Strong decays of the explicitly exotic doubly charmed DDK bound state

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Nowadays, it is generally accepted that the DK interaction in isospin zero is strongly attractive, and the D_{s0}^* (2317) can be described as a DK molecular state. Recent studies have shown that the three-body DDK system binds as well with a binding energy about 60–70 MeV. The DDK bound state has an isospin of $1/2$ and a spin-parity of 0^- . If discovered either experimentally or in lattice QCD, it will not only provide further support for the molecular nature of the D_{s0}^* (2317), but also provide a way to understand other exotic hadrons expected to be of a molecular nature. In the present work, we study its two-body strong decay widths via triangle diagrams. We find that the partial decay width into $DD_s\pi$ is at the order of 2–3 MeV, which seems to be within the reach of the current experiments such as Belle II. As a result, we strongly recommend that this decay channel of the DDK bound state is searched for experimentally.

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I. INTRODUCTION

In 2003, the *BABAR* Collaboration observed a narrow state in the inclusive $D_s^+\pi^0$ invariant mass distribution of the e^+e^- collision at energies near 10.6 GeV [1], i.e., the D_{s0}^* (2317) (D_{s0} for short in the present work), which was later confirmed by the *CLEO* Collaboration [2] and the *Belle* Collaboration [3]. Because the low mass, small width, and decay mode of the D_{s0} are quite different from those of a conventional $J^P = 0^+ c\bar{s}$ state in the naive quark model, its

nature has remained a topic of tremendous theoretical interest ever since its discovery [4–37]. In recent years, the importance of the DK interaction in forming the D_{s0} has been confirmed by lattice QCD simulations [38–42]. See Ref. [43] for a short summary of the theoretical, experimental, and lattice QCD supports for the molecular interpretation of the D_{s0} as a DK bound state.

If the D_{s0} is indeed (dominantly) a DK bound state, a natural question to ask is whether the DDK three-body system is still bound. In Ref. [44], by describing the $D_{s0}D$ interaction via a one kaon exchange (OKE), it was shown that the OKE interaction is strong enough to form a $D_{s0}D$ molecular state with a binding energy of 50–60 MeV, regardless whether the $D_{s0}DK$ coupling is determined by treating the D_{s0} as a $c\bar{s}$ state or a DK molecule. In Ref. [45], a study was done by explicitly considering the three-body $D(DK - D_s\pi - D_s\eta)$ system and by solving the Faddeev equations using the two-body inputs provided by the unitarized chiral perturbation theory and the local hidden

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symmetry approach. A three-body bound state was found in this latter work, with a total mass around 4140 MeV, which is an isospin doublet containing two states (R^{++}, R^+). In a more recent work [46], using the Gaussian expansion method, the existence of this state has been further confirmed though with a slightly smaller binding energy of $\sim 60\text{--}70$ MeV, and it has been found that even the $DDDK$ or DDD_{s0} system is bound. It is interesting to note that the $D\bar{D}^*K$ [47–49], the DKK and $DK\bar{K}$ [50], as well as the $DK\bar{N}$ [51] systems bind as well because of the strong attraction between D and K .

As pointed out in Ref. [45], the three-body DDK bound state can decay strongly via diagrams such as those shown in Fig. 1. In the present work, we calculate explicitly the partial decay widths from such processes, aiming to provide further motivation for the experimental search for this state. The present manuscript is organized as follows. The theoretical formalism is explained in Sec. II. The predicted partial decay widths are presented in Sec. III, followed by a short summary in Sec. IV.

II. THEORETICAL FORMALISM

In the following, we focus on the doubly charged state R^{++} . Due to isospin symmetry, the decay width of its isospin partner R^+ can be calculated analogously, and only small differences are expected because of the slightly different masses of its molecular components. As mentioned in Ref. [45], though the R^{++} is a bound state of the DDK system or the $D_{s0}D$ system, it is possible for such a state to decay strongly. Keeping in mind that the D_{s0}^+ is observed in the inclusive $D_s^+\pi^0$ invariant mass distribution, which violates isospin, the R^{++} can decay via $R^{++} \equiv D_{s0}^+D^+ \rightarrow (D_s^+\pi^0)D^+$. An alternative process, without involving isospin breaking, is via triangle diagrams such as those shown in Fig. 1. These processes conserve isospin

FIG. 1. Triangle diagrams representing the decay of the R^{++} state to $D_s^+D^{*+}$ and $D^+D_s^{*+}$ (a)–(b), and R^+ state to $D_s^+D^{*0}$ and $D^0D_s^{*+}$ (c)–(d).

and therefore, should be the dominant ones, as compared to the ones that violate isospin.

It should be noted that here we consider only η and K exchanges because DK and $D_s\eta$ are the only two relevant channels in the dynamical generations of the $D_{s0}^*(2317)$ (see, e.g., Refs. [38–42] and references cited therein). The replacement of the η/K by ϕ/K^* is not allowed by angular momentum and parity conservations. In addition, the $R^{++/+}$ can not decay into a baryon-antibaryon pair such as $\Xi_{cc}^{++}\bar{\Lambda}$ because there is no phase space for such decays. In principle, one may wonder if the D^*D_s and DD_s^* play a role in the dynamical generation of the $R^{++/+}$, but they do not. This can be understood from two perspectives. First, the $R^{++/+}$ state is dynamically generated from $D(DK) \sim DD_{s0}^*(2317)$ interactions. Second, the thresholds of D^*D_s and DD_s^* are about 160 MeV below the pole position of the $R^{++/+}$, which further implies that these two channels play a marginal role in the dynamical generation of the $R^{++/+}$.

In the following, we explain how to calculate the four diagrams shown in Fig. 1.

In order to calculate the Feynman diagrams shown in Fig. 1, we need to determine the relevant vertices. For the vertex of $R^{++}D_{s0}^+D^+$, since the R^{++} can be treated as a bound state of $D_{s0}^+D^+$ [44], this coupling can be determined by the Weinberg compositeness condition. In the present work, we adopt the method developed in Refs. [52–68]. In this framework, the interacting Lagrangian between R , D_{s0} , and D can be written as [52,53]

$$\mathcal{L}_R(x) = g_{RD_{s0}D}(x)R^T(x) \int dy \Phi_R(y^2) D_{s0}(x + \omega_D y) \times D(x - \omega_{D_{s0}} y) + \text{H.c.}, \quad (1)$$

where $\omega_i = m_i/(m_i + m_j)$ is a kinematical parameter with m_i and m_j being the masses of the involved mesons. In the Lagrangian of Eq. (1), an effective correlation function $\Phi(y^2)$ is introduced to reflect the distribution of the two constituents, $D_{s0}^+(2317)$ and $D^+(D^0)$, in the hadronic molecular $R^{++}(R^+)$ state. The introduced correlation function also serves the purpose of making the Feynman diagrams ultraviolet finite. Here, we choose the Fourier transformation of the correlation function in terms of a Gaussian form,

$$\Phi(p^2) \doteq \exp(-p_E^2/\Lambda^2), \quad (2)$$

where $\Lambda \sim 1.0$ GeV [52–68] is a size parameter, which characterizes the distribution of the molecular components inside the molecule, and p_E is the Euclidean Jacobi momentum [52–68]. In the present work, we take $\Lambda = 1$ GeV, unless otherwise stated.

The coupling constant $g_{RD_{s0}D}$ in Eq. (1) could be determined by the compositeness condition [52,53], where the renormalization constant of the composite particle should be zero, i.e.,

$$Z_R \equiv 1 - \Sigma_R(m_R^2) = 0, \quad (3)$$

with $\Sigma_R(m_R^2)$ being the derivative of the mass operator of the R . The concrete form of the mass operator of the DDK bound state R corresponding to the diagram in Fig. 2 is

$$\mathcal{W}_R(k_0) = \frac{g_{RD_{s0}D}^2}{16\pi^2} \int_0^\infty d\alpha \int_0^\infty d\beta \frac{1}{z^2} \exp\left\{-\frac{1}{\Lambda^2} \times \left[-2k_0^2\omega_{D_{s0}}^2 + \alpha m_{D_{s0}}^2 + \beta(-k_0^2 + m_D^2) + \frac{\Delta_M^2}{4z}\right]\right\}, \quad (4)$$

where $z = 2 + \alpha + \beta$, $\Delta_M = -4\omega_{D_{s0}}k_0 - 2\beta k_0$, and $k_0^2 = m_R^2$ with k_0, m_R denoting the four momenta and mass of the R , respectively. Here, we set $m_R = m_{D_{s0}} + m_D - E_b$ with E_b being the binding energy of R , k_1 and $m_{D_{s0}}$ are the four momenta and mass of the D_{s0} , and m_D is the mass of the D meson, respectively.

In the present work, we calculate the two-body decay width of the R via the triangle diagrams shown in Fig. 1. To evaluate the diagrams, in addition to the Lagrangian of Eq. (1), the following effective Lagrangian terms, responsible for the interactions between heavy-light pseudoscalar and vector mesons, are needed as well [33]:

$$\mathcal{L}_{P\phi P^*} = ig\langle P_\mu^* u^\mu P^\dagger - P u^\mu P_\mu^* \rangle, \quad (5)$$

where $P = (D^0, D^+, D_s^+)$ and $P^* = (D^{*0}, D^{*+}, D_s^{*+})$, u^μ is the axial vector combination of the pseudoscalar-meson fields and their derivatives,

$$u^\mu = i(u^\dagger \partial^\mu u - u \partial^\mu u^\dagger), \quad (6)$$

where $u^2 = U = \exp(i\frac{\phi}{f_0})$, $f_0 = 92.4$ MeV, and the pseudoscalar-meson octet ϕ is represented by the 3×3 matrix,

$$\phi = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (7)$$

FIG. 2. Self-energy of the R^{++} and R^+ states.

From Eqs. (5)–(7), one can easily obtain the interaction vertices $\eta DD^*, KDD_s^*$, and πD^*D . The coupling constant g can be determined from the strong decay width $\Gamma(D^{*+} \rightarrow D^0 \pi^+) = 56.46 \pm 1.22$ keV, together with the branching ratio $\text{BR}(D^{*+} \rightarrow D^0 \pi^+) = (67.7 \pm 0.5)\%$ [69]. With the help of Eq. (5), the two body decay width $\Gamma(D^{*+} \rightarrow \pi^+ D^0)$ is related to g via

$$\Gamma(D^{*+} \rightarrow \pi^+ D^0) = \frac{1}{12\pi} \frac{g^2 |\vec{p}_\pi|^3}{f_0^2 M^2}, \quad (8)$$

where $\vec{p}_\pi$ is the three momentum of π^+ in the rest frame of the decaying vector meson D^{*+} . Using the corresponding experimental strong decay width and the masses of the relevant particles given in Table I [69], we obtain $g = 1.097 \pm 0.012$ GeV.

In the chiral unitary approaches [20,21,33,70], the D_{s0} is found to be dynamically generated from the DK and $D_s \eta$ S -wave interactions. As a result, the vertices $D_{s0} DK$ and $D_{s0} \eta D_s$ can be easily written as

$$\mathcal{L}_{D_{s0}DK} = g_{D_{s0}DK} D_{s0} DK, \quad (9)$$

$$\mathcal{L}_{D_{s0}D_s\eta} = g_{D_{s0}D_s\eta} D_{s0} D_s \eta, \quad (10)$$

where the coupling of the D_{s0} to DK and $D_s \eta$ states, $g_{D_{s0}DK}$ and $g_{D_{s0}D_s\eta}$, can be obtained from the coupling constant of the D_{s0} to the DK and ηD_s channels in isospin zero, which are found to be $g_{D_{s0}DK} = 10.21$ GeV (10.203 GeV) and $g_{D_{s0}D_s\eta} = 6.40$ GeV (5.876 GeV) in Ref. [21] ([20]), multiplied by the appropriate Clebsch-Gordan (CG) coefficients, namely, $g_{D_{s0}^+ D^+ K^0} = g_{D_{s0}^+ D^0 K^+} = -g_{D_{s0}DK}/\sqrt{2}$ and $g_{D_{s0}^+ D_s^+ \eta} = g_{D_{s0}D_s\eta}$.

With the above vertices, the amplitudes of the triangle diagrams of Fig. 1, evaluated in the center of mass frame of final states, are

$$-i\mathcal{M}_\eta^a = g_{R^{++}D_{s0}^+D^+} g_{D_{s0}^+D_s^+\eta} \frac{-g}{\sqrt{3}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi[(k_1\omega_{D^+} - k_2\omega_{D_{s0}^+})^2] \epsilon_\mu(p_2) q^\mu \frac{1}{q^2 - m_\eta^2} \frac{1}{k_1^2 - m_{D_{s0}^+}^2} \frac{1}{k_2^2 - m_{D^+}^2}, \quad (11)$$

TABLE I. Masses of the relevant particles in the present work (in units of MeV) [69].

D^+	D^0	η	D^{*0}	$D^{*\pm}$
1869.65	1864.83	547.862	2006.85	2010.26
K^0	$D_s^{*\pm}$	$D_{s0}^{*\pm}$	$D_s^\pm$	$K^\pm$
497.611	2112.2	2317.0	1968.34	493.677

$$\begin{aligned}
-i\mathcal{M}_{K^0}^b &= g_{R^{++}D_{s0}^+D^+}g_{D_{s0}^+D^+K^0} \\
&\times \frac{-\sqrt{2}g}{f_0} \int \frac{d^4q}{(2\pi)^4} \Phi[(k_1\omega_{D^+} - k_2\omega_{D_{s0}^+})^2] \\
&\times \epsilon_\mu(p_2)q^\mu \frac{1}{q^2 - m_{K^0}^2} \frac{1}{k_1^2 - m_{D_{s0}^+}^2} \frac{1}{k_2^2 - m_{D^+}^2}, \quad (12)
\end{aligned}$$

$$\begin{aligned}
-i\mathcal{M}_\eta^c &= g_{R^{++}D_{s0}^+D^+}g_{D_{s0}^+D^+\eta} \\
&\times \frac{-g}{\sqrt{3}f_0} \int \frac{d^4q}{(2\pi)^4} \Phi[(k_1\omega_{D^0} - k_2\omega_{D_{s0}^+})^2] \\
&\times \epsilon_\mu(p_2)q^\mu \frac{1}{q^2 - m_\eta^2} \frac{1}{k_1^2 - m_{D_{s0}^+}^2} \frac{1}{k_2^2 - m_{D^0}^2}, \quad (13)
\end{aligned}$$

$$\begin{aligned}
-i\mathcal{M}_{K^+}^d &= g_{R^{++}D_{s0}^+D^+}g_{D_{s0}^+D^0K^+} \\
&\times \frac{-\sqrt{2}g}{f_0} \int \frac{d^4q}{(2\pi)^4} \Phi[(k_1\omega_{D^+} - k_2\omega_{D_{s0}^+})^2] \\
&\times \epsilon_\mu(p_2)q^\mu \frac{1}{q^2 - m_{K^+}^2} \frac{1}{k_1^2 - m_{D_{s0}^+}^2} \frac{1}{k_2^2 - m_{D^0}^2}. \quad (14)
\end{aligned}$$

The corresponding partial decay width then reads

$$d\Gamma[R \rightarrow] = \frac{1}{2J+1} \frac{1}{32\pi^2} \frac{|\vec{p}_1|}{m_R^2} |\overline{\mathcal{M}}|^2 d\Omega, \quad (15)$$

where $J = 0$ is the total angular momentum of the initial R state, the overline indicates the sum over the polarization vectors of final hadrons, and $|\vec{p}_1|$ is the three momentum of the decay products in the rest frame of the (R^{++}, R^+) states. Then the total decay widths of the (R^{++}, R^+) states are

$$\Gamma_{R^{++}} = \Gamma[R^{++} \rightarrow D_s^+ D^{*+}] + \Gamma[R^{++} \rightarrow D^+ D_s^{*+}], \quad (16)$$

$$\Gamma_{R^+} = \Gamma[R^+ \rightarrow D_s^+ D^{*0}] + \Gamma[R^+ \rightarrow D^0 D_s^{*+}]. \quad (17)$$

III. RESULTS AND DISCUSSIONS

To estimate the partial decay widths of the R , we first need to determine the coupling constants related to the molecular state and its components.

In Refs. [44–46], the R^{++} state is found to have a binding energy about 15–45 MeV, with respect to the $D_{s0}^+ D^+$ threshold. In this mass range, the coupling constant is dependent on the mass of the bound state R as shown in Fig. 3. One finds that the coupling constant $g_{R^{++}D_{s0}^+D^+}$ decreases with $m_{R^{++}}$. With a value of the mass $m_{R^{++}} = 4140$ MeV, the corresponding coupling constant is $g_{R^{++}D_{s0}^+D^+} = 9.02$ GeV.

We show the dependence of the total decay width on the masses of the bound state R^{++} in Fig. 4. One can see that the total decay width increases slightly with the mass of the

FIG. 3. The coupling constant $g_{R^{++}D_{s0}^+D^+}$ as a function of the mass of R^{++} .

bound state R^{++} from 4.13 to 4.17 GeV. The predicted total decay width is small and found to be $\Gamma_{R^{++}} = 2.5\text{--}2.6$ MeV.

In Fig. 4, we also show the partial decay widths of the $R^{++} \rightarrow D^+ D_s^{*+}$ and $D_s^+ D^{*+}$ as a function of the mass of the bound state R . The corresponding partial decay widths for several masses of the bound state are listed in Table II. We note that the transition $R^{++} \rightarrow D^+ D_s^{*+}$ is the main decay channel, almost saturating the total width. The corresponding partial decay widths are $\Gamma_{R^{++} \rightarrow D^+ D_s^{*+}} = 2.30\text{--}2.50$ MeV and $\Gamma_{R^{++} \rightarrow D_s^+ D^{*+}} = 0.26\text{--}0.29$ MeV, which yields a total decay width of 2.6–2.8 MeV. We note that the results depend only moderately on the cutoff. For instance, varying the cutoff from 0.5 to 1.5 GeV, the total decay width changes from 1.6 to 3.4 MeV within the R^{++} mass range of 4.13 to 4.17 GeV.

We find that the contribution from the η meson exchange is very small, because the $\eta D^0 D^{*0}$ vertex, which involves the creation or annihilation of an additional $s\bar{s}$ quark pair, is

FIG. 4. Total decay width (black solid line), partial decay widths of the $R^{++} \rightarrow D^+ D_s^{*+}$ (red dashed line), $R^{++} \rightarrow D_s^+ D^{*+}$ (blue dash-dot-dotted line), and the ratio of the partial decay widths (dark cyan dash dotted line) as a function of the mass of the R^{++} .

TABLE II. Partial decay widths of $R^{++} \rightarrow D_s^+ D^{*+}$ and $D^+ D_s^{*+}$ for different R^{++} masses (in units of MeV).

Mode	$\Lambda(\text{GeV})$	4130	4140	4150	4160	4170
$R^{++} \rightarrow D_s^+ D^{*+}$	1.0	0.27	0.28	0.29	0.29	0.26
$R^{++} \rightarrow D^+ D_s^{*+}$	1.0	2.33	2.43	2.49	2.47	2.30

strongly suppressed. Moreover, the main component of the $D_{s0}(2317)$ is DK [20,21,33,70], and the coupling constant related to this vertex is larger than the others. These two factors make the contribution from the K meson exchange the most important one.

In Fig. 4, we also show the ratio of the partial decay widths into $D^+ D_s^{*+}$ and $D_s^+ D^{*+}$. The ratio of branching fractions is found to be of the order of 8–9 and is almost independent of the mass of the bound state R .

IV. SUMMARY

In this work, inspired by the recent series of studies that showed the likely existence of a DDK bound state, we have studied its partial decay widths into $D_s D^*$ and DD_s^* . Such a decay involves the treatment of the R state as a quasibound state of $D_{s0}^*(2317)D$ and utilizing the Weinberg compositeness condition to determine the corresponding coupling. Our studies find a relative small total decay width, of the

order of 2–3 MeV, mainly to DD_s^* , and the results depend only moderately on the single parameter of the method, the cutoff Λ .

The predicted decay width seems to suggest that it is possible to observe such a state at Belle or Belle II, e.g., via the inclusive invariant mass distribution $D^+ D_s^+ \pi^0$, which is quite similar to the experimental discovery of the $D_{s0}^*(2317)$ by BABAR, Belle, and CLEO. On the other hand, its production yields at these experimental setups remain to be studied.

Recent lattice QCD studies of compact tetraquark states, see, e.g., Refs. [71,72], suggest that a study of the DDK bound state in terms of its minimal quark content $cc\bar{q}\bar{s}$ might be within the reach of the state of the art of lattice QCD simulations, even taking explicitly into account its three-meson molecular nature [73].

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