

Positivity bounds without boosts: New constraints on low energy effective field theories from the UV

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We derive new positivity bounds for low-energy effective field theories (EFTs) that are not invariant under Lorentz boosts. “Positivity bounds” are the low-energy manifestation of certain fundamental properties in the UV—to date, they have been used to constrain a wide variety of EFTs; however, since existing bounds require Lorentz invariance they are not directly applicable when this symmetry is broken, such as for most cosmological and condensed matter systems. Assuming that the low-energy degrees of freedom can be extended into the UV in a way consistent with the fundamental axioms of unitarity, analyticity, and locality, we derive an infinite family of bounds that (derivatives of) the $2 \rightarrow 2$ EFT scattering amplitude must satisfy even when Lorentz boosts are broken. We apply these bounds to the leading-order EFT of both a superfluid and the scalar fluctuations produced during inflation, comparing in the latter case with the current observational constraints on primordial non-Gaussianity, demonstrating that these bounds open a new window into the high-energy physics that lies beneath our low-energy measurements.

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I. INTRODUCTION

From the early ages of our universe to present day condensed matter physics, the natural world favors solutions to its fundamental laws that break Lorentz invariance. For instance, both the inflationary spacetime background [1–4] and many-body systems at high density [5–11] provide a preferred reference frame. Low-energy fluctuations in these systems are accurately captured by effective field theories (EFTs), which are not invariant under Lorentz boosts. This EFT framework, by providing a simple parametrization of the possible low-energy interactions (in which coupling constants are fixed by comparing with data), has allowed us to make great progress in analyzing our low-energy observations even in cases where we do not fully understand the underlying UV physics (such as inflation).

However, care must be taken when adopting an EFT parametrization, since not all of the EFT parameter space is equally viable. For Lorentz-invariant systems, over the past two decades, powerful connections have been developed between low-energy EFTs and their underlying high-energy UV completions, known as *positivity bounds* [12–23]. These bounds constrain the sign or relative size

of the EFT interactions, and when they are violated, it signals that no new high-energy physics, no matter how ingenious our model building, can restore unitarity, analyticity, and locality beyond the EFT cutoff—thus identifying values of the EFT parameters that have no standard UV completion. These connections between the UV and the IR have been applied to great effect in a huge variety of different settings: constraining new physics beyond the Standard Model [24–38] and beyond general relativity [39–70]. However, the main limitation of these bounds is that they rely on Lorentz invariance at all energy scales—for all their successes, they cannot be used to constrain everyday systems in which Lorentz invariance is broken. In this paper, we derive new positivity bounds, which can be applied to EFTs in which Lorentz invariance is broken (either spontaneously or explicitly).

II. PROPERTIES OF THE AMPLITUDE

Let us begin by describing the properties of the UV physics that we will use to derive positivity bounds in the IR. The central object of our derivation is the scattering amplitude $\mathcal{A}_{\pi_1\pi_2\rightarrow\pi_3\pi_4}$ between scalar fluctuations π (which represent any light degree of freedom in the low-energy EFT).

A. Unbroken symmetries

We will assume that the Poincaré invariance of the system is broken by a single preferred (timelike) direction n^μ . This breaks enough symmetry (Lorentz boosts) to

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capture condensed matter systems such as fluids and cosmological systems such as inflation, but preserves enough symmetry (spatial rotations and space-time translations) to retain a well-defined S -matrix description. We additionally assume that the free propagation of π is described by

$$\omega^2 = c_\pi^2 |\mathbf{k}|^2 + m^2. \quad (1)$$

The asymptotic states for our scattering process are labeled by their spatial momentum, $\mathbf{k}_a$, with frequencies ω_a fixed by (1), and a basis of kinematic variables that are invariant under the unbroken symmetries is $s_{ab} = (\omega_a + \omega_b)^2 - c_\pi^2 |\mathbf{k}_a + \mathbf{k}_b|^2$ and $\omega_{ab} = \omega_a + \omega_b$ [71]. Only five of these are independent, and we choose to work with

$$\begin{aligned} \omega_s &= \omega_{12}, & \omega_t &= \omega_{13}, & \omega_u &= \omega_{14}, \\ s &= s_{12}, & t &= s_{13}, \end{aligned} \quad (2)$$

where $u = s_{14}$ is fixed by $s + t + u = 4m^2$. We refer to $\pi_1 \pi_2 \rightarrow \pi_3 \pi_4$ as the s -channel process, with amplitude $\mathcal{A}_s(s, t, \omega_s, \omega_t, \omega_u)$. Real physical momenta correspond to

$$s > -t > 0 \quad \text{and} \quad \omega_s \geq \sqrt{s + (|\omega_u| + |\omega_t|)^2}, \quad (3)$$

which we refer to as the s -channel region (see Supplemental Material [72] for the explicit map from (2) to the s -channel momenta).

B. Crossing

In addition to the s -channel region, there are other values of $(s, t, \omega_s, \omega_t, \omega_u)$ for which all of the momenta can become real. In particular, the u -channel region, $u > -t > 0$ and $\omega_u > \sqrt{u + (|\omega_s| + |\omega_t|)^2}$, corresponds to the u -channel process $\pi_1 \pi_4 \rightarrow \pi_3 \pi_2$, and the associated amplitude is $\mathcal{A}_u(u, t, \omega_u, \omega_t, \omega_s)$. For the scattering of identical scalar fluctuations, the s - and u -channels are physically equivalent. This leads to “crossing relations” between amplitudes at different kinematics,

$$\mathcal{A}_s(s, t, \omega_s, \omega_t, \omega_u) = \mathcal{A}_u(u, t, \omega_u, \omega_t, \omega_s), \quad (4)$$

which must hold at all energies (in both EFT and UV).

C. Unitarity

Unitarity of the S -matrix leads to the well-known optical theorem for scattering amplitudes,

$$2 \operatorname{Im} \mathcal{A}_{\pi_1 \pi_2 \rightarrow \pi_1 \pi_2} = \sum_n |\mathcal{A}_{\pi_1 \pi_2 \rightarrow n}|^2, \quad (5)$$

which relates the imaginary part of the forward-limit amplitude to a sum over amplitudes for the $2 \rightarrow n$ processes. We will require that the underlying high-energy

physics that UV completes the EFT is unitary—in particular, the optical theorem then guarantees that $\operatorname{Im} \mathcal{A}_s \geq 0$ for forward-limit scattering in the UV.

D. Analyticity

In classical physics, the *causal* propagation of signals in the time domain is encoded in the *analyticity* of the response function in the frequency domain. Within the framework of quantum field theory, while analyticity of the S -matrix can be derived from microcausality in certain limiting cases [73–75], there are few general implementations of causality beyond perturbation theory. Consequently, bottom-up studies of the S -matrix typically postulate analyticity in place of a causality condition, since the former is always mathematically precise (see, for instance, the discussion in [76]). Here, we shall do the same and assume that the physical boost-breaking amplitude is a real boundary value of an analytic function with known poles and branch cuts (required by unitarity). In the Supplemental Material [72], we describe a particular limiting case in which this analyticity can be derived from the micro-causality of the underlying UV theory, but just like for the Lorentz-invariant S -matrix, this is by no means a rigorous proof that analyticity and causality are always connected (and we will similarly assume analyticity even in cases where it is not clear how to implement causality at the level of the S -matrix).

Analyticity is a crucial ingredient in deriving Lorentz-invariant positivity bounds, used to relate the EFT amplitude (at small s) to the underlying UV amplitude (at large s) via complex analysis. The key difference when boosts are broken is that the amplitude is now also a function of $\{\omega_s, \omega_t, \omega_u\}$, and one must specify how these additional variables are to be held fixed when analytically continuing into the complex s -plane. Previous studies of scattering amplitudes in cosmology [77–80] have fixed these energies so that $\mathbf{k}_1 + \mathbf{k}_2 = 0$ (i.e., the *center-of-mass frame*), since this corresponds to an incoming two-particle state which is rotationally invariant, allowing the optical theorem (5) to be simplified using a partial wave expansion. However, the resulting amplitude, $\mathcal{A}_s(s, t, \sqrt{s}, 0, 0)$ is *not* suitable for positivity arguments for two reasons:

- (i) The nonanalytic $\omega_s = \sqrt{s}$ can introduce additional branch cuts in the complex s -plane [77],
- (ii) Crossing is no longer manifest, since (4) maps $\mathcal{A}_s(s, t, \sqrt{s}, 0, 0)$ to $\mathcal{A}_u(u, t, 0, 0, \sqrt{s})$.

To overcome these issues, we have identified a way to fix the energies that is analytic in s and invariant under $s \leftrightarrow u$ crossing so that the resulting amplitude is amenable to positivity arguments. We begin by noting that the conventional proofs of analyticity use Lorentz boosts to fix $\mathbf{k}_1 - \mathbf{k}_3 = 0$ (i.e., the *Breit frame*), which corresponds to fixing the energies,

$$\omega_s + \omega_u = \sqrt{4m^2 - t}, \quad \omega_s - \omega_u = \frac{s - u}{2\sqrt{4m^2 - t}}. \quad (6)$$

This condition fixes the energies as analytic functions of s and is preserved under $s \leftrightarrow u$ crossing (which swaps s with u and ω_s with ω_u) so the resulting amplitude enjoys a trivial crossing relation. For Lorentz-invariant theories, one can move freely from the center-of-mass frame (in which unitarity is simplest) to the Breit frame (in which causality and crossing are simplest) by applying a boost—but when boosts are broken, these are different physical processes (different particle energies $\{\omega_s, \omega_t, \omega_u\}$). For general kinematics, the energies need not take the form (6); however, we can always parametrize them as

$$\omega_s + \omega_u = 2M\gamma, \quad \omega_s - \omega_u = \frac{s - u}{4M}, \quad (7)$$

where ω_s and ω_u have been written in terms of two new variables γ and M . Mathematically, (7) fixes the energies in a way that is analytic in s (at fixed t, M, ω_t, γ) and invariant under $s \leftrightarrow u$ crossing, and so the resulting amplitude $\tilde{\mathcal{A}}_s(s, t, M, \omega_t, \gamma)$ [$= \mathcal{A}_s$ with energies fixed as in (7)] enjoys all of the good properties of $\mathcal{A}_s^{(\text{Breit})}(s, t)$, in particular, a trivial crossing relation,

$$\tilde{\mathcal{A}}_s(s, t, M, \omega_t, \gamma) = \tilde{\mathcal{A}}_u(u, t, M, \omega_t, \gamma). \quad (8)$$

Physically, varying s with $\{\gamma, M, \omega_t\}$ held fixed corresponds to changing the interaction energy while maintaining a constant velocity relative to the Breit frame. The s -channel region (3) is given by $M > 0$ and $(\gamma - 1)s + \gamma t/2 \geq \omega_t^2 + 2|\omega_u \omega_t|$, which reduces to simply $\gamma \geq 1$ in the forward limit. In the Supplemental Material [72], we show that analyticity of $\tilde{\mathcal{A}}_s$ in s at fixed $\{t, \gamma, M, \omega_t\}$ is implied by the condition that operators commute outside of the π -cone for large values of s .

E. Polynomial boundedness

We also require a bound on the high-energy growth of the amplitude:

$$\lim_{s \rightarrow \infty} |\tilde{\mathcal{A}}_s(s, t, M, \omega_t, \gamma)| < s^2, \quad (9)$$

where s is taken large at fixed $\{t, M, \omega_t, \gamma\}$. In the Lorentz-invariant case, the classic results of Froissart and Martin [81,82] show that polynomial boundedness of the amplitude, $\lim_{s \rightarrow \infty} |\mathcal{A}_s(s, t)| < s^2$, follows from *locality*. We briefly review this connection in the Supplemental Material [72] and show that the analogous steps that lead to the Froissart-Martin bound in the Lorentz-invariant case can also be performed without Lorentz boosts, and so we will refer to property (9) as locality of the UV physics.

F. Asymptotic states

Finally, let us point out a further assumption which, unlike the previous properties (A)–(E), does not have an analog in the usual Lorentz-invariant positivity bounds. We assume that the low-energy degrees of freedom, π , can be smoothly connected to high-energy degrees of freedom that obey the above properties so that the object $\mathcal{A}_{\pi_1 \pi_2 \rightarrow \pi_3 \pi_4}$ is still well-defined in the UV. This assumption has also been made in many earlier works (e.g., [77,83]), and given an explicit UV completion, one can always verify whether this assumption holds (see, e.g., [84]). Since our bottom-up approach is to remain as agnostic as possible about the UV (i.e., state the fewest assumptions about the UV that would lead to predictive IR consequences), we add property (F) as an assumption that must be made about how the EFT degrees of freedom are embedded into the UV completion at high energies.

III. POSITIVITY BOUNDS

Armed with the above properties, we can construct a *dispersion relation* that relates the amplitude $\tilde{\mathcal{A}}_s(s, t, M, \omega_t, \gamma)$ in the EFT to the underlying UV completion, producing a number of new positivity relations that constrain low-energy EFTs.

In the complex s -plane [with $\{t, M, \omega_t, \gamma\}$ held fixed], $\tilde{\mathcal{A}}_s(s)$ has the same analytic structure as in a Lorentz-invariant theory—it is analytic everywhere except for the poles and branch cuts on the real s -axis required by unitarity. We can therefore evaluate the integral $\oint_C d\mu \tilde{\mathcal{A}}_s(\mu)/(\mu - s)^{n+1}$ first along a contour C that closely encircles the pole (giving $\partial_s^n \tilde{\mathcal{A}}_s$ by Cauchy's residue theorem) and then along a pair of semicircular contours in the upper and lower half-planes (closed above and below the real axis respectively) whose radii go to infinity. These are equal by property (D) analyticity,

$$\frac{1}{n!} \partial_s^n \tilde{\mathcal{A}}_s(s) = C_\infty + \int_{-\infty}^{\infty} \frac{d\mu}{\pi} \frac{\text{Im} \tilde{\mathcal{A}}_s(\mu)}{(\mu - s - i\epsilon)^{n+1}}, \quad (10)$$

where we have suppressed the dependence on $\{t, M, \omega_t, \gamma\}$ and replaced $\tilde{\mathcal{A}}_s(\mu + i\epsilon) - \tilde{\mathcal{A}}_s(\mu - i\epsilon)$ with $2i \text{Im} \tilde{\mathcal{A}}_s(\mu)$. C_∞ is the contribution from the arcs at infinity and vanishes by property (E) locality for all $n \geq 2$. Property (B) crossing can be used to relate the branch cut on the negative real axis to that along the positive real axis so that there is a single integral $\int_{2m^2 - t/2}^{\infty} d\mu$ over the kernel,

$$P_n(\mu, s) = \frac{\text{Im} \tilde{\mathcal{A}}_s(\mu)}{(\mu - s)^{n+1}} - \frac{\text{Im} \tilde{\mathcal{A}}_u(\mu)}{(u - \mu)^{n+1}}. \quad (11)$$

Following the philosophy of [15,49,67], the strongest positivity bounds are obtained by subtracting as much of the IR (EFT) information as possible so that the bounds

represent only our ignorance of the UV. This corresponds to subtracting the portion of the branch cut up to the scale s_b at which the EFT is no longer reliable,

$$\tilde{\mathcal{A}}_s^{(n)}(s) := \frac{\partial_s^n \tilde{\mathcal{A}}_s(s)}{n!} - \int_{2m^2-t/2}^{s_b} \frac{d\mu}{\pi} P_n(\mu, s). \quad (12)$$

$\tilde{\mathcal{A}}_s^{(n)}$ is the n th derivative of the EFT amplitude with low-energy branch cuts removed and is related to the underlying UV completion by the dispersion relation,

$$\tilde{\mathcal{A}}_s^{(n)}(s) = \int_{s_b}^{\infty} \frac{d\mu}{\pi} P_n(\mu, s). \quad (13)$$

In the forward limit, we can use the final property (C) unitarity to establish that $P_n(\mu, s) \geq 0$ for all $\mu \geq s_b$ and any even $n \geq 2$, providing that $|s| < s_b$ is within the EFT's regime of validity and that $M > 0$, $\gamma \geq 1$ have been chosen so that both cuts have physical kinematics. This gives our final positivity bounds,

$$\tilde{\mathcal{A}}_s^{(2N)} \Big|_{\substack{t=0 \\ \omega_l=0}} \geq 0, \quad (14)$$

for all $N \geq 1$ and for any values of s , $M > 0$ and $\gamma \geq 1$, which lie within the regime of validity of the EFT. This is the analog of the positivity bounds $\partial_s^{2n} \mathcal{A}(s, t=0) > 0$ in Lorentz-invariant theories. Even without explicit knowledge of the underlying UV completion, we have shown that the fundamental properties of unitarity, causality, and locality require that the low-energy theory [i.e., $\mathcal{A}_s(\mu)$ computed at low μ in the EFT] obeys (14).

In practice, we compute the EFT amplitude in perturbation theory, where loop corrections are suppressed by powers of s/Λ^2 and ω/Λ , where Λ is the EFT cutoff. The exact positivity bound (14) therefore implies a perturbative bound,

$$\frac{1}{2} \partial_s^2 \tilde{\mathcal{A}}_s^{\text{tree}} \Big|_{\substack{s=0 \\ t=0}} \geq \frac{2}{\pi} \int_{2m^2-t}^{s_b} \frac{d\mu}{\mu^3} \text{Im} \tilde{\mathcal{A}}_s^{1\text{-loop}} \Big|_{\substack{s=0 \\ t=0}}, \quad (15)$$

providing s_b is chosen sufficiently low that higher order loop corrections can be ignored [85]. If the theory is weakly coupled, then we can trust the loop expansion even at energies near the cutoff (since there is an additional small parameter that suppresses these corrections), and so the entirety of the EFT branch cut may be subtracted (to any desired order in this weak coupling).

IV. APPLICATIONS

The positivity bounds (14) and their perturbative form (15) can be applied to *any* low-energy EFT with rotation and translation invariance, and if they are violated, it signals that there can be no UV completion, which respects unitarity, analyticity, and locality (as defined above). To

illustrate the power of these bounds, we will now focus on the following EFT,

$$S[\pi] = \int d^4x c_\pi^{-3} \left(-\frac{1}{2} (\partial\pi)^2 + \frac{\alpha_1}{\Lambda^2} \dot{\pi}^3 - \frac{\alpha_2}{\Lambda^2} \dot{\pi} (\partial\pi)^2 + \frac{\beta_1}{\Lambda^4} \dot{\pi}^4 - \frac{\beta_2}{\Lambda^4} \dot{\pi}^2 (\partial\pi)^2 + \frac{\beta_3}{\Lambda^4} (\partial\pi)^4 \right), \quad (16)$$

with corresponding amplitude,

$$\Lambda^4 \mathcal{A}_s = 2\beta_3(s^2 + t^2 + u^2) + (2\beta_2 - 4\alpha_2^2)(s\omega_s^2 + t\omega_t^2 + u\omega_u^2) + 24\omega_1\omega_2\omega_3\omega_4 \left[(\beta_1 - 4\alpha_1\alpha_2) - \frac{3}{2}\alpha_1^2 \left(\frac{\omega_s^2}{s} + \frac{\omega_t^2}{t} + \frac{\omega_u^2}{u} \right) \right], \quad (17)$$

where $\dot{\pi} = n^\mu \partial_\mu \pi$ and $(\partial\pi)^2 = -\dot{\pi}^2 + c_s^2 \delta^{ij} \partial_i \pi \partial_j \pi$ is contracted using the effective metric that determines the free propagation (1), $\{\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3\}$ are constant Wilson coefficients, and the overall factor of the sound speed ensures canonical normalization [86,87]. This action captures the low-energy degree of freedom of a superfluid and also the scalar fluctuations produced during inflation in the slow-roll decoupling limit.

A. Superfluid

In the context of a superfluid, the scalar π is the Goldstone boson which transforms nonlinearly under the spontaneously broken boosts. This nonlinear symmetry fixes all but one Wilson coefficient at each order in π , and the coefficients appearing in (16) and (17) can all be fixed in terms of $\{c_s, \alpha_1, \beta_1\}$ only,

$$\alpha_2 = \frac{1 - c_s^2}{2c_s^2}, \quad \beta_2 = \frac{3}{2c_s^2} \alpha_1 + \frac{(1 - c_s^2)^2}{2c_s^4}, \quad \beta_3 = \frac{1 - c_s^2}{8c_s^4}. \quad (18)$$

The only nonzero $\tilde{\mathcal{A}}_s^{(2N)}$ in the forward limit at this order is $N = 1$, which we write as $f = \Lambda^4 \tilde{\mathcal{A}}_s^{(2)} \Big|_{\omega_l=0, t=0}$,

$$f(\gamma) = 4\beta_3 + 2(\beta_2 - 2\alpha_2^2)\gamma + \frac{3}{2}(\beta_1 - 4\alpha_1\alpha_2)\gamma^2 - \frac{9}{4}\alpha_1^2\gamma^3. \quad (19)$$

The positivity bound (14) requires that $f \geq 0$ for all values of $\gamma \geq 1$ that lie within the regime of validity of the EFT. At large γ , the $\dot{\pi}^3$ interaction eventually violates this bound, leading to an EFT cutoff in γ , as previously observed in [87]. Below this cutoff, the bound can be written as $\beta_1 \geq \beta_1^{\min}(c_s, \alpha_1)$, where

$$\beta_1^{\min} = \frac{3\alpha_1^2\gamma}{2} + \frac{2\alpha_1}{c_s^2} \left(1 - c_s^2 - \frac{1}{\gamma} \right) - \frac{4(1 - c_s^2)}{12c_s^4\gamma^2}. \quad (20)$$

It is not possible, no matter how ingenious our UV model building, to arrange for large effects at quadratic and cubic order in π (c_s and α_1) without also generating large effects at quartic order (β_1), unless we sacrifice one of the above fundamental properties.

The one-loop contribution to the positivity bound (15) can be evaluated from the tree-level amplitude in (17) using the optical theorem and is a simple polynomial in s/M^2 . At large s ,

$$\text{Im}\tilde{\mathcal{A}}_s^{1\text{-loop}} = \frac{\beta^2}{2048\pi\Lambda^8 M^4} \left(1 + \mathcal{O}\left(\frac{M^2}{s}\right) \right), \quad (21)$$

where $\beta^2 = \frac{3}{5}(\beta_1 + \beta_2 - (\frac{3}{2}\alpha_1 + \alpha_2)(\alpha_1 + 2\alpha_2))^2$. Subtracting this branch cut up to an energy scale $\omega_b = s_b/4M$ gives a loop-improved positivity bound,

$$f|_{\gamma=1} \geq \frac{\beta^2}{16\pi^2\Lambda^4} \left(1 + \mathcal{O}\left(\frac{M}{\omega_b}\right) \right), \quad (22)$$

which is strictly stronger than the previous bound, $f \geq 0$. Crucially, (22) depends nonlinearly on β_1 , and so there are some values of α_1 and c_s for which there are *no* allowed values for β_1 —this improved bound can only be satisfied if

$$\left(1 - c_s^2 + \frac{3}{2}\alpha_1 c_s^2 \right) \frac{\omega_b^4}{\Lambda^4} + 30\pi^2 c_s^4 \geq 0, \quad (23)$$

which allows us to bound α_1 in terms of c_s only.

B. Inflation

During inflation, the expanding spacetime background spontaneously breaks temporal diffeomorphisms. Following the seminal work of [1], these time diffeomorphisms can be restored (nonlinearly realized) by introducing a single scalar degree of freedom. In the decoupling and subhorizon limits,

$$\omega \gg \frac{\Lambda^2}{M_P}, \quad \text{and} \quad \omega \gg H, \quad (24)$$

where H is the background Hubble rate, and the scalar sector of the theory is described by (16) with the tuning (18). In this context, c_s is the sound speed of scalar perturbations produced during inflation, α_1 (β_1) controls the primordial bi- (tri)spectrum, and the Planck measurements of the power spectrum fix the symmetry-breaking scale $\Lambda = (58.89 \pm 0.21)H$ at 68% confidence [88].

The positivity bounds derived here provide a qualitatively new way to analyze these primordial signals. For instance, Eq. (20) for $\beta_1^{\min}$ is a robust lower bound on the size of the trispectrum g_{NL} , given measurements of the

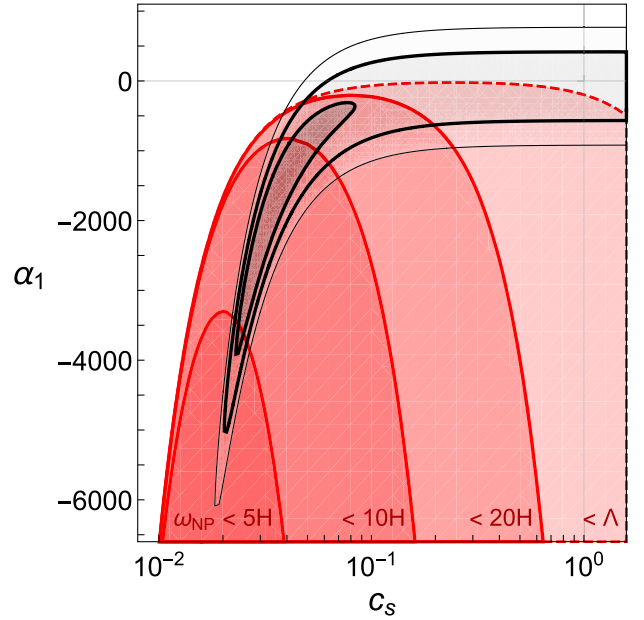


FIG. 1. Assuming a unitary, causal, and local UV completion, (23) sets a scale ω_{NP} beyond which inflation can no longer be single field and weakly coupled. The red shaded regions correspond to $\omega_{\text{NP}} < 5H, 10H, 20H$ and $\Lambda = 58.89H$, and the gray contours show the 68%, 95%, and 99.7% confidence intervals from the Planck observations of the bispectrum.

bispectrum f_{NL} and c_s . Taking for illustration the central values of the Planck 2018 contours [88], $f_{\text{NL}}^{\text{eq}} \approx -26$ and $f_{\text{NL}}^{\text{or}} \approx -38$, the bound $f \geq 0$ gives $g_{\text{NL}} \geq 12400$ [89] for the trispectrum from the $\dot{\pi}^4$ interaction [90,91]. This rules out over 70% of the observationally allowed parameter space $-2.7 \times 10^6 < g_{\text{NL}} < +1.1 \times 10^6$. Of course, since these numbers are sensitive to the relatively large uncertainties in c_s and α_1 , they should be taken only as an illustration: The point is that, in the future, as our measurements of the bispectrum improve, positivity bounds will provide a new way to “bootstrap” information about the trispectrum (without the need to directly detect g_{NL}).

Finally, our bound (23) can be used to place a restriction on the values of α_1 and c_s , which are compatible with the UV properties of Sec. II. Indeed, we can use (23) to define a scale $\omega_{\text{NP}}(c_s, \alpha_1)$, which is the largest possible energy at which inflation can remain single field and weakly coupled [i.e., described by (16) and (17)]. At this scale, new UV physics (new degrees of freedom beyond π) or nonperturbative effects *must* become important if we are to preserve unitarity, analyticity, and locality. We show in Fig. 1 how this scale compares with current Planck constraints on $\{c_s, \alpha_1\}$. The majority of the 68% confidence region requires new physics within an order of magnitude of H .

V. CONCLUSION

In summary, we have constructed new positivity bounds—constraints placed on a low-energy effective

field theory by the consistency of its underlying UV completion—when boosts are broken. The results presented here open up several avenues for future work. First and foremost, there are a number of systems in which Lorentz boosts are broken and the free propagation at low energies has the form $\omega \sim c_\pi k$, and our bounds can immediately be applied to the corresponding low-energy EFTs—examples include various condensed matter systems [92–99] and the EFT of dark energy [100] for late-Universe cosmology. Second, in covariant theories that possess both a Lorentz-invariant and a Lorentz-breaking background, it would be interesting to compare the bounds one would infer from the standard Lorentz-invariant arguments about the Lorentz-invariant background (see, e.g., [69,70]) from the bounds derived here about the Lorentz-breaking background. Finally, although we have dispensed with Lorentz boosts, our positivity bounds still required time translation invariance—it would be interesting, in light of the progress made here, to reconsider positivity arguments for interactions that also break time translations

(such as inflationary perturbations on superhorizon scales) [101]. Ultimately, there is no shortage of systems in which we only have experimental access to the low-energy fluctuations around a particular boost-breaking background, and here, we have provided the first set of positivity bounds that can be used to probe the underlying UV physics and that can translate basic UV properties into new theoretical priors which constrain the EFT parameter space.

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