

Doubly heavy tetraquark multiplets as heavy antiquark-diquark symmetry partners of heavy baryons

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Symmetries play important roles in the understanding of hadron structures and spectroscopy. Motivated by the discovery of the doubly charmed tetraquark $T_{cc}^+(3875)$, we study the ground states of the doubly heavy tetraquarks with the QCD inspired heavy antiquark-diquark symmetry in the constituent quark model. Six ground states of $T_{QQ}(Q = c, b)$ are predicted and the lightest T_{cc} state has a mass of 3875.8 ± 7.6 MeV and spin-parity 1^+ , which are consistent with those of the observed $T_{cc}^+(3875)$. In addition, the magnetic moments of the predicted tetraquarks $T_{cc}^+(3876)$ and $T_{bb}^-(10396)$ are also estimated in the same model, which provide further information to distinguish the structures of the T_{QQ} states.

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I. INTRODUCTION

Since 2003, many exotic hadronic states that cannot be well understood by the conventional constituent quark model have been observed. Typically, in the heavy flavor sector, the observations of the so-called X , Y , Z states, the P_c pentaquarks and so on, have extended the hadron spectrums and deepen our understanding on the strong interactions between quarks in the nonperturbative region (see, e.g., Refs. [1–9] for reviews). In 2017, the doubly heavy baryon Ξ_{cc}^{++} was discovered in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum by the LHCb collaboration with a mass 3621.4 MeV [10]. Recently, the LHCb Collaboration reported a narrow state in the $D^0 D^0 \pi^+$ invariant mass spectrum [11,12], with $J^P = 1^+$ and a mass 3875 MeV very close to $D^{*+} D^0$ threshold, namely the $T_{cc}^+(3875)$. These first reported doubly heavy baryon and tetraquark arouse the interests of studies on the doubly heavy hadrons both theoretically and experimentally [13–32].

Since, in a doubly heavy hadron, the heavy quark is almost near its mass shell, it is natural to expect that the heavy quark limit is applicable. In the heavy quark limit, the heavy diquark in a baryon is almost at rest and can be regarded as a spectator, therefore it can be viewed as a compact object. Since the light quark is in color triplet in a

heavy baryon, the heavy diquark should be in color antitriplet. Therefore, the heavy diquark $\bar{X} = [QQ]$ in a doubly heavy hadron can be regarded as a compact object without radical excitation and belongs to the $\bar{3}_c$ color representation, the same as the one for the antiheavy quark $\bar{Q}$.¹ Then, in the heavy quark limit, the color interactions are common for $\bar{X}$ and $\bar{Q}$, which leads to the superflavor symmetry, or the heavy antiquark-diquark symmetry (HADS), for the heavy quark sector [33–35]. The HADS sets up a relation between the hadrons with the same light quarks but different number of heavy quarks. There are two charm quarks in the doubly charmed baryon $\Xi_{cc}^{(*)}$, so it can be related to the $D^{(*)}$ meson through the HADS [15,36,37]. With the HADS, one can easily derive a relation between the mass splitting of the doubly charmed baryon doublet and that of the charmed meson doublet, $m_{\Xi_{cc}^*} - m_{\Xi_{cc}} = \frac{3}{4}(m_{\bar{D}^*} - m_{\bar{D}})$ [15,38,39], which has been numerically confirmed by a series of lattice QCD simulations [40–43]. Again with the HADS, the systematic spectrum of the doubly heavy baryons is estimated by using the chiral partner structure and heavy quark spin-flavor symmetry [15].

In this work, we studied the spectrum of the doubly heavy tetraquarks using a constituent quark model with respect to the HADS in which the doubly heavy tetraquarks can be regarded as the HADS partners of the heavy baryons. Six ground tetraquark states with different quantum number configurations in both charm and bottom sectors are predicted. The predicted mass of the lowest

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¹In this work, we change the notation $X \sim [QQ]$ used in [15] to $\bar{X} \sim [QQ]$ considering that $[QQ]$ is in the $\bar{3}_c$ representation.

doubly charmed tetraquark state, as a counterpart of $\bar{\Lambda}_c$ in the HADS, is consistent with the observed $T_{cc}^+(3875)$. Although, whether the nature of $T_{cc}^+(3875)$ is a molecular state composed of D and D^* mesons or compact tetraquark is still up for debate [44–50]. Since the HADS exists in the compact doubly heavy tetraquark picture but not in that of hadronic molecule, the doubly heavy tetraquarks predicted as the HADS partners of $\Sigma_Q^{(*)}$ can be easily distinguished from that predicted using the hadronic molecular picture. As well as the mass relation between the doubly heavy baryons and heavy mesons mentioned above, HADS indicates specific relations between doubly heavy tetraquarks and heavy baryons. The observation of the predicted tetraquarks related to $\Sigma_Q^{(*)}$ baryons can be taken as an evidence of the tetraquark nature of $T_{cc}^+(3875)$.

Moreover, with the same model, the magnetic moments of the lowest doubly heavy tetraquarks are also estimated and compared to the calculation without the HADS. The results with and without HADS agree well, which indicates that the HADS conserves well in these models for doubly heavy systems. Along this line, considering that the observations of heavy baryons and doubly charmed tetraquark $T_{cc}^+(3875)$, it is very probable that existing doubly heavy tetraquarks are HADS partners of the $\Sigma_Q^{(*)}$ baryon.

II. MODEL DESCRIPTION

We employ the constituent quark model proposed in [51] to predict the masses of the doubly heavy tetraquarks. This model can successfully describe the ground states of light mesons and baryons. In Refs. [52,53], the model is developed to describe the heavy baryons and successfully predict the Ξ_{cc} mass. With the developments of these works, in Ref. [45], the spin-parity 1^+ doubly charmed tetraquark T_{cc} is related to the doubly charmed Ξ_{cc} baryon and the predicted mass is found to be consistent with the $T_{cc}^+(3875)$ observed in 2021. In this work, we calculate the doubly heavy tetraquarks (T_{cc} and T_{bb}) spectrum using the HADS, which relates the doubly heavy tetraquarks with the heavy antibaryons ($\bar{\Lambda}_c$, $\bar{\Sigma}_c^{(*)}$). According to the HADS [33–35], the heavy antidiquark $[\bar{Q}\bar{Q}]$ can be regarded as a heavy object X in 3_c representation. Along this line, there should exist doubly heavy tetraquarks $T_{[qq][\bar{Q}\bar{Q}]}$ ($q = u, d$ and $Q = c, b$) that can be regarded as the counterparts of Λ_Q and $\Sigma_Q^{(*)}$ baryons, see Fig. 1.

Therefore, thanks to the HADS, to obtain the spectrum of the doubly heavy tetraquarks, we only need to calculate the three-body system made of the heavy antidiquark X and the two light quarks instead of the system made of four quarks directly. The potential of this three-body system can be determined by using the constituent quark model for heavy baryon after substituting the heavy quark with X .

The mass formula of baryons in the constituent quark model reads

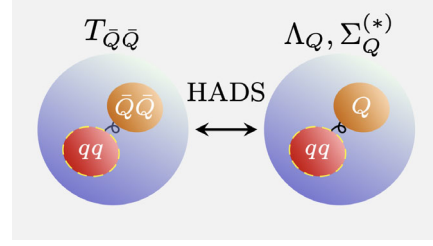


FIG. 1. Cartoon of the HADS relations of $T_{\bar{Q}\bar{Q}}$ and $\Lambda_Q, \Sigma_Q^{(*)}$.

$$M = \sum_i m_i + \sum_{i<j} (F_i \cdot F_j) [B_{ij} + (\sigma_i \cdot \sigma_j) \alpha_{ij} / m_i m_j], \quad (1)$$

where $i, j = 1, 2, 3$. Here F_i, σ are the color and spin operators, respectively. The factor of color operator between two quarks in baryons is $-2/3$ ($\bar{3}_c$ representation). B_{ij} is the binding between a heavy quark (c and b) and a flavored quark (s, c , and b) for describing the heavy baryons, for which we need only B_{cs} and B_{bs} to describe the baryons listed in Table II. m_i is the i th quark mass and α is the quark-quark spin hyperfine interaction coupling. Note that the spin split of diquarks $\frac{\alpha}{m_i^2}$ decreases with quark masses (m_q, m_s, m_c, m_b), which is consistent with the convergence of heavy quark symmetry. The quark-quark spin hyperfine coupling α is the same for the light-light quarks and heavy-light quarks but is flavor dependent for the heavy diquark QQ . For detailed discussions, we refer to Refs. [52,53].

The mass formula we used to calculate the baryon masses is first proposed for the light quark systems in Ref. [51] with three parameters (m_q, m_s , and a coupling a describing the spin splitting). The model was developed in Refs. [52,53] with six extra parameters ($m_c, m_b, a_{cs}, a_{bs}, B_{cs}$, and B_{bs}) successfully describing the heavy baryon spectrum. In this paper, we do not use the ready-made parameters or fit the light baryons and heavy baryons step by step, instead we use a global best-fit strategy to determine the parameters simultaneously. As a result, we need only seven parameters (Table I) describing all the already discovered light and heavy baryons (Table II) equivalently well. These seven parameters including four constituent quark masses ($m_q^b, m_s^b, m_c^b, m_b^b$, the superscript “b” here and there in the text means effective constituent quark mass in baryons), two bindings (B_{cs}, B_{bs}) and a coupling α , which can be determined from the ground baryon spectrum. We fix these parameters using a global best fit for the masses of all the ground baryons discovered so far (except Ξ_{cc} which is taken as an input to constraint the binding between the two charm quarks). The global best

TABLE I. Masses, bindings, and hyperfine couplings of quarks in baryons (in unit of MeV).

Parameters	m_q^b	m_s^b	m_c^b	m_b^b	$\alpha/(m_q^b)^2$	B_{cs}	B_{bs}
Values	364.3	536.2	1715.9	5047.3	-76.8	53.4	62.6

TABLE II. Predicted masses of ground baryons with parameters in Table I compared with experiment values (in unit of MeV).

State	$I(J^+)$	$\sum_i m_i + \sum_{i < j} (F_i \cdot F_j) [B_{ij} + (\sigma_i \cdot \sigma_j) \alpha_{ij} / m_i m_j]$	This model	PDG [54]
N	1/2(1/2 ⁺)	$3m_q^b + 2\alpha/(m_q^b)^2$	939.3	938.9
Δ	1/2(3/2 ⁺)	$3m_q^b - 2\alpha/(m_q^b)^2$	1246.5	1232
Λ	0(1/2 ⁺)	$2m_q^b + m_s^b + 2\alpha/(m_q^b)^2$	1111.2	1115.68
Σ	1(1/2 ⁺)	$2m_q^b + m_s^b - \frac{2}{3} [\alpha/(m_q^b)^2 - 4\alpha/(m_q^b m_s^b)]$	1176.9	1193.1
Σ^*	1(3/2 ⁺)	$2m_q^b + m_s^b - \frac{2}{3} [\alpha/(m_q^b)^2 + 2\alpha/(m_q^b m_s^b)]$	1385.6	1384.6
Ξ	1/2(1/2 ⁺)	$2m_s^b + m_q^b - \frac{2}{3} [\alpha/(m_s^b)^2 - 4\alpha/(m_q^b m_s^b)]$	1321.2	1318.3
Ξ^*	1/2(3/2 ⁺)	$2m_s^b + m_q^b - \frac{2}{3} [\alpha/(m_s^b)^2 + 2\alpha/(m_q^b m_s^b)]$	1529.9	1533.4
Ω	0(3/2 ⁺)	$3m_s^b - 2\alpha/(m_s^b)^2$	1679.5	1672.45
Λ_c	0(1/2 ⁺)	$2m_q^b + m_c^b + 2\alpha/(m_q^b)^2$	2290.9	2286.46
Σ_c	1(1/2 ⁺)	$2m_q^b + m_c^b - \frac{2}{3} [\alpha/(m_q^b)^2 - 4\alpha/(m_q^b m_c^b)]$	2452.2	2453.54
Σ_c^*	1(3/2 ⁺)	$2m_q^b + m_c^b - \frac{2}{3} [\alpha/(m_q^b)^2 + 2\alpha/(m_q^b m_c^b)]$	2517.4	2518.13
Ξ_c	1/2(1/2 ⁺)	$m_c^b + m_q^b + m_s^b - \frac{2}{3} [B_{cs} - 3\alpha/(m_s^b m_q^b)]$	2476.4	2469.1
Ξ_c'	1/2(1/2 ⁺)	$m_c^b + m_q^b + m_s^b - \frac{2}{3} [B_{cs} + \alpha/(m_s^b m_q^b) - 2\alpha/(m_c^b m_s^b) - 2\alpha/(m_c^b m_q^b)]$	2579.1	2578.5
Ξ_c^*	1/2(3/2 ⁺)	$m_c^b + m_q^b + m_s^b - \frac{2}{3} [B_{cs} + \alpha/(m_s^b m_q^b) + \alpha/(m_c^b m_s^b) + \alpha/(m_c^b m_q^b)]$	2633.8	2645.63
Ω_c	0(1/2 ⁺)	$m_c^b + 2m_s^b - \frac{2}{3} [2B_{cs} + \alpha/(m_s^b)^2 - 4\alpha/(m_c^b m_s^b)]$	2711.2	2695.2
Ω_c^*	0(3/2 ⁺)	$m_c^b + 2m_s^b - \frac{2}{3} [2B_{cs} + \alpha/(m_s^b)^2 + 2\alpha/(m_c^b m_s^b)]$	2755.5	2765.9
Λ_b	0(1/2 ⁺)	$2m_q^b + m_b^b + 2\alpha/(m_b^b)^2$	5622.3	5619.6
Σ_b	1(1/2 ⁺)	$2m_q^b + m_b^b - \frac{2}{3} [\alpha/(m_b^b)^2 - 4\alpha/(m_q^b m_b^b)]$	5812.3	5813.1
Σ_b^*	1(3/2 ⁺)	$2m_q^b + m_b^b - \frac{2}{3} [\alpha/(m_b^b)^2 + 2\alpha/(m_q^b m_b^b)]$	5834.5	5832.53
Ξ_b	1/2(1/2 ⁺)	$m_b^b + m_q^b + m_s^b - \frac{2}{3} [B_{bs} - 3\alpha/(m_s^b m_q^b)]$	5801.7	5794.5
Ξ_b'	1/2(1/2 ⁺)	$m_b^b + m_q^b + m_s^b - \frac{2}{3} [B_{bs} + \alpha/(m_s^b m_q^b) - 2\alpha/(m_b^b m_s^b) - 2\alpha/(m_b^b m_q^b)]$	5928.4	5935.02
Ξ_b^*	1/2(3/2 ⁺)	$m_b^b + m_q^b + m_s^b - \frac{2}{3} [B_{bs} + \alpha/(m_s^b m_q^b) + \alpha/(m_b^b m_s^b) + \alpha/(m_b^b m_q^b)]$	5947.1	5955.32
Ω_b	0(1/2 ⁺)	$m_b^b + 2m_s^b - \frac{2}{3} [2B_{bs} + \alpha/(m_s^b)^2 - 4\alpha/(m_b^b m_s^b)]$	6049.8	6046.1

fit strategy means that we determine the values of the parameters by minimizing the mass difference between the predicted and experimental values of all the 23 ground baryons. The parameters we obtained are listed in Table I. With these few parameters, the predicted masses in comparison with experimental values of ground baryon spectrum are shown in Table II with a smallest mass difference

$$\chi_{\text{Model}} = \sqrt{\sum_{i=1}^{23} (m_{\text{pred}} - m_{\text{exp}})^2 / 23} = 7.6 \text{ MeV}, \quad (2)$$

which shows a very good consistency.

III. MASSES OF DOUBLY HEAVY TETRAQUARKS

To study the mass spectrum of the doubly heavy tetraquarks, we first denote their configurations as

$$\psi_T(C_d, S_d, C_D, S_D, I, J) = [[qq]_{S_d}^{C_d} [\bar{Q}\bar{Q}]_{S_D}^{C_D}]_J^I, \quad (3)$$

where $q = u, d$ is the light quark, $Q = c, b$ is the heavy quark, and $C_d(C_D)$ and $S_d(S_D)$ are the color and spin quantum numbers of the light and heavy diquarks, respectively. I is the total isospin and J is the total angular momentum of the tetraquark. Considering ground states of the doubly heavy tetraquarks, with the constraint of Pauli principle, we express the allowed configurations of $T_{[qq][\bar{Q}\bar{Q}]}$ as

$$\begin{aligned} \psi_{T[(0(1^+))]} &= [[qq]_0^3 [\bar{Q}\bar{Q}]_1^3]_0^0, \\ \psi_{T[(1(0^+))]} &= [[qq]_1^3 [\bar{Q}\bar{Q}]_1^3]_0^1, \\ \psi_{T[(1(1^+))]} &= [[qq]_1^3 [\bar{Q}\bar{Q}]_1^3]_1^1, \\ \psi_{T[(1(2^+))]} &= [[qq]_1^3 [\bar{Q}\bar{Q}]_1^3]_{12}^1, \\ \psi_{T'[(0(1^+))]} &= [[qq]_1^6 [\bar{Q}\bar{Q}]_0^6]_0^0, \\ \psi_{T'[(1(0^+))]} &= [[qq]_0^6 [\bar{Q}\bar{Q}]_0^6]_0^1. \end{aligned} \quad (4)$$

In these six configurations, if we regard the heavy antiquark pair $X = [\bar{Q}\bar{Q}]$ as a compact color source, in the sense of the HADS, the first tetraquark with isospin 0 and spin-parity 1^+ should be the counterpart of Λ_Q and the second to the fourth ones with isospin 1 and spin-parity 0^+ , 1^+ , 2^+ are the counterparts of $\Sigma_Q^{(*)}$, because both these heavy tetraquarks and baryons have the same quantum number configurations of the light diquark. While for the last two tetraquarks, they are in the novel color structures that do not appear in mesons and baryons because of the color confinement, which could be good subjects to study the interactions and properties in new color structures.

To calculate the mass of $T_{[qq][\bar{Q}\bar{Q}]}$ with mass formula in Eq. (1) by regarding the heavy antidiquark $X = [\bar{Q}\bar{Q}]$ as a heavy bosonlike quark, we need two more parameters, α_{QQ} and B_{QQ} , and the coefficients of color and spin operators.

TABLE III. Predicted masses (in unit of MeV) of doubly heavy tetraquarks.

State	$I(J^P)$	Configuration	Mass formula	T_{cc}	T_{bb}
T_{QQ}	$0(1^+)$	$[[\bar{q}\bar{q}]_0^3[QQ]_1^3]_1^0$	$2m_q^b + m_{[QQ]_1^3} + 2\alpha/(m_q^b)^2$	3875.8 ± 7.6	10396.0 ± 7.6
T_{QQ}	$1(0^+)$	$[[\bar{q}\bar{q}]_1^3[QQ]_1^3]_0^1$	$2m_q^b + m_{[QQ]_1^3} - \frac{2}{3}[\alpha/(m_q^b)^2 - 8\alpha_{[QQ]_1^3 q}/(m_q^b m_{[QQ]_1^3})]$	4035.4 ± 13.6	10585.6 ± 8.5
T_{QQ}	$1(1^+)$	$[[\bar{q}\bar{q}]_1^3[QQ]_1^3]_1^1$	$2m_q^b + m_{[QQ]_1^3} - \frac{2}{3}[\alpha/(m_q^b)^2 - 4\alpha_{[QQ]_1^3 q}/(m_q^b m_{[QQ]_1^3})]$	4058.0 ± 9.5	10593.2 ± 7.8
T_{QQ}	$1(2^+)$	$[[\bar{q}\bar{q}]_1^3[QQ]_1^3]_2^1$	$2m_q^b + m_{[QQ]_1^3} - \frac{2}{3}[\alpha/(m_q^b)^2 + 4\alpha_{[QQ]_1^3 q}/(m_q^b m_{[QQ]_1^3})]$	4103.2 ± 9.5	10608.3 ± 7.8
T'_{QQ}	$0(1^+)$	$[[\bar{q}\bar{q}]_0^6[QQ]_0^6]_1^0$	$2m_q^b + m_{[QQ]_0^6} + \frac{1}{3}\alpha/(m_q^b)^2$	$4228.6 \pm 7.6^*$	$10949.8 \pm 7.6^*$
T'_{QQ}	$1(0^+)$	$[[\bar{q}\bar{q}]_0^6[QQ]_0^6]_0^1$	$2m_q^b + m_{[QQ]_0^6} - \alpha/(m_q^b)^2$	$4331.0 \pm 7.6^*$	$11052.2 \pm 7.6^*$

The α_{cc} can be determined by the masses of charmonium J/ψ and η_c as $\alpha_{cc}/m_c^2 = -3/16(m_{J/\psi} - m_{\eta_c}) = -21.2$ MeV. And B_{cc} can be estimated through the mass formula $m_{\Xi_{cc}} = 2m_c^b + m_q^b - 2/3[B_{cc} + \alpha_{cc}/(m_q^b)^2 - 4\alpha/(m_q^b m_c^b)]$ as 217.7 MeV. Similarly, the parameter α_{bb} can be determined as $\alpha_{bb}/m_b^2 = -3/16(m_{\Upsilon_b} - m_{\eta_b}) = -11.6$ MeV. While for B_{bb} , since there is no experimental data of a doubly bottomed baryon, we instead use the heavy bottomonium data, $B_{bb} = -3/16[(3m_{\Upsilon_b} + m_{\eta_b}) - 8m_b^b] = 422.0$ MeV [53]. With the parameters and mass formula in Eq. (1), the masses of the doubly heavy baryons are predicted to be $\Xi_{cc}^* = 3686.8$ MeV, $\Xi_{bb} = 10170.5$ MeV, and $\Xi_{bb}^* = 10192.6$ MeV. The color operator coefficients in Eq. (1) are trial, and the spin operator coefficients serve $\sum_{i<j} \sigma_i \cdot \sigma_j = 4 \sum_{i<j} S_i \cdot S_j = 2(S^2 - \sum_j S_j^2)$. With $\sigma_{q_1} \cdot \sigma_{q_2}$ and $\sigma_{\bar{Q}_3} \cdot \sigma_{\bar{Q}_4}$ clarified, $\sigma_q \cdot \sigma_{\bar{Q}} \bar{Q}$ can be calculated by $(\sum \sigma_i \cdot \sigma_j - \sigma_{q_1} \cdot \sigma_{q_2} - \sigma_{\bar{Q}_3} \cdot \sigma_{\bar{Q}_4})/2$.

For the heavy diquark $\bar{X} = [QQ]_S^C$ that can be regarded as a heavy antiquark in the sense of HADS, its mass is expressed as

$$m_{[QQ]_S^C} = 2m_Q + (F_1 \cdot F_2)[B_{QQ} + (\sigma_1 \cdot \sigma_2)\alpha_{QQ}/m_Q^2].$$

With the parameters in Table I, we have $m_{[cc]_1^3} = 3300.8$ MeV, $m_{[cc]_0^6} = 3525.6$ MeV, $m_{[bb]_1^3} = 9821.0$ MeV, and $m_{[bb]_0^6} = 10246.8$ MeV. For the coupling between heavy diquarks $[QQ]_S^C$ and light quark q , according to HADS in heavy quark limit, it is the same as α_{Qq} , so we have $\alpha_{[QQ]_S^C q} = \alpha$.

Once the masses of heavy diquarks and the coupling are determined, we can use Eq. (1) to calculate the mass spectrum of the doubly charmed tetraquarks T_{cc} and the doubly bottomed tetraquarks T_{bb} . The predicted masses and their expressions of different quantum number configurations are summarized in Table III. It should be noted that the predicted tetraquarks in the color-6 states (results with superscript * in Table III) are not that robust as the color-antitriplet one, since the heavy diquarks in color-6 configuration may have repulsive interactions and not verified experimentally. However, if these states indeed exist, then they could provide us with novel information in color interactions.

The lowest mass of the predicted doubly charmed tetraquark T_{cc} is found to be 3875.8 ± 7.6 MeV, which is consistent with the discovered mass of $T_{cc}^+(3875)$ [11,12]. The heavy quark spin symmetry indicates that the splits of the spin multiplets of the doubly heavy tetraquarks converge when the heavy quark limit is taken. The predicted mass spectrum also shows this convergence—the mass splittings between T_{bb} isovector multiplets is about one third of that between T_{cc} isovector multiplets, which is actually the mass ratio of constituent charm and bottom quarks ($\frac{m_b}{m_c} = 2.94$). Same as the relation between the mass splitting of doubly charmed baryons and that of the charmed mesons, i.e., $m_{\Xi_{cc}^*} - m_{\Xi_{cc}} = \frac{3}{4}(m_{\bar{D}^*} - m_{\bar{D}})$ [15,38,39], one can deduce a relation between the mass splitting of the isovectorial heavy baryons and that of the doubly heavy tetraquarks

$$m_{T_{QQ}[1(2^+)]} - m_{T_{QQ}[1(0^+)]} = m_{\Sigma_Q^*} - m_{\Sigma_Q}, \quad (5)$$

which is exactly the case of the predictions of this model. Moreover, one can also deduce the mass relation between the isovectorial and isoscalar states of the ground doubly heavy tetraquarks and the ground heavy baryons as

$$2m_{T_{QQ}[(1(2^+))]} + m_{T_{QQ}[1(0^+)]} - 3m_{T_{QQ}[0(1^+)]} = 2m_{\Sigma_c^*} + m_{\Sigma_c} - 3m_{\Lambda_c}. \quad (6)$$

These mass relations are based on the HADS and not shared with the molecular picture. Therefore they cannot only be used to clarify their nature but also be evidence to verify or defuse the molecular nature of the observed $T_{cc}^+(3875)$, in case these predicted doubly heavy tetraquarks are discovered in future experiments.

We next consider the uncertainties of the calculation which essentially come from two aspects, the uncertainty of the model and the breaking of HADS. The uncertainty of the model can be estimated by the average mass difference χ_{Model} between the predictions and experimental values presented in Table II, ~ 7.6 MeV. The breaking of HADS is at the level of $\Lambda_{\text{QCD}}/(m_Q v)$ [33], where v is the velocity of the heavy quark pair. Following Refs. [38,55], we consider a 25% breaking of HADS. The breaking of HADS only affects the hyperfine coupling $\alpha_{[QQ]_S^C q}$, thus the uncertainty of HADS breaking can be estimated by

$$\chi_{\text{HADS}} = 0.25(F_{[QQ]_S^c} \cdot F_q)(\sigma_{[QQ]_S^c} \cdot \sigma_q) \frac{\alpha_{[QQ]_S^c q}}{m_q^b m_{[QQ]_S^c}^c}, \quad (7)$$

which yields ~ 11.3 and ~ 3.8 MeV for T_{cc} and T_{bb} in $1(0^+)$ state and the half of that in $1(1^+)$ and $1(2^+)$ states, respectively. Assuming these two uncertainties are independent, the total uncertainty is then estimated by $\chi = \sqrt{\chi_{\text{Model}}^2 + \chi_{\text{HADS}}^2}$.

Another interesting thing is that the HADS is naturally conserved in the present calculation for $T_{QQ}[0(1^+)]$, $T'_{QQ}[0(1^+)]$, and $T'_{QQ}[1(0^+)]$ states because the HADS affected term $\alpha_{[QQ]_S^c q}/m_Q^2$ does not appear in their mass expressions. This means that these states can be directly predicted from the mass formula in Eq. (1) by extending it to four body.

IV. MAGNETIC MOMENT

The magnetic moment of a hadron is expressed as

$$\mu_H = \sum_i \langle H \uparrow | 2\mu_i s_{zi} | H \uparrow \rangle, \quad (8)$$

where $\mu_i = g \frac{q_i e}{2m_i} s$ is the magnetic moment of quark i with charge $q_i e$ and $g = 2$ for a point particle. s_{zi} is the z -axis component of the spin operator. $|H \uparrow\rangle$ is the flavor spin wave function of the hadron. With expression (8), the magnetic moment of Λ_Q is then predicted to be $\mu_{\Lambda_Q} = \mu_Q = \frac{q_Q e}{2m_Q}$. With HADS, the heavy diquark $\bar{X} = [QQ]_1^{\bar{3}}$ can be viewed as a heavy antiquark with color $\bar{3}$ and spin 1. The same as the Λ_Q , the magnetic moment of the predicted $0(1^+)$ doubly tetraquark $T_{[[\bar{u}\bar{d}]_0^3[QQ]_1^{\bar{3}}]}$ can be estimated as $\mu_{T_{[QQ]_1^{\bar{3}}}} = \mu_{[QQ]_1^{\bar{3}}} = \frac{q_{[QQ]_1^{\bar{3}} e}}{m_{[QQ]_1^{\bar{3}}}}$. With the mass of $[QQ]_1^{\bar{3}}$ obtained above, the magnetic moments of $T_{cc}^+(3876)$ and $T_{bb}^-(10396)$ are

$$\mu_{T_{cc}^+(3876)} = 0.759\mu_N, \quad \mu_{T_{bb}^-(10396)} = -0.127\mu_N,$$

which are consistent with the light-cone QCD sum rule calculation of the $J^P = 1^+$ tetraquark state using the diquark-antidiquark picture, $\mu_{T_{cc}^+ - \text{Di}} = 0.66_{-0.23}^{+0.34}$ [56].

As the diquark $[QQ]_1^{\bar{3}}$ is not a genuine pointlike quark, we can also use Eq. (8) to calculate the magnetic moments of the lowest T_{cc}^+ and T_{bb}^- with quantum numbers $0(1^+)$. The flavor-spin wave function reads

$$|T_{[[\bar{u}\bar{d}]_0^3[QQ]_1^{\bar{3}}]} \uparrow\rangle = \left(\frac{1}{\sqrt{2}} |\bar{u}\bar{d} - \bar{d}\bar{u}\rangle \frac{1}{\sqrt{2}} |\uparrow\downarrow - \downarrow\uparrow\rangle \right) \times |QQ\uparrow\uparrow\rangle. \quad (9)$$

The magnetic moment of $T_{[[\bar{u}\bar{d}]_0^3[QQ]_1^{\bar{3}}]}$ is therefore obtained as $2\mu_Q = \frac{q_Q e}{m_Q}$. With the mass of heavy quark m_Q in

Table I, the magnetic moments of $T_{cc}^+(3876)$ and $T_{bb}^-(10396)$ are

$$\mu_{T_{cc}^+(3876)} = 0.732\mu_N, \quad \mu_{T_{bb}^-(10396)} = -0.124\mu_N,$$

which are very close to the HADS prediction.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we construct a simple but effective model based on the QCD inspired heavy HADS to study the doubly heavy tetraquarks. Thanks to the HADS, the doubly heavy tetraquarks are related to the heavy baryons, of which the masses are predicted with an explicit mass formula. Six ground states of both doubly charmed and bottomed tetraquarks are predicted.

The results show that the predicted mass of the lowest doubly charmed tetraquarks $T_{cc}(3876)$ is well consistent with the observed $T_{cc}^+(3875)$ by LHCb. The isoscalar $T_{QQ}[0(1^+)]$ can be viewed as the HADS partner of $\bar{\Lambda}_Q$, while the isovectorial multiplets $T_{QQ}[1(0^+)]$, $T_{QQ}[1(1^+)]$, and $T_{QQ}[1(2^+)]$ are the HADS partners of $\bar{\Sigma}_Q^{(*)}$, for which we propose a high possibility that they could exist. For the $T'_{QQ}[0(1^+)]$ and $T'_{cc}[1(0^+)]$, they are in novel color structures that do not occur in mesons and baryons, which may provide further understandings about the strong interactions between quarks if discovered experimentally.

Within the HADS, the mass relations between the doubly heavy tetraquarks and the heavy baryons are also studied, which can be tested by future experiments. Due to heavy quark flavor symmetry, these studies are also suitable for doubly heavy tetraquarks that contain both charm and bottom quarks. Although we do not discuss the doubly heavy tetraquarks T_{bc} in this work because of the absence of doubly heavy baryons $\Xi_{bc}^{(*)}$ and the lack of bottom-charmed mesons, it is easy to follow these studies when the poor situations of data between charm and bottom quarks have changed.

By using the same model, we also estimated the magnetic moments of the isoscalar tetraquarks $T_{cc}^+(3876)$ and $T_{bb}^-(10396)$ states and compared to the calculation without the HADS. The results show good consistence. The predicted magnetic moments are also consistent with other previous studies in diquark-antidiquark picture but differ from the hadronic molecular picture. Thus the magnetic moment provides further information to distinguish the nature of $T_{cc}^+(3875)$. Along this line, considering that the observations of heavy baryons and doubly charmed tetraquark $T_{cc}^+(3875)$, it is very probable doubly heavy tetraquark multiplets exist that are HADS partners of the $\Sigma_Q^{(*)}$ baryon. We encourage our colleagues to search for these states in the current and upcoming facilities.

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