Solving the strong CP problem with horizontal gauge symmetry

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We present a solution to the strong CP problem that relies on the horizontal gauge symmetry and CP invariance in a full theory. Similar to other Nelson-Barr-type solutions, CP violation in both the strong and weak sectors of the Standard Model (SM) is attributed to the condensation of complex scalars Φ in the model. The model is differentiated from others in that it explains the hierarchy in the quark-Higgs Yukawa coupling in the SM based on a series of sequential breakings of the horizontal $SU(3)_f$ gauge symmetry. The experimental constraint $\bar{\theta} \lesssim 10^{-10}$ requires $\langle \Phi \rangle \lesssim 10^{13} - 10^{14}$ GeV (the vacuum expectation value of complex scalars) and $\lambda \lesssim 10^{-6}$ (the scalar quartic coupling). We show that this small coupling is natural in the sense of 't Hooft naturalness. Compared to other Nelson-Barr-type models with a CP -breaking scale $\Lambda_{CP} \lesssim 10^8$ GeV, our model is more advantageous in terms of consistency with thermal leptogenesis.

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I. INTRODUCTION

The smallness of a parameter in a theory can be considered natural provided certain additional symmetries are restored in the limit where the parameter is sent to zero [1]. This sense of naturalness, however, finds an unnaturally small parameter when applied to the QCD sector of the Standard Model (SM), i.e., $\bar{\theta} = \theta_0 + \text{Arg det}(Y_u Y_d)$. Here θ_0 is the QCD vacuum angle parametrized by a coefficient of the term $\sim F_{\mu\nu} \tilde{F}^{\mu\nu}$ in the QCD sector and Y_q is the Yukawa coupling matrix. The parameter enters in the expression for the neutron electric dipole moment (NEDM) $d_n = 3.6 \times 10^{-16} \bar{\theta} \text{ e cm}$ [2], of which the current experimental constraint $d_n < 3 \times 10^{-26} \text{ e cm}$ [3] yields $\bar{\theta} < 10^{-10}$. Setting $\bar{\theta} = 0$ still does not undo the breaking of CP symmetry because of the nonzero Kobayashi-Maskawa (KM) phase in the weak sector of the SM.

Several explanations for the smallness of $\bar{\theta}$ have been suggested. These include, for instance, the possibility of having a massless up quark and introducing a global $U(1)$ symmetry with a color anomaly [4–7]. For the purpose of setting $\bar{\theta} = 0$, the field redefinition of the up quark and the vacuum expectation value (VEV) of the pseudo-Nambu-Goldstone boson arising from the breaking of the

anomalous global $U(1)$ can be used for the former and latter cases, respectively. The lattice computation of the up-quark mass shows significant deviation from zero [8], and thus this simplest solution seems less likely (however, see, e.g., Ref. [9]). For the latter solution [4–7, 10–13], a variety of experimental searches for the pseudo-Nambu-Goldstone boson (the axion) have been suggested and performed, and more are under active investigation (see, e.g., Ref. [14]).

Another class of solution concerns the spontaneous breaking of CP symmetry. The most well-known of these solutions is the Nelson-Barr model [15, 16]. The model begins with the assumption that CP transformation is a symmetry of the model, giving rise to $\theta_0 = 0$. Furthermore, all of the interaction coefficients in the Lagrangian become real and the model is constructed in a way that the determinant of the fermion mass matrix is rendered real as long as CP is conserved. The model assumes a complex scalar sector, and the vacuum thereof breaks CP . It is the VEV of this complex scalar that makes the next-to-leading-order contribution to the fermion mass matrix complex, thereby inducing the CP -violating KM phase in the weak sector of the SM. One of the features that distinguishes solutions of this kind from others is that CP violations in the strong and weak sectors are attributed to fundamentally identical physics.

Along with an unknown fundamental origin of CP -violating parameters, i.e., $\bar{\theta}$ and the KM phase, the underlying physics responsible for the fermion mass hierarchy remains a mystery in the SM as well. On the other hand, the KM phase and the mass hierarchy are related to each other in that both are associated with Yukawa coupling matrices in the SM. Given this situation, if one aims to resolve the

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strong CP problem by relying on the physics of spontaneous CP violation, it would be natural to suspect that the underlying origins of $\bar{\theta}$, the KM phase, and the hierarchy of fermion masses may be very closely related to one another.

In this work, as an answer to such a suspicion, we present a model that contains new heavy quarks and complex scalars apart from the SM particle content. Also, we extend the gauge group of the SM by introducing a horizontal $SU(3)_f$ gauge symmetry and by assuming three Z_2 discrete gauge symmetries. The complex scalars obtain VEVs of different scales, which results in not only the spontaneous breaking of CP symmetry, but also a series of sequential breakings of $SU(3)_f$. Within the model, the Yukawa coupling structure of the SM is explained as well as the smallness of $\bar{\theta}$.

II. MODEL

Apart from the SM gauge group, the model has a horizontal (flavor) gauge symmetry $SU(3)_f$ as an additional gauge group [17–19] and three discrete gauge symmetries $Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)}$. The particle content of the model is extended by introducing into the SM particles three complex scalars Φ_{ai} and six heavy Dirac fermions Ψ_α^u and Ψ_α^d , where α and i are indices for $SU(3)_f$ and $Z_2^{(i)}$, respectively ($\alpha, i = 1, 2, 3$). The quantum numbers of the particle content of the model are presented in Table I. We see that there is no gauge anomaly if we introduce the lepton sector with three right-handed neutrinos [19].

Within the model, a gauged CP symmetry [20,21] is assumed, resulting in real interaction coefficients and $\theta_0 = 0$ (or π). When the complex scalars Φ_i acquire VEVs both

CP and $SU(3)_f$ are spontaneously broken.¹ Without loss of generality, we can write the vacuum of the scalar sector as

$$\langle \Phi_1 \rangle = \begin{bmatrix} 0 \\ 0 \\ X_1 \end{bmatrix}, \quad \langle \Phi_2 \rangle = \begin{bmatrix} 0 \\ Y_2 \\ X_2 \end{bmatrix}, \quad \langle \Phi_3 \rangle = \begin{bmatrix} Z_3 \\ Y_3 \\ X_3 \end{bmatrix}, \quad (1)$$

where X_1 and Y_2 are real, and the rest are complex. It is assumed that $V_1 > V_2 > V_3$, where $|\langle \Phi_i \rangle| \equiv V_i$.

The renormalizable $SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(3)_f \times (Z_2)^3$ invariant Yukawa coupling of the model reads

$$\mathcal{L}_{Yuk} = \mathcal{L}_q + \mathcal{L}_Q + \mathcal{L}_{qQ}, \quad (2)$$

with

$$\mathcal{L}_q = a^u H^\dagger q_\alpha \bar{u}_\alpha + a^d H q_\alpha \bar{d}_\alpha + \text{H.c.}, \quad (3)$$

$$\mathcal{L}_Q = M^U U_\alpha \bar{U}_\alpha + M^D D_\alpha \bar{D}_\alpha + \text{H.c.}, \quad (4)$$

and

$$\mathcal{L}_{qQ} = b^u H^\dagger q_\alpha \bar{U}_\alpha + b^d H q_\alpha \bar{D}_\alpha + \text{H.c.}, \quad (5)$$

where the interaction coefficients are real due to CP invariance and $SU(2)_L$ indices are suppressed.² Similar models were considered in Refs. [22,23]. Then, the mass matrices of each of the up- and down-sector fermions in the model become

$$\mathcal{M}^u = \begin{bmatrix} \mathcal{M}_{11}^u & \mathcal{M}_{12}^u \\ \mathcal{M}_{21}^u & \mathcal{M}_{22}^u \end{bmatrix} = \begin{bmatrix} a^u H_0^* I_{3 \times 3} & b^u H_0^* I_{3 \times 3} \\ 0 & M^U I_{3 \times 3} \end{bmatrix}, \quad (6)$$

$$\mathcal{M}^d = \begin{bmatrix} \mathcal{M}_{11}^d & \mathcal{M}_{12}^d \\ \mathcal{M}_{21}^d & \mathcal{M}_{22}^d \end{bmatrix} = \begin{bmatrix} a^d H_0 I_{3 \times 3} & b^d H_0 I_{3 \times 3} \\ 0 & M^D I_{3 \times 3} \end{bmatrix}, \quad (7)$$

where $\mathcal{M}^u$ and $\mathcal{M}^d$ are made up of four 3×3 block matrices and H_0 is the neutral component of the Higgs $SU(2)_L$ doublet. At the electroweak symmetry breaking (EWSB) vacuum, H_0 has a VEV, $|\langle H_0 \rangle| \simeq 246$ GeV. The block matrices shown in Eqs. (6) and (7) are proportional to the identity matrix because of $SU(3)_f$. At the renormalizable and tree level, the complex scalars Φ_i as a source of CP violation do not appear in the fermion mass matrices since the fermions are even under discrete symmetries, while the scalars are odd.

The determinant of the fermion mass matrix reads [24]

$$\det \mathcal{M} = [\det \mathcal{M}_{11}] [\det (\mathcal{M}_{22} - \mathcal{M}_{21} \mathcal{M}_{11}^{-1} \mathcal{M}_{12})] \quad (8)$$

$$= [\det \mathcal{M}_{22}] [\det (\mathcal{M}_{11} - \mathcal{M}_{12} \mathcal{M}_{22}^{-1} \mathcal{M}_{21})]. \quad (9)$$

¹To avoid the domain-wall problem arising from the breaking of CP , we assume that CP violation precedes inflation.

²Yukawa couplings in the model remain finite at all orders with the renormalization of a^q and b^q ($q = u, d$) in Eqs. (3) and (5).

TABLE I. Quantum numbers of the scalar bosons and fermions of the model. $U(D)$ and $\bar{U}(\bar{D})$ are the Weyl spinors that form the Dirac fermions $\Psi^u = (U, \bar{U}^*)^T$ and $\Psi^d = (D, \bar{D}^*)^T$. The subscript of a complex scalar indicates under which Z_2 the field is odd. Notice that the quantum numbers of $\bar{u}$ and $\bar{U}$ are completely identical. We define $\bar{U}$ as the partner of U for the mass term and $\bar{u}$ as an orthogonal direction to $\bar{U}$. The same applies for $\bar{d}$ and $\bar{D}$.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(3)_f$	$Z_2^{(1)}$	$Z_2^{(2)}$	$Z_2^{(3)}$
q	$\square$	$\square$	$+1/6$	$\square$	$+$	$+$	$+$
$\bar{u}$	$\bar{\square}$	1	$-2/3$	$\bar{\square}$	$+$	$+$	$+$
$\bar{d}$	$\bar{\square}$	1	$+1/3$	$\bar{\square}$	$+$	$+$	$+$
H	1	$\square$	$-1/2$	1	$+$	$+$	$+$
U	$\square$	1	$+2/3$	$\square$	$+$	$+$	$+$
$\bar{U}$	$\bar{\square}$	1	$-2/3$	$\bar{\square}$	$+$	$+$	$+$
D	$\square$	1	$-1/3$	$\square$	$+$	$+$	$+$
$\bar{D}$	$\bar{\square}$	1	$+1/3$	$\bar{\square}$	$+$	$+$	$+$
Φ_1	1	1	0	$\square$	$-$	$+$	$+$
Φ_2	1	1	0	$\square$	$+$	$-$	$+$
Φ_3	1	1	0	$\square$	$+$	$+$	$-$

The complex phase of $\langle H_0 \rangle$ gets canceled in the product $\mathcal{M}^u \mathcal{M}^d$. Along with Eqs. (6) and (7), this shows that $\det \mathcal{M}^u \mathcal{M}^d$ is real at the tree level. In the next section, we examine additional contributions to the fermion mass matrices which lead to $\delta \text{Arg}[\det \mathcal{M}^u \mathcal{M}^d] \neq 0$.

III. NONZERO CONTRIBUTION TO $\bar{\theta}$

In this section, we investigate a nonzero contribution to $\bar{\theta}$ that arises as a consequence of the structure of the model. With the constraint $\bar{\theta} < 10^{-10}$, a nonvanishing contribution to the β function of $\bar{\theta}$ arises at seven-loop order [25] in the SM. Furthermore, $\bar{\theta} = 0$ holds in a higher energy scale than $\langle \Phi \rangle$ in the model due to the CP symmetry. Thus, we assume that the renormalization group evolution of $\bar{\theta}$ is negligible. With this, we apply the experimental constraint $\bar{\theta} \lesssim 10^{-10}$ from measurements of the NEDM to the energy scale for the breaking of CP and $SU(3)_f$. This will constrain the VEV of Φ_i and the quartic couplings of the complex scalars in the model.

A. Contributions from higher-dimensional operators

The interaction between the complex scalars and fermions in the model might be induced by UV physics, which can be studied using Planck-suppressed higher-dimensional operators. In this section, we probe possible higher-dimensional operators allowed by the symmetries of the model. Then we figure out which operators could potentially spoil the reality of $\det \mathcal{M}^u \mathcal{M}^d$. These dangerous operators can be used to constrain the vacuum of the scalar sector.

We start with the dimension-five operators. For the up sector, the operators contributing to the mass matrices after the complex scalar condensation are

$$\begin{aligned} \mathcal{O}_{21}^{(u,5)} &= \sum_{i=1}^3 c_{1,i}^{(u,5)} \frac{\Phi_{\beta i}^\dagger \Phi_{\beta i}}{M_P} U_\alpha \bar{u}_\alpha + \sum_{i=1}^3 c_{2,i}^{(u,5)} \frac{\Phi_{\alpha i}^\dagger \Phi_{\beta i}}{M_P} U_\alpha \bar{u}_\beta \\ &+ \sum_{i=1}^3 c_{3,i}^{(u,5)} \frac{H^\dagger H}{M_P} U_\alpha \bar{u}_\alpha, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathcal{O}_{22}^{(u,5)} &= \sum_{i=1}^3 d_{1,i}^{(u,5)} \frac{\Phi_{\beta i}^\dagger \Phi_{\beta i}}{M_P} U_\alpha \bar{U}_\alpha + \sum_{i=1}^3 d_{2,i}^{(u,5)} \frac{\Phi_{\alpha i}^\dagger \Phi_{\beta i}}{M_P} U_\alpha \bar{U}_\beta \\ &+ \sum_{i=1}^3 d_{3,i}^{(u,5)} \frac{H^\dagger H}{M_P} U_\alpha \bar{U}_\alpha, \end{aligned} \quad (11)$$

where $M_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass and the repeated indices for $SU(3)_f$ are assumed to be summed over. Here the subscript of an operator indicates the block matrix position in $\mathcal{M}^u$ to which the operator contributes [see Eq. (6)]. The superscript specifies the up sector and the operator's mass dimension. The operator coefficients are real due to CP invariance. The contributions to each mass matrix from dimension-five operators

are Hermitian, as can be seen in Eqs. (10) and (11). The same operators can be found for the down sector with U and $\bar{u}$ replaced with D and $\bar{d}$. Note that $\mathcal{M}_{11}$ and $\mathcal{M}_{12}$ are identity matrices up to the dimension-five-operator level. Hermiticity is maintained under addition and inversion, and thus it can be inferred from Eq. (8) that the reality of $\det \mathcal{M}^u \mathcal{M}^d$ is protected up to the dimension-five-operator level. Put another way, $\text{Arg}(\det \mathcal{M}^u \mathcal{M}^d) = 0$ (or π) holds up to the dimension-five-operator level.³

Next, the dimension-six operators allowed by the symmetries of the model are given as

$$\begin{aligned} \mathcal{O}_{11}^{(u,6)} &= \sum_{i=1}^3 a_{1,i}^{(u,6)} \frac{\Phi_{\beta i}^\dagger \Phi_{\beta i}}{M_P^2} H^\dagger q_\alpha \bar{u}_\alpha \\ &+ \sum_{i=1}^3 a_{2,i}^{(u,6)} \frac{\Phi_{\alpha i}^\dagger \Phi_{\beta i}}{M_P^2} H^\dagger q_\alpha \bar{u}_\beta \\ &+ \sum_{i=1}^3 a_{3,i}^{(u,6)} \frac{H^\dagger H}{M_P^2} H^\dagger q_\alpha \bar{u}_\alpha, \end{aligned} \quad (12)$$

$$\begin{aligned} \mathcal{O}_{12}^{(u,6)} &= \sum_{i=1}^3 b_{1,i}^{(u,6)} \frac{\Phi_{\beta i}^\dagger \Phi_{\beta i}}{M_P^2} H^\dagger q_\alpha \bar{U}_\alpha \\ &+ \sum_{i=1}^3 b_{2,i}^{(u,6)} \frac{\Phi_{\alpha i}^\dagger \Phi_{\beta i}}{M_P^2} H^\dagger q_\alpha \bar{U}_\beta \\ &+ \sum_{i=1}^3 b_{3,i}^{(u,6)} \frac{H^\dagger H}{M_P^2} H^\dagger q_\alpha \bar{U}_\alpha. \end{aligned} \quad (13)$$

Again, the coefficients of operators are real due to CP invariance. Together with these contributions, $\mathcal{M}_{11}^u$ and $\mathcal{M}_{12}^u$ are no longer proportional to the identity matrix, but instead are Hermitian. In general, a product of Hermitian matrices is Hermitian only when the component Hermitian matrices commute with each other. The Wilson coefficients of the operators in Eqs. (12) and (13) are arbitrary unknowns, and thus it remains undecided whether the block matrices in Eq. (6), $\mathcal{M}_{rs}^u$ ($r, s = 1, 2$), commute with each other. Therefore, we conclude that the breaking of the reality of $\det \mathcal{M}^u \mathcal{M}^d$ starts from the dimension-six-operator level.

By using Eqs. (24) and (27) (the details will be discussed in Sec. IV), the application of the experimental constraint $\bar{\theta} \lesssim 10^{-10}$ to the ratio of the leading contributions to the imaginary and real parts of $\det \mathcal{M}$ in Eq. (9) yields

$$\begin{aligned} \delta \bar{\theta} &\sim a_2^{(d,6)} b_2^{(d,6)} c_2^{(d,5)} b^d \frac{|\Phi_1|^2 |\Phi_2|^2 |\Phi_3|^2}{M^D M_P^5} \times |y_b|^{-3} \\ &\simeq a_2^{(d,6)} b_2^{(d,6)} 10^{-20} \times |y_b|^{-2} \ll 10^{-10}, \end{aligned} \quad (14)$$

³ $\bar{\theta} \simeq 0$ may be favored by observations [2].

where we used

$$|y_b| \simeq b^d c_{2,i=1}^{(d,5)} \frac{|X_1|^2}{M^D M_P}. \quad (15)$$

The value 10^{-20} is estimated from the VEVs of the complex scalars, which will be obtained in the following discussion about the dimension-seven-operators' contribution to $\bar{\theta}$. The dominant contribution to $\delta\bar{\theta}$ from the down sector is traced to $|y_b|^{-2} > |y_t|^{-2}$. From Eq. (14), we can infer that the nonzero $\delta\bar{\theta}$ arising from the fermion mass matrices up to the dimension-six operators causes CP violation, albeit not large enough to be used to constrain the parameters of the model.

Nonzero contributions to $\text{Arg}(\det \mathcal{M}^u \mathcal{M}^d)$ can also occur in the following parts of the dimension-seven operators:

$$\begin{aligned} \mathcal{O}_{21}^{(u,7)} \ni & \sum_{i,j=1}^3 c_{1,ij}^{(u,7)} \frac{\Phi_{\gamma i}^\dagger \Phi_{\gamma i} \Phi_{\alpha j}^\dagger \Phi_{\beta j}}{M_P^3} U_\alpha \bar{U}_\beta \\ & + \sum_{i,j=1}^3 c_{2,ij}^{(u,7)} \frac{\Phi_{\gamma i}^\dagger \Phi_{\gamma j} \Phi_{\alpha i}^\dagger \Phi_{\beta j}}{M_P^3} U_\alpha \bar{U}_\beta \\ & + \sum_{i,j=1}^3 c_{3,ij}^{(u,7)} \frac{\Phi_{\gamma i}^\dagger \Phi_{\gamma j} \Phi_{\alpha j}^\dagger \Phi_{\beta i}}{M_P^3} U_\alpha \bar{U}_\beta, \end{aligned} \quad (16)$$

$$\begin{aligned} \mathcal{O}_{22}^{(u,7)} \ni & \sum_{i,j=1}^3 d_{1,ij}^{(u,7)} \frac{\Phi_{\gamma i}^\dagger \Phi_{\gamma i} \Phi_{\alpha j}^\dagger \Phi_{\beta j}}{M_P^3} U_\alpha \bar{U}_\beta \\ & + \sum_{i,j=1}^3 d_{2,ij}^{(u,7)} \frac{\Phi_{\gamma i}^\dagger \Phi_{\gamma j} \Phi_{\alpha i}^\dagger \Phi_{\beta j}}{M_P^3} U_\alpha \bar{U}_\beta \\ & + \sum_{i,j=1}^3 d_{3,ij}^{(u,7)} \frac{\Phi_{\gamma i}^\dagger \Phi_{\gamma j} \Phi_{\alpha j}^\dagger \Phi_{\beta i}}{M_P^3} U_\alpha \bar{U}_\beta, \end{aligned} \quad (17)$$

where the repeated indices for $SU(3)_f$ are assumed to be summed over. Corresponding operators of a similar form can be found in the down sector. In Eqs. (16) and (17), we only showed $\mathbf{1} \otimes \mathbf{8} \otimes \mathbf{8}$ -type operators, although there are three more types of operators, including $\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8}$, $\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{1}$, and $\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1}$. For the current purpose of estimating the order of magnitude of $\delta\bar{\theta}$ due to dimension-seven operators, it suffices to study $\mathbf{1} \otimes \mathbf{8} \otimes \mathbf{8}$ -type operators below. Up to the dimension-seven-operator level, the up-sector fermion mass matrix is given by

$$\begin{aligned} \mathcal{M}^u &= \begin{bmatrix} \mathcal{M}_{11}^u & \mathcal{M}_{12}^u \\ \mathcal{M}_{21}^u & \mathcal{M}_{22}^u \end{bmatrix} \\ &= \begin{bmatrix} a^u H_0^* I_{3 \times 3} + \mathcal{O}_{11}^{(u,6)} & b^u H_0^* I_{3 \times 3} + \mathcal{O}_{12}^{(u,6)} \\ \mathcal{O}_{21}^{(u,5)} + \mathcal{O}_{21}^{(u,7)} & M^U I_{3 \times 3} + \mathcal{O}_{22}^{(u,5)} + \mathcal{O}_{22}^{(u,7)} \end{bmatrix}. \end{aligned} \quad (18)$$

At the dimension-seven-operator level, differing from the previous lower-dimensional cases, we see that irremovable complex phases arise in the diagonal elements of $\mathcal{M}_{21}^u$ and $\mathcal{M}_{22}^u$. Namely, $\mathcal{M}_{21}^u$ and $\mathcal{M}_{22}^u$ are no longer Hermitian matrices. This is because $c_{2,ij}^{(u,7)} = c_{2,ji}^{(u,7)}$, $c_{3,ij}^{(u,7)} = c_{3,ji}^{(u,7)}$, $d_{2,ij}^{(u,7)} = d_{2,ji}^{(u,7)}$, and $d_{3,ij}^{(u,7)} = d_{3,ji}^{(u,7)}$ are not ensured for the second- and third-type contributions to $\mathcal{O}_{21}^{(u,7)}$ and $\mathcal{O}_{22}^{(u,7)}$ in Eqs. (16) and (17). The same applies for the down sector. Now this fact makes the complexity of $\det \mathcal{M}^u \mathcal{M}^d$ manifest.

To estimate the $\delta\bar{\theta}$ arising at the level of the dimension-seven operators, we compare the dominant contributions to the real and imaginary parts of $\det \mathcal{M}^u \mathcal{M}^d$ by referring to Eq. (9). Up to the dimension-five-operator level, the dominant contribution to the real part comes from $\det \mathcal{M}_{21}$, which reads $\sim (|y_t| M^U)^3$. On the other hand, we find the dominant contribution to the imaginary part to be $\sim b^u (|y_t| M^U)^2 \mathcal{O}_{21}^{(u,7)}$. The same applies for the down sector. Then, the ratio of these two produces $\delta\bar{\theta} \sim (b^u \mathcal{O}_{21}^{(u,7)}) / (|y_t| M^U) + (b^d \mathcal{O}_{21}^{(d,7)}) / (|y_b| M^D)$. Thus, we obtain

$$\delta\bar{\theta} \sim \frac{b^q c_{2,i=1,j=2}^{(q,7)} |X_1|^2 |X_2|^2}{|y_{q3}| M^Q M_P^3} \lesssim 10^{-10}, \quad (19)$$

where Eq. (19) shows a greater contribution in the up and down sectors, and q_3 is either t or b . With the use of Eq. (27), we can rewrite Eq. (19) as

$$\delta\bar{\theta} \sim \frac{c_{2,i=1,j=2}^{(q,7)} |X_2|^2}{c_2^{(q,5)} M_P^2} \lesssim 10^{-10}, \quad (20)$$

where q can be either u or d , depending on which makes the greater contribution. Taking $c_{2,i=1,j=2}^{(q,7)} / c_2^{(q,5)} = 10^P$, we obtain the upper bound on $|X_2| \lesssim 10^{(26-P)/2}$ GeV. For instance, for $P=0$ and $P=-2$, the upper bound is 10^{13} and 10^{14} GeV, respectively.

Having obtained the upper bound on $|X_2|$ in terms of values of the Wilson coefficients of the dimension-five and -seven operators, we realize that $\delta\bar{\theta}$ due to dimension-six operators hardly exceeds 10^{-10} unless we fine-tune the Wilson coefficients of dimension-seven operators. Hence, we conclude that $\bar{\theta} \lesssim 10^{-10}$ constrains the VEVs of the complex scalars at the dimension-seven-operator level. Yet, CP violation starts at the dimension-six-operator level.

Before ending this section, it is worth reconsidering the physical reason for breaking of reality of $\det \mathcal{M}^u \mathcal{M}^d$ at the dimension-six-operator level. In other words, why is CP violated at the dimension-six-operator level? The fact that $\mathcal{M}_{11}^q$ and $\mathcal{M}_{12}^q$ ($q = u, d$) remain proportional to the identity matrix was the reason to make $\det \mathcal{M}^u \mathcal{M}^d$ real

up to the dimension-five-operator level [see Eq. (8)]. This was possible due to the horizontal $SU(3)_f$ gauge symmetry. Also, the fact that the heavy fermions including $\bar{U}$ are an $SU(2)_L$ singlet disallows the dimension-five operator contribution to $\mathcal{M}_{12}$. In this way, we may understand that the reality of $\det \mathcal{M}^u \mathcal{M}^d$ up to the dimension-five-operator level is related to the horizontal $SU(3)_f$ gauge symmetry and $SU(2)_L$ singlet heavy fermions. Nonetheless, $\mathcal{M}_{11}^q$ and $\mathcal{M}_{12}^q$ ($q = u, d$) are no longer proportional to the identity matrix starting from the dimension-six-operator level, resulting in a nonzero contribution to $\bar{\theta}$.

B. One-loop contribution to $\bar{\theta}$

In the previous section, we observed that the tree-level nonvanishing contribution to $\bar{\theta}$ arises from dimension-seven operators, which constrains the symmetry-breaking scale for both CP and the $SU(3)_f$ horizontal gauge symmetry. In this section, we will investigate how the scalar sector within the model is constrained by the one-loop radiative correction to $\bar{\theta}$. To this end, we begin with the following renormalizable scalar potential which respects $SU(3)_f \times (Z_2)^3$:

$$\begin{aligned}
 V(\Phi) = & -\sum_{i=1}^3 \frac{1}{2} m_{\Phi_i}^2 |\Phi_{ai}|^2 + \frac{\lambda_0}{4} \sum_{i,j=1}^3 |\Phi_{ai}|^2 |\Phi_{\beta j}|^2 \\
 & + \frac{\lambda_+}{4} \sum_{i,j=1}^3 \Phi_{ai}^\dagger \Phi_{aj} \Phi_{\beta i}^\dagger \Phi_{\beta j} \\
 & + \frac{\lambda_-}{4} \sum_{i,j=1}^3 \Phi_{ai}^\dagger \Phi_{aj} \Phi_{\beta j}^\dagger \Phi_{\beta i} \\
 & + \text{higher order terms},
 \end{aligned} \tag{21}$$

where the repeated $SU(3)_f$ indices are assumed to be summed over. Notice that the higher-order terms are non-negligible so as to determine all VEVs for Φ_i , since $\lambda_{\pm}$ are very small (as shown below). The determination of the VEVs of Φ_i is beyond the scope of this paper. Again, the CP invariance renders all of the interaction coefficients in $V(\Phi)$ in Eq. (21) real.

For contributions to $\delta\bar{\theta}$ from higher-dimensional operators, we observe that a significant imaginary part of $\det \mathcal{M}^u \mathcal{M}^d$ arises at the dimension-seven-operator level. We observe that the dominant contribution to the imaginary part of $\det \mathcal{M}^u \mathcal{M}^d$ is attributable to $\det \mathcal{M}_{21}^q$ ($q = u, d$) in Eq. (9). This implies that the leading radiative correction to $\det \mathcal{M}_{21}^q$ ($q = u, d$) with four scalar condensation external lines must also be constrained by the experimental constraint $\bar{\theta} \lesssim 10^{-10}$.

With that being said, our aim is to ascertain whether one-loop corrections to the block matrices of $\mathcal{M}_{21}^u$ and $\mathcal{M}_{21}^d$ spoil the Hermiticity after the condensation of Φ and thus

induce $\text{Arg}(\det \mathcal{M}^u \mathcal{M}^d) \neq 0$. If the Hermiticity is broken in $\mathcal{M}_{21}^q$, then the loop correction must be constrained. Recall that $c_{2,ij}^{(q,7)} \neq c_{2,ji}^{(q,7)}$ and $c_{3,ij}^{(q,7)} \neq c_{3,ji}^{(q,7)}$ ($q = u, d$) were the essential points to make the diagonal components of $\mathcal{M}_{21}^u$ and $\mathcal{M}_{21}^d$ complex when we discussed dimension-seven operators [see the second- and third-type contributions to $\mathcal{O}_{21}^{(u,7)}$ in Eq. (16)]. Hence, testing the Hermiticity at the loop level reduces to checking whether the loop corrections to $c_{2,ij}^{(q,7)}$ and $c_{2,ji}^{(q,7)}$ ($c_{3,ij}^{(q,7)}$ and $c_{3,ji}^{(q,7)}$) are identical.

For each loop correction to $c_{2,ij}^{(q,7)}$ ($c_{3,ij}^{(q,7)}$), if there exists a corresponding identical correction to $c_{2,ji}^{(q,7)}$ ($c_{3,ji}^{(q,7)}$) up to a loop factor, then the one-loop corrections do not break the Hermiticity of $\mathcal{M}_{21}^u$ and $\mathcal{M}_{21}^d$ because $\delta c_{2,ij}^{(q,7)} = \delta c_{2,ji}^{(q,7)}$ ($\delta c_{3,ij}^{(q,7)} = \delta c_{3,ji}^{(q,7)}$). As a matter of fact, we find that this is not the case as we observe the breaking of the one-to-one correspondence. In the following, we demonstrate this by showing that the correction to $c_{2,ij}^{(q,7)}$ does not have a corresponding correction to $c_{2,ji}^{(q,7)}$. The same can be observed for $c_{3,ij}^{(q,7)}$ and $c_{3,ji}^{(q,7)}$ as well.

Among the many different contributions, in Fig. 1 we show loop corrections to $c_{2,ij}^{(u,7)}$ and $c_{2,ji}^{(u,7)}$ with the same internal lines in the right and left panels, respectively. Provided the scalar quartic vertex factors are different, we may conclude that the loop correction to $c_{2,ij}^{(u,7)}$ in the right panel lacks its corresponding partner for $c_{2,ji}^{(u,7)}$. The same applies for the down sector.

In accordance with Eq. (21), we see that the vertex factors for scalar quartic interactions read $(\lambda_- \lambda_+)$ and $(\lambda_+ \lambda_0)$ for the left and right panels in Fig. 1, respectively. This proves that the one-loop corrections to $c_{2,ij}^{(q,7)}$ and $c_{2,ji}^{(q,7)}$ are different and thus the diagonal components of $\mathcal{M}_{21}^u$ and $\mathcal{M}_{21}^d$ assume complex values. One may wonder what happens if we modify the kind of scalar fields found in the internal lines such that we can have the same scalar quartic interaction factors for the left and right panels in Fig. 1. However, in those cases the momentum-space

FIG. 1. One-loop correction to the dimension-seven operator with coefficients $c_{2,ij}^{(u,7)}$. The right panel corresponds to a correction of the dimension-seven operator with coefficients $c_{2,ij}^{(u,7)}$. The left panel corresponds to a correction of the dimension-seven operator with coefficients $c_{2,ji}^{(u,7)}$. The blobs correspond to the dimension-five operators with the coefficient $c_2^{(u,5)}$. The arrow on the internal line is directed from $\Phi^\dagger$ to Φ .

integral for the loop changes because of the different scalar masses for the different kinds of scalars in the internal lines. To prevent the one-loop correction from making $\delta\bar{\theta}$ exceed 10^{-10} , we demand that

$$(\delta\bar{\theta})_{1\text{-loop}} \simeq \frac{\delta\mathcal{M}_{21,\alpha=3\beta=3}^q}{\mathcal{M}_{21,\alpha=3\beta=3}^q} \simeq \frac{b^q c^{(q,5)} \lambda^2}{|y_{q3}|} \frac{|X_1|^2 |X_2|^2}{16\pi^2 m_\Phi^2 M_Q M_P} \lesssim 10^{-10}, \quad (22)$$

where Eq. (22) provides the dominant contribution to the up and down sectors. For simplicity, we do not specify the kind of λ in Eq. (22). Using $|X_1| = 10^R |X_2|$ and Eq. (27), we can further simplify Eq. (22) to obtain

$$(\delta\bar{\theta})_{1\text{-loop}} \simeq 10^{-2R} \frac{\lambda^2}{16\pi^2} \frac{|X_1|^2}{m_\Phi^2} \lesssim 10^{-10}, \quad (23)$$

where m_Φ is the mass of the heaviest scalar particle in the loop. For λ^2 , there are three possibilities: λ_0^2 , $\lambda_0 \lambda_\pm$, or $\lambda_\pm \lambda_\pm$. We find that the first case does not violate the Hermiticity of the one-loop corrections to the block matrix $\mathcal{M}_{21}^u$. For the other two cases, we find that $m_\Phi^2 \simeq |X_1|^2 \lambda_0$ provided at least one of the three internal lines corresponds to a massive scalar mode.⁴ Therefore, we may argue that both nonzero λ_+ and λ_- are responsible for spoiling the Hermiticity of the one-loop corrections to $\bar{\theta}$ and thus are subject to the upper bound on λ obtained above, while constraining λ_0 is not necessary to fulfill $\bar{\theta} \lesssim 10^{-10}$. From Eq. (23), the upper bound on $\lambda_\pm$ is obtained to be $\lambda_\pm \lesssim 10^{-8+2R}$. For the exemplary case with $R = 1$, the constraint becomes $\lambda_\pm \lesssim 10^{-6}$.

Having $\lambda_\pm \lesssim 10^{-8+2R}$, we realize that the Wilson coefficients of the dimension-five operators we discussed in Sec. III A can be constrained. The dimension-five operators with the coefficients $c_2^{(q,5)}$ and $d_2^{(q,5)}$ can induce a radiative correction to λ_- . For example, the diagram shown in Fig. 2 contributes a radiative correction to λ_- of $\sim (c_{2,i}^{(u,5)})^2 / (16\pi^2)$. Thus, the constraint on λ_- transforms into $c_{2,i}^{(u,5)} \lesssim 10^{(-6+2R)/2}$ and $d_2^{(q,5)} \lesssim 10^{(-6+2R)/2}$. For instance, the $R = 1$ case results in $c_{2,i}^{(u,5)} \lesssim 10^{-2}$ and $d_2^{(q,5)} \lesssim 10^{-2}$.

Thus far we have discussed one-loop corrections to dimension-seven operators of $\mathbf{1} \otimes \mathbf{8} \otimes \mathbf{8}$ type only. However, for $\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1}$ -, $\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{1}$ -, and $\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8}$ -type dimension-seven operators, we find that one-loop corrections with four external scalar lines are Hermitian

⁴For $\lambda_0 \gg \lambda_\pm$, the complex scalar Φ_i contains two classes of bosons. One has heavier masses of $|X_i|^2 \lambda_0$ ($i = 1, 2, 3$) and the other has lighter masses of $|X_i|^2 \lambda_\pm$ ($i = 2, 3$). However, we checked that our conclusion does not change.

FIG. 2. The diagram for the one-loop radiative correction to the λ_- quartic interaction of the complex scalar fields defined in Eq. (21). The blobs correspond to the dimension-five operator with the coefficient $c_{2,i}^{(u,5)}$ defined in Eq. (10).

whenever they are proportional to λ_0^2 . Therefore, we conclude that there is no constraint on λ_0 as far as one-loop corrections to dimension-seven operators are concerned.

The upper bounds on $\lambda_\pm$, $c_{2,i}^{(q,5)}$, and $d_{2,i}^{(q,5)}$ ($q = u, d$) now change our understanding of the smallness of $\bar{\theta}$. The smallness of $\bar{\theta}$ turns out to originate from the smallness of $\lambda_\pm$, $c_{2,i}^{(q,5)}$, and $d_{2,i}^{(q,5)}$ ($q = u, d$). So at first glance it seems that what the model achieves is just to convert the form of the smallness. This latter smallness, however, can be considered distinguished from the former in that the symmetry of the model is enhanced to include $SU(3)_f^{(1)} \times SU(3)_f^{(2)} \times SU(3)_f^{(3)}$ in the limit $\lambda_\pm, c_{2,i}^{(q,5)}, d_{2,i}^{(q,5)} \rightarrow 0$. Here the superscript on $SU(3)_f^{(i)}$ specifies to which the complex scalar field Φ_i $SU(3)_f^{(i)}$ applies. This enables us to understand the smallness of $\lambda_\pm$, $c_{2,i}^{(q,5)}$ and $d_{2,i}^{(q,5)}$ ($q = u, d$) natural in the sense of 't Hooft [1]. In sum, the model succeeds in converting the unnatural smallness of $\bar{\theta}$ into other natural smallness.

IV. EFFECTIVE YUKAWA COUPLING

In this section, we study how the model can produce the effective Yukawa coupling in the SM. For this purpose, it turns out that we need constraints on the Wilson coefficients of higher-dimensional operators, as we shall see below.

If we assume that $d^{(q,5)}$ is small enough to satisfy the condition

FIG. 3. Diagram that produces a low-energy effective SM quark-Higgs Yukawa coupling after spontaneous breaking of $SU(3)_f$ and CP . The sum over the Z_2 index i is assumed. The blob corresponds to the higher-dimensional operator in Eq. (10). The cross represents the mass insertion of the heavy fermion.

$$d^{(q,5)} \frac{\Phi_{ai}^\dagger \Phi_{ai}}{M_P} \ll M^Q, \quad (24)$$

where $q = u, d$ and $Q = U, D$, then for the energy scale between the EWSB scale and the heavy fermion mass scale M^Q , the effective SM quark-Higgs Yukawa coupling can be obtained by integrating out the heavy fermions Ψ^u and Ψ^d . The relevant diagram of the UV physics contribution to the SM Yukawa coupling is shown in Fig. 3. With this, the effective Yukawa coupling induced in the low-energy region reads

$$Y_{\alpha\beta}^u \simeq a^u + \sum_{i=1}^3 b^u c_{1,i}^{(u,5)} \frac{\langle \Phi_{ai}^\dagger \rangle \langle \Phi_{\beta i} \rangle}{M^U M_P} \delta_{\alpha\beta} + \sum_{i=1}^3 b^u c_{2,i}^{(u,5)} \frac{\langle \Phi_{ai}^\dagger \rangle \langle \Phi_{\beta i} \rangle}{M^U M_P}, \quad (25)$$

$$Y_{\alpha\beta}^d \simeq a^d + \sum_{i=1}^3 b^d c_{1,i}^{(d,5)} \frac{\langle \Phi_{ai}^\dagger \rangle \langle \Phi_{\beta i} \rangle}{M^D M_P} \delta_{\alpha\beta} + \sum_{i=1}^3 b^d c_{2,i}^{(d,5)} \frac{\langle \Phi_{ai}^\dagger \rangle \langle \Phi_{\beta i} \rangle}{M^D M_P}, \quad (26)$$

where a^q, b^q ($q = u, d$), the heavy fermion masses are defined in Eqs. (6) and (7), and $c^{(q,5)}$ ($q = u, d$) is from Eq. (10). The Hermitian Yukawa coupling in the low-energy region turns out to be one of the features of the model.

Depending on the value of a^q ($q = u, d$), there can be two different situations for the effective SM Yukawas. In the first situation, a^q can be $\mathcal{O}(1)$ so as to be the leading contribution. In this case, with other parameters, a^q needs to be tuned as a free parameter of the model to reproduce the SM quark masses [22]. Since the structure of the SM Yukawa will be dominated by a^q and other contributions, the VEVs of Φ_i cannot explain the hierarchical structure of the SM Yukawas without the parameter tuning. In the second situation, as a free parameter, a^q can be small enough to be negligible in comparison with other

contributions [23]. For example, an accidental symmetry can be introduced that is not respected by the terms in Eq. (3) in order to suppress a^q . Then, the Y^q in Eqs. (25) and (26) are dominated by dimension-five operators and thus the hierarchical structure can be explained by the hierarchy of the VEVs of the complex scalars Φ_i . Since Y^q given in Eqs. (25) and (26) are Hermitian, a^q do not affect the Cabibbo-Kobayashi-Maskawa (CKM) matrix for both cases. In our work, we consider the second scenario.

For $\alpha = 3, \beta = 3$, Y^q is matched to the SM top-Higgs and bottom-Higgs Yukawa couplings. Then, for the energy scale between the EWSB scale and M^Q , we have

$$|y_{q3}| \simeq b^q (c_{1,i=1}^{(q,5)} + c_{2,i=1}^{(q,5)}) \frac{|X_1|^2}{M^Q M_P}, \quad (27)$$

where $q_3 = t, b$ and $Q = U, D$. In particular, for $q_3 = t$ we demand that $|y_t| \simeq \mathcal{O}(1)$. Recall that $|X_1| = 10^R |X_2|$ and $c_{2,i=1}^{(u,5)} \lesssim 10^{(-6+2R)/2}$. If $|b^u| \lesssim |y_t|$ holds at the scale of M_Q ,⁵ then with the constraint that $|X_2| \lesssim 10^{(26-P)/2}$ GeV, the assumption that $c_{1,i=1}^{(u,5)}$ and $c_{2,i=1}^{(u,5)}$ are comparable gives $M^U \lesssim 10^{5-P+3R}$ GeV. As an example, for $P = 0$ and $R = 1$, M^U cannot be greater than 10^8 GeV.⁶ On the other hand, if $|b^u| > |y_t|$ holds at the scale of M_Q , then we see that a larger value of b^u gives rise to a larger value of M^U in order to match Eq. (27). However, notice that there must be an upper bound for b^u at the scale of M_Q to avoid the breaking of perturbativity at the $SU(3)_f$ breaking scale. Then, with Eqs. (24) and (27), we realize that the condition $d^{(u,5)} < c^{(u,5)}$ should be satisfied even in this case.

Now if UV physics allows $c_{1,i}^{(q,5)} \lesssim c_{2,i}^{(q,5)}$ ($q = u, d$), then we may understand the hierarchical structure in the SM Yukawas from the hierarchy of the complex scalar VEVs, i.e., $\langle \Phi_1 \rangle > \langle \Phi_2 \rangle > \langle \Phi_3 \rangle$. Assuming that the conditions $a^u, a^d \ll 1$ and $c_{1,i}^{(q,5)} \lesssim c_{2,i}^{(q,5)}$ ($q = u, d$) can hold in a UV physics sector, the SM quark mass matrices can be approximately described by the 14 free parameters that appear in Eqs. (25) and (26). First, the ratios of the VEVs Φ_2 and Φ_3 in Eq. (1) to $\langle \Phi_1 \rangle = X_1$ multiplied by the ratio of the coefficients produce nine free real parameters, namely,

$$p_{aj} \equiv \sqrt{\frac{c_{2,j}^{(u,5)} \langle \Phi_{aj} \rangle}{c_{2,i=1}^{(u,5)} |X_1|}}, \quad (28)$$

⁵In this case, the mixing between $\bar{t}$ and $\bar{U}_3$ becomes non-negligible and thus the observed $\bar{t}'$ is a mixture of $\bar{t}$ and $\bar{U}_3$. We need more precise calculations for the up-type quark mass matrix than that estimated below.

⁶A search for the heavy quarks with masses in this regime at the LHC or future colliders would be one way to test the model.

where $(\alpha, j) = (2, 2)$ provides one real parameter and $(3, 2)$, $(1, 3)$, $(2, 3)$, and $(3, 3)$ provide another four complex parameters. Second, the ratios between the Wilson coefficients $c_{2,i}^{(d,5)}/c_{2,i}^{(u,5)}$ with $i = 1, 2, 3$ give an additional three free parameters. Last, the ratios b^u/M^U and b^d/M^D give two more parameters. The last two are irrelevant for the CKM quark mixing matrix, but they play the role of scaling factors for each individual quark mass eigenvalue.

As we were curious about the hierarchy of the VEVs of the complex scalars Φ_i , we calculated the four quark mass ratios, three mixing angles, and Jarlskog invariant in the SM. Interestingly, assuming $c_{2,j}^{(u,5)}/c_{2,i=1}^{(u,5)} \sim \mathcal{O}(1)$, we obtained the ratios $|Y_2|/|X_1| \simeq 0.08$ and $|X_2|/|X_1| \simeq 0.6$.

V. CONCLUSIONS

In this paper we presented a model designed to resolve the strong CP problem. The SM particle content is extended by including additional complex scalars Φ and heavy fermions Ψ^u (Ψ^d). Furthermore, the model introduces the horizontal $SU(3)_f$ gauge symmetry and discrete gauge symmetries $Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)}$ into the SM gauge group. The quantum numbers of the particle content of the model can be found in Table. I.

Beginning as a gauged CP -invariant theory, spontaneous CP violation is triggered by the condensation of the complex scalar field Φ_i . Simultaneously, the horizontal $SU(3)_f$ is spontaneously broken around the energy scale $\sim 10^{13} - 10^{14}$ GeV. This is in contrast with other Nelson-Barr-type models where the CP breaking occurs for $\Lambda_{CP} \lesssim 10^8$ GeV by dimension-five operators [26]. The higher breaking scale of our model is better able to avoid the tension with thermal leptogenesis [27, 28]. We found that, provided the scalar sector of the model features a small enough quartic self-interaction at this scale, i.e., $\lambda \lesssim 10^{-6}$, the radiatively induced CP -violating parameter in the QCD sector $\bar{\theta}$ can be small enough to avoid the current

experimental constraint $\bar{\theta} \lesssim 10^{-10}$. We emphasize that in spite of this small magnitude, the model predicts a nonzero $\bar{\theta}$ value. Improved future experimental measurements of the electric dipole moments of the neutron, other particles, or atoms could be used to test the possibility of a nonvanishing $\bar{\theta}$ and hence the model. The upper bound $\lambda \lesssim 10^{-6}$ further constrains the Wilson coefficients of dimension-five operators to be smaller than 10^{-2} . The newly obtained small values of the other parameters in the model (compared to $\bar{\theta}$) turns out to be technically natural, enhancing the symmetry of the model [1].

On the other hand, the quark-Higgs Yukawa coupling structure is explained as a consequence of the model. The sequential breaking (so-called tumbling) of the horizontal $SU(3)_f$ gauge symmetry by the different VEVs of three complex scalars leads to the hierarchical structure of the effective Yukawa coupling in the SM [18]. Within the model, the CKM matrix is determined by the scalar field sector dynamics and the interplay between fermions and scalars communicated by UV physics. Therefore, in this work we found that CP violation in the strong and weak sectors and the hierarchical structure of the Yukawa coupling in the SM originate from the common underlying physics of the breaking of CP and $SU(3)_f$ induced by the scalar field dynamics. An extension that includes the lepton sector will be given elsewhere.

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