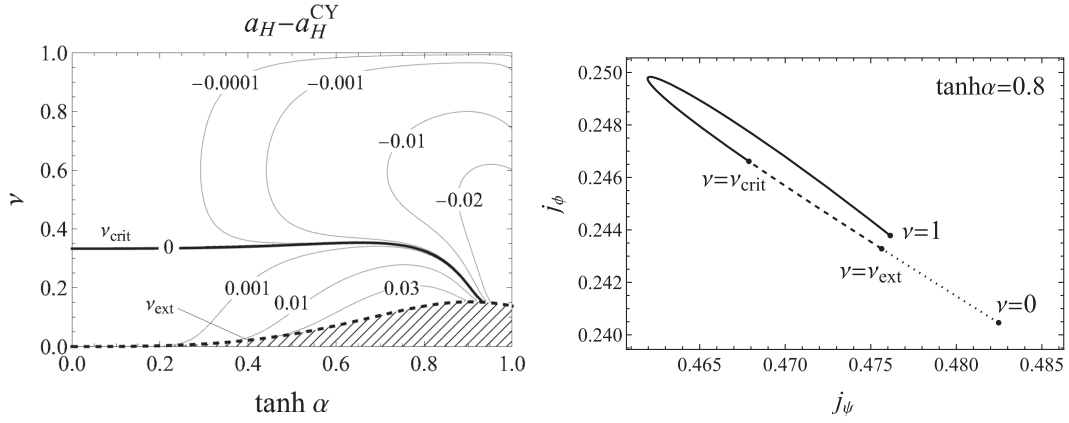


FIG. 3. The comparison of the dimensionless horizon areas with the Cvetič-Youm black hole for the same conserved charges (M, J_ψ, J_ϕ, Q) . Left: contour plots of $a_H - a_H^{\text{CY}}$ in the $\tanh \alpha - \nu$ plane. For $\nu_{\text{ext}} < \nu < 1$ (the white region), a Cvetič-Youm black hole with the same charges exists for each $(\tanh \alpha, \nu)$, while for $0 < \nu < \nu_{\text{ext}}$ (the hatched region), there is no Cvetič-Youm black hole with the same charges. Right: the curve corresponding to $\tanh \alpha = 0.8$ plotted in the $j_\psi - j_\phi$ plane as an example. For $\tanh \alpha < 0.940...$, the capped black hole with $\nu_{\text{ext}} < \nu < \nu_{\text{crit}}$ ($\nu_{\text{crit}} < \nu < 1$) has larger (smaller) entropy than the Cvetič-Youm black hole, whereas for $0.940... < \tanh \alpha < 1$, the former always has smaller entropy than the latter.

under the assumption that the horizon cross section has the topology of S^3 . In particular, for (ii), the capped black hole has larger entropy than the Cvetič-Youm black hole, and hence the capped black hole with a large bubble becomes thermodynamically more stable. As seen from the left panel of Fig. 3, this regime disappears for $\tanh \alpha > 0.940...$. This also implies that too much amount of the electric charge destabilizes the bubble structure.

It is worth noting that, due to the disk-shaped bubble, there would be more general solutions with five independent parameters of the mass, two angular momenta, electric charge, and magnetic flux [15].

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