

Predictions for feed-down enhancements at the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ thresholds via the triangle and box singularities

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We demonstrate that triangle singularity (TS) and box singularity (BS) mechanisms can produce unique narrow enhancements at the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ thresholds in the invariant mass spectra of $J/\psi p$ and $J/\psi p \pi$, respectively. Taking into account that such mechanisms only depend on the initial $\Sigma_c^{(*)} \bar{D}^{(*)}$ interactions near threshold within the TS or BS kinematic regimes, the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ threshold enhancements can also acquire contributions from both the heavier pentaquark decays and the $\Sigma_c^{(*)} \bar{D}^{(*)}$ scatterings of the continuum as a feed-down phenomenon. A search for these structures in the $J/\psi p$ and $J/\psi p \pi$ spectra in both exclusive and semi-inclusive processes will provide a crucial evidence for the hadronic molecule nature of these observed pentaquarks and clarify the role played by the TS and BS in the near-threshold dynamics.

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Introduction. The observation of hidden charm pentaquarks by the LHCb collaboration in 2015 [1] provided a strong evidence for QCD exotics in the baryon sector and has initiated tremendous activities in both experiment and theory to understand their nature. Based on the run-1 data sample, two states, $P_c(4380)$ and $P_c(4450)$, were observed in the $J/\psi p$ invariant mass spectrum in $\Lambda_b^0 \rightarrow J/\psi p K^-$ [1]. Inspired by the results, many different solutions, such as the hadronic molecules, clustering diquark models, kinematic singularities, and so on, were proposed in the literature (see, e.g., review articles [2–12]). With the run-2 data samples, the reanalysis of $\Lambda_b^0 \rightarrow J/\psi p K^-$ exposes more detailed structures in the $J/\psi p$ invariant mass spectrum [13]. $P_c(4450)$ splits into two states, $P_c(4440)$ and $P_c(4457)$, which fit well the $J^P = 1/2^-$ and $3/2^-$ doublet formed by $\Sigma_c \bar{D}^*$, and a new state $P_c(4312)$, which fits the $\Sigma_c \bar{D}$ molecular state, is identified [14–24]. Meanwhile, signals for $P_c(4380)$, formerly assigned as the $\Sigma_c \bar{D}$ molecule in the run-1 analysis, become obscure in the run-2 analysis. It turns out to be broad and can be even absorbed by the background [13]. It should be noted that the P_c spectra obtained by the run-2 analysis fit well the $\Sigma_c^{(*)} \bar{D}^{(*)}$ molecular scenario of the S -wave

interactions which was also the view of some earlier theoretical predictions [25–28].

The LHCb run-2 result provides an opportunity for understanding the $\Sigma_c^{(*)} \bar{D}^{(*)}$ interactions near threshold in the molecular picture. However, as warned by the LHCb analysis, alternative descriptions could not be excluded, in particular if one of the states happens to have positive parity [13]. Even for the most favored molecular picture, there still exist crucial issues to be understood. In the molecular picture the favored one boson exchange dynamics describes an attractive interaction between $\Sigma_c^{(*)}$ and $\bar{D}^{(*)}$, but repulsive between $\Lambda_c^{(*)}$ and $\bar{D}^{(*)}$ [14–24,26,27]. This can explain the P_c states as the $\Sigma_c \bar{D}^{(*)}$ molecules and no enhancements are observed at the $\Lambda_c \bar{D}^{(*)}$ thresholds. However, a crucial question must be raised for the molecular pentaquark $\Sigma_c \bar{D}^{(*)}$ production in the Λ_b decays. Note that the $\Sigma_c^{(*)} \bar{D}^{(*)}$ configurations in the Λ_b decays would be either produced via the color-suppressed process, or via their subleading couplings to $\Lambda_c^{(*)} \bar{D}^{(*)}$ through the kinematic singularity [i.e. triangle singularity (TS) [29,30]] enhanced transitions as pointed out in Refs. [31,32]. In contrast, the $\Lambda_c \bar{D}^{(*)}$ configurations can be accessed by both the direct emission processes and TS enhanced transitions. The absence of enhancements around the thresholds of $[\Lambda_c \bar{D}](4150)$ and $[\Lambda_c \bar{D}^*](4290)$ seems to confirm that those observed hadronic molecule states are genuine states driven by the $\Sigma_c^{(*)} \bar{D}^{(*)}$ interactions, while the repulsive nature of the $\Lambda_c^{(*)} \bar{D}^{(*)}$ interactions would not produce apparent threshold structures at leading order in the

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exclusive decays of Λ_b . The production mechanisms seem to be strongly correlated with the nature of P_c s. As discussed in Ref. [31], the TS mechanism plays a crucial role to promote the P_c production amplitude in $\Lambda_b \rightarrow K^- \Lambda_c^{(*)} \bar{D}^{(*)} \rightarrow K^- J/\psi p$. Thus, further evidence is demanded for resolving the paradox, namely, how the subleading production mechanism for $\Sigma_c^{(*)} \bar{D}^{(*)}$ would produce leading predominant structures in the $J/\psi p$ spectrum.

In Ref. [33] it was proposed to search for these heavy pentaquark states in J/ψ photoproduction where the cross sections at large scattering angles should serve as a direct probe for the production of the pentaquarks as genuine states. Although these hidden charm pentaquark states are searched at GlueX based on their 12 GeV data, the present statistics do not allow a conclusion for their existence at this moment [34]. In this work, based on the hadronic molecule picture, we identify the crucial role played by the kinematic singularities in the pentaquark three-body and four-body decays into $J/\psi p\pi$ and $J/\psi p\pi\pi$, respectively. As the consequence, two narrow enhancements at the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ thresholds in the $J/\psi p$ and $J/\psi p\pi$ invariant mass spectra, respectively, are predicted. This mechanism implies a feed-down process to the semi-inclusive $J/\psi p$ and $J/\psi p\pi$ channel which will further enhance the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ threshold peaks. Confirmation of these structures in the semi-inclusive production channels of $J/\psi p$ and $J/\psi p\pi$ should be a direct evidence for the role played the kinematic singularities, and explain the promoted P_c production in Λ_b decays as the hadronic molecules.

Formalism. Given the productions of these P_c states via various processes, their decays into $J/\psi p$ can be via direct two-body decays into $J/\psi p$, three-body decays into $J/\psi p\pi$, or four-body decays into $J/\psi p\pi\pi$. An illustration of these transitions is presented in Fig. 1. Note that in both Figs. 1(b) and 1(c) the transitions satisfy the TS condition [29,30], i.e. all the internal particles can approach their on-shell condition simultaneously. As the leading singularity in the loop functions, it may provide non-negligible partial widths for these P_c states via their $J/\psi p\pi$ three-body decays. In comparison with the two-body decay amplitude which counts an order of v , where v is the typical nonrelativistic on-shell velocity of the intermediate heavy hadrons, the triangle loop of Figs. 1(a) and 1(b) counts $(v^5/v^6)v \sim \mathcal{O}(1)$.

The transitions of Fig. 1(d) fulfill the so-called “box singularity” (BS), where all the internal particles can also approach their on-shell conditions simultaneously. The amplitudes of Fig. 1(d) count $(v^5/v^8)v^2 \sim 1/v$ which appears to be more leading than 1(b) and 1(c). However, taking into account the limited phase space, contributions from $P_c(4440)$ and $P_c(4457) \rightarrow J/\psi p\pi\pi$ will be suppressed compared to those three-body decays.

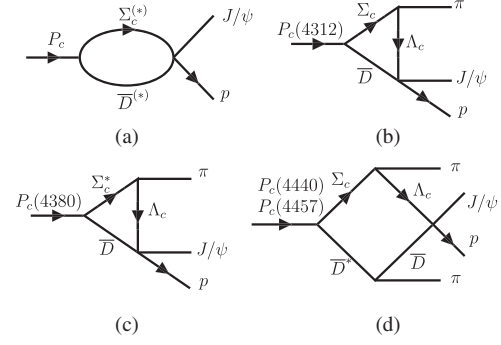


FIG. 1. Feynman diagrams illustrating the P_c states decay into $J/\psi p$, $J/\psi p\pi$, and $J/\psi p\pi\pi$. Parts (b)–(d) manifest the TS and BS mechanisms that can feed down to the semi-inclusive $J/\psi p$ and $J/\psi p\pi$ spectra, respectively.

In the feed-down scenario the semi-inclusive spectra of $J/\psi p$ or $J/\psi p\pi$ will be measured. It means that all the transitions in Fig. 1 will contribute and sum from the $J/\psi p$ or $J/\psi p\pi$ threshold to higher energies. As the unique prediction, the TS and BS will lead to the threshold enhancements at the mass of $\Lambda_c \bar{D}$ in the $J/\psi p$ invariant mass spectrum, and at the thresholds of $\Lambda_c \bar{D}^*$ and $\Sigma_c \bar{D}$ in the $J/\psi p\pi$ invariant mass spectrum, respectively.

To proceed, we employ an effective Lagrangian approach to evaluate the feed-down mechanisms. The S -wave interactions between P_c and $\Sigma_c^{(*)} \bar{D}^{(*)}$ can be expressed as [35]

$$\begin{aligned} \mathcal{L}_{P_c(4312)\Sigma_c\bar{D}} &= g_{P_c(4312)} \bar{D}^\dagger \bar{\Sigma}_c P_c, \\ \mathcal{L}_{P_c(4380)\Sigma_c^*\bar{D}} &= g_{P_c(4380)} \bar{D}^\dagger \bar{\Sigma}_c^* \cdot P_c, \\ \mathcal{L}_{P_c(4440)\Sigma_c\bar{D}^*} &= g_{P_c(4440)} \bar{D}^{*\mu\dagger} \bar{\Sigma}_c \gamma_5 \tilde{\gamma}_\mu (p_A) P_c, \\ \mathcal{L}_{P_c(4457)\Sigma_c\bar{D}^*} &= g_{P_c(4457)} \bar{\Sigma}_c \bar{D}^{*\dagger} \cdot P_c, \end{aligned} \quad (1)$$

where $\bar{\Sigma}_c^{(*)}$ and $\bar{D}^{(*)\dagger}$ denote the field operators for Σ_c^* and $\bar{D}^{(*)}$, respectively, and P_c denotes the spinor field of P_c states; g_{P_c} are the coupling constants between P_c and corresponding $\Sigma_c^{(*)} \bar{D}^{(*)}$ channels. They are determined in the molecular scenario by reproducing the mass and width of the P_c states. We adopt the coupling constants $g_{P_c(4312)} = 2.16$, $g_{P_c(4380)} = 2.19$, $g_{P_c(4440)} = 2.6$, and $g_{P_c(4457)} = 1.7$ determined from Ref. [36]. $\tilde{\gamma}_\mu(p_A)$ is a momentum-dependant function, which is defined as $\tilde{\gamma}^\mu(p_A) = (-g^{\mu\nu} + p_A^\mu p_A^\nu / p_A^2) \gamma_\nu$ [35]. The projection operator for the spin-3/2 particle can be written as

$$\begin{aligned} \sum u^\nu \bar{u}^\alpha &= (\not{p} + m) \left[-g^{\nu\alpha} + \frac{1}{3} \gamma^\nu \gamma^\alpha + \frac{1}{3m} (\gamma^\nu p^\alpha - \gamma^\alpha p^\nu) \right. \\ &\quad \left. + \frac{2}{3m^2} p^\nu p^\alpha \right], \end{aligned} \quad (2)$$

with which the propagator of Σ_c^* in Fig. 1(b) can be obtained. Lagrangians for other vertices in Fig. 1 are collected as follows:

$$\begin{aligned}
\mathcal{L}_{\Lambda_c \bar{D} J/\psi p} &= ig_x \psi^\mu \bar{N} \gamma_5 \gamma_\mu \Lambda_c \bar{D}, \\
\mathcal{L}_{\Sigma_c \Lambda_c \pi} &= g_{\Sigma_c \Lambda_c \pi} \bar{\Lambda}_c \partial^\mu \pi \gamma_5 \gamma_\mu \Sigma_c, \\
\mathcal{L}_{\Sigma_c^* \Lambda_c \pi} &= g_{\Sigma_c^* \Lambda_c \pi} \bar{\Lambda}_c \partial^\mu \pi \Sigma_{c\mu}^*, \\
\mathcal{L}_{D^* D \pi} &= ig_{D^* D \pi} D_\mu^* \bar{D} \partial^\mu \pi,
\end{aligned} \quad (3)$$

where the coupling constants $g_{\Sigma_c \Lambda_c \pi}$, $g_{\Sigma_c^* \Lambda_c \pi}$, and $g_{D^* D \pi}$ are determined by the partial decay widths of the corresponding processes [37]. Coupling g_x describes the contact interaction for the $\Lambda_c \bar{D} J/\psi p$ vertex. It can be estimated by a t -channel D meson exchange near threshold as shown in Fig. 2.

The Lagrangians involved in the determination of g_x in Fig. 2(a) are

$$\begin{aligned}
\mathcal{L}_{\Lambda_c D p} &= ig_{DN\Lambda_c} D^\dagger \bar{N} \gamma_5 \Lambda_c, \\
\mathcal{L}_{J/\psi D \bar{D}} &= g_{J/\psi D \bar{D}} (D \partial_\mu \bar{D} - \partial_\mu D \bar{D}) \psi^\mu,
\end{aligned} \quad (4)$$

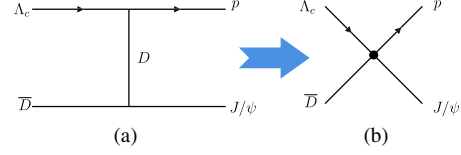


FIG. 2. The near-threshold interaction between $\Lambda_c \bar{D}$ and $J/\psi p$ is described by a t -channel D meson exchange from which the effective contact coupling g_x is extracted.

where the coupling constant $g_{DN\Lambda_c} = 13.2$ is estimated the same as the coupling of $KN\Lambda$ by employing the $SU(4)$ symmetry [38–41]; Coupling $g_{J/\psi D \bar{D}} = 2g_2 \sqrt{m_{J/\psi} m_D m_{\bar{D}}}$ with $g_2 = \sqrt{m_\psi}/(2m_D f_{J/\psi})$ and $f_{J/\psi} = 0.405$ GeV [42] is adopted from the heavy quark symmetry relation. By equalling Figs. 2(a) and 2(b), we extract $|g_x| = 3.0$ GeV⁻¹ at the threshold of $\Lambda_c \bar{D}$.

With the above Lagrangians, the triangle and box loop amplitudes in Fig. 1 can be expressed as

$$\begin{aligned}
\mathcal{M}^{P_c(4312) \rightarrow J/\psi p \pi} &= \int \frac{d^4 p_{\Lambda_c}}{(2\pi)^4} \bar{u}_p \gamma_5 \gamma_\mu \epsilon_{J/\psi}^{*\mu} (\not{p}_{\Lambda_c} + m_{\Lambda_c}) \gamma_5 \gamma_\nu p_\pi^\mu (\not{p}_{\Sigma_c} + m_{\Sigma_c}) u_{P_c} \frac{g_{P_c(4312)} g_{\Sigma_c \Lambda_c \pi} g_x \mathcal{F}^2(p_{\Lambda_c}^2)}{(p_{\Sigma_c}^2 - m_{\Sigma_c}^2)(p_D^2 - m_D^2)(p_{\Lambda_c}^2 - m_{\Lambda_c}^2)}, \\
\mathcal{M}^{P_c(4380) \rightarrow J/\psi p \pi} &= \int \frac{d^4 p_{\Lambda_c}}{(2\pi)^4} \bar{u}_p \gamma_5 \gamma_\mu \epsilon_{J/\psi}^{*\mu} (\not{p}_{\Lambda_c} + m_{\Lambda_c}) p_{\pi\nu} (\not{p}_{\Sigma_c} + m_{\Sigma_c}) \\
&\quad \times \left[-g^{\nu\alpha} + \frac{1}{3} \gamma^\nu \gamma^\alpha + \frac{1}{3m_{\Sigma_c}} (\gamma^\nu p^\alpha - \gamma^\alpha p^\nu) + \frac{2}{3p_{\Sigma_c}^2} p_{\Sigma_c}^\nu p_{\Sigma_c}^\alpha \right] u_{P_c} \frac{g_{P_c(4380)} g_{\Sigma_c^* \Lambda_c \pi} g_x \mathcal{F}^2(p_{\Lambda_c}^2)}{(p_{\Sigma_c^*}^2 - m_{\Sigma_c^*}^2)(p_D^2 - m_D^2)(p_{\Lambda_c}^2 - m_{\Lambda_c}^2)}, \\
\mathcal{M}^{P_c(4440) \rightarrow J/\psi p \pi \pi} &= \int \frac{d^4 p_D}{(2\pi)^4} \left(-g^{\alpha\rho} + \frac{p_D^\alpha p_D^\rho}{m_{D^*}^2} \right) p_{\pi\rho} \bar{u}_p \gamma_5 \gamma_\mu \epsilon_{J/\psi}^{*\mu} (\not{p}_{\Lambda_c} + m_{\Lambda_c}) \gamma_5 \gamma_\nu p_\pi^\nu (\not{p}_{\Sigma_c} + m_{\Sigma_c}) \gamma_5 \tilde{\gamma}_\alpha u_{P_c} \\
&\quad \times \frac{g_{P_c(4440)} g_{\Sigma_c^* \Lambda_c \pi} g_{D^* D \pi} \mathcal{F}^2(p_{\Lambda_c}^2)}{(p_{\Sigma_c}^2 - m_{\Sigma_c}^2)(p_{D^*}^2 - m_{D^*}^2)(p_{\Lambda_c}^2 - m_{\Lambda_c}^2)(p_D^2 - m_D^2)}, \\
\mathcal{M}^{P_c(4457) \rightarrow J/\psi p \pi \pi} &= \int \frac{d^4 p_D}{(2\pi)^4} \bar{u}_p \gamma_5 \gamma_\mu \epsilon_{J/\psi}^{*\mu} (\not{p}_{\Lambda_c} + m_{\Lambda_c}) \gamma_5 \gamma_\nu p_\pi^\nu (\not{p}_{\Sigma_c} + m_{\Sigma_c}) u_{P_c} \left(-g^{\alpha\beta} + \frac{p_D^\alpha p_D^\beta}{m_{D^*}^2} \right) p_{\pi\beta} \\
&\quad \times \frac{g_{P_c(4457)} g_{\Sigma_c^* \Lambda_c \pi} g_{D^* D \pi} \mathcal{F}^2(p_{\Lambda_c}^2)}{(p_{\Sigma_c}^2 - m_{\Sigma_c}^2)(p_{D^*}^2 - m_{D^*}^2)(p_{\Lambda_c}^2 - m_{\Lambda_c}^2)(p_D^2 - m_D^2)}.
\end{aligned} \quad (5)$$

In the kinematic regions of the TS and BS, the internal particles are in the vicinity of their on-shell conditions. It allows us to further simplify the integrands by adopting the following eigenfunction relations for the spin-1/2, spin-1, and spin-3/2 particles: $(p + m) \rightarrow 2m$, $(-g^{\mu\nu} + p^\mu p^\nu/p^2) \rightarrow \delta^{ij}$, and $(p + m)[-g^{\mu\nu} + \frac{1}{3} \gamma^\mu \gamma^\nu + \frac{1}{3m} (\gamma^\mu p^\nu - \gamma^\nu p^\mu) + \frac{2}{3m^2} p^\mu p^\nu] \rightarrow 2m[-g^{\mu\nu} + \frac{1}{3} \gamma^\mu \gamma^\nu + \frac{1}{3} (\gamma^\mu g^{\nu 0} - \gamma^\nu g^{\mu 0}) + \frac{2}{3} g^{\mu 0} g^{\nu 0}]$.¹ Meanwhile, to include the effects from the finite size of the hadrons in the vertex couplings, the monopole form factor $\mathcal{F}(q^2) = (\Lambda^2 - m_q^2)/(q^2 - m_q^2)$ is employed in our calculations.

For the relatively broad states $\Sigma_c^{(*)}$, complex masses with the widths are adopted [43–46], while for D^* we have ignored its width effects since they do not cause significant changes to the lineshapes of the $J/\psi p$ or $J/\psi p \pi$ invariant mass spectra.

¹In our numerical calculations, we have checked the influence of the on-shell approximation. It shows that the lineshapes of the TS/BS peaks are almost intact due to the nonrelativistic nature of these transitions.

Results and discussions. Proceeding to the numerical calculations, apart from the contributions from the initial genuine pentaquark decays, nonresonance initial-state $\Sigma_c^{(*)}$ and $\bar{D}^{(*)}$ scatterings can also feed down to the production of the threshold enhancements via the TS or BS mechanisms. Recognizing that the feed-down mechanism actually involves all possible genuine states and continuum contributions, our estimate of the threshold structures due to the TS and BS structures, respectively, can serve as a lower limit for experimental search. This unique feature makes the semi-inclusive measurements of the $J/\psi p$ and $J/\psi p\pi$ spectra interesting and useful for understanding not only the nature of the $\Lambda_c \bar{D}^{(*)}$ interactions, but also the role played by the TS and BS mechanisms in the promoted productions of P_c s in the Λ_b decays.

In Fig. 3 we plot the $J/\psi p$ invariant mass spectra for four observed P_c states in their three-body or four-body decays into either $J/\psi p\pi$ or $J/\psi p\pi\pi$. Clear and predominant peaks appear at the $\Lambda_c \bar{D}$ threshold due to the TS and BS mechanisms. Meanwhile, it shows that $P_c(4380)$ has the largest contributions to the $J/\psi p$ spectrum. It is due to the relatively large phase space for $\Sigma_c^* \rightarrow \Lambda_c \pi$ within the TS kinematic region that strongly enhances the decay rate of $P_c(4380) \rightarrow J/\psi p\pi$. Such an effect actually will add additional partial decay widths to the $P_c(4380)$ decay.²

It should be clarified that the semi-inclusive $J/\psi p$ spectrum should contain contributions from the direct production of $\Lambda_c \bar{D}^{(*)}$ as the intermediate states in $\Lambda_b \rightarrow \Lambda_c \bar{D}_s^{(*)}/\Lambda_c \bar{D}_{s1} \rightarrow K^- J/\psi p$ [1,13]. However, as shown in Ref. [31], the threshold of $\Lambda_c \bar{D}^{(*)}$ will only appear as a subleading two-body CUSP because the mass of Λ_b is far above the thresholds of $\Lambda_c \bar{D}_s^*$ and $\Lambda_c \bar{D}_{s1}$ and the TS condition cannot be satisfied. Similarly, the $\Lambda_c \bar{D}^*$ threshold will only appear as a CUSP in $\Lambda_b \rightarrow \Lambda_c \bar{D}_s^{(*)}/\Lambda_c \bar{D}_{s1} \rightarrow K^- J/\psi p\pi$. This situation actually helps to clarify the role played by the TS and BS mechanisms since, e.g., a narrow peak at the threshold of $\Lambda_c \bar{D}$ in the semi-inclusive $J/\psi p$ spectrum will indicate the leading TS and/or BS mechanisms instead of a CUSP effect [47].

The feed-down process via the BS mechanism also gives access to the $\Lambda_c \bar{D}^*$ threshold which can manifest as a narrow threshold enhancement as shown by the invariant mass spectrum of $J/\psi p\pi$ in Fig. 4. We stress that although such a threshold enhancement is illustrated by the four-body

²We note that the TS also occurs in another situation in the three-body and four-body decays into $\Lambda_c \bar{D}\pi$ and $\Lambda_c \bar{D}\pi\pi$, respectively. Different from Fig. 1, the S -wave coupling of $\bar{D}_0(2400) \rightarrow \bar{D}\pi$ suggests that the decay of $P_c(4380) \rightarrow \Lambda_c \bar{D}_0 \rightarrow \Lambda_c \bar{D}\pi$ could be an important channel for $P_c(4380)$. These processes will also contribute to the total width of $P_c(4380)$ although it cannot be as large as 100 MeV suggested by the LHCb fits [13]. More detailed studies of such processes will be reported elsewhere.

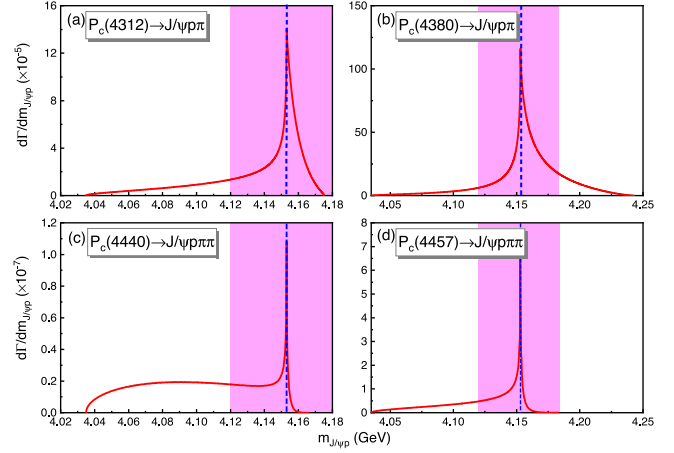


FIG. 3. The $J/\psi p$ invariant mass spectra (red solid lines) from different P_c state decays. The dashed vertical lines indicate the $\Lambda_c \bar{D}$ threshold.

decays of $P_c(4440)$ and $P_c(4457)$, any processes involving the near-threshold $\Sigma_c^{(*)} \bar{D}^*$ S -wave scatterings may feed down to the $J/\psi p\pi\pi$ channel and produce the $\Lambda_c \bar{D}^*$ threshold enhancement. Thus, this phenomenon also serve as a unique probe for the role played by the BS mechanism and can be searched in experiment.

In Fig. 1(c) in addition to the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ in the invariant mass spectra of $J/\psi p$ and $J/\psi p\pi$, respectively, the $\Sigma_c^* \bar{D}$ threshold can also be accessed due to the BS mechanism. However, since the $\Sigma_c^* \bar{D}$ and $J/\psi p\pi$ thresholds are close to each other, the $\Sigma_c^* \bar{D}$ threshold structure will be strongly suppressed by the phase space.

We can estimate the partial decay width of $P_c(4380) \rightarrow J/\psi p\pi$ as a reference for the significance of the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ peaks in the molecular scenario. We extract $\mathcal{R} = \Gamma[P_c(4380) \rightarrow J/\psi p]/\Gamma[P_c(4380) \rightarrow J/\psi p\pi] = 118 \text{ keV}/24 \text{ keV} \simeq 5$, where the D/D^* are included as

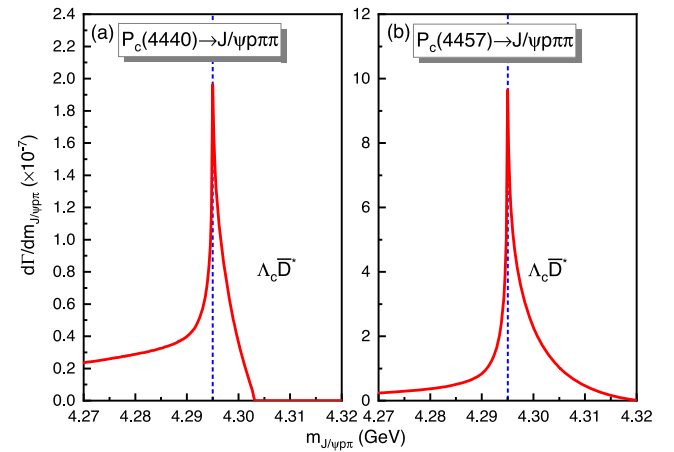


FIG. 4. The $J/\psi p\pi$ invariant mass spectra (red solid lines) from $P_c(4440)$ (left) and $P_c(4457)$ (right) decays. The $\Lambda_c \bar{D}^*$ threshold is indicated by the dashed vertical lines.

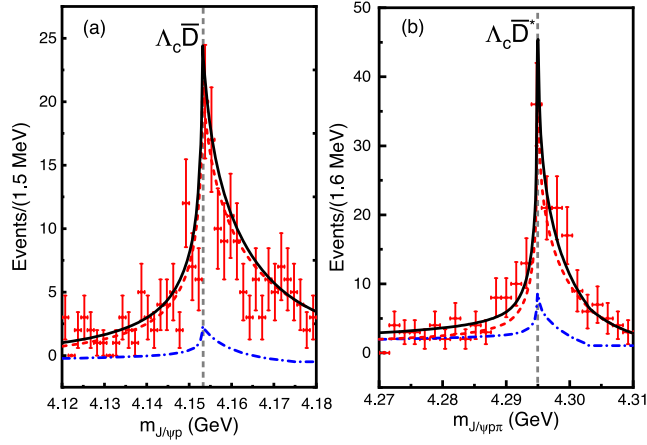


FIG. 5. The accumulated $J/\psi p \pi$ (left) and $J/\psi p$ (right) invariant mass spectra from all different P_c states are shown by the black solid lines in (a) and (b). For (a), the red dashed and blue dot dashed lines represent the results from $P_c(4380)$ and the sum of all the others, respectively. For (b), the red dashed and blue dot dashed lines represent the results from $P_c(4457)$ and $P_c(4440)$, respectively.

the exchanged particles in $P_c \rightarrow J/\psi p$. Then, based on the present 246k Λ_b events at LHCb, the events of the TS peak at the $\Lambda_c \bar{D}$ threshold in $\Lambda_b \rightarrow K^- J/\psi p \pi^0$ will be only about 1/5 of the present events for $P_c(4380)$. Although this is challenging for identifying P_c s, the production of the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ peaks is unnecessary to go through the P_c decays. As emphasized earlier, the feed-down mechanisms suggest that any processes involving the near-threshold $\Sigma_c^{(*)} \bar{D}$ ($\Sigma_c^{(*)} \bar{D}^*$) S -wave scatterings may feed down to the $J/\psi p \pi$ ($J/\psi p \pi \pi$) channel and produce the $\Lambda_c \bar{D}$ ($\Lambda_c \bar{D}^*$) threshold peak in the $J/\psi p$ ($J/\psi p \pi$) spectrum. It is possible that many channels can contribute to the $\Sigma_c^{(*)} \bar{D}$ ($\Sigma_c^{(*)} \bar{D}^*$) channels. Therefore, we estimate that the semi-inclusive cross sections for the $\Lambda_c \bar{D}$ ($\Lambda_c \bar{D}^*$) threshold peak will be enhanced by about 10 times of the signals based on the present 246k Λ_b decays. It means that the total semi-inclusive events can produce a significance comparable to the signals of P_c s in $\Lambda_b \rightarrow K^- J/\psi p$ with the factor $10 \times \mathcal{R} \simeq 2$ and with the same detection efficiency. Just for demonstration, we plot the $J/\psi p$ and $J/\psi p \pi$ spectra in Figs. 5(a) and 5(b) which corresponds to the $P_c(4380)$ production with about 100 times of the present Λ_b events. However, considering the feed-down mechanisms from other channels, about 10 times of the present total semi-inclusive $J/\psi p$ events will produce the same significance of the $\Lambda_c \bar{D}$ peak as plotted in Fig. 5(a).

In addition, the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ threshold kinematics turn out to be unique. The signals originated from the TS and BS mechanisms will drop smoothly in the higher energy end which will avoid significant interferences with other processes. It means that these feed-down signals will stand predominantly on the incoherent background in the semi-inclusive measurement.

Summary. We study the feed-down transitions via the observed P_c decays or the S -wave $\Sigma_c^{(*)} \bar{D}^{(*)}$ rescatterings into $J/\psi p$ and/or $J/\psi p \pi$, and predict that predominant narrow structures can be produced at the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ thresholds due to the TS and BS mechanisms. Since the S -wave interactions between Λ_c and $\bar{D}^{(*)}$ are repulsive in the hadronic molecule picture, a search for the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ threshold enhancements can provide supportive evidences for the hadronic molecule interpretation for these P_c states including $P_c(4380)$. Meanwhile, we stress that it will also provide a unique way for pinning down the role played by the TS and BS mechanisms. Since such feed-down transitions, in principle, can always occur given the production of near-threshold $\Sigma_c^{(*)} \bar{D}^{(*)}$, we anticipate that the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ threshold enhancements can be accessed in the semi-inclusive $J/\psi p$ and $J/\psi p \pi$ spectra in various processes, such as hadron-hadron collisions at LHCb, e^+e^- annihilations at Belle-II, and J/ψ photoproduction reactions at GlueX.

Note added.—Recently, the GlueX collaboration published their J/ψ photoproduction data [48], where two structures at the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ thresholds are identified. Since the signals for the P_c states are insignificant there, these two structures indicate the importance of the $\Lambda_c \bar{D}$ and $\Lambda_c \bar{D}^*$ open thresholds as CUSPs, and imply that more predominant structures can be expected in view of the TS (BS) mechanisms in $\gamma p \rightarrow J/\psi p \pi$ ($J/\psi p \pi \pi$) in the invariant mass spectrum of $J/\psi p$ ($J/\psi p \pi$). Future experimental searches for them are strongly recommended.

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