

Better angle on hadron transverse momentum distributions at the Electron-Ion Collider

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We propose an observable q_* sensitive to transverse momentum dependence (TMD) in $eN \rightarrow ehX$, with q_*/E_N defined purely by lab-frame angles. In 3D measurements of confinement and hadronization this resolves the crippling issue of accurately reconstructing small transverse momentum P_{hT} . We prove factorization for $d\sigma_h/dq_*$ for $q_* \ll Q$ with standard TMD functions, enabling q_* to substitute for P_{hT} . A double-angle reconstruction method is given which is exact to all orders in QCD for $q_* \ll Q$. q_* enables an order-of-magnitude improvement in the expected experimental resolution at the EIC.

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I. INTRODUCTION

A deeper understanding of the emergent properties of the nucleon, such as confinement and hadronization, has been a frontier of nuclear and particle physics research since the inception of quantum chromodynamics (QCD) five decades ago. An important one-dimensional view of the nucleon is provided by the deep-inelastic scattering (DIS) process $e^-(\ell) + N(P) \rightarrow e^-(\ell') + X$, where the scattering is mediated by an off-shell photon of momentum $q = \ell - \ell'$ (with $Q^2 \equiv -q^2 > 0$). Confinement is probed by measurements of $x = Q^2/(2P \cdot q)$, the momentum fraction carried by the colliding parton inside the nucleon N . A more intricate view is obtained by identifying a hadron h in semi-inclusive DIS (SIDIS), $e^-(\ell) + N(P) \rightarrow e^-(\ell') + h(P_h) + X$. Here measurements of the longitudinal momentum fraction $z = (P \cdot P_h)/(P \cdot q)$ that the hadron retains when forming from the struck quark give insight into the complex dynamics of hadronization. Measuring the hadron's transverse momentum $\vec{P}_{hT}$ relative to $\vec{q}$ gives access to a three-dimensional view of the confinement and hadronization processes for N and h , together with spin correlations that probe these processes.

The region most sensitive to these dynamics occurs for small transverse momentum, $P_{hT} \sim \Lambda_{\text{QCD}} \ll Q$, where

Λ_{QCD} is the QCD confinement scale. Here the cross section obeys a rigorous factorization theorem [1], with the confinement and hadronization dynamics encoded in universal transverse momentum-dependent (TMD) parton distribution functions (PDFs) and fragmentation functions (FFs). SIDIS cross sections have been extensively studied experimentally at HERMES [2–4], COMPASS [5–7], RHIC [8,9], and JLab [10–12] and together with the Drell-Yan process have enabled extractions of TMD PDFs and FFs (TMDs) by various groups, e.g. [13–15]. A key scientific goal of the upcoming Electron-Ion-Collider (EIC) [16] is to study SIDIS with enormous beam luminosities to determine TMD PDFs and FFs with unprecedented precision. Progress has also been made toward calculations of TMD PDFs from lattice QCD [17–22].

A key challenge in experimental studies of TMDs is that measurements of $\vec{P}_{hT}$ require reconstructing the photon momentum (or Breit frame) to great accuracy to avoid loss of precision on $P_{hT} = |\vec{P}_{hT}| \ll Q$. A misreconstruction of $\vec{\ell}'$ by $\mathcal{O}(\Delta)$ leads to a misreconstruction of $\vec{q}$ and therefore $\vec{P}_{hT}$ by $\mathcal{O}(\Delta)$, which is a large uncertainty for $P_{hT} \ll Q$. For example, for a nominal measurement at $P_{hT}/z = 1$ GeV with $Q = 20$ GeV, a typical detector resolution of $\Delta = 0.5$ GeV leads to a 50% uncertainty. This puts in peril the EIC physics program to unveil the dynamics of hadronization and confinement in the kinematic region with the largest sensitivity.

In this paper we construct a novel SIDIS observable, q_* , designed to be maximally resilient against resolution effects while delivering the same sensitivity to TMD dynamics as $\vec{P}_{hT}$. The key insight is that while the magnitude of the electron and hadron three momentum is subject to limited detector resolution, modern tracking detectors deliver near-perfect resolution on the angles of

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charged particle tracks. We will therefore construct q_* to satisfy the following three criteria: (i) it is purely defined in terms of lab-frame angles and the beam energies; (ii) at small values $q_* \ll Q$, the differential cross section $d\sigma/dq_*$, including spin correlations, still satisfies a rigorous factorization theorem in terms of the standard TMD PDFs and FFs; (iii) it does not dilute the statistical power of the available event sample. Our construction is inspired by, but features key differences to, the Drell-Yan ϕ_η^* observable in hadron-hadron collisions [23], which has enabled tests of perturbative QCD from permil-level Z-pole measurements at the Tevatron and LHC [24–29].

Below we define q_* in detail, prove the factorization theorem for q_* with standard TMDs, and evaluate the expected detector resolution, statistical power, and resilience against systematic biases of q^* versus $\vec{P}_{hT}$.

II. CONSTRUCTING q_*

Consider the target rest frame shown in Fig. 1(a) where the nucleus N is at rest and the z -axis is along the incoming lepton beam. The lepton momenta $\vec{\ell}$ and $\vec{\ell}'$ define the lepton plane as the x - z plane. We wish to take advantage of the high-precision reconstruction of polar angles (rapidities) and azimuthal angles in the EIC lab frame. Here we give results in terms of EIC frame rapidities in the light target mass limit $M \ll Q$, with full M dependence in [30]. The acoplanarity angle in the target rest frame, $\phi_{\text{acop}}^{\text{rest}}$, is defined by $\tan \phi_{\text{acop}}^{\text{rest}} = -P_{h,y}/P_{h,x}$, where $P_{h,x}$ and $P_{h,y}$ are components of $\vec{P}_h$. From Fig. 1(a), it is obvious that $\tan \phi_{\text{acop}}^{\text{rest}} \propto \sin \phi_h P_{hT}$, where ϕ_h is the azimuthal angle of $\vec{P}_h$ in the Trento frame. We may thus use ϕ_{acop} as a precision probe of the hadron transverse momentum P_{hT} .¹

To work out the full relation between $\phi_{\text{acop}}^{\text{rest}}$ and P_{hT} , consider now the leading-power (LP) kinematics illustrated in Fig. 1(b), where $\lambda \sim P_{hT}/(zQ) \ll 1$. We find

$$\tan \phi_{\text{acop}}^{\text{rest}} = \frac{\sin \phi_h P_{hT}}{zQ\sqrt{1-y}} + \mathcal{O}(\lambda^2). \quad (1)$$

We now wish to express Q and $y = (P \cdot q)/(P \cdot \ell)$ in terms of final-state angles in the EIC frame, which is defined by a 180° rotation about our rest frame y axis and then a boost along the z -axis, so $\phi_{\text{acop}}^{\text{EIC}} = -\phi_{\text{acop}}^{\text{rest}}$. From Fig. 1(b), momentum conservation gives $\ell'_x = \ell'_x$, $q_x = -\ell'_x$, and $P_{hT} \ll Q$ implies $\theta_h + \theta_e + \alpha = \pi/2$. We find $y = 1 - \sin \theta_h / \cos \alpha$ and $Q^2 = (\ell_{\text{rest}}^0)^2 [\frac{\sin^2 \theta_e}{\cos^2 \alpha} - (1 - \frac{\sin \theta_h}{\cos \alpha})^2]$. Boosting to the EIC frame:

¹Lab-frame acoplanarity angles are also useful as a measure of transverse momentum in jet production in DIS [31,32]. Unlike this work, the observable of Refs. [31,32] does not feature the same experimental improvements since traditional jet axis reconstruction is not angular, and it also has nonglobal logarithms that are nonperturbative in the TMD region of interest.

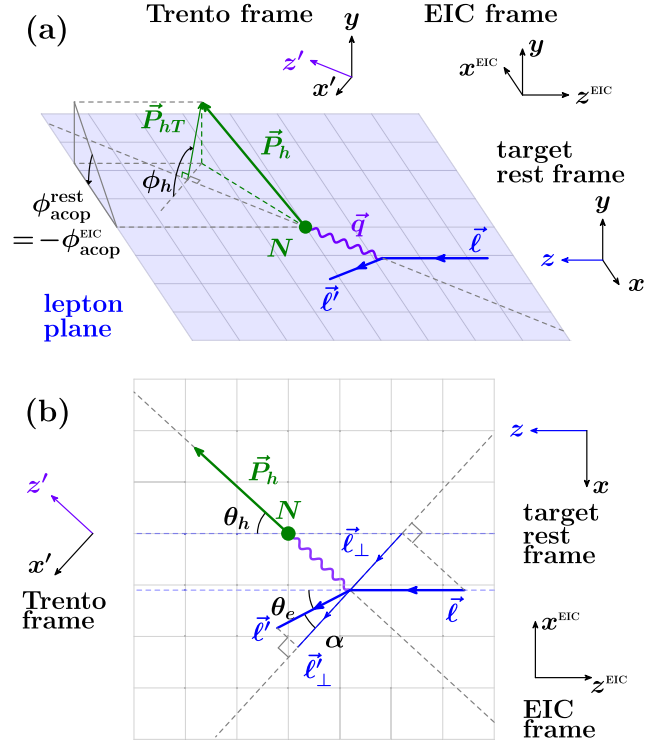


FIG. 1. (a) Definition of $\phi_{\text{acop}}^{\text{rest}}$ needed to construct q^* . Momenta are not to scale. We define the conventional Trento frame [33] for SIDIS, as well as the target rest frame and the EIC frame. ϕ_h is the azimuthal separation between $\vec{P}_h$ and $\vec{\ell}$ in the Trento frame. The acoplanarity angle in the target rest frame is $\phi_{\text{acop}}^{\text{rest}} \equiv \pi - \Delta\phi^{\text{rest}}$, where $\Delta\phi^{\text{rest}}$ is the azimuthal separation between $\vec{P}_h$ and $\vec{\ell}'$. (b) In-plane geometry for leading-power kinematics $P_{h,\perp}/Q \ll 1$, where we can approximate that $\vec{P}_h$ is along the same direction as $\vec{q}$. This geometry yields the double-angle formulas in Eq. (2) Q , x , y with angular measurements.

$$Q^2 = (2P_{\text{EIC}}^0)^2 \frac{e^{\eta_e + \eta_h}}{1 + e^{\Delta\eta}} + \mathcal{O}(\lambda), \quad y = \frac{1}{1 + e^{\Delta\eta}} + \mathcal{O}(\lambda^2),$$

$$x = [(2P_{\text{EIC}}^0)^2/s] e^{\eta_e + \eta_h} + \mathcal{O}(\lambda), \quad (2)$$

where η_i are the EIC frame pseudorapidities of the outgoing lepton $i = e$ and hadron $i = h$, $\Delta\eta \equiv \eta_h - \eta_e$, and $s = (P + \ell)^2$. This construction agrees with the double-angle formula in Ref. [34]. However, Ref. [34] uses the struck quark angle in a tree-level picture, while our Eq. (2) uses the hadron angle and holds to all orders in α_s , and up to power corrections in $P_{hT}/(zQ)$, which controls the distance to the Born limit. The $\mathcal{O}(\lambda)$ corrections to Eq. (2) are given in [30].

To exploit the proportionality in Eq. (1) to probe P_{hT} , we define an optimized observable:

$$q_* \equiv 2P_{\text{EIC}}^0 \frac{e^{\eta_h}}{1 + e^{\Delta\eta}} \tan \phi_{\text{acop}}^{\text{EIC}}. \quad (3)$$

Expanding in $P_{hT} \ll zQ$ it has a simple LP limit

$$q_*^{\text{LP}} = \sin \phi_h \frac{P_{hT}}{z}. \quad (4)$$

Thus for TMD analyses, Q^2 , x , y , and q^* can all be measured from the beam energy P_{EIC}^0 and angular variables. We may also define a dimensionless variable,

$$\phi_{\text{SIDIS}}^* = \sqrt{\frac{e^{\Delta\eta}}{1 + e^{\Delta\eta}}} \tan \phi_{\text{acop}}^{\text{EIC}} = \frac{q_*}{Q} + \mathcal{O}(\lambda^2). \quad (5)$$

This is analogous to the setup for the ϕ_{η}^* observable in Drell-Yan [23]. We expect the purely angular observables q_* and ϕ_{SIDIS}^* to be measured to much higher relative precision compared to the transverse momentum P_{hT} .

III. FACTORIZATION

Consider the standard factorization theorem for polarized SIDIS [1,35–37],

$$\begin{aligned} \frac{d\sigma}{dx dy dz d^2\vec{P}_{hT}} = \sigma_0 \bigg\{ & W_{UU,T} + \lambda_e S_L \sqrt{1 - \epsilon^2} W_{LL} + \epsilon \cos(2\phi_h) W_{UU}^{\cos(2\phi_h)} + S_L \epsilon \sin(2\phi_h) W_{UL}^{\sin(2\phi_h)} \\ & + S_T \sin(\phi_h - \phi_S) W_{UT,T}^{\sin(\phi_h - \phi_S)} + \epsilon S_T \left[\sin(\phi_h + \phi_S) W_{UT}^{\sin(\phi_h + \phi_S)} + \sin(3\phi_h - \phi_S) W_{UT}^{\sin(3\phi_h - \phi_S)} \right] \\ & + \lambda_e S_T \sqrt{1 - \epsilon^2} \cos(\phi_h - \phi_S) W_{LT}^{\cos(\phi_h - \phi_S)} \bigg\}, \end{aligned} \quad (6)$$

where $\sigma_0 \equiv \alpha^2 \pi y \kappa_\gamma / [zQ^2(1 - \epsilon)]$, α is the fine-structure constant, λ_e is the lepton beam helicity, $S^\mu = (0, S_T \cos \phi_S, S_T \sin \phi_S, -S_L)$ is the nucleon spin vector in the Trento frame [37], $\epsilon = (1 - y)/(1 - y + y^2/2)$, and $\kappa_\gamma = 1$ for $M \ll Q$. We have only kept the structure functions W that are nonzero at leading power in λ . They can be written in terms of the hard function $\mathcal{H}(Q^2)$ and various b_T -space TMD PDFs $\tilde{g}(x, b_T)$ and TMD FFs $\tilde{D}(z, b_T)$ [38]:

$$\begin{aligned} W_{PP'}^{\text{ang}} &\propto \mathcal{F}[\mathcal{H}\tilde{g}^{(n)}\tilde{D}^{(m)}] \\ &\equiv 2z \int_0^\infty \frac{db_T b_T}{2\pi} \mathcal{I}[\mathcal{H}\tilde{g}^{(n)}\tilde{D}^{(m)}] J_{n+m}(b_T P_{hT}/z) \end{aligned} \quad (7)$$

with $\mathcal{I}[\mathcal{H}\tilde{g}^{(n)}\tilde{D}^{(m)}] \equiv (Mb_T)^n (-Mb_T)^m \sum_f \mathcal{H}_f \tilde{g}_f^{(n)} \tilde{D}_f^{(m)}$, where f sums over quarks and antiquarks. For example, $W_{UU}^{\cos(2\phi_h)} = -\mathcal{F}[\mathcal{H}\tilde{h}_1^{\perp(1)}\tilde{H}_1^{\perp(1)}]$, where $\tilde{h}_1^{\perp(1)}$ and $\tilde{H}_1^{\perp(1)}$ are the Boer-Mulders [39] and Collins [40] functions. For details on our notation and conventions, see Ref. [41].

To compute the spectrum differential in x, y, z and q_* , we insert the leading-power measurement $\delta(q_* + \sin \phi_h P_{hT}/z)$ and analytically perform the integral over $d^2\vec{P}_{hT} = dP_{hT} P_{hT} d\phi_h$. As an explicit example, we work out the contribution from $W_{UU}^{\cos(2\phi_h)}$. Using Eq. (7):

$$\begin{aligned} &\int_0^\infty dP_{hT} P_{hT} \int_0^{2\pi} d\phi_h \delta\left(q_* + \sin \phi_h \frac{P_{hT}}{z}\right) \cos(2\phi_h) W_{UU}^{\cos(2\phi_h)} \\ &= -\frac{2z^3}{\pi} \int db_T \mathcal{I}[\mathcal{H}\tilde{h}_1^{\perp(1)}\tilde{H}_1^{\perp(1)}] \int_0^{2\pi} \frac{d\phi_h}{\sin^2 \phi_h} \Theta\left(-\frac{q_*}{\sin \phi_h}\right) \cos(2\phi_h) \frac{b_T |q_*|}{2} J_2\left(\frac{b_T q_*}{\sin \phi_h}\right) \\ &= -\frac{2z^3}{\pi} \int db_T \mathcal{I}[\mathcal{H}\tilde{h}_1^{\perp(1)}\tilde{H}_1^{\perp(1)}] \cos(q_* b_T). \end{aligned} \quad (8)$$

The ϕ_h integral, which is specific to the structure function, can only depend on $q_* b_T$ by dimensional analysis, and in this case yields a simple $\cos(q_* b_T)$.

In total, the LP cross section differential in q_* is

$$\begin{aligned} \frac{d\sigma}{dx dy dz dq_*} = \frac{2z^3}{\pi} \sigma_0 \int_0^\infty db_T \bigg\{ & \cos(q_* b_T) \left(\mathcal{I}[\mathcal{H}\tilde{f}_1\tilde{D}_1] - \epsilon \mathcal{I}[\mathcal{H}\tilde{h}_1^{\perp(1)}\tilde{H}_1^{\perp(1)}] + \lambda_e S_L \sqrt{1 - \epsilon^2} \mathcal{I}[\mathcal{H}\tilde{g}_{1L}\tilde{D}_1] \right) \\ & + \cos \phi_S \sin(q_* b_T) S_T \left(\mathcal{I}[\mathcal{H}\tilde{f}_{1T}^{\perp(1)}\tilde{D}_1] + \epsilon \mathcal{I}[\mathcal{H}\tilde{h}_1\tilde{H}_1^{\perp(1)}] + \frac{\epsilon}{4} \mathcal{I}[\mathcal{H}\tilde{h}_{1T}^{\perp(2)}\tilde{H}_1^{\perp(1)}] \right) \\ & - \sin \phi_S \sin(q_* b_T) \lambda_e S_T \sqrt{1 - \epsilon^2} \mathcal{I}[\mathcal{H}\tilde{g}_{1T}^{\perp(1)}\tilde{D}_1] \bigg\}. \end{aligned} \quad (9)$$

We stress that the TMD PDFs and FFs $\tilde{f}_1, \tilde{D}_1, \tilde{h}_1^{\perp(1)}, \dots$ are the *same* as in the standard factorization for the P_{hT} spectrum and TMD spin correlations. This is analogous to the role of the unpolarized [42,43] and Boer-Mulders [44] TMD PDFs in the Drell-Yan ϕ_η^* . The factorization theorem can equivalently be written in terms of momentum-space TMDs, see [30]. Crucially, definite subsets of these TMDs contribute to the even and odd parts of the spectrum under $q_* \rightarrow -q_*$. The odd parts are accessible through the asymmetry $d\sigma(q_* > 0) - d\sigma(q_* < 0)$. Contributions can be further disentangled experimentally through their unique dependence on λ_e, S^μ and ϵ , i.e., by taking asymmetries with opposite beam polarizations and by measuring cross sections as a function of y .² E.g., the double asymmetry for $q_* \rightarrow -q_*$ and $\lambda_e \rightarrow -\lambda_e$ as a function of x and $|q_*|$ gives direct access to the worm-gear T function $\tilde{g}_{1T}^\perp(x, b_T)$.

IV. EXPERIMENTAL SENSITIVITY

To show the improvement that q_* makes for TMD analyses, we use PYTHIA 8.306 [45] to simulate $e^-p \rightarrow e^-X$ at 18×275 GeV², disabling QED corrections. We select on $h = \pi^+$ and apply the following cuts (“SIDIS cuts”):

$$x > 0.001, \quad 0.01 < y < 0.95, \quad z > 0.05, \\ Q^2 > 16 \text{ GeV}^2, \quad W^2 = (P + q)^2 > 100 \text{ GeV}^2. \quad (10)$$

Scaled to an integrated EIC luminosity of 10 fb^{-1} , this results in a sample with $N_{\pi^+} = 4.18 \times 10^8$. We first assess the expected detector resolution of q_* compared to P_{hT} . We apply Gaussian smearing to the final-state electron and hadron momenta, assuming a tracking detector that matches the performance given in Ref. [16]: a resolution of $\sigma_p/p = 0.05\% p/\text{GeV} \oplus 0.5\%$ on the momentum $p = |\vec{p}|$ of charged particles in the central barrel region $|\eta| < 1$, $0.05\% p/\text{GeV} \oplus 1\%$ in the inner endcap $1 < |\eta| < 2.5$, and $0.1\% p/\text{GeV} \oplus 2\%$ in the outer endcap $2.5 < |\eta| < 3.5$. A particle is forward (backward) if it has $1 < |\eta| < 3.5$ and $\eta > 0$ ($\eta < 0$). We assume a fixed angular resolution of $\sigma_\theta = \sigma_\phi = 0.001$. We ignore the electromagnetic calorimeter as its e^- energy resolution is expected to be a factor of two worse than the tracker [16]. Our key results for the detector resolution on q_* compared to P_{hT} are shown in Fig. 2 for the case of a backward e^- and

²We recommend reconstructing S^μ using a rotation by θ_h to maintain a purely angular measurement, see [30], which is justified at LP. Note that the transversity and pretzelosity PDFs have a degenerate contribution $\epsilon S_T(h_1 + h_{1T}^\perp/4)$ to q_* in Eq. (9), while the worm-gear L function $h_{1L}^\perp$ drops out, due to q_* being even under $\phi_h \leftrightarrow \pi - \phi_h$. Encouragingly, the subleading-power Cahn effect $\propto \cos(\phi_h)$, which pollutes standard $\tilde{P}_{hT}$, also drops out for the same reason.

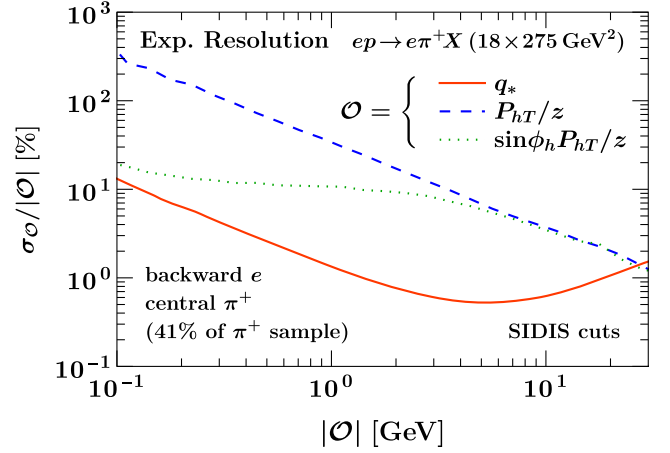


FIG. 2. Expected event-level detector resolution σ_O for different SIDIS TMD observables O . We show relative resolutions as a function of the magnitude of $O = q_*$ (solid red), P_{hT}/z (dashed blue), and $\sin \phi_h P_{hT}/z$ (dotted green).

a central h , which accounts for the largest share (41%) of the event sample. We see that q_* improves over the resolution of P_{hT}/z by an *order of magnitude* across the strongly confined TMD region $q_*, P_{hT}/z \lesssim 2$ GeV. Similar results are obtained for other detector regions, see [30]. It is interesting to compare q_* to a direct measurement of $\sin \phi_h P_{hT}/z$, to which it reduces at leading power. The latter has improved resolution over P_{hT}/z at small values thanks to picking up on the same acoplanarity of the event, which is stable against the electron momentum resolution, but cannot outperform q_* since it is not a pure angular variable.

To verify the statistical sensitivity of q_* to TMD physics we perform a Bayesian reweighting analysis of the unpolarized cross section $d\sigma/(dx dz dQ^2 dO)$ for $O = P_{hT}/z$ and $|q_*|$. We assume that the O spectrum is measured in twenty equidistant bins between $0 \leq O \leq 4$ GeV inside 1000 three-dimensional bins in x, z, Q^2 with equal statistics $N_{\pi^+}/1000$ in each, and for definiteness consider a bin centered on $x = 0.1, z = 0.15, Q = 20$ GeV in the following. To account for the fact that the factorized dependence on x, z, Q^2 is determined from all bins at equal x, z, Q^2 simultaneously, we multiply the available statistics by another factor 100, arriving at an effective sample size of $N_{\text{eff}} = N_{\pi^+}/10 = 4.18 \times 10^7$. At fixed x, z, Q^2 , a common model for the nonperturbative TMDs is [46]

$$\tilde{f}_1^{\text{NP}} = e^{-\omega_1 b_T^2}, \\ \tilde{D}_1^{\text{NP}} = \alpha e^{-\omega_2 b_T^2} + (1 - \alpha)(1 - \omega_3 b_T^2) e^{-\omega_3 b_T^2}, \quad (11)$$

where the ω_i encode the width of the TMDs. We are interested in how much better the three free parameters ω_i can be determined at the EIC using either q_* or P_{hT} . (We hold the parameter α fixed for simplicity.) We assume a Gaussian prior probability density $\pi(\omega_i)$ based on the

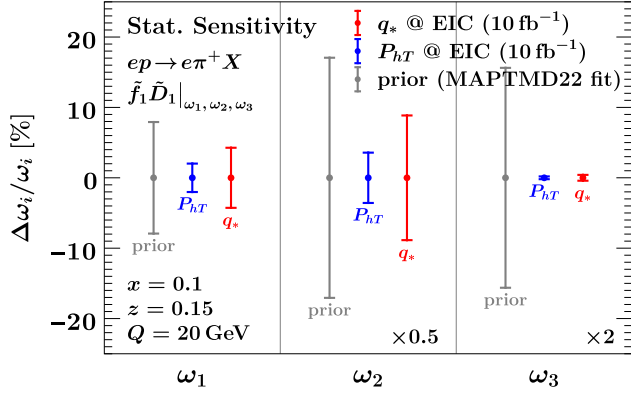


FIG. 3. Statistical sensitivity to TMD nonperturbative model coefficients at the 10 fb^{-1} EIC when measuring P_{hT} (blue) or q_* (red) compared to the prior (MAPTMD22 fit [46], gray). Despite its superior resolution, q_* enjoys comparable statistical sensitivity to P_{hT} .

central values and standard deviations from [46]. We combine Eq. (11) with leading-logarithmic TMD evolution and tree-level matching in SCETlib [47], and insert it into Eqs. (6) and (9) to generate EIC pseudodata d_n for bin n in $\mathcal{O}$, using the central ω_i . In the same way, we generate theory replicas $t_n(\omega_i)$ distributed according to the prior. By normalizing $\sum d_n = \sum t_n = 1$ to the sum over bins at fixed x and z , collinear PDFs and FFs drop out at this order. (For details on the theory calculation, see [30].) We then sample the posterior parameter probability distribution

$$\pi(\omega_i | d_n) \propto \exp \left[-\sum_n \left(\frac{d_n - t_n(\omega_i)}{\sigma_n} \right)^2 \right] \pi(\omega_i) \quad (12)$$

using a standard χ^2 likelihood function, where $\sigma_n = \sqrt{d_n/N_{\text{eff}}}$ is the Poisson error on pseudodata bin n . Our results for the mean and variance of the posterior distribution are shown in Fig. 3 compared to those of the prior. Comparing P_{hT} and q_* , we find that the superior experimental resolution of q_* only requires giving up a minor amount of statistical sensitivity to the ω_i . In particular, there is more than a factor 10 improvement in uncertainty on the dominant fragmentation parameter ω_3 in either case. The choice of binning at small q_* should be optimized in the future to exploit its excellent resolution, but we stress that we have not done so here.

The same setup can be used to assess the robustness of q_* against systematic uncertainties. We use PYTHIA [45] to generate biased pseudodata d_n^{bias} subject to either (i) a momentum miscalibration, $p \rightarrow (1 + \delta_p)p$, or (ii) a shape effect from a nonuniform detector response (encoded e.g. in an efficiency) that changes at a slow rate $\Delta\epsilon_X$ as a function

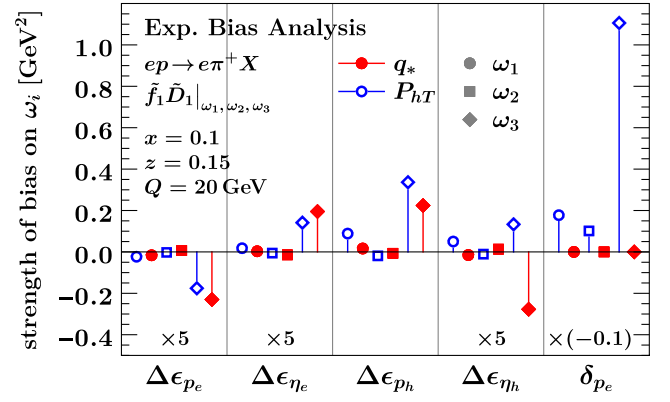


FIG. 4. Strength of bias on TMD nonperturbative model coefficients from potential sources of systematic uncertainty. By construction q_* (red) is robust against the large calibration uncertainty δ_{p_e} that impacts P_{hT} (blue). Both q_* and P_{hT} exhibit similar susceptibility to nonuniform detector response, modeled by $\Delta\epsilon_X$.

of $X = \{p_e, p_h, \eta_e, \eta_h\}$ across the x, z, Q^2 bin at hand. (The absolute value of the efficiency cancels in the normalized d_n^{bias} .) Repeating the reweighting analysis, we evaluate the partial derivatives of the posterior's mean ω_i with respect to the bias parameters, which we dub the “strength” of the bias, as shown in Fig. 4. As anticipated, we find that an analysis using P_{hT} is severely susceptible to the electron momentum calibration δ_{p_e} , while the calibration uncertainty using q_* vanishes exactly due to its purely angular nature. (Both P_{hT}/z and q_* are independent of δ_{p_h} at observable level for $M_h \ll Q$.) Figure 4 also shows that P_{hT} and q_* have comparable susceptibility to nonuniform detector response, despite the exponential factors of $\eta_{e,h}$ appearing in q_* , demonstrating the robustness of q_* against these sources of bias.

By replacing measurements of $d\sigma/dP_{hT}d\phi_h$ by $d\sigma/dq_*$ with angular reconstruction of Q, x, y , the prospects for precisely mapping the 3D structure of hadronization and confinement with TMDs are bright. We anticipate a follow-up campaign to aid this endeavor by discovering other useful angular observables that resolve the remaining TMD PDFs and by studying theoretical ingredients, like the convergence of the known higher-order QCD corrections for these cross sections.

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