

Extending the nonrelativistic string AdS coset

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Inspired by Lie algebra expansion, we consider an extension of the algebra introduced in A. Fontanella and S. J. van Tongeren [Coset space actions for nonrelativistic strings, *J. High Energy Phys.* **06** (2022) 80.] for the nonrelativistic string coset action in $\text{AdS}_5 \times S^5$. We show that the extended algebra admits a nondegenerate inner product which is adjoint invariant under the full extended algebra. Furthermore, we provide a finite-dimensional representation of the extended algebra.

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I. INTRODUCTION

Nonrelativistic (NR) strings may be regarded potentially interesting from the holographic point of view, since they propagate in string Newton-Cartan geometries, which are non-Lorentzian. Therefore, they represent a novel arena where to study non-anti-de Sitter (AdS) holography, which is currently still poorly understood. NR strings have first been introduced in flat space [1,2], and later also in $\text{AdS}_5 \times S^5$ [3]. NR strings are Weyl anomaly free in ten-dimensional spacetimes [4], and setting to zero the beta function produces NR gravity equations of motion for the background geometry. For a recent review of NR strings, see e.g., [5]. Few topics of NR strings in $\text{AdS}_5 \times S^5$ have been studied so far, such as classical solutions [6], semiclassical expansion in light-cone gauge [7], and the coset formulation of the action and its Lax representation [8].

In this letter, we focus on further understanding the coset action for NR strings in AdS proposed in [8]. We show that the particular Lie algebra expansion considered in [9] naturally suggests considering noncentral extensions, denoted by $Z_{ab}, Z_{a'b'}$, in the coset construction. Although the fields introduced by such extensions are unphysical, since they appear both in the coset numerator and denominator, the extended algebra admits a nondegenerate inner product which is adjoint invariant under the full algebra. It is worthwhile noticing that an inner product with such nice

properties cannot be constructed for the original algebra without the extensions $Z_{ab}, Z_{a'b'}$. Furthermore, inspired by Lie algebra expansion, we are able to provide a representation for the generators of the extended algebra.

II. THE COSET ACTION OF [8]

The action for NR strings in $\text{AdS}_5 \times S^5$, in the set of coordinates originally used in [3], is

$$S = -\frac{T}{2} \int d^2\sigma [\gamma^{\alpha\beta} (x^a x_a (-\partial_\alpha x^0 \partial_\beta x^0 + \cos^2 x^0 \partial_\alpha x^1 \partial_\beta x^1) + \partial_\alpha x^a \partial_\beta x_a + \partial_\alpha x^{a'} \partial_\beta x_{a'}) + \epsilon^{\alpha\beta} ((\lambda_+ e_\alpha^+ + \lambda_- e_\alpha^-) \partial_\beta x^0 + (\lambda_+ e_\alpha^+ - \lambda_- e_\alpha^-) \cos x^0 \partial_\beta x^1)], \quad (1)$$

where T is the string tension, $\sigma^\alpha = (\tau, \sigma)$, with $\alpha = 0, 1$, are the string world sheet coordinates, $\gamma^{\alpha\beta} \equiv \sqrt{-h} h^{\alpha\beta}$ is the Weyl invariant combination of the inverse world-sheet metric $h^{\alpha\beta}$ and $h = \det(h_{\alpha\beta})$, and $e_\alpha^\pm$ are (the light-cone components of) the worldsheet zweibein. x^A , with $A = 0, 1$, are longitudinal coordinates originating from AdS_5 , while x^a and $x^{a'}$, with $a, b, \dots = 2, 3, 4$ and $a', b', \dots = 1, \dots, 5$, are transverse coordinates originating from AdS_5 and S^5 respectively, which are contracted with $\delta_{ab}, \delta_{a'b'}$. $\lambda_\pm$ are nondynamical scalar Lagrange multiplier fields. The idea now is to rewrite the degrees of freedom (d.o.f.) of this action in terms of a coset sigma model.

Typically, the coset numerator G is taken to be the isometry group of the target space. Such group is not yet fully determined, although known to be infinite dimensional [10]. In the flat space case, the isometry group is also infinite dimensional but, on the contrary of the AdS case, it is fully known [11]. Since it is impractical to work with

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infinite dimensional algebras, it is convenient to restrict to a consistent finite truncation. In the flat space, a particular extension of the string Bargmann algebra can be identified as the finite subalgebra we are interested in,¹ as it contains (a subalgebra of) the local symmetry expected on a NR string action. This subalgebra can also be generated by a particular technique known as Lie algebra expansion. This same technique may then be applied to the AdS case, and fix the coset numerator without requiring any particular knowledge of the infinite dimensional isometry algebra. The coset denominator H is then fixed accordingly by a counting of degrees of freedom.

Concretely, the coset numerator algebra $\mathfrak{g}$ considered in the AdS case of [8] is the direct sum of a particular extension of a string Newton-Hooke and an Euclidean algebra. The extended string Newton-Hooke algebra is spanned by a longitudinal boost M , longitudinal translations H_A , transverse rotations J_{ab} , transverse translations P_a , string-Galilei boosts G_{Ab} , and noncentral extensions Z_A and Z . These generators have the following nonvanishing commutation relations

$$\begin{aligned}
[J_{ab}, J_{cd}] &= \delta_{bc}J_{ad} - \delta_{ac}J_{bd} + \delta_{ad}J_{bc} - \delta_{bd}J_{ac}, \\
[J_{ab}, P_c] &= \delta_{bc}P_a - \delta_{ac}P_b, \\
[J_{ab}, G_{Ac}] &= \delta_{bc}G_{Aa} - \delta_{ac}G_{Ab}, \\
[G_{Aa}, G_{Bb}] &= \delta_{ab}\varepsilon_{AB}Z, \\
[M, G_{Aa}] &= -\varepsilon_A{}^B G_{Ba}, \\
[G_{Aa}, P_b] &= \delta_{ab}Z_A, \\
[G_{Aa}, H_B] &= -\eta_{AB}P_a, \\
[H_A, Z_B] &= -\varepsilon_{AB}Z, \\
[M, Z_A] &= -\varepsilon_A{}^B Z_B, \\
[Z, H_A] &= -\varepsilon_A{}^B Z_B, \\
[M, H_A] &= -\varepsilon_A{}^B H_B, \\
[H_A, H_B] &= -\varepsilon_{AB}M, \\
[H_A, P_b] &= G_{Ab}.
\end{aligned} \tag{2}$$

The Euclidean algebra is spanned by spatial translations $P_{a'}$ and spatial rotations $J_{a'b'}$, with commutation relations

$$\begin{aligned}
[P_{a'}, J_{b'c'}] &= \delta_{a'b'}P_{c'} - \delta_{a'c'}P_{b'}, \\
[J_{a'b'}, J_{c'd'}] &= \delta_{b'c'}J_{a'd'} - \delta_{a'c'}J_{b'd'} + \delta_{a'd'}J_{b'c'} - \delta_{b'd'}J_{a'c'}.
\end{aligned} \tag{3}$$

¹There is at least another finite subalgebra, known as string Newton-Cartan algebra, which one may consider in order to construct a NR string coset action in flat space. However, such choice gives equations of motion less suitable for a Lax representation.

The coset denominator algebra $\mathfrak{h}$ is taken to be everything that generates the numerator except for $H_A, P_a, P_{a'}$.

As shown in [8], the action (1) may be rewritten in a coordinate-free language in terms of a Maurer Cartan (MC) 1-form $A = g^{-1}dg$ with $g \in G$, an external current $\Lambda_\alpha = \lambda_- e_\alpha^- Z_+ + \lambda_+ e_\alpha^+ Z_-$ which encodes the $\lambda_\pm$ fields, and a symmetric bilinear inner product $\langle \cdot, \cdot \rangle$ on $\mathfrak{g}$,

$$S^{G/H} = -\frac{T}{2} \int d^2\sigma \gamma^{\alpha\beta} \langle J_\alpha^{(1)}, J_\beta^{(1)} \rangle, \tag{4}$$

where J_α is defined as $J_\alpha = A_\alpha + \gamma_{\alpha\beta} \varepsilon^{\beta\gamma} \Lambda_\gamma$, and $J^{(1)}$ is the projection of J into the grading 1 subspace with respect to the $\mathbb{Z}_2$ automorphism, which induces the split²

$$\mathfrak{g} = \mathfrak{g}^{(0)} \oplus \mathfrak{g}^{(1)}. \tag{5}$$

The inner product is demanded to be adjoint invariant under $\mathfrak{g}^{(0)} \equiv \mathfrak{h} \setminus \{Z_A\}$. The most generic symmetric bilinear with such property is

$$\begin{aligned}
\langle P_a, P_b \rangle &= \omega_1 \delta_{ab}, & \langle H_\pm, Z_\mp \rangle &= -\omega_1/2, \\
\langle H_+, H_- \rangle &= \omega_2, & \langle M, M \rangle &= \omega_3, \\
\langle J_{ab}, J_{cd} \rangle &= \omega_4 \delta_{[a[c} \delta_{b]d]}, & \langle P_{a'}, P_{b'} \rangle &= \omega_6 \delta_{a'b'}, \\
\langle J_{a'b'}, J_{c'd'} \rangle &= \omega_5 \delta_{[a'[c'} \delta_{b']d']}.
\end{aligned} \tag{6}$$

In order for (4) to reproduce precisely (1), one needs to set³ $\omega_2 = 0$ and choose the set of coordinates

$$g = g_l g_t, \quad g_l = e^{x^1 H_1} e^{x^0 H_0}, \quad g_t = e^{x^a P_a + x^{b'} P_{b'}}. \tag{7}$$

Although (6) is degenerate in the full algebra $\mathfrak{g}$, it is nondegenerate in the generators carrying physical d.o.f., i.e., in $\mathfrak{g} \setminus \mathfrak{h}$. It is worthwhile noticing that enforcing adjoint invariance under a larger algebra than $\mathfrak{g}^{(0)}$ makes the inner product degenerate also in the $\mathfrak{g} \setminus \mathfrak{h}$ subspace.

The equations of motion coming from the action (4) admit a Lax pair,

$$\mathcal{L}_\alpha = \ell_0 A_\alpha^{(0)} + \ell_1 A_\alpha^{(1)} + \ell_2 \gamma_{\alpha\beta} \varepsilon^{\beta\gamma} J_\gamma^{(1)}, \tag{8}$$

where the parameters need to satisfy the relation $\ell_1^2 - \ell_2^2 = 1$. The Lax representation of the equations of motion holds on the constraint surface defined by the equations of motion for $\lambda_\pm$.

²In contrast with relativistic strings in AdS, where $\mathfrak{g}^{(0)} = \mathfrak{h}$ and $\mathfrak{g}^{(1)} = \mathfrak{g} \setminus \mathfrak{h}$, in the NR case the coset structure is not aligned with grading, but $\mathfrak{g}^{(0)} = \mathfrak{h} \setminus \{Z_A\}$ and $\mathfrak{g}^{(1)} = \{H_A, P_a, P_{a'}, Z_A\}$.

³This condition is coordinate-free, and required by the expected string Newton-Cartan structure of a NR string action.

III. EXTENDING THE ALGEBRA

Although the algebra given in (2) and (3) is obtained from Lie algebra expansion of $\mathfrak{so}(2,4) \oplus \mathfrak{so}(6)$, it is not the most generic algebra that can be obtained from this procedure. A more general Lie algebra expansion was given in [9], schematically

$$\begin{aligned} J_{AB} &\rightarrow \epsilon_{AB}(M + \epsilon^2 Z), \\ J_{Aa} &\rightarrow \epsilon G_{Aa}, \\ J_{ab} &\rightarrow J_{ab} + \epsilon^2 Z_{ab}, \\ P_A &\rightarrow H_A + \epsilon^2 Z_A, \\ P_a &\rightarrow \epsilon P_a, \\ J_{a'b'} &\rightarrow J_{a'b'} + \epsilon^2 Z_{a'b'}, \\ P_{a'} &\rightarrow \epsilon P_{a'}, \end{aligned} \quad (9)$$

where on the left-hand side are the generators of $\mathfrak{so}(2,4) \oplus \mathfrak{so}(6)$, and ϵ is the expansion parameter. The commutation relations for the expanded algebra are inherited from the commutation relations of the parental algebra via power matching in ϵ , together with consistency conditions of the flatness of MC 1-forms.

The expansion (9) generates an algebra, $\hat{\mathfrak{g}}$, which contains the same set of generators as the one considered in the previous section, supplemented with $\{Z_{ab}, Z_{a'b'}\}$. The non-vanishing commutation relations of $\hat{\mathfrak{g}}$ are given by (2) and (3) after replacing the old $[P_a, P_b]$, $[G_{Aa}, G_{Bb}]$, $[P_{a'}, P_{b'}]$ by

$$\begin{aligned} [P_a, P_b] &= Z_{ab}, \\ [G_{Aa}, G_{Bb}] &= \delta_{ab}\epsilon_{AB}Z - \eta_{AB}Z_{ab}, \\ [P_{a'}, P_{b'}] &= -Z_{a'b'}, \end{aligned} \quad (10)$$

and supplementing them with

$$\begin{aligned} [J_{ab}, Z_{cd}] &= \delta_{bc}Z_{ad} - \delta_{ac}Z_{bd} + \delta_{ad}Z_{bc} - \delta_{bd}Z_{ac}, \\ [J_{a'b'}, Z_{c'd'}] &= \delta_{b'c'}Z_{a'd'} - \delta_{a'c'}Z_{b'd'} + \delta_{a'd'}Z_{b'c'} - \delta_{b'd'}Z_{a'c'}. \end{aligned} \quad (11)$$

For the new algebra $\hat{\mathfrak{g}}$, we construct the most generic bilinear inner product which is adjoint invariant under the full $\hat{\mathfrak{g}}$,

$$\begin{aligned} \langle M, Z \rangle &= \phi_1, & \langle H_{\pm}, Z_{\mp} \rangle &= -\phi_1/2, \\ \langle H_+, H_- \rangle &= -\phi_2/2, & \langle M, M \rangle &= \phi_2, \\ \langle J_{ab}, Z_{cd} \rangle &= -2\delta_{[a[c}\delta_{b]d]}\phi_1, & \langle G_{\pm a}, G_{\mp b} \rangle &= \delta_{ab}\phi_1/2, \\ \langle J_{ab}, J_{cd} \rangle &= \delta_{[a[c}\delta_{b]d]}\phi_3, & \langle P_a, P_b \rangle &= \delta_{ab}\phi_1, \\ \langle J_{a'b'}, J_{c'd'} \rangle &= \delta_{[a'[c'}\delta_{b']d']}\phi_4, & \langle P_{a'}, P_{b'} \rangle &= \delta_{a'b'}\phi_5, \\ \langle J_{a'b'}, Z_{c'd'} \rangle &= 2\delta_{[a'[c'}\delta_{b']d']}\phi_5, \end{aligned} \quad (12)$$

where ϕ_i are arbitrary constants. Although both $\mathfrak{g}$ and $\hat{\mathfrak{g}}$ are not semi-simple, in contrast to what happens to $\mathfrak{g}$, the inner product (12) is (i) adjoint invariant under the full algebra $\hat{\mathfrak{g}}$, and (ii) nondegenerate in $\hat{\mathfrak{g}}$.

IV. A MATRIX REPRESENTATION

Inspired by the Lie algebra expansion (9), we are able to construct a finite-dimensional matrix representation for the generators of $\hat{\mathfrak{g}}$. Each generator has a level given by its power in ϵ coming from the expansion. In particular, we can split $\hat{\mathfrak{g}}$ into three subspaces $\hat{\mathfrak{g}} = \hat{\mathfrak{g}}_0 \oplus \hat{\mathfrak{g}}_1 \oplus \hat{\mathfrak{g}}_2$,

$$\begin{aligned} \hat{\mathfrak{g}}_0 &= \{M, H_A, J_{ab}, J_{a'b'}\}, \\ \hat{\mathfrak{g}}_1 &= \{G_{Aa}, P_a, P_{a'}\}, \\ \hat{\mathfrak{g}}_2 &= \{Z, Z_A, Z_{ab}, Z_{a'b'}\}. \end{aligned} \quad (13)$$

Note that the commutation relations are graded with respect to this level,

$$[\hat{\mathfrak{g}}_n, \hat{\mathfrak{g}}_m] \subset \hat{\mathfrak{g}}_{n+m}, \quad (14)$$

where $\hat{\mathfrak{g}}_n = \emptyset$ for $n > 2$. This level grading should not be confused with the $\mathbb{Z}_2$ grading of $\mathfrak{so}(2,4) \oplus \mathfrak{so}(6)$ and induced on the expanded algebra (9), which appears in the coset action and splits the algebra as $\hat{\mathfrak{g}} = \hat{\mathfrak{g}}^{(0)} \oplus \hat{\mathfrak{g}}^{(1)}$, where $\hat{\mathfrak{g}}^{(1)} = \{H_A, P_a, P_{a'}, Z_A\}$ and $\hat{\mathfrak{g}}^{(0)}$ is its complement.

A representation $\hat{\rho}$ for $\hat{\mathfrak{g}}$ can be constructed by taking a representation for $\mathfrak{so}(2,4) \oplus \mathfrak{so}(6)$, indicated by ρ , in tensor product with linear maps ω_i acting on a three-dimensional vector space whose basis carry the generator level. These linear maps are required to respect the grading condition (14) and are chosen to be

$$\begin{aligned} \omega_0 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \omega_1 &= \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\ \omega_2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (15)$$

Concretely, for a given generator $\hat{X} \in \hat{\mathfrak{g}}_i$, its representation is given by

$$\hat{\rho}(\hat{X}) \equiv \omega_i \otimes \rho(X), \quad (16)$$

where $X \in \mathfrak{so}(2,4) \oplus \mathfrak{so}(6)$ is the parent generator associated with $\hat{X}$ from the expansion (9).

If we choose ρ to be the spinorial representation, see e.g., [12] for our conventions, an inner product on $\hat{\rho}$, invariant under the adjoint action of $\hat{\mathfrak{g}}$, is constructed as follows:

$$\langle \hat{\rho}(\hat{X}), \hat{\rho}(\hat{Y}) \rangle \equiv \text{STr}[(a\omega_0 + b\omega_1 + c\omega_2)^t \otimes \mathbf{1}] \hat{\rho}(\hat{X}) \hat{\rho}(\hat{Y}), \quad (17)$$

where the prefactors $\omega_i^t \otimes \mathbf{1}$ are responsible for undoing the level shift, and a , b , and c are arbitrary constants. The supertrace ‘STr’ is defined as

$$\text{STr}(m \otimes \mathcal{M}) \equiv \text{Tr}(m) \text{STr}(\mathcal{M}), \quad (18)$$

where the supertrace in the spinorial representation is computed in the usual way, i.e., $\text{STr}(\mathcal{M}) = \text{Tr}_{\text{AdS}}(\mathcal{M}) - \text{Tr}_S(\mathcal{M})$. For this specific inner product, the values of ϕ_i are

$$\phi_1 = \phi_5 = c, \quad \phi_2 = 3a, \quad -\phi_3 = \phi_4 = 6a. \quad (19)$$

V. THE COSET ACTION BASED ON $\hat{\mathfrak{g}}$

The same coset construction of the action for NR strings in AdS given in [8] may be repeated by taking $\hat{\mathfrak{g}}$ as the Lie algebra of the coset numerator, and $\hat{\mathfrak{h}}$ chosen to be everything in $\hat{\mathfrak{g}}$ except for $\{H_A, P_a, P_{a'}\}$. The new coset action retains the shape of (4), but now any algebraic input information belongs to $\hat{\mathfrak{g}}$. The MC 1-form $A = g^{-1}dg$ now requires $g \in \hat{G}$, and the inner product is given in (12). In order to match the general structure of the action of NR strings in string Newton-Cartan backgrounds, one is required to set⁴ $\phi_2 = 0$ and $\phi_1 = \phi_5$, which leaves the inner product still nondegenerate. The action (1) is then reproduced by the same choice of coordinates as in (7). In the coordinate-free language, the equations of motion obtained by an arbitrary variation of the group element g , $\delta g = g\xi$ with $\xi \in \hat{\mathfrak{g}}$, can now be derived immediately, without the need of considering component by component as in [8], thanks to adjoint invariance of the inner product under the full $\hat{\mathfrak{g}}$. The equations of motion derived in this way are the same as the one found in [8], which may be checked by looking at the variation induced on the MC 1-form $\delta A_\alpha = \partial_\alpha \xi + [A_\alpha, \xi]$, together with the commutation relations of the additional generators $Z_{ab}, Z_{a'b'}$. With the same reasoning, the gauge transformations of the MC 1-form remain the same as before, except that now the MC 1-form has new components in the $Z_{ab}, Z_{a'b'}$ directions, which however are unphysical. As a

⁴In the matrix representation, this corresponds to $a = 0$. The inner product still remains nondegenerate.

consequence of this gauge invariance, Noether identities allow us to disregard the equations of motion of unphysical d.o.f. in the same way as in [8]. Finally, the Lax pair argument also remains unaltered.

VI. CONCLUSIONS

In this letter, we showed that Lie algebra expansion naturally suggests that one should consider the noncentral extensions $Z_{ab}, Z_{a'b'}$ in the coset action construction of NR strings in AdS. By extending the algebra, the inner product gains nicer properties, such as adjoint invariance under the full algebra and nondegeneracy. This makes a drastic simplification in the derivation of the equations of motion from the coset action, rendering any further use of the coset action more manageable. Moreover, when the generators $Z_{ab}, Z_{a'b'}$ are included, Lie algebra expansion gives the hint on how to construct a finite dimensional representation based on levels, which, on the other hand, cannot be carried out when $Z_{ab}, Z_{a'b'}$ are removed. The gauge fields associated with $Z_{ab}, Z_{a'b'}$ are unphysical, as they play no role in the coset action, leaving the equations of motion, Lax pair, and gauge invariance unchanged. The isometry algebra of NR strings in AdS has not been identified yet, and it would be interesting to know if the $Z_{ab}, Z_{a'b'}$ generators are part of it.⁵

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⁵The hint comes from the fact the inner product gains nice properties. For NR strings in flat space, where the isometry algebra is known, including Z_{ab} does not make the inner product nicer. However, in that case, it is also easy to show that Z_{ab} cannot be embedded into the global symmetry algebra, as fulfilling the $[G_{Aa}, G_{Bb}]$ relations would imply $[Z_{ab}, J_{cd}] = 0$ in the embedding.

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