

Lighter QCD axion from anarchy

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We introduce the *anarchic axion*, a class of axion models which solves the strong *CP* problem within current nEDM constraints with a lighter than usual QCD axion, thus populating new parameter space that ongoing and future experiments target. The anarchic axion is driven light by a soft breaking of the Peccei-Quinn symmetry, which also predicts a residual neutron electric dipole moment. We introduce a novel measure to quantify the tuning required for large deviations from the usual QCD axion band. In addition to motivating searches for unusually light axions, this work establishes a new target for axion effective field theory.

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I. INTRODUCTION

The Peccei-Quinn (PQ) solution [1,2] to the strong charge-parity (*CP*) problem of quantum chromodynamics (QCD) predicts relationships between the axion mass m_a , decay constant f_a and axion-photon coupling $g_{a\gamma\gamma}$. In this Letter, we introduce a new class of multi-scalar axion models—the *anarchic axion*—so dubbed as to emphasize that the PQ symmetry arises accidentally.

The $U(1)_{\text{PQ}}$ symmetry is softly broken leading to a qualitative solution to the Strong *CP* problem that deviates from the traditional QCD axion band, importantly populating a region of parameter space targeted by many ongoing experiments. Related work to solving the strong *CP* problem departing from the canonical band include Refs. [3–11]. Furthermore, the new parameter space corresponds to an *almost* perfect alignment between the soft breaking vacuum and the θ_{QCD} vacuum. This provides a unique handle on the axion quality problem [3,12–17] allowing the introduction of a novel measure for quantifying the fine tuning.

In this paper we first introduce the field content and the potential of an anarchic axion model. Given an appropriate basis choice, we compute the m_a , f_a and $g_{a\gamma\gamma}$ relations, and present the new parameter space where experiments can

hunt for the anarchic axion. We then discuss the quality of this soft solution to the Strong *CP* problem and introduce a fine tuning measure to quantify the axion quality. We conclude by commenting on possible natural UV completions and future directions.

II. THE ANARCHIC AXION

The particle content and charge assignments of the model, summarized in Table I, consists of three complex scalars: H_1 and H_2 which can be identified with the Higgs doublet fields in DFSZ axion constructions [18,19], and a gauge singlet Φ . Standard Model (SM) fermions coupling to H_1 and H_2 will mediate the requisite effective operator coupling the anarchic axion and the anomalous $G\tilde{G}$ QCD term. The scalar potential is

$$V = \sum_{i=1,2} (\mu_i^2 |H_i|^2 + \lambda_i |H_i|^4) + \lambda |H_1|^2 |H_2|^2 + \lambda' |H_1 H_2|^2 + \mu_3^2 |\Phi|^2 + \lambda_3 |\Phi|^4 + \lambda_{13} |H_1|^2 |\Phi|^2 + \lambda_{23} |H_2|^2 |\Phi|^2, \quad (1a)$$

$$V_{\text{break}}^{C_\lambda} = -C_\lambda H_1 H_2 \Phi + \text{H.c.}, \quad (1b)$$

TABLE I. The field content of the anarchic axion model.

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$\mathbb{Z}_5$	$U(1)_{\text{PQ}}$
Q_L^i	3	2	1/6	0	X_Q
u_R^i	3	1	2/3	1	$X_Q - X_1$
d_R^i	3	1	−1/3	0	$X_Q - X_2$
L_L^i	1	2	−1/2	0	X_L
e_R^i	1	1	−1	0	$X_L - X_2$
H_1	1	2	−1/2	4	X_1
H_2	1	2	1/2	0	X_2
Φ	1	1	0	1	X_3

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which is invariant under the global $U(1)_{H_1} \times U(1)_{H_2} \times U(1)_\Phi$ symmetry up to a gauge-invariant term with a coupling C_λ of mass dimension 1. Additional gauge symmetry preserving terms are forbidden by invoking a $\mathbb{Z}_5$ symmetry at high scales. The $\mathbb{Z}_5$ allows a term Eq. (1b) which breaks the global symmetry down to $U(1)_Y \times U(1)_X$, where $U(1)_Y$ is identified with SM hypercharge and $U(1)_X$ will be identified with the *accidentally* arising PQ symmetry, with $X_3 = -X_1 - X_2$.

We choose the parameters of the potential such that all three complex scalar fields acquire a vacuum expectation value (vev) $v_{1,2,3}$. Writing the electrically neutral fields in a nonlinear representation, we have $\sqrt{2}H_i^0 = (v_i + h_i)e^{ia_i/v_i}$ and $\sqrt{2}\Phi = (v_3 + h_3)e^{ia_3/v_3}$, where h_i define the CP even radial modes and a_i define the CP odd angular modes.

Given an appropriate choice of basis, the angular modes can be rewritten as two Goldstone fields a and A . We will then derive a basis-invariant anomalous CP -violating (CPV) phase in the QCD sector, providing a mass for the Goldstone modes and allowing the identification of one mode a with the axion which solves the strong CP problem in the $\mathbb{Z}_5$ symmetric phase while the other mode A is heavy.

The interesting phenomenology of the anarchic axion is the deviation from the canonical QCD axion band in $\{m_a, f_a\}$ parameter space (and similarly in the axion-photon effective coupling), which results from the introduction of soft PQ breaking at low scales. Specifically,

$$V_{\text{break}}^{B_\mu} = -B_\mu H_1 H_2 + \text{H.c.}, \quad (2)$$

which further breaks the symmetry down to $U(1)_Y$. The B_μ coupling is of mass dimension 2 and breaks the $\mathbb{Z}_5$ symmetry explicitly. The magnitude of B_μ can be generated by a $\mathbb{Z}_5$ symmetric UV completion and would therefore be suppressed by a heavy UV scale.

We parameterize the symmetry breaking couplings as $B_\mu = |B_\mu|e^{-i\theta_\mu}$ and $C_\lambda = |C_\lambda|e^{-i\theta_\lambda}$. The original global symmetry can be used to render θ_λ unphysical, and hence C_λ is a real parameter.

III. THE GOLDSTONE BASIS

Following the procedure discussed in the Supplementary Material [20], we perform a basis transformation to align the Goldstone fields with the $U(1)_Y \times U(1)_X$ symmetries, which isolates the Goldstone eaten by the gauging of hypercharge. In this new basis, the angular potential is

$$V_{\text{ang}} = -|B_\mu| \left[\prod_{i=1}^2 (v_i + h_i) \right] \cos \left(\frac{a}{v_a} + \frac{A}{v_A} \delta^2 - \theta_\mu \right) - \frac{|C_\lambda|}{\sqrt{2}} \left[\prod_{i=1}^3 (v_i + h_i) \right] \cos \left(\frac{A}{v_A} (1 + \delta^2) - \theta_\lambda \right), \quad (3)$$

where $\delta = v_A/v_a$, $v_1 v_2 = v v_a / \sqrt{1 + \delta^2}$ and $v_3 = v_A / \sqrt{1 + \delta^2}$, with $v = \sqrt{v_1^2 + v_2^2} \approx 246$ GeV is the electroweak vev.

Next, we include the correction to the potential arising from QCD instantons induced by the coupling of SM quarks to H_1 and H_2 . The effective Lagrangian in the anarchic axion model is

$$\mathcal{L}_{G\tilde{G}} \supset \frac{g_s^2}{32\pi^2} \left(\bar{\theta}_{\text{SM}} - N_g \left(\frac{a}{v_a} + \delta^2 \frac{A}{v_A} \right) \right) G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad (4)$$

where N_g is the number of quark generations and $\bar{\theta}_{\text{SM}} \equiv \theta_{\text{QCD}} + \arg \det Y_u Y_d$ for the SM quark Yukawa matrices.

By applying global $U(1)$ transformations on H_1 , H_2 , Φ and the SM quarks, we can reshuffle the separate phases from Eq. (3) and Eq. (4) into a new $\bar{\theta}$ defined via $\bar{\theta}_{\text{SM}} - N_g \theta_\mu \equiv N_g \bar{\theta}$. We may choose $U(1)$ phases such that $\bar{\theta}$ is only in the B_μ contribution to the potential, and hence below Λ_{QCD} the corresponding a and A fields experience the instanton potential,

$$\mathcal{L}_{G\tilde{G}} \supset \Lambda_{\text{QCD}}^4 \cos \left(N_g \left(\frac{a}{v_a} + \delta^2 \frac{A}{v_A} \right) \right), \quad (5)$$

where $\Lambda_{\text{QCD}}^4 \equiv \frac{m_u m_d}{(m_u + m_d)^2} m_\pi^2 f_\pi^2$. Via the Peccei-Quinn mechanism, $\bar{\theta}$ is relaxed to the *observable* CPV parameter $\bar{\theta}_{\text{eff}}$, seen as the tadpoles effects of a and A in Eq. (5). Consequently, the total angular potential for a and A fields is now

$$\begin{aligned} -V_{\text{ang}} &= \Lambda_{\text{QCD}}^4 \cos(N_g(\alpha + \alpha' \delta^2)) \\ &+ \Lambda_{\text{QCD}}^4 \frac{v_a}{v_{\text{max}}} \cos(\alpha + \alpha' \delta^2 + \bar{\theta}) \\ &+ \frac{|C_\lambda| v v_A^2}{\sqrt{2} \delta (1 + \delta^2)} \cos(\alpha' (1 + \delta^2)), \end{aligned} \quad (6)$$

where we have introduced $\alpha \equiv a/v_a$, $\alpha' \equiv A/v_A$ as convenient notation for the fields, and we have

$$v_{\text{max}} \equiv \frac{\Lambda_{\text{QCD}}^4}{|B_\mu| v} \sqrt{1 + \delta^2} \quad (7)$$

as the extremal value of the PQ vev v_a . For $v_a > v_{\text{max}}$ the hierarchy of the terms in the angular potential would flip and the field a could not be used anymore to relax $\bar{\theta}_{\text{eff}}$.

IV. RELAXATION AND HEAVY A MASS

To ensure A is heavy, we will necessarily take $|C_\lambda| \gg |B_\mu|^{1/2}$, Λ_{QCD} , such that the mass of A arises dominantly from the C_λ contribution to the potential, yielding

$$m_A^2 = \frac{|C_\lambda|v}{\sqrt{2}} \left(\frac{1}{\delta} + \delta + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{|C_\lambda|}\right) \right), \quad (8)$$

setting the scale of A well above the electroweak scale. The v_{max} scale is the energy where the soft PQ breaking parameter must be nearly aligned to the $\bar{\theta}_{\text{SM}}$ to avoid neutron electric dipole moment (nEDM) constraints [21]. Pragmatically, as will be shown below, v_{max} corresponds to the largest possible scale suppression in $g_{a\gamma\gamma}$ for $\bar{\theta} \approx \pi$.

V. THE ANARCHIC AXION PARAMETERS

We now derive the m_a , f_a and $g_{a\gamma\gamma}$ relations. After spontaneous breaking of the PQ symmetry by v_a , the Goldstone bosons a and A acquire tadpoles α_0 and α'_0 , respectively. Importantly, the unphysical θ_λ also allows us to shift α'_0 purely into α_0 , leaving α'_0 unobservable, as seen in Eq. (6). Consequently, the tadpole α_0 entirely generates $\bar{\theta}_{\text{eff}} = -\alpha_0$, giving a potentially measurable nEDM. Note that in this basis, α_0 entirely captures the deviation from the canonical DFSZ due to nonvanishing B_μ .

Expanding Eq. (6) about the minimum and dropping constant terms and the heavy A field, yields

$$-\frac{V_{\text{ang}}}{\Lambda_{\text{QCD}}^4} = \alpha \left(N_g \sin(-N_g \bar{\theta}_{\text{eff}}) + \frac{v_a}{v_{\text{max}}} \sin(\bar{\theta} - \bar{\theta}_{\text{eff}}) \right) + \frac{1}{2} \alpha^2 \left(N_g^2 \cos(N_g \bar{\theta}_{\text{eff}}) + \frac{v_a}{v_{\text{max}}} \cos(\bar{\theta} - \bar{\theta}_{\text{eff}}) \right). \quad (9)$$

For $|C_\lambda| \gg |B_\mu|^{1/2}$, Λ_{QCD} , a is already in its mass basis, where m_a is given up to $\mathcal{O}(\Lambda_{\text{QCD}}/|C_\lambda|)$ corrections by

$$m_a^2 = \frac{\Lambda_{\text{QCD}}^4}{v_a^2} \left(N_g^2 \cos(N_g \bar{\theta}_{\text{eff}}) + \frac{v_a}{v_{\text{max}}} \cos(\bar{\theta} - \bar{\theta}_{\text{eff}}) \right). \quad (10)$$

Using Eq. (10), the axion decay constant is

$$\frac{1}{f_a} \equiv \frac{N_g}{v_a} = -\frac{\cos(\bar{\theta} - \bar{\theta}_{\text{eff}})}{2N_g v_{\text{max}} \cos(N_g \bar{\theta}_{\text{eff}})} + \sqrt{\frac{m_a^2}{\Lambda_{\text{QCD}}^4 \cos(N_g \bar{\theta}_{\text{eff}})} + \left(\frac{\cos(\bar{\theta} - \bar{\theta}_{\text{eff}})}{2N_g v_{\text{max}} \cos(N_g \bar{\theta}_{\text{eff}})} \right)^2}. \quad (11)$$

Note that in the limit $v_a \ll v_{\text{max}}$, we recover the canonical relation $m_a^2 f_a^2 = \Lambda_{\text{QCD}}^4$.

Finally, to evaluate the axion-diphoton coupling, we partition the irreducible $U(1)_{\text{em}}$ anomaly shared by a_1 and a_2 into the mass eigenstate a , giving

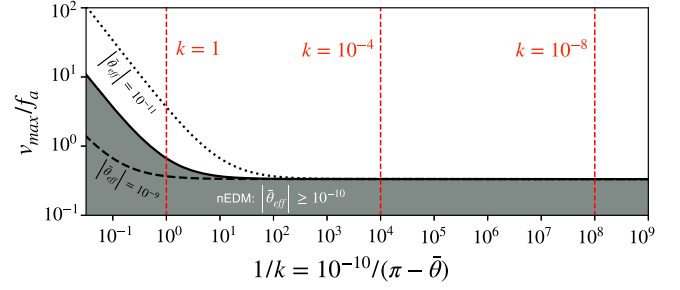


FIG. 1. Contours correspond to values of physical CPV $|\bar{\theta}_{\text{eff}}|$, obtained from Eq. (13). The gray region corresponds to parameter space constrained by the nEDM measurement.

$$g_{a\gamma\gamma} = \frac{e^2}{8\pi^2} \left(\frac{E}{N} - 1.92 \right) \frac{1}{f_a} \times \left(1 - \frac{m_a^2}{\sqrt{2}v|C_\lambda|} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{|C_\lambda|^2}\right) \right), \quad (12)$$

where $E/N = 8/3$, analogous to the DFSZ case [22].

VI. CP VIOLATION AND nEDM

We now return to the derivation of the tadpole α_0 acquired by a , and the resulting observable CPV $\bar{\theta}_{\text{eff}}$ constrained by measurements of the nEDM. To make contact with phenomenology, it will be useful to consider the leading order contribution to $\bar{\theta}_{\text{eff}}$. The first term of Eq. (9) encodes the residual CPV $\bar{\theta}_{\text{eff}}$ in the anarchic axion model, where the leading contribution up to $\mathcal{O}((\pi - \bar{\theta})^2)$ is given by

$$\bar{\theta}_{\text{eff}} = \frac{2(\pi - \bar{\theta})}{-1 + \sqrt{1 + \frac{4N_g^2 m_a^2 v_{\text{max}}^2}{\Lambda_{\text{QCD}}^4}}}. \quad (13)$$

Contours of $\bar{\theta}_{\text{eff}}$ are shown in Fig. 1. We define $k \equiv (\pi - \bar{\theta})/10^{-10}$ to capture the sensitivity to the deviation of $\bar{\theta}$ from π , i.e. for $k \lesssim 1$ a tuning will be required and we saturate at $v_{\text{max}}/f_a \rightarrow 1/N_g$. The white region is allowed by the nEDM bound. For $k \gtrsim 1$, we recover the DFSZ solution to strong CP , relaxing the required tuning.

VII. RESULTS

In Fig. 2, we display $g_{a\gamma\gamma}$ vs. m_a for the anarchic axion. The DFSZ axion line is shown in yellow. Regions targeted by experimental searches or constrained by astrophysical considerations are shaded out in gray [22]. For fixed values of m_a and v_{max} , the blue contours are computed by plugging in $1/f_a$ from Eq. (11) into Eq. (12) for $g_{a\gamma\gamma}$, and using Eq. (13) to fix $\bar{\theta}_{\text{eff}}$ as a function of k , m_a and v_{max} . We also use Eq. (13) to enforce the nEDM constraint $|\bar{\theta}_{\text{eff}}| \leq 10^{-10}$. Choosing a specific k denoted by a red

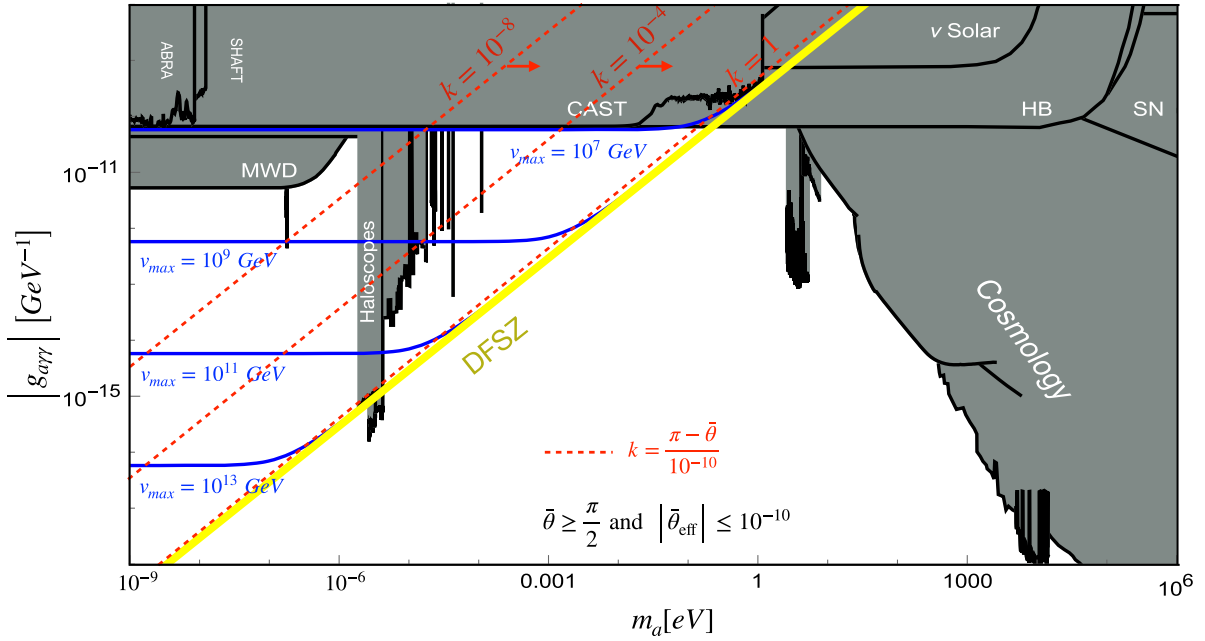


FIG. 2. Parameter space for the axion-diphoton coupling in the anarchic axion model consistent with current nEDM constraint [21]. Experimental limits, shown by the gray shaded regions are extracted from Ref. [23]. We show representative values of $v_{\max}$ and k that highlight the accessible light axion parameter space probed by ongoing haloscope and microwave cavity experiments.

dotted line, we can access lighter axion masses along a given blue contour of fixed $v_{\max}$ up to the intersection point. Explicitly, accessing smaller axion masses requires a small k and hence tuning $\bar{\theta}$ closer to π .

As Eq. (11) suggests for sufficiently small m_a (i.e., $m_a \ll |\Lambda_{\text{QCD}}^2 \cos(\bar{\theta} - \bar{\theta}_{\text{eff}})/(2N_g v_{\max})|$), $1/f_a$ becomes insensitive to m_a , and approaches $1/f_a \simeq |\cos(\bar{\theta} - \bar{\theta}_{\text{eff}})/(N_g v_{\max})|$. Physically, the vev shift from the B_μ term begins to dominate in this regime; corresponding to the kink in the blue lines of Fig. 2 upon their intersection with the $k = 1$ line. In the $v_a \lesssim v_{\max}$ region we can evaluate the corresponding magnitude of B_μ by

$$|B_\mu| \lesssim \frac{\Lambda_{\text{QCD}}^4}{v v_a} \sqrt{1 + \delta^2} = \frac{2\Lambda_{\text{QCD}}^4}{\sin(2\phi) v^2}, \quad (14)$$

with ϕ being the mixing angle between v_1 and v_2 , leading to an estimate of $|B_\mu| \sin(2\phi) \lesssim 10^{-9} \text{ GeV}^2$.

In Fig. 2 we have enforced $\bar{\theta} \in [\frac{\pi}{2}, \pi]$ leading to light Anarchic Axion masses populating the region to the left of the DFSZ band. Note that heavier masses can populate the region to the right of the DFSZ line for $\bar{\theta} < \pi/2$. We leave the details of the heavy anarchic axion to future work [24].

VIII. THE QUALITY PROBLEM

All axion models suffer from a high scale quality problem, i.e. higher dimensional operators, which are generally present from gravity effects, break the PQ symmetry at high scales and shift the axion vev.

The quality problem of the anarchic axion manifests as an alignment of $\bar{\theta}$ with π once the soft breaking B_μ term starts to dominate the vev shift, as is evident from the low m_a plateau in Fig. 2. To quantify the quality problem, we introduce a measure Δ_{BG} of fine tuning, following [24,25]:

$$\Delta_{\text{BG}}(\bar{\theta}_{\text{eff}}) \equiv \left| \frac{\bar{\theta}}{\bar{\theta}_{\text{eff}}} \frac{\partial \bar{\theta}_{\text{eff}}}{\partial \bar{\theta}} \right|. \quad (15)$$

Note that a large value of Δ_{BG} implies a large tuning. Indeed, in Fig. 3 we observe that Δ_{BG} grows as $\bar{\theta} \rightarrow \pi$. We emphasize that the region where the anarchic axion exhibits a lighter than usual axion is characterized by a nontrivial fine tuning. This approach of *quantifying* the quality

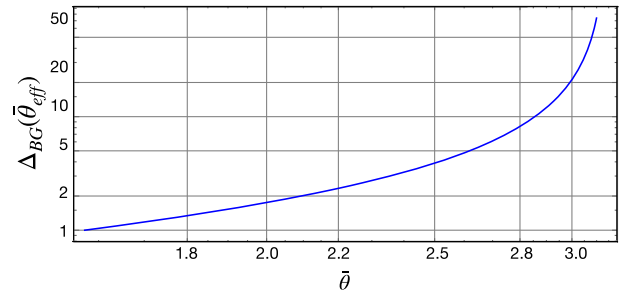


FIG. 3. A measure of tuning for the variable $\bar{\theta}$ consistent with nEDM constraints on $\bar{\theta}_{\text{eff}}$ given in Eq. (13), fixing $m_a = 10^{-6} \text{ eV}$ and $v_{\max} = 10^7 \text{ GeV}$. As the figure demonstrates, the tuning exponentially increases as $\bar{\theta} \rightarrow \pi$, and this tuning is negligibly sensitive to alternate m_a and $v_{\max}$ values.

problem can be potentially applied to other axion models where the PQ symmetry breaking enters explicitly at high scales. Note that this is only possible since the residual CPV $\bar{\theta}_{\text{eff}}$ is calculable.

We also mention the possibility that the soft breaking B_μ term of the anarchic axion model can arise as the leading low energy operator matched to a Planck-suppressed PQ breaking term in canonical axion models. We will reserve a study of the matching requirements of soft PQ breaking terms in high-quality axion models for the future.

IX. DISCUSSION

In this paper we have introduced the anarchic axion model which solves the QCD Strong CP problem parametrically while populating new regions of parameter space. We close by mentioning that these regions with fine tuning can be motivated by, e.g. a clockwork-like ultraviolet model [26–28]. For instance, consider a $U(1)_{\text{PQ}}$ bulk gauge symmetry in a 5D warped geometry with a bulk electroweak singlet scalar, where PQ charges will be carried by a brane-localized Higgs and right-handed up-type quarks. Given an appropriate choice of boundary conditions for the bulk field, the Anarchic Axion model then arises as an effective description with a global $U(1)_{\text{PQ}}$ symmetry. A discrete $\mathbb{Z}_5$ symmetry can be identified with a remnant of the bulk gauge symmetry. We leave the details of such UV completions to future work.

While the interesting phenomenology of the anarchic axion model discussed in this paper arose due to a soft PQ breaking term, other variations can produce similar phenomenology. For instance, replacing the soft breaking B_μ

term with a Φ^3 term would result in a potential which is protected by an accidental and global $\mathbb{Z}_3$ symmetry. We leave the exploration of variants of anarchic axion models to future work. We expect the cosmological production of the anarchic axion to proceed through a variation of the canonical misalignment mechanisms and also leave the detailed implications of anarchic axion dark matter to future study.

The experimental observation of an exceptionally light axion deviating from the canonical QCD axion band would be evidence for an anarchic axion solution to the strong CP problem.

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Note added.—When this work was in preparation, a related preprint appeared [29], focusing on generating high-quality axion solutions by suppressing Planck-scale operators using chiral gauged $U(1)$ symmetries.

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