Schur index of the $\mathcal{N}=4$ $U(N)$ supersymmetric Yang-Mills theory via the AdS/CFT correspondence

Reona Arai,^{*} Shota Fujiwara[†], Yosuke Imamura[‡] and Tatsuya Mori[§]

Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

(Received 13 February 2020; accepted 28 March 2020; published 17 April 2020)

We calculate the Schur index of the $\mathcal{N}=4$ $U(N)$ supersymmetric Yang-Mills theory with finite N via the AdS/CFT correspondence as the contribution of D3-branes wrapped on contractible cycles in S^5 on some assumptions motivated by preliminary analyses. As far as we have checked numerically it agrees with the index calculated on the gauge theory side. In a certain limit it reproduces the analytic result given by Bourdier, *et al.*

DOI: [10.1103/PhysRevD.101.086017](https://doi.org/10.1103/PhysRevD.101.086017)

I. INTRODUCTION

The Schur index [1] is a specialization of the superconformal index [2] that is defined for $\mathcal{N}=2$ superconformal theories. It is a function of a universal fugacity q , and if the theory has a flavor symmetry we can also introduce additional flavor fugacities. For a Lagrangian theory we can calculate it by the localization formula, which gives the index as a matrix integral. In some cases we can use other methods. The IR formula [3] enables us to calculate it from the Bogomolnyi-Prasad-Sommerfield (BPS) spectrum in the Coulomb branch. For a class S theory it is given as a correlation function on a Riemann surface [4,5]. It is also known that for an arbitrary $\mathcal{N}=2$ superconformal theory there is a corresponding chiral algebra and the Schur index is given as the vacuum character of the chiral algebra [6]. By using these different methods complementarily we can obtain nonperturbative information of the theory. In this work we propose another method to calculate the Schur index of the $\mathcal{N}=4$ $U(N)$ supersymmetric Yang-Mills theory based on the AdS/CFT correspondence [7].

The $\mathcal{N}=4$ theory, regarded as a special $\mathcal{N}=2$ theory, has the flavor symmetry $SU(2)_F \subset SU(4)_R$, and we introduce the flavor fugacity u . The Schur index is defined by

$$\mathcal{I}(q, u) = \text{tr}_{\text{BPS}}(e^{2\pi i(J+\bar{J})} q^{H+J+\bar{J}} u^{R_x-R_y}), \quad (1)$$

where the trace is taken over states saturating certain bounds. See [8] for our conventions. The localization formula is

$$\mathcal{I}_{U(N)}(q, u) = \int d\mu_N \text{Pexp}(i_V(q, u) \chi_N(z_a)), \quad (2)$$

where $d\mu_N$ is the $U(N)$ Haar measure of the integral over gauge fugacities z_a ($a=1, \dots, N$) and $\chi_N(z_a)$ is the character of the $U(N)$ adjoint representation,

$$d\mu_N = \frac{1}{N!} \prod_{a=1}^N \frac{dz_a}{2\pi i z_a} \prod_{a \neq b} \left(1 - \frac{z_a}{z_b}\right), \quad \chi_N(z_a) = \sum_{a,b=1}^N \frac{z_a}{z_b}. \quad (3)$$

We define the plethystic exponential Pexp by

$$\text{Pexp} f = \prod_i \frac{1}{(1-f_i)^{c_i}}, \quad (4)$$

for a function f with the series expansion $f = \sum_i c_i f_i$, where c_i are numerical coefficients and f_i are monomials made of the fugacities. The letter index i_V is

$$i_V(q, u) = \frac{q(u + \frac{1}{u}) - 2q^2}{1 - q^2}. \quad (5)$$

For the special case with $u=1$ Bourdier, *et al.* analytically carried out the integral in (2) and obtain [9]

$$\left. \frac{\mathcal{I}_{U(N)}}{\mathcal{I}_{U(\infty)}} \right|_{u \rightarrow 1} = \sum_{n=0}^{\infty} \mathcal{I}_n^{\text{BDF}}, \quad \mathcal{I}_n^{\text{BDF}} = (-1)^n ({}_{N+n}C_N + {}_{N+n-1}C_N) q^{nN+n^2}, \quad (6)$$

where ${}_nC_k = \frac{n!}{k!(n-k)!}$ is the binomial coefficient.

^{*}r.arai@th.phys.titech.ac.jp

[†]s.fujiwara@th.phys.titech.ac.jp

[‡]imamura@phys.titech.ac.jp

[§]t.mori@th.phys.titech.ac.jp

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The purpose of this paper is to reproduce (2) and (6) for finite N on the anti-de Sitter (AdS) side. In the large N limit we can analytically evaluate the integral by the saddle point analysis [2] and obtain

$$\mathcal{I}_{U(\infty)}(q, u) = \text{Pexp} \left(\frac{uq}{1-ug} + \frac{u^{-1}q}{1-u^{-1}q} - \frac{q^2}{1-q^2} \right). \quad (7)$$

On the AdS side this is reproduced as the index of Kaluza-Klein modes of the supergravity multiplet in the dual spacetime $\text{AdS}_5 \times S^5$. If N is finite, as the parameter relation $N = L^4 T_{D3}$ including the AdS radius L and the D3-brane tension T_{D3} implies, we should take account of D3-branes extended in $\text{AdS}_5 \times S^5$, and we can guess that the ratio $\mathcal{I}_{U(N)}/\mathcal{I}_{U(\infty)}$ expresses the contribution of D3-branes. What brane configurations should we take into account to calculate the index? In the case of BPS partition function it is possible to reproduce the exact result by the geometric quantization of 1/8 BPS brane configurations [10]. An 1/8 BPS configuration is given as the intersection of a holomorphic surface $h(X, Y, Z) = 0$ and $S^5 = \{(X, Y, Z) | |X|^2 + |Y|^2 + |Z|^2 = 1\}$ [11]. In the calculation of the superconformal index in [8] such configurations were treated as excitations of “rigid D3-branes.” A rigid D3-brane here means a D3-brane wrapped on a large S^3 in S^5 given by the linear equation $aX + bY + cZ = 0$. The collective motion of a rigid D3-brane is described by the moduli space $\mathbb{CP}^2$ with the projective coordinates (a, b, c) . Corresponding to the fact that $\mathbb{CP}^2$ is covered by three coordinate patches we can treat all rigid brane configurations and excitations of them as excitations of three specific brane configurations: $X = 0$, $Y = 0$, and $Z = 0$. In the case of Schur index only two configurations $X = 0$ and $Y = 0$ give nontrivial contributions, and a part of the finite N correction of the Schur index was correctly reproduced as the contribution from a single D3-brane wrapped on $X = 0$ and $Y = 0$. See [8] for more details. Although we do not have any proof this fact seems to suggest that some localization mechanism works for D3-brane configurations. By assuming this mechanism keeps working for multiple-brane configurations we propose the relation,

$$\frac{\mathcal{I}_{U(N)}(q, u)}{\mathcal{I}_{U(\infty)}(q, u)} = \sum_{n_1, n_2=0}^{\infty} \mathcal{I}_{(n_1, n_2)}(q, u; N), \quad (8)$$

where $\mathcal{I}_{(n_1, n_2)}$ is the contribution from the configuration with n_1 D3-branes wrapped on $X = 0$ and n_2 D3-branes wrapped on $Y = 0$.

Classically, the energy of the brane system is $(n_1 + n_2)N$ in the unit of L^{-1} and expected to give $\mathcal{O}(q^{(n_1+n_2)N})$ terms in the index. By comparing this with (6) it is natural to identify $\mathcal{I}_n^{\text{BDF}}$ with the contribution of brane systems with $n_1 + n_2 = n$. Namely,

$$\mathcal{I}_n^{\text{BDF}}(q; N) = \lim_{u \rightarrow 1} \sum_{k=0}^n \mathcal{I}_{(n-k, k)}(q, u; N). \quad (9)$$

In the following we calculate $\mathcal{I}_{(n-k, k)}$ and numerically confirm that (8) and (9) indeed hold.

II. GAUGE THEORY ON WRAPPED BRANES

The brane system giving $\mathcal{I}_{(n-k, k)}$ consists of $n-k$ D3-branes wrapped on $X = 0$ and k D3-branes wrapped on $Y = 0$. These two 3-cycles intersect in S^5 along S^1 , and a bifundamental hypermultiplet arises on the intersection. Namely, the theory realized on the brane system is the $U(n-k) \times U(k)$ gauge theory with a bifundamental hypermultiplet. The index is

$$\mathcal{I}_{(n-k, k)} = (uq)^{(n-k)N} (u^{-1}q)^{kN} \int d\mu_{n-k} \int d\mu'_k \text{Pexp} f_{\text{tot}}, \quad (10)$$

where the prefactors $(uq)^{(n-k)N}$ and $(u^{-1}q)^{kN}$ are the classical contributions of the D3-branes wrapped on the two cycles [8]. The total letter index f_{tot} is

$$\begin{aligned} f_{\text{tot}} = & f_V(q, u) \chi_{n-k}^{\text{adj}}(z) + f_H(q, u) \chi_{n-k, k}^{\text{bf}}(z, z') \\ & + f_V(q, u^{-1}) \chi_k^{\text{adj}}(z'), \end{aligned} \quad (11)$$

where $f_V(q, u)$ and $f_H(q, u)$ are the letter indices for a vector multiplet on $X = 0$ and a half hypermultiplet on the intersection, respectively. The letter index of the vector multiplet on $Y = 0$ is obtained from that for $X = 0$ by the $SU(2)_F$ Weyl reflection $u \rightarrow u^{-1}$. $\chi_{n-k, k}^{\text{bf}}$ is the character of the bifundamental representation,

$$\chi_{n-k, k}^{\text{bf}}(z, z') = \sum_{a=1}^{n-k} \sum_{b=1}^k \left(\frac{z_a}{z'_b} + \frac{z'_b}{z_a} \right). \quad (12)$$

We can easily determine the BPS spectrum of the hypermultiplet by using the supersymmetry algebra, and we obtain the letter index,

$$f_H = \frac{1}{q} - q. \quad (13)$$

Fortunately, the plethystic exponential of $f_H \chi_{n-k, k}^{\text{bf}}$ is quite simple,

$$\text{Pexp}(f_H(q) \chi_{n-k, k}^{\text{bf}}(z, z')) = q^{2(n-k)k}. \quad (14)$$

Because this is independent of the gauge fugacities the integral in (10) is factorized into the $U(n-k)$ part and the $U(k)$ part, and $\mathcal{I}_{(n-k, k)}$ is given by

$$\begin{aligned} \mathcal{I}_{(n-k, k)} = & (uq)^{(n-k)N} F_{n-k}(q, u) \cdot q^{2(n-k)k} \\ & \cdot (u^{-1}q)^{kN} F_k(q, u^{-1}). \end{aligned} \quad (15)$$

$F_n(q, u)$ is the index of the $U(n)$ gauge theory realized on $X = 0$, which is given by

$$F_n(q, u) = \int d\mu_n \text{Pexp}(f_V(q, u)\chi_n(z_a)). \quad (16)$$

As is pointed out in [8] the letter index f_V is obtained from i_V in (5) by the variable change,

$$f_V(q, u) = i_V(q^{\frac{1}{2}}u^{-\frac{1}{2}}, q^{-\frac{3}{2}}u^{-\frac{1}{2}}) = \frac{\frac{1}{uq} - \frac{2}{u}q + q^2}{1 - \frac{1}{u}q}. \quad (17)$$

Correspondingly, F_n is related to $\mathcal{I}_{U(n)}$ by $F_n(q, u) = \mathcal{I}_{U(n)}(q^{\frac{1}{2}}u^{-\frac{1}{2}}, q^{-\frac{3}{2}}u^{-\frac{1}{2}})$. Unfortunately, we cannot directly obtain F_n in the form of q -expansion by using this relation as far as $\mathcal{I}_{U(n)}$ is also given as the q -expansion. We need to calculate F_n separately by performing the integral in (16). When we calculate $\mathcal{I}_{U(N)}$ by (2) we usually assume $|q| < 1$ and pick up poles in the unit circle on z_a -planes. For the integral in (16) we have to pick up the corresponding poles obtained by the variable change, which are not always in the unit circle. We show the first few terms of F_n ($n \leq 4$),

$$\begin{aligned} F_0(q, u) &= 1, \\ F_1(q, u) &= \frac{u^3}{1-u^2}q + (1-u^2)q^2 + \left(\frac{1}{u^3} - u^3\right)q^3 + \left(\frac{1}{u^6} - \frac{1}{u^2} + 1 - u^4\right)q^4 + \dots, \\ F_2(q, u) &= \frac{u^{10}(2-u^2)}{(1-u^2)(1-u^4)}q^4 + u^5(2-u^4)q^5 + (2+2u^6-u^{12})q^6 + \dots, \\ F_3(q, u) &= \frac{u^{21}(5-3u^2-3u^4+2u^6)}{(1-u^2)(1-u^4)(1-u^6)}q^9 + \frac{u^{14}(5-3u^4-3u^6+2u^{10})}{1-u^4}q^{10} + \dots, \\ F_4(q, u) &= \frac{u^{36}(14-9u^2-10u^4-2u^6+6u^8+7u^{10}-5u^{12})}{(1-u^2)(1-u^4)(1-u^6)(1-u^8)}q^{16} + \dots. \end{aligned} \quad (18)$$

We point out that the leading terms of F_n are given by $F_n(q, u) = q^{n^2}u^{2n^2}G_n(u) + \mathcal{O}(q^{n^2+1})$ with the functions G_n generated by

$$\sum_{n=0}^{\infty} G_n(u)t^n = \exp\left(\sum_{p=1}^{\infty} \frac{(2p-1)!}{(p!)^2} \frac{t^p}{u^{-p} - u^p}\right). \quad (19)$$

III. COMPARISONS

Let us introduce the notation $A^{(\leq m)}$ to mean the q -expansion of A up to the q^m term. F_n contributes to the q^{nN+n^2} or higher order terms in (15). To do a nontrivial check for the leading term of F_4 in (18) for $U(1)$ theory we need to calculate the both hand sides of (15) up to q^{20} terms. For this purpose we calculated $F_n^{(\leq 20-n)}$ ($n \leq 4$), and by substituting them into the conjectural relation (8) we obtained $(\mathcal{I}_{U(N)}/\mathcal{I}_{U(\infty)})^{(\leq 19+N)}$ for $N = 1, 2, 3, 4$. We have found the complete agreement with the results obtained from (2). [See the Appendix for the first few terms of $\mathcal{I}_{U(N)}/\mathcal{I}_{U(\infty)}$ calculated by (2) for small N .] For $N = 0$, the “ $U(0)$ ” gauge theory is the trivial theory with no excitation, and the index is $\mathcal{I}_{U(0)} = 1$. Although the physical interpretation on the gravity side is not clear we have found that (8) with $N = 0$ correctly gives $(1/\mathcal{I}_{U(\infty)})^{(\leq 19)}$. We also found that the right-hand side of (8) vanishes for $N = -1$. Namely, (8) formally gives $\mathcal{I}_{U(-1)} = 0$.

We also confirmed (9) by taking the $u \rightarrow 1$ limit. Note that the limit must be taken after the summation with respect to k because functions F_n have poles at $u = 1$. If we sum up $\mathcal{I}_{(n-k,k)}$ over $k = 0, \dots, n$ the poles at each order cancel, and we obtain the leading term (6) as well as vanishing subleading terms. We have confirmed that (9) correctly reproduces $(\mathcal{I}_n^{\text{BDF}})^{(\leq n(N-1)+20)}$ for $n = 1, 2, 3, 4$ and arbitrary N .

IV. DISCUSSIONS

We proposed the relations (8) and (9) for the Schur index of the $\mathcal{N} = 4$ $U(N)$ supersymmetric Yang-Mills theory, and numerically confirmed that they correctly reproduce the results obtained on the gauge theory side.

Our calculation was based on some assumptions. We assumed the localization of the path integral to the special configurations consisting of branes wrapped on the two specific cycles $X = 0$ and $Y = 0$. We also assumed that quantum gravity corrections do not spoil our calculation.

There are many directions of extension. There seems no essential difficulty to generalize our analysis to the superconformal index. It would be also possible to apply our method to other examples of AdS/CFT. An analytic formula of the Schur index for a class of $\mathcal{N} = 2$ was obtained in [12], and it may be possible to reproduce it by the D3-brane analysis. There were some analysis of

single-brane configurations for S-fold theories [8], orbifold theories [13], and toric gauge theories [14]. It would be interesting to extend these results to multiple-brane configurations.

ACKNOWLEDGMENTS

We would like to thank Daisuke Yokoyama for valuable discussions and comments.

APPENDIX: RESULTS ON THE GAUGE THEORY SIDE

In this appendix we show the explicit form of q -expansion of $\mathcal{I}_{U(\infty)}$ and $\mathcal{I}_{U(N)}/\mathcal{I}_{U(\infty)}$ calculated on the gauge theory side.

The q -expansion of the index in the large N limit (7) is

$$\begin{aligned}\mathcal{I}_{U(\infty)}(q, u) = & 1 + \chi_1 q + (2\chi_2 - 2)q^2 + (3\chi_3 - 2\chi_1)q^3 + (-4\chi_2 + 5\chi_4 + 1)q^4 + (\chi_1 - 5\chi_3 + 7\chi_5)q^5 \\ & + (3\chi_2 - 9\chi_4 + 11\chi_6 - 1)q^6 + (\chi_1 + 2\chi_3 - 11\chi_5 + 15\chi_7)q^7 + (-2\chi_2 + 6\chi_4 - 18\chi_6 + 22\chi_8 + 4)q^8 \\ & + (2\chi_3 + 5\chi_5 - 23\chi_7 + 30\chi_9)q^9 + \cdots,\end{aligned}\quad (\text{A1})$$

where $\chi_n = (u^{n+1} - u^{-(n+1)})/(u - u^{-1})$ is the $SU(2)$ character. The inverse of (A1) is

$$\begin{aligned}1/\mathcal{I}_{U(\infty)}(q, u) = & 1 - \chi_1 q + (3 - \chi_2)q^2 + (5 - \chi_2)q^4 + (-\chi_1 - \chi_3 + \chi_5)q^5 + (-\chi_2 - \chi_4 + 8)q^6 + (\chi_7 - 2\chi_3)q^7 \\ & + (-2\chi_2 - \chi_6 + 13)q^8 + (-\chi_1 - 2\chi_3 + \chi_7)q^9 + (-3\chi_2 - \chi_4 - \chi_6 + 21)q^{10} \\ & + (2\chi_7 - 4\chi_3)q^{11} + (-3\chi_2 - \chi_4 - 2\chi_6 + \chi_{10} - \chi_{12} + 30)q^{12} + \cdots.\end{aligned}\quad (\text{A2})$$

The ratio of $\mathcal{I}_{U(N)}$ calculated by using the localization formula (2), and the large N limit (A1) is given for small N as follows:

$$\begin{aligned}\mathcal{I}_{U(1)}(q, u)/\mathcal{I}_{U(\infty)}(q, u) = & 1 - \chi_2 q^2 + (2\chi_1 - \chi_3)q^3 + (2\chi_2 - \chi_4 - 1)q^4 + (2\chi_2 - 1)q^6 + (\chi_3 - 2\chi_5 + \chi_7)q^7 \\ & + (-\chi_2 - \chi_6 + \chi_8 + 1)q^8 + (-2\chi_1 + 4\chi_3 - \chi_5 - 2\chi_7 + \chi_9)q^9 + (\chi_2 - \chi_6 - \chi_8 + \chi_{10} + 2)q^{10} \\ & + (\chi_5 - \chi_7 - \chi_9 + \chi_{11})q^{11} + (2\chi_4 - \chi_6 - \chi_8 - 1)q^{12} + \cdots,\end{aligned}\quad (\text{A3})$$

$$\begin{aligned}\mathcal{I}_{U(2)}(q, u)/\mathcal{I}_{U(\infty)}(q, u) = & 1 - \chi_3 q^3 + (2\chi_2 - \chi_4 - 1)q^4 + (\chi_1 + \chi_3 - \chi_5)q^5 + (2\chi_4 - \chi_6 - 3)q^6 + q^8(\chi_2 + \chi_4 + 1)q^8 \\ & + (-3\chi_1 + \chi_3 - \chi_7 + \chi_9)q^9 + (-2\chi_2 + \chi_4 + \chi_6 - 2\chi_8 + \chi_{10} + 1)q^{10} \\ & + (\chi_1 + 2\chi_3 - \chi_5 - \chi_7 - 2\chi_9 + 2\chi_{11})q^{11} + \cdots,\end{aligned}\quad (\text{A4})$$

$$\begin{aligned}\mathcal{I}_{U(3)}(q, u)/\mathcal{I}_{U(\infty)}(q, u) = & 1 - \chi_4 q^4 + (-\chi_1 + 2\chi_3 - \chi_5)q^5 + (\chi_4 - \chi_6 + 2)q^6 + (-3\chi_1 + 2\chi_3 + \chi_5 - \chi_7)q^7 \\ & + (-\chi_4 + 2\chi_6 - \chi_8)q^8 + (3\chi_2 - \chi_4 + 2\chi_6 - 4)q^{10} + (-2\chi_1 + \chi_3 + \chi_5 - \chi_7 - \chi_9 + \chi_{11})q^{11} \\ & + (-\chi_2 - 3\chi_4 + 2\chi_6 - \chi_{10} + \chi_{12} + 2)q^{12} + \cdots,\end{aligned}\quad (\text{A5})$$

$$\begin{aligned}\mathcal{I}_{U(4)}(q, u)/\mathcal{I}_{U(\infty)}(q, u) = & 1 - \chi_5 q^5 + (-\chi_2 + 2\chi_4 - \chi_6)q^6 + (\chi_1 + \chi_5 - \chi_7)q^7 + (-\chi_2 + \chi_4 + \chi_6 - \chi_8)q^8 \\ & + (-\chi_3 + \chi_5 + \chi_7 - \chi_9)q^9 + (-2\chi_2 + \chi_4 - \chi_6 + 2\chi_8 - \chi_{10} + 1)q^{10} \\ & + (-2\chi_2 + 4\chi_4 - 2\chi_6 + 2\chi_8 + 2)q^{12} + \cdots.\end{aligned}\quad (\text{A6})$$

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