

Soft metric fluctuations during inflation

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The conservation of the long wavelength fluctuations of the metric plays a vital role in cosmology as the link between quantum fluctuations during inflation and late time observations. This is a well-known property of the classical evolution equations, but demonstrating that it is robust to quantum correction involves a number of technical arguments. In this paper, we will use effective field theory (EFT) techniques to demonstrate the all orders conservation of the superhorizon scalar and tensor fluctuations of the metric during inflation. We show how to construct an EFT for these soft modes, in analogy with soft de Sitter effective theory. We pay particular attention to how the breaking of time diffeomorphisms by the inflationary background introduces new timescales that alter the structure of the EFT. In this description, the all orders conservation of the metric fluctuations is a direct consequence of symmetries and power counting that cannot be altered by loop corrections. We further show that this holds when the inflaton (or metric fluctuations) is coupled to additional heavy fields, as in quasi-single-field inflation. We match this behavior to several calculations in the ultraviolet (UV) theory and show how the Mellin representation enables a more transparent connection between the UV and the EFT descriptions.

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I. INTRODUCTION

The seeds of structure in our Universe are well described by long-wavelength metric fluctuations produced during inflation [1,2]. The anisotropies in the cosmic microwave background and inhomogeneities in the distribution of matter encode the statistics of a scalar metric (adiabatic) fluctuation produced at much earlier times. Current observations are consistent with the predictions for these scalar fluctuations from many inflationary models [3]. Moreover, a coming generation of surveys will increase the sensitivity to CMB B modes, which encode the signal of gravitational waves (tensor modes) produced during the inflationary epoch [4–7].

The precise relationship between quantum fluctuations during inflation and the statistics of density fluctuations in the later universe is potentially sensitive to quantum corrections [8,9]. It is a remarkable feature of general relativity in accelerating cosmologies that metric fluctuations do not evolve at long distances [10]. Importantly, this ensures a trivial relationship (classically) between the early and late universe in single field inflation. The conservation of the adiabatic mode is also deeply tied to the single field

consistency conditions [11,12], which offer one of the most powerful windows into the dynamics of inflation [13].

Despite its importance, the conservation of the metric fluctuations quantum mechanically rests on surprisingly technical arguments. It is generally appreciated that conservation is tied to the nonlinearly realized symmetries of the long wavelength modes [14–16]. Unfortunately, symmetry alone is insufficient to prove conservation and must be supplemented with a variety of diagrammatic arguments [17–19]. Moreover, due to their complexity, these arguments have not been extended to the tensor modes (although similar arguments are expected to apply).

The central challenge with many approaches to loops in cosmology is that the expected size of a given correction is not easily determined. For example, although the action contains positive powers of the scale factor, $a(t)$, it was shown in [9] that quantum corrections are, at most, logarithmic in $a(t)$. In this case, a detailed argument was used to show that the positive powers are always canceled by inverse powers of the scale factor from the form of the propagators. Although much progress has been made in understanding the nature of loop corrections in de Sitter space and inflation [20–35], the physical simplicity of the final results is not manifest in the mathematical framework for calculating cosmological correlators.

Effective field theory (EFT) [36,37] is a promising strategy for understanding the conservation of the metric fluctuation [38]. The goal of defining an EFT is to make the size of corrections explicit (typically in the action or equations of motion) by choosing the appropriate variables

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and a renormalization scheme for the physical regime of interest. This approach has been successfully applied to long wavelength fluctuations of scalar fields in de Sitter space in soft de Sitter effective theory (SdSET) [39]. In this approach, one defines the degrees of freedom in the regime $k/(aH) \ll 1$ where k is the length of the comoving wave vector and H is the Hubble parameter, so that the fields behave as scaling operators under rescaling of the coordinate distance (dilatations). In this framework, loop correlations take a familiar scaleless form and can be regulated using generalizations of dimensional regularization. The size of perturbative corrections is organized into powers of $k/(aH)$ that are manifest in the action. The interactions of light fields do generate large logs that can be resummed by conventional renormalization group techniques to give the framework of stochastic inflation [40–42] (at leading order) and corrections thereof [43–45].

Our goal in this paper is to write the EFT for soft metric fluctuations during inflation. This presents two nontrivial challenges beyond SdSET. First, the inflationary background breaks the de Sitter isometries, allowing for explicit time dependence in the action and for non-Lorentz invariant interactions [46]. Second, the action of the canonical metric fluctuations, after gauge fixing and imposing constraints, is not manifestly local [11]. While it is desirable to write down the EFT for these degrees of freedom directly, the possibility of nonlocal terms adds some nuance to the usual EFT playbook.

The appeal of this approach is that it will make the conservation of both the scalar and tensor modes of the metric manifest from power counting and symmetries. Ultimately, symmetries allow us to circumvent the most acute challenges of working with the metric fluctuations during inflation. They define a robust set of observables [47] that are subject to nontrivial Ward identities [48–51], without appealing to locality or de Sitter invariance. These symmetries dictate that one of the EFT degrees of freedom is time independent in the free EFT, and that interactions decouple at late times (are formally irrelevant). This holds, not only in single field inflation, but also for couplings of the inflaton or metric to additional fields that redshift away, such as quasi-single-field inflation [52–54]. As a result, one can see directly from the construction of the EFT that conservation of the metric fluctuations survives quantum corrections, much like how Wilsonian Renormalization Group (RG) and EFT clarify the meaning of renormalizability in terms of dimensional analysis [55].

Like SdSET, matching to the UV description requires determining both the initial conditions and the couplings of the effective action. We use the Mellin representation [56–61] of the full one-loop power spectrum to demonstrate how the EFT matching connects to the full microscopic calculation. Mellin space has the useful property of organizing the UV calculation into an infinite sum over power laws, which makes the connection to the EFT transparent. This approach also allows us to directly compare

(and contrast) the single field inflation results with the nontrivial anomalous dimensions associated with heavy fields. Principal series fields are known to have anomalous dimensions [62–66], and arise from nondynamical operators that must be added to the EFT [61,67]. We show that no such operators are needed in single field inflation, thus ensuring conservation follows from power counting.

This paper is organized as follows: In Sec. II, we will introduce the EFT of the soft scalar metric fluctuations and how it is organized by symmetries and renormalization. In Sec. III, we will use power counting of the EFT to demonstrate all orders conservation. We show explicitly how this arises at one-loop in the UV theory by matching. In Sec. IV, we extend the EFT and all orders conservation results to include tensor modes. We conclude in Sec. V. This paper includes three appendixes: In Appendix A, we include details of the derivation of the EFT. Appendix B details the one-loop matching calculation. Finally, Appendix C provides a flat space example of the scheme dependence of one-loop calculations.

II. EFT OF SCALAR MODES

We will start by working out the basic structure of the EFT of the soft scalar mode of the metric during single-field inflation. Like SdSET, much of the structure can be understood starting from the classical description of the metric in the limit of vanishing wavenumber. We will then use this classical intuition to choose an ansatz for the field operators in the soft limit and construct the action of the EFT.

A. Inflationary background

When one thinks of EFT and single-field inflation, it is natural to start from the EFT of inflation [46,68] as our UV starting point. In this description, the homogeneous universe is described by an Friedmann–Robertson–Walker (FRW) metric $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$, where the Hubble parameter $H(t)$ obeys

$$H(t) = \frac{\dot{a}(t)}{a(t)} - \dot{H}(t) \ll H^2(t). \quad (2.1)$$

Here $H(t)$ is some function determined by the unknown UV theory.

During inflation, the background field defines a preferred time slicing where each spacelike surface is defined by constant field, $\phi(\vec{x}, t) = \phi(t)$. Furthermore, the evolution of the field is approximately linear in time, $\phi \propto t$, so that time translations are broken by the background. In the EFT of inflation, one describes the fluctuations around this background by introducing the Goldstone boson π associated with the broken time translations (in the flat space limit). It is useful to define the action in terms of

$$U(\vec{x}, t) = t + \pi(\vec{x}, t), \quad (2.2)$$

where $U(\vec{x}, t)$ transforms as a scalar under diffeomorphisms, but $\pi(\vec{x}, t)$ transforms nonlinearly. This field is coupled to a dynamical metric, now in the Arnowitt–Deser–Misner (ADM) form

$$ds^2 = -N^2 dt^2 + a(t)^2 [e^{2\zeta+2\gamma}]_{ij} (dx^i + N^i dt)(dx^j + N^j dt). \quad (2.3)$$

The most general possible action for π can be constructed by treating U as a scalar field and then expanding in π and the metric components [69]. For calculating correlators of the scalar metric fluctuation, ζ , at horizon crossing, it is advantageous to use the gauge where $\zeta = 0$ and the Goldstone boson equivalence theorem [70,71] ensures we can neglect dynamical gravity. One can then return to the variable ζ , from π via the transformation

$$\zeta = -H\pi + \dots \quad (2.4)$$

This is sufficient to calculate the leading observational signal of a wide range of inflationary models.

However, the long distance behavior of an inflating universe is not well described in terms of π , but instead is more manifest in terms of ζ and γ directly [72,73]. For example, the fact that $\zeta(\vec{k}, t)$ and $\gamma_{ij}(\vec{k}, t)$ are constant when $k \ll aH$ is an essential ingredient used across cosmology when relating the observed maps of the universe to the dynamics during inflation [10]. This observation can be traced to the statement that ζ nonlinearly realizes the group of de Sitter (dS) isometries [15,48]

$$\begin{aligned} \vec{x} &\rightarrow e^\lambda \vec{x} & \zeta(\vec{x}, t) &\rightarrow \zeta(e^\lambda \vec{x}, t) + \lambda, \\ \vec{x} &\rightarrow \vec{y} = \vec{x} + \vec{b}(e^{-2Ht} H^{-2} - \vec{x}^2) + 2(\vec{b} \cdot \vec{x})\vec{x} \\ \zeta(\vec{x}, t) &\rightarrow \zeta(\vec{y}, t) - 2\vec{b} \cdot \vec{x}. \end{aligned} \quad (2.5)$$

These symmetries imply that the inflationary correlators obey a set of Ward identities, known as the single-field consistency conditions. These conditions can be directly implemented at the level of late-time observables.

Our goal in this section is to write down an EFT for ζ directly, holding $\gamma_{ij} = 0$ fixed for now. As a starting point, we will use the action for ζ derived from the top down [74,75], before understanding the same behavior from the bottom up.

B. EFT from the top down

We start with the quadratic action for the full adiabatic mode ζ

$$S_2 = \int dt d^3x a^3(t) \frac{M_{\text{pl}}^2 \dot{H}(t)}{c_s^2(t) H^2(t)} \left(-\dot{\zeta}^2 + \frac{c_s^2(t)}{a^2(t)} \partial_i \zeta \partial^i \zeta \right), \quad (2.6)$$

where we recall we can assume $\dot{H} < 0$ by enforcing the null-energy condition. Our goal here is to write an EFT that isolates the physics of the long wavelength modes, namely those with $c_s(t)k \ll aH(t)$. To understand the behavior in this limit, we can solve the equations of motion taking $k = 0$

$$\frac{\partial}{\partial t} \left(a^3 \frac{\dot{H}(t)}{c_s^2(t) H^2(t)} \dot{\zeta} \right) = 0 \rightarrow \zeta = c_0 + c_1 \int^t dt' \frac{c_s^2(t') H^2(t')}{a^3 \dot{H}(t')}. \quad (2.7)$$

To get intuition, it will occasionally be more useful to work around a specific time $t = t_\star$ (or conformal time $\tau_\star$) and treat the time evolution as a power law

$$\begin{aligned} H(t) &= H(t_\star) \left(\frac{a(t)}{a(t_\star)} \right)^{-\epsilon(t_\star)}, \\ \dot{H}(t) &= -\epsilon(t_\star) \left(\frac{a(t)}{a(t_\star)} \right)^\eta H^2(t) \end{aligned} \quad (2.8)$$

where we defined the slow roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{\epsilon H}, \quad s = \frac{\dot{c}_s}{c_s H}. \quad (2.9)$$

We can then treat ζ as an approximate power law for $t \sim t_\star$

$$\begin{aligned} \zeta(\vec{k} = 0, t) &= c_0 + c_1 \left(\frac{a(t)H(t)}{c_s(t)} \right)^{-\beta}, \\ \beta &= 3 + 2\epsilon + \eta + s + \mathcal{O}(\epsilon^2), \end{aligned} \quad (2.10)$$

where $\mathcal{O}(\epsilon^2)$ includes all terms quadratic in the slow roll parameters η , s , and ϵ . When necessary, these logs that arise from the slow-roll expansion can be resummed [76]. Here we have written our solution in terms of $a(t)H(t)/c_s(t)$ in anticipation of the EFT expansion in $c_s(t)k/[a(t)H(t)] \ll 1$.

The slow-roll expansion is a natural starting point to derive an EFT for the soft modes, in direct analogy with SdSET. Since the time dependence is nearly the same as dS, we factor out the power-law behavior in time

$$\begin{aligned} \zeta(\vec{x}, t) &= \zeta_+(\vec{x}, t) + \left(\frac{a(t)H(t)}{c_s(t)} \right)^{-\beta} \zeta_-(\vec{x}, t) \\ &\equiv \zeta_+(\vec{x}, t) + \Lambda(t)^{-\beta} \zeta_-(\vec{x}, t), \end{aligned} \quad (2.11)$$

where we defined the (comoving) UV cutoff as $\Lambda(t) \equiv a(t)H(t)/c_s(t)$. At zeroth order in slow roll, $\beta = 3$ and $H(t) = H(t_\star)$ and this is the same ansatz as a massless scalar in dS. Using

$$\begin{aligned} \dot{\zeta} &= \dot{\zeta}_+ - \beta H(t)(1 - \epsilon - s)(a(t)H(t))^{-\beta} \zeta_- \\ &\quad + (a(t)H(t))^{-\beta} \dot{\zeta}_-, \end{aligned} \quad (2.12)$$

we can substitute into the quadratic action to find

$$S = \int dt d^3x a(t)^3 \frac{M_{\text{pl}}^2 |\dot{H}(t)|}{c_s^2(t) H^2(t)} \times [\dot{\zeta}_+^2 + 2\beta(1 - \epsilon - s)H(aH)^{-\beta} \dot{\zeta}_+ \dot{\zeta}_- + (aH)^{-2\beta} \dot{\zeta}_-^2 + \dots] \quad (2.13)$$

where we have dropped the terms suppressed by k^2/a^2 or higher. Notice that, by definition,

$$\frac{\partial}{\partial t} \left(a^3 \frac{\dot{H}(t)}{c_s^2(t) H^2(t)} \beta(1 - \epsilon - s) H(aH)^{-\beta} \right) = 0, \quad (2.14)$$

so that the coefficient of the dominant kinetic term is time independent. In fact, this is a completely general consequence of our ansatz and not only a statement at low orders in the slow-roll expansion.

To see how the EFT will behave even away from the slow-roll limit, we define the coefficient of the kinetic term $\kappa(t)$ and the nontrivial solution $\rho(t)$ as

$$\kappa(t) \equiv a(t)^3 \frac{M_{\text{pl}}^2 |\dot{H}(t)|}{c_s^2(t) H^2(t)}, \quad \frac{\partial}{\partial t} (\kappa(t) \dot{\rho}(t)) = 0. \quad (2.15)$$

We will take the ansatz

$$\zeta(\vec{x}, t) = \zeta_+(\vec{x}, t) + \rho(t) \zeta_-(\vec{x}, t) \quad (2.16)$$

which also implies

$$\dot{\zeta} = \dot{\zeta}_+ + \dot{\rho} \zeta_- + \rho \dot{\zeta}_-, \quad \partial_i \zeta = \partial_i \zeta_+ + \rho \partial_i \zeta_-. \quad (2.17)$$

From the top down, we may simply substitute this ansatz into the quadratic action,

$$S = \int dt d^3x \kappa(t) \left[\dot{\zeta}^2 - \frac{c_s^2}{a^2} (\partial_i \zeta)^2 \right], \quad (2.18)$$

to determine the quadratic action of the EFT. This is given by

$$\begin{aligned} S &= \int dt d^3x \kappa(t) \left[(\dot{\zeta}_+ + \dot{\rho} \zeta_- + \rho \dot{\zeta}_-)^2 - \frac{c_s^2}{a^2} (\partial_i \zeta_+ + \rho \partial_i \zeta_-)^2 \right] \\ &= \int dt d^3x \kappa(t) \left[2\dot{\rho} \dot{\zeta}_+ \zeta_- + 2\rho \dot{\zeta}_+ \dot{\zeta}_- + \dot{\zeta}_+^2 + \rho^2 \dot{\zeta}_-^2 - 2\rho \frac{c_s^2}{a^2} \partial_i \zeta_+ \partial_i \zeta_- \right], \end{aligned} \quad (2.19)$$

where we eliminated the other terms by the same integration by parts and field redefinitions as in SdSET, but now

with the equation of motion $\dot{\kappa} \dot{\rho} + \kappa \ddot{\rho} = 0$. We then find that the leading kinetic term is $2\kappa \dot{\rho} \dot{\zeta}_+ \zeta_-$, where $\kappa(t) \dot{\rho}$ is a constant (time independent) by construction. Therefore, the leading order equations of motion become

$$\partial_t (\kappa(t) \dot{\rho} \zeta_+) = 0. \quad (2.20)$$

However, we also have second order kinetic terms. We can remove $\dot{\zeta}_+^2$ by a field redefinition of ζ_- , and we can neglect $\dot{\zeta}_-^2$ because it is suppressed by ρ^2 and can be ignored. However, when computing the $\mathcal{O}(k^2)$ correction, we cannot drop the $\dot{\zeta}_+ \dot{\zeta}_-$ term. Instead, we must treat it as a perturbation so as not to introduce additional (unphysical) degrees of freedom. The resulting interaction Hamiltonian becomes

$$H_{\text{int}} = \int d^3x \left[-2\rho(t) \kappa(t) \dot{\zeta}_+ \dot{\zeta}_- + 2\kappa \rho \frac{c_s^2}{a^2} \partial_i \zeta_+ \partial_i \zeta_- \right] \quad (2.21)$$

so that the resulting evolution of ζ_+ at subleading order is given by

$$\begin{aligned} \dot{\zeta}_+^{(2)} &= i \int dt d^3x \left[-2\rho(t) \kappa(t) \dot{\zeta}_+ [\dot{\zeta}_-, \zeta_+] + 2\kappa \rho \frac{c_s^2}{a^2} \partial_i \zeta_+ [\partial_i \zeta_-, \zeta_+] \right] \\ &= i \int dt \frac{1}{2\kappa \dot{\rho}} \left[-2\partial_t (\rho \kappa \dot{\zeta}_+) + 2\kappa \rho \frac{c_s^2}{a^2} \partial^2 \zeta_+ \right], \end{aligned} \quad (2.22)$$

where $\zeta_{\pm}$ on the rhs is just the interaction picture operator. Specializing to the case of slow-roll inflation, at leading order in the slow roll parameters, this expression gives

$$\dot{\zeta}_+ = \frac{i}{3} \left(-\frac{\ddot{\zeta}_+}{H} + \frac{c_s^2}{a^2 H} \partial^2 \zeta_+ \right). \quad (2.23)$$

We emphasize here that $\dot{\zeta}_+ \dot{\zeta}_-$ gives a correction at order k^2 that is needed to get the correct result. We can understand this result as the statement that we should use the ζ_+ equations of motion to eliminate higher time derivatives while being careful of higher time derivatives of ζ_- . These contributions are always subleading and can usually be addressed by integrating by parts first.

The canonical momentum defined with respect to ζ_+ is given by the action to be

$$\Pi = 2\kappa \dot{\rho} \zeta_- + \mathcal{O}\left(\frac{k^2}{a^2} \zeta_+\right) \quad (2.24)$$

and therefore as $k \rightarrow 0$, we have

$$\begin{aligned} [\zeta_+(\vec{x}, t), \zeta_-(\vec{x}', t)] &= \frac{i}{2\kappa \dot{\rho}} \delta(\vec{x} - \vec{x}') \rightarrow [\zeta_+(\vec{k}, t), \zeta_-(\vec{k}', t)] \\ &= \frac{i}{2\kappa \dot{\rho}} (2\pi)^3 \delta(\vec{k} + \vec{k}'). \end{aligned} \quad (2.25)$$

As with SdSET, the commutator being $\mathcal{O}(k^0)$ fixes the relation between the scaling with k of ζ_+ and ζ_- .

The initial conditions for ζ_+ at horizon crossing are not determined by the EFT itself but are fixed by matching. In the free theory, since $\zeta \rightarrow \zeta_+$ as $t \rightarrow \infty$, we can match the power spectrum to find

$$\begin{aligned} \langle \zeta_+(\vec{k}) \zeta_+(\vec{k}') \rangle' &= \frac{H(t_\star)^4}{4M_{\text{pl}}^2 \dot{H}(t_\star) c_s(t_\star)} \frac{1}{k^3} \left(\frac{k_\star}{k} \right)^{2\epsilon+\eta+s} \\ &\equiv \frac{\Delta_\zeta^2}{k^3} \left(\frac{k}{k_\star} \right)^{n_s-1}. \end{aligned} \quad (2.26)$$

The tilt of the spectrum, $n_s - 1 = -2\epsilon - \eta - s$, is directly related to $3 - \beta = n_s - 1$. This is not a coincidence, and can be understood as follows: in the $k/\Lambda(t) \rightarrow 0$ limit, we might anticipate that the mode functions take the form

$$\begin{aligned} \bar{\zeta} &\approx \frac{\Delta_\zeta}{k^{(3-(n_s-1))/2}} \left((1 + \mathcal{O}(k^2/\Lambda^2)) \right. \\ &\quad \left. + i \left(\frac{k}{\Lambda(t)} \right)^\beta (C + \mathcal{O}(k^2/\Lambda^2)) \right). \end{aligned} \quad (2.27)$$

This ansatz is based on the assumption that the corrections to the leading scaling behavior can be expressed as an expansion in $k/\Lambda(t)$. With this assumption, if we write the ζ operator as $\hat{\zeta} = \bar{\zeta} \hat{a}^\dagger + \text{H.c.}$, then the commutator must be

$$[\hat{\zeta}, \hat{\zeta}] \propto i k^{-3+(n_s-1)+\beta}. \quad (2.28)$$

However, since the commutator must vanish at spacelike separation, we know that this must be a contact term (scales as k^0) and therefore

$$-3 + (n_s - 1) + \beta = 0 \rightarrow (n_s - 1) = 3 - \beta. \quad (2.29)$$

In this precise sense, the organization of corrections into powers of $k/\Lambda(t)$ is consistent with the scaling of ζ_+ and ζ_- , and is matched by the result of an explicit calculation. Furthermore, the ansatz in Eq. (2.27) can be justified more rigorously from approximate symmetries of the UV description.

C. Bottom-up EFT

The goal of writing an EFT for the soft modes during inflation is to make transparent which interactions in the theory alter the long wavelength behavior. As with any EFT, we want to write down the most general action consistent with the symmetries. From there, we expect that the relative importance of different terms is fixed by power counting and renormalization. The power counting parameter is the (comoving) soft momentum k compared to the scale of horizon crossing

$$\Lambda(t) = \frac{a(t)H(t)}{c_s(t)} \rightarrow \frac{k}{\Lambda(t)} \ll 1. \quad (2.30)$$

Like any EFT, we will use the UV scale $\Lambda(t)$ to make the action dimensionless and, as a result, make power counting of the terms in the action explicit.

The most important symmetry to single field inflation is the dilation symmetry

$$\vec{x} \rightarrow e^\lambda \vec{x} \quad \zeta(\vec{x}, t) \rightarrow \zeta(e^\lambda x, t) + \lambda. \quad (2.31)$$

If we split ζ into the components $\zeta_\pm$, then we see that ζ_+ transforms nontrivially

$$\zeta_+(\vec{x}, t) \rightarrow \zeta_+(e^\lambda x, t) + \lambda, \quad \zeta_-(\vec{x}, t) \rightarrow \zeta_-(e^\lambda x, t). \quad (2.32)$$

Imposing this symmetry, ζ_+ must appear with derivatives or in powers of e^{ζ_+} that are fixed by scaling. For example, our leading action might contain terms of the form

$$\begin{aligned} S &= \int dt d^3x e^{3\zeta_+} \left(\kappa \dot{\rho} \dot{\zeta}_+ \zeta_- - \kappa(t) \rho(t) \Lambda(t)^{-2} e^{-2\zeta_+} \partial_i \zeta_+ \partial^i \zeta_- \right. \\ &\quad \left. + \sum_n c_n(t) \rho(t) (\Lambda(t)^{-2} e^{-2\zeta_+} \partial^2 \zeta_+)^n \zeta_- + \dots \right). \end{aligned} \quad (2.33)$$

We have not included time derivatives of ζ_+ , given that the leading equations of motion allow us to relate $\dot{\zeta}_+ = a^{-2} \partial^2 \zeta_+ + \dots$. This substitution clearly holds in perturbation theory, but can be made precise by a field redefinition.

Without any additional information, there is no particular reason to keep these terms and not the many other possible terms. For applications to a wide variety of inflationary models, we can work within the slow-roll expansion, so that $\rho \approx (a(t)H(t))^{-3}$ and the time dependence of the couplings is suppressed, e.g., $\dot{c}_n(t)/c_n(t) \ll 1$.

When the slow roll parameters are small, we can power count the interactions treating ζ_- as an operator with dimension $\beta \approx 3$. In this regime, power counting is effectively identical to SdSET with the added constraint of the nonlinearly realized symmetries. To understand the scaling, we recall that if we rescale $x \rightarrow \lambda^{-1}x$, $\vec{k} \rightarrow \lambda \vec{k}$ and $\Lambda(t) \rightarrow \lambda \Lambda(t)$, the physical wavelengths are all held fixed in units of the horizon radius. Therefore, if we count powers of k , we can restore units with powers of $\Lambda(t)$ up to the (small) time-dependence of the couplings. Defining the dimension of ζ_+ and ζ_- to be $\Delta_+ = (n_s - 1)/2$ and $\Delta_- = (n_s - 1)/2 + \beta = 3 - (n_s - 1)/2$, a given interaction term can be written

$$S_{\text{int}} = \int d^3x dt c_n(t) (\Lambda)^{3-n(\Delta_++1)-m\Delta_-} \mathcal{D}(e_+^\zeta \partial \zeta_+)^n \zeta_-^m \quad (2.34)$$

Here the power of 3 comes from power counting the measure d^3x as k^{-3} and dt as k^0 . Like SdSET, we can remove any term with $m = 0$ with a field redefinition of ζ_- . This can also be understood in perturbation theory since $[\zeta_+, \zeta_-] = 0$, we require at least one power of ζ_- to generate a correction to a ζ_+ cosmological correlator. As a result, the lowest dimension (local) operators will be

$$S_{\text{int}} = \int d^3x dt c_n(t) (\Lambda)^{-n-(n-1)(n_s-1)/2} (e^{-\zeta_+} \partial \zeta_+)^n \zeta_- . \quad (2.35)$$

The contribution of this interaction is proportional to $c_n(t) (k/\Lambda(t))^{-n-(n-1)(n_s-1)/2}$ and is therefore suppressed. In the language of RG flow, these interactions are all irrelevant by power counting and decouple at late times.

It is worth noting, at this point, that the development of this EFT could equally apply to nonattractor models, such as ultra-slow-roll inflation [77–79]. The key difference in many such examples is that the role of the decaying and growing modes is switched [80], so that ζ_- grows with time, making it the dominant contribution to the power spectrum. This gives rise to qualitatively different behavior because it is always ζ_+ that transforms nonlinearly. In addition, an effectively unstable mode will have a negative dimension, which substantially complicates the discussion of power counting. For these reasons, the nonattractor models require special care and are beyond the scope of this work.

D. Nonlocal terms

Unfortunately, the assumption that our action is local is not justified in the case of metric fluctuations. As a result, we must ensure that there are no nonlocal terms in the action that alter the power counting conclusions. Concretely, when writing the metric in the ADM form,

$$ds^2 = -N^2 dt^2 + a(t)^2 \delta_{ij} e^{2\zeta} (dx^i + N^i dt) (dx^j + N^j dt), \quad (2.36)$$

the action is only manifestly local in terms of N , N^i , and ζ . However, N and N^i are nondynamical and, at linear order in ζ , they obey the constraints [74]

$$\delta N = -\epsilon \zeta, \quad N^i = -\frac{\partial^i \zeta}{H} + \frac{\epsilon}{c_s^2} \frac{\partial^i}{\partial^2} \dot{\zeta}. \quad (2.37)$$

As a result, from local terms in the action in terms of N , N^i and ζ , we will generate nonlocal terms for the ζ action alone,

$$\mathcal{L} \supset \partial_j N^j N^i \partial_i \zeta \rightarrow \frac{\epsilon^2}{c_s^4} \dot{\zeta} \left(\frac{\partial_i}{\partial^2} \dot{\zeta} \right) \partial^i \zeta. \quad (2.38)$$

Using our ansatz, this would generate an interaction in the EFT

$$\mathcal{L}_{\text{int}} \supset c_3(t) \zeta_- \left(\frac{\partial_i}{\partial^2} \dot{\zeta}_+ \right) \partial^i \zeta_+. \quad (2.39)$$

This term is nonlocal at face value. However, we should recognize that the quadratic equation for motion for ζ_+ is

$$\dot{\zeta}_+ = c_s^2(t) \frac{k^2}{a^2(t)} \zeta_+ + \mathcal{O}\left(\frac{k^2}{a^2}\right). \quad (2.40)$$

From experience with other EFTs, this should suggest that we can perform a field redefinition that effectively replaces $\dot{\zeta}_+$ with its solution to the equation of motion. Concretely, if we have

$$S \supset \int d^3x dt \zeta_- \left(\dot{\zeta}_+ + c_s^2(t) \frac{\partial^2}{a^2} \zeta_+ \right) + c_3(t) \zeta_- \left(\frac{\partial_i}{\partial^2} \dot{\zeta}_+ \right) \partial^i \zeta_+, \quad (2.41)$$

then, under the change of variables

$$\zeta_- \rightarrow \zeta_- + c_3(t) \frac{\partial_i}{\partial^2} (\zeta_- \partial^i \zeta), \quad (2.42)$$

we get

$$S \rightarrow \int d^3x dt \left(\zeta_- \left(\dot{\zeta}_+ + c_s^2(t) \frac{\partial^2}{a^2} \zeta_+ \right) + c_3(t) \zeta_- \left(c_s^2(t) \frac{\partial_i}{a^2(t)} \dot{\zeta}_+ \right) \partial^i \zeta_+ + \frac{\partial_i}{\partial^2} (c_3(t) \zeta_- \dot{\zeta}_+ \partial^i \zeta_+) \right). \quad (2.43)$$

While it is not obvious that the last term can be treated as a total derivative, the sum over the individual terms vanishes by momentum conservation (for nonzero internal momenta).

The more general question that we must answer in an EFT is what nonlocal terms are allowed by symmetries and physical principles. While it may be clear $\dot{\zeta}$ is allowed by the shift symmetry $\zeta \rightarrow \zeta + \lambda$, it is less clear that $\nabla^{-2} \dot{\zeta}$ would be. To address this question, we can define the charge under the dilation current using the conjugate momentum

$$\Pi = \zeta_- \rightarrow \mathcal{Q} = \int d^3x \delta \zeta \Pi. \quad (2.44)$$

For this symmetry to leave the action invariant, we require $[\mathcal{Q}, S] = \delta S = 0$. We write the action in terms of individual invariant operators so that

$$S \supset \int d^4x \sqrt{g} \sum_n \mathcal{O}_n(x) \rightarrow \left[\mathcal{Q}, \int d^4x \sqrt{g} \sum_n \mathcal{O}_n(x) \right] = 0. \quad (2.45)$$

Any potentially marginal operator, $\mathcal{O}_m$, will take that form $\mathcal{O}_m = \zeta_- F(\zeta_+, \partial_t, \vec{\partial})$ where $F(\zeta_+, \partial)$ may be nonlocal in $\vec{\partial}$. Now we can Fourier transform the fields so that

$$\int d^4x \sqrt{g} \mathcal{O}_m \rightarrow c_m(t) \zeta_-(\vec{p}_0) \left(\prod_{i=1}^N \int d^3p_i \zeta_+(\vec{p}_i) \right) \times \tilde{F}(\{p_i\}) (2\pi)^3 \delta\left(\sum_{n=0}^n \vec{p}_n\right). \quad (2.46)$$

Since the shift symmetry arises from $\mathcal{Q} = \gamma \zeta_-(\vec{q} = 0) + \mathcal{O}(\zeta_+ \zeta_-)$, a single commutator will set one of the momenta of $\zeta_+(\vec{p}_i)$ to zero,

$$[\mathcal{Q}, \zeta_+(\vec{p})] \rightarrow \gamma i (2\pi)^3 \delta(\vec{p}) + \mathcal{O}(\zeta_+). \quad (2.47)$$

Therefore, after N commutators we have

$$[\mathcal{Q}, [\mathcal{Q}, \dots [\mathcal{Q}, S], \dots],] = 0 \supset \zeta_-(\vec{p}_0 = 0) F(\{\vec{p}_i\} = 0) + \mathcal{O}(\zeta_- \zeta_+), \quad (2.48)$$

where $\{p_i\} = 0$ means all the individual momenta vanish. In the perturbative regime, the first term cannot be canceled by higher orders in ζ_+ and therefore must vanish. Furthermore, by defining the charges at finite volume and taking the limit $V \rightarrow \infty$, we require that

$$\lim_{\{\vec{p}_i\} \rightarrow 0} F(\{\vec{p}_i\}) = 0, \quad (2.49)$$

for any order of limits. This is equivalent to the requirement that there are more powers of $\vec{q}$ in the numerator than in the denominator. This is sufficient to ensure that any such operator is irrelevant by power counting.

For a concrete example, let us take the nonlocal term in Eq. (2.38) that arises from the leading ADM constraints. We should be able to understand the scaling dimension of the operator without any matching, based on the combination of $\{\zeta_+, \zeta_-, \partial\}$ that are present, up to slow roll factors. Using the Equations of motion (EOM) $\dot{\zeta}_+ = a^{-2} \partial^2 \zeta_+$, we get

$$\dot{\zeta} \left(\frac{\partial_i}{\partial^2} \dot{\zeta} \right) \partial^i \zeta \supset \partial^2 \zeta_+ \partial_i \partial^{-2} \zeta_- \partial_i \zeta_+. \quad (2.50)$$

Up to slow roll factors, the dimension of the operator should be -2 , and will hence be irrelevant. Still, we want to see how this works from our symmetry based argument. Let us therefore define the operator in the action

$$\begin{aligned} \mathcal{O}(x) &= c_1 \partial^2 \zeta_+(x) \partial_i \partial^{-2} \zeta_-(x) \partial_i \zeta_+(x) \\ \rightarrow \mathcal{O}(k) &= c_1 \int \frac{d^3 q_1}{(2\pi)^3} \frac{d^3 q_2}{(2\pi)^3} \frac{q_1^2 q_{2,i} (k - q_1 - q_2)_i}{(k - q_1 - q_2)^2} \\ &\quad \times \zeta_+(q_1) \zeta_+(q_2) \zeta_-(k - q_1 - q_2). \end{aligned} \quad (2.51)$$

Now, let us act with the charge operator on $\mathcal{O}(k)$ defined at $k = 0$ (any choice of momentum is allowed). We get

$$[\mathcal{Q}, \mathcal{O}(k=0)] = c_2 \lim_{q_2 \rightarrow 0} \frac{d^3 q_1}{(2\pi)^3} \frac{q_1^2 q_{2,i} (k - q_1 - q_2)_i}{(k - q_1 - q_2)^2} \times \zeta_+(q_1) \zeta_-(k - q_1 - q_2). \quad (2.52)$$

With another commutator with the charge operator, we get,

$$[\mathcal{Q}, [\mathcal{Q}, \mathcal{O}(k=0)]] = c_3 \lim_{q_1, q_2 \rightarrow 0} \frac{q_1^2 q_{2,i} (k - q_1 - q_2)_i}{(k - q_1 - q_2)^2} \times \zeta_-(k - q_1 - q_2). \quad (2.53)$$

Now, on taking this limit, we see that we smoothly get 0, since we have more factors of momenta in the numerator than the denominator. In general, this condition will be satisfied if and only if we have more factors of momenta in the numerator than in the denominator. Since the physical momenta is q/a , this means we must have more factors of a in the denominator than in the numerator, up to slow roll corrections, and hence the nonlocal terms are irrelevant.

E. Renormalization

As with SdSET and other perturbative calculations in cosmology, dynamical renormalization plays a nontrivial role in organizing the perturbative calculation and resumming large logarithmic corrections. In pure dS (SdSET), every factor of k must appear in the combination $k/[aH]$ and this procedure is straightforward. Specifically, we regulate divergences by analytic continuation of the dimensions [43] so that we encounter divergences of the form

$$\langle \mathcal{O} \dots \rangle \supset \langle \mathcal{O} \dots \rangle_0 \times [aH]^{-2\alpha} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{(p^2 + K^2)^{3/2-\alpha}} \quad (2.54)$$

$$\rightarrow \langle \mathcal{O}' \dots \rangle \left(-\frac{1}{2\pi^2} \left(\frac{1}{2\alpha} + \log \frac{K}{[aH]} - \log 2 \right) \right), \quad (2.55)$$

where K is an IR regulator typically given by the (soft) external momenta. We then define the renormalized operator by introducing a reference time $t_\star$ and scale $K_\star$ so that

$$\mathcal{O} = Z_{\mathcal{O}} \mathcal{O}_R, \quad \text{with} \quad Z_{\mathcal{O}} - 1 \propto -\frac{1}{2\pi^2} \left(\frac{1}{2\alpha} + \log \frac{K_\star}{[aH]_\star} \right) \quad (2.56)$$

where $[aH]_\star = a(t_\star)H$. As a result, the correction to the renormalized operator is under control, so that

$$\langle \mathcal{O}_R \dots \rangle \propto \langle \mathcal{O}_R \dots \rangle_0 \times \left(\frac{1}{2\pi^2} \log \frac{[aH]_\star}{[aH]} + \log \frac{K}{K_\star} \right). \quad (2.57)$$

Since the scale $t_\star$ is unphysical, one can then resum these logs by the usual RG treatment by demanding that the bare correlators are independent of $t_\star$.

Unfortunately, the scheme defined here does not work for metric fluctuations directly (although it can be used for additional field during inflation). Diffeomorphism invariance does not allow us to change the time dependence or scaling dimensions of ζ_+ and ζ_- independently of the background. As a result, we must modify the above treatment. Fortunately, for many practical calculations, the tilt of the spectrum provides the natural regulator we need for an RG treatment. This can be easily demonstrated in the case of mixing of composite operators, where by Wick contraction we have [39]

$$\langle \zeta_+^n \dots \rangle \supset \langle \zeta_+^{n-2} \dots \rangle \times \binom{n}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{H(t_\star)^4}{4M_{\text{pl}}^2 \dot{H}(t_\star) c_s(t_\star)} \frac{1}{k^3} \times \left(\frac{k_\star}{k} \right)^{2\epsilon + \eta + s}, \quad (2.58)$$

$$= \langle \zeta_+^{n-2} \dots \rangle \binom{n}{2} \frac{H(t_\star)^4}{8\pi^2 M_{\text{pl}}^2 \dot{H}(t_\star) c_s(t_\star)} \times \left(-\frac{1}{n_s - 1} - \gamma_E + \log \frac{K}{k_\star} + \dots \right) \quad (2.59)$$

where K is an IR regulator. Suppose that $k_\star \sim K$ so that the log is already small and does not need to be renormalized. We still need to remove the $n_s - 1$ pole, so we define the renormalized (composite) operator

$$[\zeta_+]_R = [\zeta_+] + [\zeta_+^{n-2}] \binom{n}{2} \frac{H(t_\star)^4}{8\pi^2 M_{\text{pl}}^2 \dot{H}(t_\star) c_s(t_\star)} \frac{1}{n_s - 1}. \quad (2.60)$$

Now we recall that $t_\star$ is an unphysical scale, and therefore the original correlators should not depend on it. Therefore, the renormalized operator obeys

$$\frac{d}{dt_\star} [\zeta_+]_R = -[\zeta_+^{n-2}]_R \binom{n}{2} \frac{H(t_\star)^4}{8\pi^2 M_{\text{pl}}^2 \dot{H}(t_\star) c_s(t_\star)} \quad (2.61)$$

where we see that derivatives with respect to $t_\star$ cancel the $1/(n_s - 1)$ to give the same result as we would for a massless scalar.

The possible mixing of composite operators of ζ is constrained by self-consistency with the nonlinearly realized symmetries of ζ [44]. In addition, all these equations can be recast, using integration by parts, as a single RG equation for the probability distribution of ζ_+ . The result is that

$$\frac{\partial}{\partial \tau} P(\zeta_+, \tau) = \sum_{n \geq 2} (-1)^n \frac{\gamma_n}{n!} \frac{\partial^n}{\partial \zeta_+^n} P(\zeta_+, \tau), \quad (2.62)$$

where $\tau = Ht$ and we used the equations of motion to relate $t_\star$ and t , as is familiar from the Callan–Symanzik equation. Intuitively, the nonlinear dilation, $\zeta_+ \rightarrow \zeta_+ + \lambda$, only allows ζ_+ to appear with derivatives. The coefficients γ_n are determined by the mixing of $[\zeta_+^n] \rightarrow 1$.

This equation is important because it allows us to determine the allowed renormalization group evolution of $\exp(q\zeta_+)$, which appears throughout our action. Specifically, we can use integration by parts to write

$$\frac{d}{d\tau} \langle \exp(q\zeta_+) \rangle = \int d\zeta_+ \exp(q\zeta_+) \frac{\partial}{\partial \tau} P(\zeta_+, \tau) \quad (2.63)$$

$$= \int d\zeta_+ \exp(q\zeta_+) \sum_{n \geq 2} (-1)^n \frac{\gamma_n}{n!} \frac{\partial^n}{\partial \zeta_+^n} P(\zeta_+, \tau) \quad (2.64)$$

$$= \langle \exp(q\zeta_+) \rangle \sum_{n \geq 2} \frac{q^n \gamma_n}{n!}. \quad (2.65)$$

We see that $\exp(q\zeta_+)$ does not mix with other operators and therefore can be renormalized by a single counterterm

$$[\exp(q\zeta_+)]_R = Z_q [\exp(q\zeta_+)]. \quad (2.66)$$

This result can also be proved directly at the operator level, but is more straightforward and intuitive in terms of the stochastic description. The result is important because it means that all the IR divergences associated with contractions of ζ_+ that arise from expanding $\exp(q\zeta_+)$ must recombine into an exponential. Moreover, when the theory is under perturbative control, the coefficients $\gamma_n \propto \Delta_\zeta^{n/2} \ll 1$ and thus do not meaningfully alter the power counting of the EFT.

The special behavior of $e^{q\zeta_+}$ under renormalization has a nice physical interpretation in terms of the operator for the volume of the universe at the end of inflation. Specifically, it is natural to define a family of (renormalized) volume operators as

$$\hat{V}^n = L^3 a^{3n}(t) e^{3n\zeta_+} \rightarrow [\hat{V}^n]_R = L^3 a^{3n}(t) [\exp(3n\zeta_+)]_R, \quad (2.67)$$

where L is some reference comoving scale (often taken to be the size of the initial inflationary patch). These operators play an important role in characterizing the onset of eternal inflation. Specifically, $\hat{V}$ was defined in [81] as an order parameter for the phase transition, while statistics of $[\hat{V}^n]_R$ characterizes the full probability distribution [44, 82–84]. The appearance of these exponential operators in the volume is required since a constant value of $\zeta = \lambda$ is equivalent to a change of the scale factor $a(t) \rightarrow e^\lambda a(t)$.

III. ALL-ORDERS CONSERVATION

The conservation of ζ_+ at the classical level, even nonlinearly, is quite physically intuitive [10]. In any local region, the long-wavelength mode is indistinguishable from an unperturbed FRW background, as it can be absorbed into the definition of the scale factor or physical distances.¹ This physical requirement would be at odds with a time evolving ζ and therefore any source of time evolution for ζ would have to come from something other than the long wavelength behavior of ζ itself. The need of an all-orders (quantum) proof of the conservation arises because the coupling of quantum (short-wavelength) fluctuations to the long modes could be the source of such time evolution. These short modes complicate this simple physical argument. As a result, proofs of the absence of time evolution at a quantum level have only been achieved through complicated diagrammatic arguments [18,19]. Our goal in this section is to explain how the conservation of ζ survives these quantum corrections in a way that is simple and intuitive, yet is also manifest in (some) methods for performing the loop calculations.

To start, we will use the results from the EFT description to show that power counting and symmetries ensure conservation of the superhorizon ζ_+ operator, even when coupled to additional massive degrees of freedom. We will then show how explicit loop calculations in Mellin variables realize this power counting behavior in the full UV description.

A. Conservation from power counting

In the previous section, we showed that within the EFT of soft ζ in the slow-roll regime, the action can be written in the form

$$S_{\text{EFT}} = \int d^4x e^{3\zeta_+} (\kappa \dot{\rho} \zeta_+ \zeta_- - \kappa(t) \rho(t) \Lambda(t)^{-2} e^{-2\zeta_+} \partial_i \zeta_+ \partial^i \zeta_- + c_n(t_\star) (\Lambda(t))^{-n+\gamma_n(t_\star)} (e^{-\zeta_+} \partial \zeta_+)^n \zeta_- + \dots), \quad (3.1)$$

$$+ c_n(t_\star) (\Lambda(t))^{-n+\gamma_n(t_\star)} (e^{-\zeta_+} \partial \zeta_+)^n \zeta_- + \dots), \quad (3.2)$$

where $\gamma_n(t_\star) \ll 1$ is the anomalous dimension, which includes renormalization of $c_n(t_\star)$ from $\exp(q\zeta_+)$ and slow-roll suppressed terms. The ... includes terms with more derivatives per operator, nonlocal terms, and higher powers of ζ_- . As we have demonstrated, all such terms are suppressed by at least one power of $\Lambda(t)$ and thus are formally irrelevant.

The irrelevance of interactions of at long wavelengths is not limited to single field inflation, but persists in couplings to additional fields, provided that their momentum space

correlators all decay as a positive power² of $k/(a(t)H(t))$. Specifically, we will suppose there is an operator $\mathcal{O}_\chi$ associated with some additional degrees of freedom χ that obeys

$$\langle \mathcal{O}_\chi(\vec{k}_1, t) \dots \mathcal{O}_\chi(\vec{k}_n, t) \rangle \propto (a(t)H(t))^{-n\Delta_\chi} \quad (3.3)$$

when $k_i \ll aH$ for all $i = 1, \dots, n$. In this case, we can treat $\Delta_\chi = \mathcal{O}(1) > 0$ as the effective dimension of the operator and rescale $\mathcal{O}_\chi = (a(t)H(t))^{-n\Delta_\chi} \tilde{\mathcal{O}}_\chi$. We can couple this to ζ_+ and ζ_- , for example, via

$$S_{\text{int}} \supset \int d^3x dt (e^{3\zeta} c_\chi(t) (a(t)H(t))^{3-\beta-\Delta_\chi} \zeta_- \tilde{\mathcal{O}}_\chi + c'_\chi(t) e^{\zeta_+} (a(t)H(t))^{1-\Delta_\chi} \partial^2 \zeta_+ \tilde{\mathcal{O}}_\chi). \quad (3.4)$$

The first term is irrelevant by power counting while the second appears to be relevant when $\Delta_\chi < 1$ [which can still be consistent with $\Delta_\chi = \mathcal{O}(1)$]. However, the second term is redundant, as we encountered for other operators that do not include factors of ζ_- . Specifically, we use the field redefinition

$$\zeta_- \rightarrow \zeta_- + \kappa^{-1} \int^t dt' c'_\chi(t') e^{-3\zeta_+} (a(t')H(t'))^{1-\Delta_\chi} \partial^2 (e^{\zeta_+} \tilde{\mathcal{O}}_\chi). \quad (3.5)$$

After using integration by parts several times, we see that this cancels $c'_\chi(t)$, up to higher order in k (note that $\dot{\mathcal{O}}_\chi = 0$ at leading order, by definition).

In matching to the UV theory, it is possible to find contributions from contact operators [61]. These arise from the need to match contact terms generated by loops in the UV theory. They appear much like a generic coupling to an additional sector

$$S_{\text{int}} \supset \int d^3x dt (e^{3\zeta} c_\Delta(t) (a(t)H(t))^{3-\beta-\Delta_\Delta} \zeta_- \tilde{\mathcal{O}}_\Delta + c'_\Delta(t) e^{\zeta_+} (a(t)H(t))^{\Delta-2} \partial^2 \zeta_+ \tilde{\mathcal{O}}'_{3-\Delta}). \quad (3.6)$$

The important feature of these contact operators is that their power spectra vanish, but

$$\langle \tilde{\mathcal{O}}_\Delta(\vec{k}) \tilde{\mathcal{O}}'_{3-\Delta}(\vec{k}') \rangle \propto \delta(\vec{k} + \vec{k}'). \quad (3.7)$$

Because they do not have their own power spectra, they must appear in pairs and thus effectively behave as

¹This is the same as the statement that only the curvature of the metric is locally observable, which is related to gradients of the long wavelength mode that redshift away.

²We do not include factors of $c_s(t)$ here as $c_s(t)$ may not be scale relevant to noninflaton degrees of freedom. As we are working in the regime where the time dependence of any parameter is small in Hubble units, power law decay in $a(t)H(t)$ is sufficient.

operators with $\Delta = 3/2$ and are therefore irrelevant. It remains an interesting question as to whether these contact operators appear in single field inflation via matching, which we will address in the next subsection.

At this stage, we would argue that the conservation of ζ_+ is guaranteed by power counting. Since every interaction in the EFT decays with powers of $a(t)H(t)$, by the usual EFT rules, we would expect all perturbative corrections to decay like $(k/(aH))$ and thus ensure that the time independent initial conditions dominate the late time behavior. However, the observation that corrections come with inverse powers of $a(t)$ is not new and was not previously sufficient to constitute a demonstration of all-orders conservation. For example, in the all-order derivation of the conservation of ζ in [18], the vanishing of the commutator outside of the horizon was used to show that the long wavelength mode in single field inflation must obey an operator equation

$$\dot{\zeta} = \sum_n c_n (a^{-2} \partial^2 \zeta)^n + \dots, \quad (3.8)$$

where ... includes operators with more and/or different distributions of derivatives. The list of operators on the rhs of this expression are very similar to the operators that multiply ζ_- in the EFT action, as their form is constrained by the same nonlinearly realized symmetries.

In [18], a diagrammatic proof was required to show that the rhs of Eq. (3.8) vanishes as a power of $k/(aH)$. The reason that this isn't self-evident is that the fields that appear here are the full UV operators. The UV description contains a scale $H(t)$ that can be defined by horizon crossing in comoving coordinates, $k = a(t)H(t)$. This scale lies in the middle of range of loop integration and thus an integral over internal momentum p may generate a power of aH rather than k . In contrast, the EFT is constructed so that all loop corrections are power laws that can be regulated by analytic continuation of the tilt or the dimension of the operators. Furthermore, the scale $a(t)H(t)$ appears only as the UV scale that defines our couplings, but does not appear as a scale in loop corrections.

B. One-loop matching with Mellin

The EFT argument for the conservation of ζ crucially relies on the existence of scaleless regulators. This is not a physical requirement, of course, but makes the power counting results clear and avoids the breaking of symmetries common to other regulators used in dS. Like any EFT, we will still need to match the soft theory to the UV and this is most easily done if the UV theory can be regulated in the same way. We will see that this naturally leads us to introduce the Mellin representation to regulate the loops.

Let us take a concrete one loop example. We will consider quantum corrections to the classical power spectrum in single field inflation,

$$\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle = i^2 \int d^4x [H_{\text{int}}, [H_{\text{int}}, \zeta(\vec{k}) \zeta(\vec{k}')]]. \quad (3.9)$$

We will consider the case where $H_{\text{int}} = -\lambda a^3 H^{-3} \zeta_c^3$, ζ_c is the dimensionless canonically³ normalized field such that $\zeta = \Delta_\zeta \zeta_c$ and $c_s = 1$. The coupling λ can arise, for example, from

$$\mathcal{L}_{\text{int}} \supset \frac{4M_3^4}{3} \frac{\zeta_c^3}{H^3} \rightarrow \lambda = \frac{8\sqrt{2}M_3^4}{3H^3} \Delta_\zeta^3, \quad (3.10)$$

in the EFT of inflation. This example already displays all the relevant challenges, but we calculate additionally the contributions from other interactions in the Appendixes.

To proceed, we define the canonically normalized ζ_c operator as

$$\zeta_c = \int \frac{d^3k}{(2\pi)^3} (a_{\vec{k}} \bar{\zeta}_c(\vec{k}, \tau) + a_{-\vec{k}}^\dagger \bar{\zeta}_c^*(\vec{k}, \tau)) \quad (3.11)$$

where

$$\bar{\zeta}_c = -i \sqrt{\frac{\pi}{2}} e^{\frac{i}{2}\pi(\nu+1/2)} \tau^\nu H_\nu^{(1)}(-k\tau), \quad (3.12)$$

and $2\nu = 3 + 2\epsilon + \kappa = 3 - (n_s - 1)$ (remember that since $c_s = 1$, we have $s = 0$). We then insert this into the Wick contracted expression

$$\begin{aligned} \langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle &= -\lambda^2 \int_{-\infty}^{\tau} d\tau_1 a^4(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^4(\tau_2) \int \frac{d^3k}{(2\pi)^3} \\ &\times \bar{\zeta}(\vec{k}, \tau) \bar{\zeta}(\vec{k}', \tau) \dot{\zeta}^*(\vec{k}, \tau_1) \dot{\zeta}^*(\vec{k}', \tau_2) \dot{\zeta}(\vec{p}, \tau_1) \\ &\times \dot{\zeta}^*(\vec{p}, \tau_2) \dot{\zeta}(\vec{p} - \vec{k}, \tau_1) \dot{\zeta}^*(\vec{k} - \vec{p}, \tau_2) \\ &+ 3 \text{ other terms.} \end{aligned} \quad (3.13)$$

Here we are dropping the other terms associated with the two commutators, but the complete details can be found in the Appendix.

Performing loop calculations directly with these Hankel functions makes regulating the theory very difficult. We will therefore use the Mellin representation,

$$\begin{aligned} i\pi e^{i\pi\nu/2} H_\nu^{(1)}(z) &= \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \Gamma\left(s + \frac{\nu}{2}\right) \Gamma\left(s - \frac{\nu}{2}\right) \\ &\times \left(-\frac{iz}{2}\right)^{-2s}, \end{aligned} \quad (3.14)$$

³Technically speaking, we have absorbed an extra factor of $H\sqrt{2}$ into the definition of ζ_c so that Δ_ζ is the amplitude of the power spectrum and ζ_c is normalized to 1.

where c is a real number chosen so that we can enclose all the poles of the Γ functions. Since ν is real, we can choose $c > \nu/2$ and close the contour in the $\text{Re } s < 0$ region to isolate both sets of poles at $s = \pm\nu/2 - n$, where $n = 0, 1, \dots$ is an integer, with residues $(-1)^n/n!$. Using the residue theorem, one recovers the series expansion of the Hankel function around $z = 0$.

While the full calculation is quite involved (see Appendix B for complete details), the main result can be understood from a few simple observations. First, the time derivatives act on the Mellin transformed mode functions as

$$\dot{\zeta}_c = C \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} (\nu - 2s) \Gamma\left(s + \frac{\nu}{2}\right) \Gamma\left(s - \frac{\nu}{2}\right) \times \left(\frac{ik\tau}{2}\right)^{-2s} (-\tau)^\nu \quad (3.15)$$

$$= C \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} (-2) \Gamma\left(s + \frac{\nu}{2}\right) \Gamma\left(s - \frac{\nu}{2} + 1\right) \times \left(\frac{ik\tau}{2}\right)^{-2s} (-\tau)^\nu. \quad (3.16)$$

On the second line, we see that the poles of the second Γ function have been shifted to $s = \nu/2 - (n+1)$ for $n = 0, \dots, \infty$ (in particular, they all lie on the negative real axis). This is expected, since in the $k\tau \rightarrow 0$ limit, there is always a constant solution whose time derivative is zero. This is important because after loop integration, divergences will be associated with coincident poles of different Γ functions.

The second important feature of the Mellin transform is that it dramatically simplifies loop and time integrations. After inserting the Mellin transform for each mode function, we will be left with a loop integral

$$\int \frac{d^3 p}{(2\pi)^3} p^{-2s_3-2s_4} (|\vec{k} - \vec{p}|)^{-2s_5-2s_6} = -\frac{i}{2\pi} \delta\left(\frac{3}{2} - (s_3 + s_4 + s_5 + s_6)\right) + \mathcal{O}(k^2) \quad (3.17)$$

where $s_{i=3,4,5,6}$ are the Mellin variables that define the Hankel functions for the four Hankel functions in the loop integral and $s_{1,2}$ will be the Mellin variables for the fields carrying momentum $\vec{k}$ and $\vec{k}'$ contracted with the external operators. On the rhs, we have written the solution as an expansion in k . It is straightforward to check that the original integral is IR finite and therefore we can treat the integral as an expansion in k/p .

The time integrals are similarly straightforward. Prior to evaluating the Mellin integrals, we have only integrals over power laws in τ and τ' to evaluate. Importantly, unlike the loop integrals, the power law depends on s_1 and s_2 . To simplify the discussion, we will drop slow roll corrections,

so that $\beta \rightarrow 3$ and $a(\tau) \propto \tau^{-1}$. In this limit, we will encounter integrals of the form

$$\int_{-\infty}^{\tau} d\tau_1 (\tau_1)^{\frac{1}{2}} (\tau_1)^{-2(s_1+s_3+s_5)} = \frac{\tau^{\frac{3}{2}-2(s_1+s_3+s_5)}}{\frac{3}{2}-2(s_1+s_3+s_5)} \rightarrow \frac{i\pi}{2} \delta\left(\frac{3}{2}-2(s_1+s_3+s_5)\right), \quad (3.18)$$

or of the nested form

$$\int_{-\infty}^{\tau} d\tau_1 (\tau_1)^{-2(s_1+s_3+s_5)} \int_{-\infty}^{\tau_1} d\tau_2 (\tau_2)^{\frac{1}{2}} (\tau_2)^{-2(s_2+s_4+s_6)} \quad (3.19)$$

$$\rightarrow \frac{1}{\frac{3}{2}-2(s_2+s_4+s_6)} i \frac{\pi}{2} \delta(3-2(s_1+\dots+s_6)). \quad (3.20)$$

In order to get time-independent corrections, we need to saturate the δ functions associated with these time integrals as well as the δ function from the loop momentum integral.

Before taking the $\tau \rightarrow 0$ limit, the integrand will include the following factor

$$\frac{(-\tau k)^{-2s_1-2s_2}}{k^{3-(n_s-1)}} \tau^{3-2(s_3+s_4+s_5+s_6)} \delta\left(\frac{3}{2}-s_3+s_4+s_5+s_6\right) = \frac{(-\tau k)^{-2s_1-2s_2}}{k^{3-(n_s-1)}} \delta\left(\frac{3}{2}-s_3+s_4+s_5+s_6\right), \quad (3.21)$$

where the δ function is from the loop integration. This form does not depend on the nature of the time integral. Therefore, any correction that survives the $\tau \rightarrow 0$ limit must be consistent with $s_1 + s_2 = 0$.

Given the form of the integrand, it remains possible to generate a logarithmic correction, $\log k\tau$, if the poles of the Γ functions coincide when $s_1 = -s_2$. This condition arises naturally from the poles associated with the time integration. Substituting this condition, by performing the s_2 Mellin-integral for example, the integrand of the s_1 Mellin integral becomes

$$\langle \zeta^2 \rangle_{1\text{-loop}} \propto \int \frac{ds_1}{(2\pi i)} \Gamma\left(s_1 + \frac{\nu}{2}\right) \Gamma\left(s_1 - \frac{\nu}{2} + 1\right) \times \Gamma\left(-s_1 + \frac{\nu}{2}\right) \Gamma\left(-s_1 - \frac{\nu}{2} + 1\right). \quad (3.22)$$

The poles of the integrand are located at $s_1 = \pm\nu/2 \pm n$ and $s_1 = \mp(\nu/2 - 1) \pm n$ where $n = 0, 1, 2, \dots$. Since $\nu/2$ is not an integer, these poles can only coincide if

$$\nu/2 + n = (\nu/2 - 1) - n' \rightarrow n + n' = -1, \quad (3.23)$$

where $n, n' \in 0, 1, 2, \dots$. Clearly, this condition is never satisfied, and therefore no logarithmic correction arises.

C. Scheme dependence

The purpose of the Mellin calculation was to demonstrate, unambiguously, that we can regulate the one-loop correction to the power spectrum without any signs of divergences or logarithmic secular terms. The purpose of our soft ζ EFT is to explain this behavior physically. However, it does not explain why this was not apparent in other approaches to calculating the one-loop power spectrum.

As a concrete illustration, we can consider the calculation of [24], which found for $-\frac{4}{3}M_3^4\zeta^3$ the correction to the power spectrum is given by

$$\langle \zeta_{\vec{k}}(t) \zeta_{\vec{k}'}(t) \rangle'_{1\text{-loop}} = -\frac{2}{15\pi^2} \frac{H^8 M_3^8}{\dot{H}^4 M_{\text{Pl}}^8} \frac{1}{k^3} \log\left(\frac{H}{\mu}\right). \quad (3.24)$$

However, it is useful to work in a scheme where $\mu = H$ and this result vanishes.⁴ This is effectively the scheme that Mellin implements automatically, as one does not need the introduction of a scale μ , as all scales are defined in terms of $H(t_*)$. In other schemes, this log may appear as a large constant [88] that would diverge as $n_s \rightarrow 1$. If such a term were universal, it would appear in Mellin space as the collision of poles, but we have verified explicitly that that does not occur in our Mellin-based scheme.

A source of the confusion is that previous calculations have been defined in dimensional regularization, which is supposed to handle these problems well. However, when calculating in-in correlators, it is often advantageous to treat time and space separately. Since the theory is invariant under spatial translations, we can Fourier transform the spatial coordinates $\vec{x} \rightarrow \vec{k}$. However, specifying a unique time that defines an equal time correlator breaks time translations, making it less useful to Fourier transform.

In flat space, loop corrections to in-in correlators using this decomposition would require performing integrals of the form

$$\int \frac{d^3 p}{(2\pi)^3} p^n e^{impt}, \quad (3.25)$$

where the factor e^{impt} is from the time evolution of the mode functions. Notice that if $t = 0$, this is a scaleless integral with a power law divergence and would vanish,

⁴It should be emphasized that the appearance of $\log \mu/H$ is itself a nontrivial property of the full one-loop calculation in many schemes. Earlier attempts to calculate this result had found $\log k$, which would have broken important symmetries of single field inflation [8, 23, 85–87]. This should emphasize the value of regulators, like the EFT or Mellin, where the absence of such logs is manifest.

$$\int \frac{d^3 p}{(2\pi)^3} p^n = 0. \quad (3.26)$$

However, at $t \neq 0$, the integral is no longer scaleless and we find a nonzero answer [by continuing $t \rightarrow t(t - i\epsilon)$ with $t < 0$],

$$\int \frac{d^3 p}{(2\pi)^3} p^n e^{impt - \epsilon m p |t|} \rightarrow \frac{1}{2\pi^2} \frac{\Gamma[3+n]}{(-imt)^{3+n}}. \quad (3.27)$$

Importantly, this expression diverges as $t \rightarrow 0$, which demonstrates that the power law divergence at $t = 0$ is not actually regulated in this scheme.

Of course, power law divergences should be removed in any scheme by a redefinition of the couplings, without introducing any need for RG flow. Therefore, even though the bad behavior appears to be time dependent, it should be removable by a change to the action. Indeed, it was argued in [24] that the problematic behavior associated with ζ^3 can be removed by a shift of the coupling of a $\zeta^{\dots 2}$ operator. In Appendix C, we show that this same behavior can be seen when computing a flat space in-in correlator with the same interactions. In this precise sense, these different calculations differ only in their effective values of the couplings of the theory as a result of the choice of the regularization scheme.

However, the Mellin-space approach is not free from the challenge of defining a simple scheme. If we follow the conventional set of contours for the remaining Mellin integrals, we will arrive at a finite correction to the power spectrum

$$\langle \zeta(\vec{k}) \zeta(-\vec{k}) \rangle' = \frac{\Delta_\zeta k^{n_s-1}}{k^3} (1 + a\lambda^2), \quad (3.28)$$

where a is a finite constant. In Appendix B, we calculate a for the $\lambda\zeta^3$ interaction, and find $a = -9.36 \times 10^4$ for a particular choice of contours (for one subset of the diagrams). However, as described in the Appendix, there is another choice of contours that is equally physical that gives rise to $a = 0$ for all the one-loop Mellin corrections. This should be understood as the statement that this one-loop correction is equivalent to the shift of $\zeta^{\dots 3}$ coupling, and therefore we can choose a scheme where the one-loop correction is zero, and any finite correction is thus determined by this additional operator. In other schemes, the constant a may even be infinite (because the divergence is not actually regulated), and should be removed by a counterterm by hand as usual.

The issue of how to apply a self-consistent scheme in Mellin space is challenging beyond one-loop [58]. The collision between poles remains a physical effect associated with RG. However, after regulating the theory, the choice of contours will shift the final results by a finite amount. Since

these choices do not change the k or τ dependence of the final result, they are consistent with shifts of local couplings. On the other hand, the proliferation of Mellin variables introduces a significant challenge to regulating and renormalizing the theory at higher-loop orders.

IV. EFT WITH TENSOR MODES

So far, we have focused only on the scalar metric fluctuation, ζ . This might seem surprising, as the tensor modes, γ_{ij} , are present in both inflation and de Sitter spacetime, and are the more basic ingredient of dynamical gravity. However, in practice, the scalars and tensors have essentially the same behavior in perturbation theory,⁵ except that tensor structures and symmetries lead to additional constraints [98,99]. Our goal in this section is to demonstrate that EFT and conservation for the soft tensor and scalar modes are effectively identical.

A. Tensors from the top down

The tensor modes, defined via $g_{ij} = a^2(t)e^{2\zeta}[e^{2\gamma}]_{ij}$, can also be treated through the EFT techniques. The action for the tensors is given by

$$S = \frac{M_{\text{pl}}^2}{8} \int d^4x a^3(t) \left[\dot{\gamma}_{ij}^2 - \frac{1}{a^2} (\partial_i \gamma)^2 \right]. \quad (4.1)$$

Solving for the time dependence, we can decompose the tensor modes into the growing and decaying modes as well

$$\gamma_{ij} = \gamma_{+,ij} + (aH)^{-\beta} \gamma_{-,ij} \quad (4.2)$$

where we have that $\beta = 3 + 2\epsilon$ for the case of the tensors. We will have

$$\dot{\gamma} = \dot{\gamma}_{ij} - \beta H(t)(1 - \epsilon)(a(t)H(t))^{-\beta} \gamma_{-,ij} + (a(t)H(t))^{-\beta} \dot{\gamma}_{-,ij}. \quad (4.3)$$

Then, we can go through the same exercise, and we will find that

$$S = \frac{M_{\text{pl}}^2}{4} \int d^4x \left[-\beta H \dot{\gamma}_{+,ij} \gamma_{-,ij} + \dot{\gamma}_{+,ij} \dot{\gamma}_{-,ij} - \frac{1}{a^2} \partial_i \gamma_{+,ij} \partial_i \gamma_{-,ij} \right] \quad (4.4)$$

to lowest order in slow roll parameters. Again, the canonical momenta is given by

⁵The observational signatures of tensor modes [89,90] and their statistics [91–97] are importantly very different from the scalar fluctuations. These differences only indirectly impact the long wavelength theory, and not the structure of the EFT.

$$\Pi_{\gamma_+} = -\frac{M_{\text{pl}}^2}{4} \beta H \gamma_- \quad (4.5)$$

with the relation

$$\begin{aligned} [\gamma_{+,ij}(\vec{x}, t), \gamma_{-,lm}(\vec{x}', t)] &= -4 \frac{i}{M_{\text{pl}}^2 \beta H} \delta(\vec{x} - \vec{x}') \delta_{il} \delta_{jm} \\ \rightarrow [\gamma_{+,ij}(\vec{k}, t), \gamma_{-,lm}(\vec{k}', t)] &= -\frac{4i}{M_{\text{pl}}^2 \beta H} \delta_{il} \delta_{jm} (2\pi)^3 \delta(\vec{k} + \vec{k}'). \end{aligned} \quad (4.6)$$

Much of the story of ζ follows through into the tensors. The canonical momenta relations enforce that the leading order interactions will be of the form $\supset \gamma_+^n \gamma_-$. Moreover, as we saw in the case of ζ , $[e^\gamma]_{ij}$ forms a nice operator that renormalizes into itself under RG. The only subtlety arises when we try to understand the operators that are allowed under symmetry transformations for γ_{ij} , since γ_{ij} has its own symmetry transformations. This is required to conclusively show that any operator that arises in the theory is irrelevant. In particular, we would want to show that no nonlocal terms can give rise to operators that become relevant outside the horizon, as we did with ζ . Let us turn to that next.

B. Symmetries of scalar and tensors

The tensor symmetries are different from the scalar symmetries [15]. Tensors change linearly under dilatation,

$$\delta \gamma_{ij} = \lambda x^k \partial_k \gamma_{ij}, \quad (4.7)$$

while the Special Conformal Transformations (SCTs) are not a good symmetry of the tensors. Instead, tensors have their own set of nonlinearly realized symmetries. Under a general gauge transformation, $x^i \rightarrow x^i + \xi^i$, the tensors transform according to

$$\delta \gamma_{ij} = \partial_i \xi_j + \partial_j \xi_i \quad (4.8)$$

for time independent ξ_i . In terms of the soft modes, this breaks up as

$$\delta \gamma_{+,ij} = \partial_i \xi_j + \partial_j \xi_i, \quad \delta \gamma_{-,ij} = 0. \quad (4.9)$$

Although any ξ_i represents a valid diffeomorphism, only those that leave the gauge unchanged and are continuously connected to physical modes are of interest for understanding the EFT. Concretely, for our choice of gauge, we will only include the transformations that keep the tensors transverse and traceless. This construction is outlined in [16].

Now we want to understand the structural constraints imposed on γ and ζ when taking into account the tensor symmetries. Instead of taking a dilatation, let us take a

gauge transformation $x^i \rightarrow x^i + \xi^i$, where $\xi_i = M_{ij}^j x_j$, where $M_{ij} = M_{ji}$ is a constant symmetric matrix. We can then capture the nonlinear shifts in the field as

$$\delta\zeta_+ = \frac{1}{3}\partial_i\xi_i, \quad \delta\gamma_{+,ij} = \partial_i\xi_j + \partial_j\xi_i. \quad (4.10)$$

The corrections are of higher order in ζ_+, γ_+ . Physically, $\delta\zeta_+$ captures the dilatation, while $\delta\gamma_+$ captures anisotropic scaling. The charge corresponding to this symmetry is given in terms of the operator

$$\begin{aligned} Q &= \int d^3x \partial_i \xi_j \left(\frac{1}{3} \delta^{ij} \Pi_{\zeta_+} + 2\Pi_{\gamma_+}^{ij} \right) \\ &= \int d^3x M_{ij} \left(\frac{1}{3} \delta^{ij} \Pi_{\zeta_+} + 2\Pi_{\gamma_+}^{ij} \right). \end{aligned} \quad (4.11)$$

Fourier transforming, and using ζ_-, γ_- as conjugate momenta, we have

$$Q = \int d^3q \left(\frac{C_1}{3} M_{ii} \zeta_-(\vec{q}) + 2C_2 M_{ij} \gamma_-^{ij}(\vec{q}) \right). \quad (4.12)$$

Now, playing the same game as with ζ_+ , we can demand that the action is invariant under transformations generated by the charge by definition. We will then have that

$$S \supset \int d^4x \sqrt{g} \sum_n \mathcal{O}_n(x) \rightarrow \left[Q, \int d^4x \sqrt{g} \sum_n \mathcal{O}_n(x) \right] = 0. \quad (4.13)$$

Any potentially marginal operator, $\mathcal{O}_m$, will take that form $\mathcal{O}_m = \zeta_- F(\zeta_+, \gamma_{+,ij}, \partial_t, \vec{\partial})$ or $\mathcal{O}_m = \gamma_{-,ij} F^{ij}(\zeta_+, \gamma_{+,ij}, \partial_t, \vec{\partial})$ where $F(\zeta_+, \gamma_{+,ij}, \partial_t, \vec{\partial})$, $F^{ij}(\zeta_+, \gamma_{+,ij}, \partial_t, \vec{\partial})$ may be nonlocal in $\vec{\partial}$. Without loss of generality, we can take the former case. Now we can Fourier transform the fields so that

$$\begin{aligned} &\int d^4x \sqrt{g} \mathcal{O}_m \\ &\rightarrow c_m(t) \zeta_-(\vec{p}_0) \left(\prod_{i=1}^N \int d^3p_i \zeta_+(\vec{p}_i) \prod_{j=1}^N \int d^3q_j \gamma_+(\vec{q}_j) \right) \\ &\quad \times \tilde{F}(\{p_i, q_j\}) (2\pi)^3 \delta\left(\sum_{n=0}^n \vec{p}_n + \vec{q}_n\right). \end{aligned} \quad (4.14)$$

Since the shift symmetry arises from $Q = \alpha\zeta_-(\vec{q}=0) + \beta_{ij}\gamma_{-,ij} + \mathcal{O}(\zeta_+\zeta_-)$, a single commutator will set one of the momenta of $\zeta_+(\vec{p}_i)$ and $\gamma_{+,ij}(\vec{q}_i)$ to zero,

$$\begin{aligned} [Q, \zeta_+(\vec{p})] &\rightarrow \alpha i(2\pi)^3 \delta(\vec{p}) + \mathcal{O}(\zeta_+), \\ [Q, \gamma_{+,ij}(\vec{q})] &\rightarrow \beta_{ij} i(2\pi)^3 \delta(\vec{q}) + \mathcal{O}(\gamma_+). \end{aligned} \quad (4.15)$$

Therefore, after N commutators we have

$$\begin{aligned} [Q, [Q, \dots [Q, S], \dots],] &= 0 \supset \zeta_-(\vec{p}_0=0) F(\{\vec{p}_i, q_i\}=0) \\ &\quad + \mathcal{O}(\zeta_-\zeta_+). \end{aligned} \quad (4.16)$$

Now, in order for the charge to be a symmetry, we need this to vanish in the limit that $p_i, q_j = 0$. This then requires that there are more derivatives in the numerator than in the denominator, and hence the nonlocal terms must be power counting suppressed even when taking both ζ, γ into account.

C. All-orders conservation

The all-orders conservation of both γ_+ and ζ_+ is again a result of power counting. As every interaction must appear with a factor of ζ_- or γ_- as before, we have

$$\begin{aligned} &\int d^4x \sqrt{g} (c_m \mathcal{O}_m + c'_m \mathcal{O}'_m) \\ &\rightarrow \int dt d^3x (c_m (aH)^{3-\beta_\zeta-\Delta_F} \zeta_- F(\zeta_+, \gamma_{+,ij} \zeta) \\ &\quad + c'_m (aH)^{3-\beta_\gamma-\Delta_{F_{ij}}} \gamma_{-,ij} F^{ij}(\zeta_+, \gamma_{+,ij} \zeta)) \end{aligned} \quad (4.17)$$

where Δ_F and $\Delta_{F_{ij}}$ and the scaling dimensions of the composite operators F and F^{ij} . As we saw in the previous section, because there must be a net positive number of derivatives in order to maintain the symmetries of the theory, $\Delta_F, \Delta_{F_{ij}} \geq 1 + \mathcal{O}(\epsilon)$ and therefore all such operators are irrelevant.

The all orders result then follows from the scheme for calculating scaleless integrals in the EFT, as we discussed in Sec. III and in [39]. Here, the tensor structure presents no additional challenges beyond the scalar case, as the momentum integrals are effectively the same. However, the additional complication is that the speed of propagation of the scalars and tensors are different, in general, and therefore the power counting can be in both $k/(a(t)H(t))$ and $k/\Lambda(t)$ [i.e., differ by factors of $c_s(t)$]. This has no impact on the conservation of ζ or γ , provided that the time variation of the parameters is small. However, when determining the evolution of composite operators or the tilt of the spectra, there is no longer a single $t_\star$ that makes logs for both γ and ζ small for the same wavenumber $\vec{k}$, and therefore matching and RG can require additional resumming of logs [76].

Finally, the conservation of the tensor modes is more robust to the addition of light fields due to the Higuchi bound [99]. Specifically, the addition of spin-2 states is highly constrained by unitarity (positivity of the two-point

statistics), as is well known from flat space. Therefore, even in the presence of additional fields, any time evolution would take the form

$$\dot{\gamma}_{+,ij} = \mathcal{O}_{ij} \quad (4.18)$$

and would require a tensor operator $\mathcal{O}$ that is close to dimension zero. We have already seen that this is not permitted for the metric fluctuation itself, and can only arise for partially massless tensor modes. As a result, even in generic multifield models where ζ can evolve (due to isocurvature modes), the tensors will be generically conserved.

V. CONCLUSIONS

Understanding perturbative quantum gravity in accelerating spacetimes is an important step toward resolving the deepest questions about the quantum nature of our universe [100]. The breakdown of our understanding is most acute in cases like eternal inflation [101,102], where the fluctuations of the metric are large. However, we can learn a lot about these nonperturbative questions from (renormalization group improved) perturbative calculations [103]. Concretely, studies of both eternal inflation and the Hartle-Hawking wavefunction [104] have found close connections to the framework of stochastic inflation [81,105], which is amenable to perturbative techniques.

In this paper, we have offered an EFT approach to understanding the all-orders behavior of both the scalar and tensor fluctuations during inflation. We showed that both are conserved in single- and quasi-single-field inflation due to the formal irrelevance of all interactions outside the horizon. The central reason for their conservation is that the metric is subject to an infinite group of nonlinearly realized symmetries that are the result of large diffeomorphisms. These symmetries do not permit the interactions and/or operator dimensions that would lead to superhorizon time evolution (provided we do not have large time dependence of the background).

The development of this EFT is part of a larger goal of understanding the nature of the metric fluctuations in accelerating universes nonperturbatively. Within the EFT description, RG flow can be translated into a Fokker-Planck equation [42]. At leading order, this is the framework of Stochastic Inflation, but the EFT allows one to systematically improve this calculation by including higher derivative corrections. Yet, this approach also predicts the demise of the Stochastic framework for sufficiently rare fluctuations [45]. The question of how to calculate the tail of the distribution [106–108] has a variety of interesting implications both for observational implications, such as primordial black holes [109], and conceptually, in terms of the meaning of the wavefunction of the universe and the nature of eternal inflation. Putting our perturbative understanding of quantum gravity in these cosmologies on a more straightforward foundation, both technically and

conceptually, is hopefully a useful first step toward addressing these challenging problems.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: DERIVING THE EFT

The derivation of our soft EFT for the metric fluctuations requires a number of technical steps and matching to the full UV theory. In this appendix, we explain some of these details that we bypassed in the main text.

1. Stochastic Initial Conditions

As emphasized in the main text, the initial conditions for $\zeta_{\pm}$ are not determined by the EFT and require matching to the full theory. We can derive the Gaussian initial condition from the quadratic action,

$$S = \frac{1}{2} \int d^4x \left(2 \frac{M_{\text{pl}}^2 |\dot{H}|}{c_s^2 H^2(t)} a^3(t) \right) [\zeta^2 - c_s^2 (\partial_i \zeta)^2]. \quad (\text{A1})$$

Assuming we can use the slow-roll expansion, the equations of motion for the mode functions are

$$\partial_{\tau}^2 \bar{\zeta} - \frac{1}{\tau} (1 + \epsilon) (2 + \eta - 2s) \partial_{\tau} \bar{\zeta} + c_s^2(\tau) k^2 \bar{\zeta} = 0, \quad (\text{A2})$$

and can be solved directly to first order in slow-roll, $\mathcal{O}(\epsilon)$, to find

$$\bar{\zeta} = \Delta_{\zeta} \zeta_c = \Delta_{\zeta} A \tau^{(3+2\epsilon+\eta-2s)/2} H_{\nu}^{(1)}(c_s(\tau) k \tau) + (\text{c.c.}), \quad (\text{A3})$$

where $H_{\nu}^{(1)}$ is the Hankel function of the first kind, and $\nu = (3 + 2\epsilon + \kappa + s)/2$. To determine the normalization of the mode function, A , we take $k\tau \rightarrow \infty$ using

$$H_{\nu}^{(1)}(c_s(\tau) k \tau) \rightarrow \sqrt{\frac{2}{\pi c_s(\tau) k \tau}} e^{i c_s(\tau) k \tau - \frac{1}{4} i \pi (-3+2\nu)}. \quad (\text{A4})$$

As a result, the canonically normalized field obeys

$$\zeta_c \rightarrow -A \tau^{(2+2\epsilon+\eta-2s)/2} \sqrt{\frac{2}{\pi c_s(\tau) k}} e^{i c_s(\tau) k \tau - \frac{1}{4} i \pi (-3+2\nu)}. \quad (\text{A5})$$

We can now normalize to plane waves using the Wentzel-Kramers-Brillouin (WKB) approximation⁶ so that

$$A = i\sqrt{\frac{\pi}{2}} e^{\frac{1}{2}i\pi(-3+2\nu)} = -i\sqrt{\frac{\pi}{2}} e^{\frac{1}{2}i\pi(\nu+1/2)}. \quad (\text{A6})$$

So overall, we have that

$$\bar{\zeta}_c = -i\sqrt{\frac{\pi}{2}} e^{\frac{1}{2}i\pi(\nu+1/2)} \tau^{(3+2\epsilon+\eta-2s)/2} H_\nu^{(1)}(c_s(\tau)k\tau), \quad (\text{A7})$$

and the ζ operator is given by

$$\zeta = \bar{\Delta}_\zeta \int \frac{d^3k}{(2\pi)^3} (a_k \zeta_c + a_k^\dagger \bar{\zeta}_c^*) \quad \bar{\Delta}_\zeta^2 = c_s^3 \Delta_\zeta^2 = \frac{c_s^2 H^4}{4M_{\text{pl}}^2 |\dot{H}|}, \quad (\text{A8})$$

where $a_k, a_k^\dagger$ have the commutation relation,

$$[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k - k'). \quad (\text{A9})$$

Now, in the soft limit $c_s k\tau \rightarrow 0$, we have

$$H_\nu^{(1)}(c_s(\tau)k\tau) \rightarrow \left(-i(c_s(\tau)k\tau)^{-\nu} \frac{2^\nu \Gamma(\nu)}{\pi} + (c_s(\tau)k\tau)^\nu \frac{2^{-\nu}(1+i\cot(\nu\pi))}{\Gamma(1+\nu)} \right). \quad (\text{A10})$$

Writing $\tau^{(3+2\epsilon+\eta-2s)/2} = \tau^{\nu-3s/2}$ and $c_s(\tau) = c_{s,0} \tau^{-s}$, we have

$$\bar{\zeta}_c \rightarrow e^{\frac{1}{2}i\pi(\nu+1/2)} A + i e^{\frac{1}{2}i\pi(\nu+1/2)} \tau^{2\nu-3s} B (1 + i\cot(\nu\pi)), \quad (\text{A11})$$

where

$$A = \frac{(c_{s,0}k)^{-\nu} 2^\nu \Gamma(\nu)}{\sqrt{2\pi}}, \quad B = \sqrt{\frac{\pi}{2}} \frac{(c_{s,0}k)^\nu 2^{-\nu}}{\Gamma(1+\nu)}. \quad (\text{A12})$$

As a result, in the limit $\vec{k} \rightarrow 0$, our fundamental field operator takes the form,

$$\begin{aligned} \zeta \rightarrow \bar{\Delta}_\zeta \int \frac{d^3k}{(2\pi)^3} & \left(A(e^{\frac{1}{2}i\pi(\nu+1/2)} a_k + e^{-\frac{1}{2}i\pi(\nu+1/2)} a_k^\dagger) \right. \\ & + iB(1+i\cot(\nu\pi)) \tau^{2\nu-3s} (e^{\frac{1}{2}i\pi(\nu+1/2)} a_k \\ & \left. + e^{2i\nu\pi} e^{-\frac{1}{2}i\pi(\nu+1/2)} a_k^\dagger) \right). \end{aligned} \quad (\text{A13})$$

⁶We absorbed an extra factor of $\sqrt{2}$ from the unusual definition so that Δ_ζ^2 is the amplitude of the power spectrum.

Since this is the operator in the long wavelength limit, it can be expressed in terms of the long wavelength fields, so that

$$\begin{aligned} \zeta = \bar{\Delta}_\zeta \int \frac{d^3k}{(2\pi)^3} & [\bar{\zeta}_+(e^{i\delta_\nu} a_k + e^{-i\delta_\nu} a_k^\dagger) \\ & + i\tau^{2\nu-3s} \bar{\zeta}_-(e^{-i\delta_\nu} a_k - e^{i\delta_\nu} a_k^\dagger)], \end{aligned} \quad (\text{A14})$$

where we have defined $\delta_\nu = \pi(\nu + 1/2)/2$, and we have

$$\begin{aligned} \bar{\zeta}_+ &= A \bar{\Delta}_\zeta = \bar{\Delta}_\zeta \frac{(c_{s,0}k)^{-\nu} 2^\nu \Gamma(\nu)}{\sqrt{2\pi}}, \\ \bar{\zeta}_- &= \bar{\Delta}_\zeta \frac{B}{\sin(\nu\pi)} = \bar{\Delta}_\zeta \sqrt{\frac{\pi}{2}} \frac{(c_{s,0}k)^\nu 2^{-\nu}}{\sin(\nu\pi) \Gamma(1+\nu)}. \end{aligned} \quad (\text{A15})$$

Now, if we define

$$\tilde{a}_k = e^{i\delta_\nu} a_k + e^{-i\delta_\nu} a_k^\dagger, \quad \tilde{b}_k = i(e^{-i\delta_\nu} a_k - e^{i\delta_\nu} a_k^\dagger), \quad (\text{A16})$$

and $\zeta_+ = \bar{\zeta}_+ \tilde{a}_k$ and $\zeta_- = \bar{\zeta}_- \tilde{b}_k$, then the power spectra become

$$\langle \zeta_+ \zeta_+ \rangle = \bar{\Delta}_\zeta^2 \frac{(c_{s,0}k)^{-2\nu} 2^{2\nu} \Gamma^2(\nu)}{2\pi} (2\pi)^3 \delta^3(k+k'), \quad (\text{A17})$$

$$\langle \zeta_- \zeta_- \rangle = \bar{\Delta}_\zeta^2 \frac{\pi}{2} \frac{(c_{s,0}k)^{2\nu} 2^{-2\nu}}{\sin^2(\nu\pi) \Gamma^2(1+\nu)} (2\pi)^3 \delta^3(k+k'), \quad (\text{A18})$$

with the tilt being $n_s - 1 = -(2\epsilon + \eta + s)$ which reproduces the well-known results. Taking $\epsilon, \eta, s \rightarrow 0$ (and $\nu \rightarrow 3/2$) one recovers the expected result

$$\langle \zeta_+ \zeta_+ \rangle' = \frac{\bar{\Delta}_\zeta^2}{c_s^3 k^3} = \frac{\Delta_\zeta^2}{k^3} = \frac{H^4}{4M_{\text{pl}}^2 \dot{H} c_s} \frac{1}{k^3}. \quad (\text{A19})$$

We could have arrived at this result more straightforwardly by taking the slow-roll parameters to vanish from the outset, but the tilt plays a nontrivial role in loop corrections, and therefore we want to be able to derive this result from the most complicated expressions.

2. Deriving the soft action from the top down

In addition to matching the initial conditions, it is useful to understand the origin of the EFT directly from the structure of the long wavelength degrees of freedom. We therefore want to derive the soft action for ζ from the top down, following the same strategy as the derivation of SdSET. By construction, the long distance theory is defined in terms of

$$\zeta = \zeta_+ + \left(\frac{aH}{c_s} \right)^{-\beta} \zeta_-. \quad (\text{A20})$$

Note here that $H = H(t)$ and $c_s(t)$ are time dependent. As discussed in the main text, symmetry enforces that there is no time dependence associated with ζ_+ . At this point, β is undetermined, but we find that β is directly given by the tilt for the theory to be consistent. To streamline the derivation, we will assume a local quadratic action for ζ and substitute

$$\dot{\zeta} = \dot{\zeta}_+ - \beta(H - \epsilon H - s_c H) \left(\frac{aH}{c_s} \right)^{-\beta} \zeta_- + \left(\frac{aH}{c_s} \right)^{-\beta} \dot{\zeta}_-, \quad (\text{A21})$$

into the quadratic action to find

$$S = \frac{1}{2} \int d^4x \left(2 \frac{\epsilon}{c_s^2} a^3(t) \right) [\dot{\zeta}^2 - a^{-2} (\partial_i \zeta)^2] \\ = \frac{1}{2} \int d^4x \left(2 \frac{\epsilon}{c_s^2} a^3(t) \right) \left[(\dot{\zeta}_+ - \beta(H - \epsilon H - s_c H) \left(\frac{aH}{c_s} \right)^{-\beta} + \left(\frac{aH}{c_s} \right)^{-\beta} \dot{\zeta}_-)^2 - a^{-2} \left(\partial_i \left(\zeta_+ + \left(\frac{aH}{c_s} \right)^{-\beta} \zeta_- \right) \right)^2 \right]. \quad (\text{A22})$$

We have to remove all the unwanted degrees of freedom, so we are left with an action for the physical degrees of freedom. We want to only write down terms that couple the two modes ζ_+, ζ_- together. Repeating a similar set of field redefinitions discussed in the main text, we arrive at

$$S = \frac{M_{\text{pl}}^2}{2} \int d^4x 2 \frac{\epsilon}{c_s^2} a^{3-\beta} H^{-\beta} \left[-2\beta H(1 - \epsilon - s_c) \dot{\zeta}_+ \zeta_- + 2\dot{\zeta}_+ \dot{\zeta}_- - 2 \frac{1}{a^2} \partial_i \zeta_+ \partial_i \zeta_- \right] \\ = \int d^4x 2 \frac{\epsilon}{c_s^2} a^{3-\beta} H^{-\beta} \left[-\beta H(1 - \epsilon - s_c) \dot{\zeta}_+ \zeta_- + \dot{\zeta}_+ \dot{\zeta}_- - \frac{1}{a^2} \partial_i \zeta_+ \partial_i \zeta_- \right]. \quad (\text{A23})$$

To further simplify the action, we will define the time evolution of the parameters such that

$$\epsilon = \epsilon(t^*) \left(\frac{aH(t)}{aH(t^*)} \right)^\eta, \quad H(t) = H(t^*) \left(\frac{aH(t)}{aH(t^*)} \right)^{-\epsilon}, \quad c_s(t) = c_s(t^*) \left(\frac{aH(t)}{aH(t^*)} \right)^{s_c}. \quad (\text{A24})$$

As a result, the time dependence is

$$\epsilon(t) c_s^{-(2-\beta)} a^{3-\beta}(t) H^{1-\beta}(t) \rightarrow \epsilon(t^*) \left(\frac{aH}{a_* H_*} \right)^\eta c_s^{-(2-\beta)}(t^*) \left(\frac{aH}{a_* H_*} \right)^{s_c(\beta-2)} \\ a^{3-\beta}(t) H^{1-\beta}(t^*) \left(\frac{aH}{a_* H_*} \right)^{-\epsilon(1-\beta)} \\ \rightarrow a^{3-\beta+\eta+s_c(\beta-2)-\epsilon(1-\beta)}(t) f(t_*), \quad (\text{A25})$$

where we are working to first order in the slow roll parameters in the action. We get that $\beta = 3 + 2\epsilon + \eta + s_c$, which is again the tilt. Here, we have defined $f(t_*)$ as

$$f(t_*) = \epsilon(t_*) H(t_*)^{-2} (k_*)^{-(\eta+\kappa+2\epsilon)} c_s^{\beta-2}(t_*). \quad (\text{A26})$$

Finally, the free action is given by

$$S_2 = -2 \int d^4x \beta \epsilon_* H_*^{-2} k_*^{(n_s-1)} c_s^{\beta-2} [\dot{\zeta}_+ \zeta_-], \quad (\text{A27})$$

where ϵ_*, H_* are evaluated at t_* . The conjugate momenta becomes

$$\Pi_+ = -2\beta(H_*)^{-2} \epsilon_*(k_*)^{n_s-1} c_s^{\beta-2} \zeta_- \quad (\text{A28})$$

and hence we have that

$$[\zeta_+, \zeta_-] = -\frac{1}{2\beta(H_*)^{-2} \epsilon_*(k_*)^{n_s-1} c_s^{\beta-2}} \delta^3(x-y). \quad (\text{A29})$$

Finally, we need to supplement the theory with initial conditions. The Gaussian initial condition reads

$$\langle \zeta_+ \zeta_+ \rangle = \frac{H_*^4}{4M_{\text{pl}}^2 \dot{H}_* c_s} \frac{1}{k^3} \left(\frac{k_*}{k} \right)^{2\epsilon+\eta+s}, \quad (\text{A30})$$

which is just the power spectrum.

APPENDIX B: UV CALCULATIONS USING THE MELLIN REPRESENTATION

1. Preliminaries

We will be using the Hankel function's Mellin representation

$$i\pi e^{i\pi\nu/2} H_\nu^{(1)}(z) = \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \Gamma\left(s + \frac{\nu}{2}\right) \Gamma\left(s - \frac{\nu}{2}\right) \left(-\frac{iz}{2}\right)^{-2s}, \quad (\text{B1})$$

for a suitable choice of c (namely one that allows us to enclose all the poles of the Γ functions). We apply this formula to the mode function for ζ ,

$$\bar{\zeta} = \Delta_\zeta \bar{\zeta}_c, \quad \bar{\zeta}_c = -i\sqrt{\frac{\pi}{2}} e^{i\pi(\nu+1/2)} \tau^\nu H_\nu^{(1)}(-k\tau), \quad (\text{B2})$$

with $2\nu = 3 + 2\epsilon + \kappa = 3 - (n_s - 1)$ and $c_s = 1$. These mode functions appear throughout typical in-in calculations and are now expressed in Mellin space. In these variables, the problem of calculating loop corrections or relating the results back to the momentum and time boils down to the identification of poles in Mellin space. This representation

of the Hankel function has left poles located at negative values of s . However, after performing loop integration, some poles will be introduced at positive values of s as well. Nevertheless, as long as the poles are simple and nonoverlapping, the regulated theory is one where we choose a contour that keeps only the negative poles, and we arrive at a convergent answer. As with other regulating schemes, this eliminates any power law divergences, but retains log divergences through the appearance of coinciding poles.

It will be important that our interactions of ζ always involve derivatives (as is required by symmetry). As a result, we are often not using ζ directly, but instead must take derivatives thereof. For spatial derivatives, this is just multiplying by powers of $k\tau$. The behavior of time derivatives is less apparent, so we will derive it as follows. We start from the mode function

$$\begin{aligned} \bar{\zeta}_c &= -ie^{i(\nu+\frac{1}{2})\frac{\pi}{2}} \frac{\sqrt{\pi}}{2} (-\tau)^{3/2-(n_s-1)/2} H\left(\frac{3}{2} - \frac{(n_s-1)}{2}, k\tau\right) \\ &= -\frac{1}{2\sqrt{\pi}} (-\tau)^{\frac{3-(n_s-1)}{2}} \int \frac{ds}{2\pi i} \Gamma\left(s + \frac{3-(n_s-1)}{4}\right) \Gamma\left(s - \frac{3-(n_s-1)}{4}\right) \left(-\frac{ik\tau}{2}\right)^{-2s}. \end{aligned} \quad (\text{B3})$$

A time derivative is just the logarithmic derivative with respect to τ , and therefore we find

$$\begin{aligned} \dot{\bar{\zeta}}_c &= -\frac{1}{2\sqrt{\pi}} (-\tau)^{\frac{3-(n_s-1)}{2}} \int \frac{ds}{2\pi i} \left(-2s + \frac{3}{2} - \frac{n_s-1}{2}\right) \cdot \Gamma\left(s + \frac{3-(n_s-1)}{4}\right) \\ \Gamma\left(s - \frac{3-(n_s-1)}{4}\right) \left(-\frac{ik\tau}{2}\right)^{-2s} &= \frac{1}{\sqrt{\pi}} (-\tau)^{\frac{3-(n_s-1)}{2}} \int \frac{ds}{2\pi i} \Gamma\left(s + \frac{3-(n_s-1)}{4}\right) \Gamma\left(s + \frac{1-(n_s-1)}{4}\right) \left(-\frac{ik\tau}{2}\right)^{-2s}. \end{aligned} \quad (\text{B4})$$

It is important that the poles of both Γ functions are located only at negative values of s . In the limit $k\tau \rightarrow 0$, this is just the statement that $\dot{\zeta} \rightarrow 0$, as we know from the EFT, but this is a statement that extends to any $k\tau$. This will be a key property of the Mellin representation that plays a very important role in the complete calculation.

We will compute the one loop power spectra

$$\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle = i^2 \int d^4x [H_{\text{int}}, [H_{\text{int}}, \zeta(\vec{k}) \zeta(\vec{k}')]] \quad (\text{B5})$$

where

$$S_{\text{int}} = \int dt d^3x \left[-\frac{2\lambda}{H^3} a^3 \dot{\zeta}_c^3 + \frac{\bar{\lambda}}{H^3} a \dot{\zeta}_c (\partial_i \zeta_c)^2 \right]. \quad (\text{B6})$$

This expression is just the interaction picture expanded out to second order. Let us look at various permutations for this term and analyze each of them separately via the Mellin representation. Analytic continuation in the Mellin variables will achieve the same result as continuing to $t \rightarrow t(1 \pm i\epsilon)$.

2. One loop from $\dot{\zeta}^3$

In the first example, we will include only the interaction

$$H_{\text{int}} = -\mathcal{L}_{\text{int}} = -\lambda a^3 \dot{\zeta}^3. \quad (\text{B7})$$

In practice, calculating the commutators is less desirable than calculating the contributions from the different operator orderings. We will refer to the orderings in terms of how many factors of H_{int} appear on each side of the operator, so that the (n, m) term has n factors to the left and m factors to the right.

a. (0, 2) leg

We will start with the term

$$\begin{aligned} \langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle_{(0,2)} &\equiv - \int_{-\infty}^{\tau} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \langle \zeta(\vec{k}) \zeta(\vec{k}') \\ &\times H_{\text{int}}(\tau_1) H_{\text{int}}(\tau_2) \rangle, \end{aligned} \quad (\text{B8})$$

which we will refer to as the (0, 2) piece. The other pieces are the (1, 1) and the (2, 0) pieces.

Now we can determine this contribution using Wick contractions and the mode functions to find

$$\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle = -\lambda^2 \int_{-\infty}^{\tau} d\tau_1 a^4(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^4(\tau_2) \int \Pi_i \frac{d^3 p_i}{(2\pi)^3} \delta^3(p_{123}) \delta^3(p_{456}) \langle \zeta(\vec{k}) \zeta(\vec{k}') \dot{\zeta}_{123} \dot{\zeta}_{456} \rangle. \quad (\text{B9})$$

Here $\dot{\zeta}_{123}$ involves three factors of $\dot{\zeta}_c$ carrying momenta labeled by 1, 2, 3. After the appropriate contractions, we get

$$\begin{aligned} \langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle &= -\lambda^2 \int_{-\infty}^{\tau} d\tau_1 a^4(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^4(\tau_2) \\ &\quad \int \frac{d^3 p}{(2\pi)^3} \zeta(\vec{k}, \tau) \zeta(\vec{k}', \tau) \dot{\zeta}_c^*(\vec{k}, \tau_1) \dot{\zeta}_c^*(\vec{k}', \tau_2) \dot{\zeta}_c(\vec{p}, \tau_1) \dot{\zeta}_c^*(\vec{p}, \tau_2) \dot{\zeta}_c(\vec{k} - \vec{p}, \tau_1) \dot{\zeta}_c^*(\vec{k} - \vec{p}, \tau_2) \\ &= -\lambda^2 \zeta(\vec{k}, \tau) \zeta(\vec{k}', \tau) \int_{-\infty}^{\tau} d\tau_1 a^4(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^4(\tau_2) \int \frac{d^3 p}{(2\pi)^3} \\ &\quad \times \left(\prod_{i=1}^6 \int \frac{ds_i}{(2\pi i)} \Gamma\left(s_i + \frac{3 - (n_s - 1)}{4}\right) \Gamma\left(s_i + \frac{1 - (n_s - 1)}{4}\right) \left(-\frac{ip_i \tau_i}{2}\right)^{-2s_i} \right) \\ &\quad \times \frac{1}{\pi^3} (-\tau_1)^{\frac{9-3(n_s-1)}{2}} (-\tau_2)^{\frac{9-3(n_s-1)}{2}} (-1)^{-2s_1-2s_2-2s_4-2s_6}. \end{aligned} \quad (\text{B10})$$

We want to simplify the integrals now. The advantage of Mellin representation comes from the fact that the p and τ integrals are now separately scaleless, which allows us to deal with them effectively in a dimensional regularization manner. Doing the momentum integral, we get,

$$\int \frac{d^3 p}{(2\pi)^3} p^{-2(s_3+s_4)} (k-p)^{-2(s_5+s_6)}. \quad (\text{B11})$$

We take the limit where $p \gg k$, or where the loop momenta are much greater than the internal momenta. In the other region, $p \ll k$, one can use the consistency conditions to prove that the integral only gives slow roll suppressed contributions. Hence, we have

$$\int \frac{d^3 p}{(2\pi)^3} p^{-2(s_3+s_4+s_5+s_6)} \rightarrow -\frac{i}{2\pi} \delta\left(\frac{3}{2} - (s_3 + s_4 + s_5 + s_6)\right), \quad (\text{B12})$$

where the δ function is understood as arising from a pole in the Mellin variables that we must include when performing future integrals. The result of including this pole shifts some of the poles from our Γ functions outside of the contour of integration.

Having dealt with the momenta integral, we now turn to the time integral. We first integrate over τ_2 to get

$$\int_{-\infty}^{\tau_1} d\tau_2 a^4(\tau_2) (\tau_2)^{\frac{9-3(n_s-1)}{2}} (\tau_2)^{-2(s_2+s_4+s_6)} = \frac{2(\tau_1)^{\frac{3-3(n_s-1)}{2}-2(s_2+s_4+s_6)}}{3-3(n_s-1)-4(s_2+s_4+s_6)}, \quad (\text{B13})$$

and then perform the τ_1 integral

$$\begin{aligned} &\int_{-\infty}^{\tau} d\tau_1 \frac{2(\tau_1)^{\frac{1-3(n_s-1)}{2}-2(s_1+s_3+s_5)} (\tau_1)^{\frac{3-3(n_s-1)}{2}-2(s_2+s_4+s_6)}}{3-3(n_s-1)-4(s_2+s_4+s_6)} \\ &= \frac{2\tau^{3-3(n_s-1)-2(s_1+s_2+\dots+s_6)}}{(3-3(n_s-1)-2(s_1+s_2+\dots+s_6))(3-3(n_s-1)-4(s_2+s_4+s_6))}. \end{aligned} \quad (\text{B14})$$

As $\tau \rightarrow 0$, this turns out to be a δ function as well,

$$\frac{\tau^{3-3(n_s-1)-2(s_1+s_2+\dots+s_6)}}{3-3(n_s-1)-2(s_1+s_2+\dots+s_6)} \rightarrow i\pi \delta\left(\frac{3}{2} - (s_1 + s_2 + \dots + s_6)\right), \quad (\text{B15})$$

enforcing the fact that we are only picking out contributions that survive in the late future. All together, this means the poles in Mellin space after integration are determined by the expression

$$\begin{aligned}
& \frac{1}{3-3(n_s-1)-4(s_2+s_4+s_6)} \delta\left(\frac{3}{2}-(s_1+s_2+\dots+s_6)\right) \\
& \times \prod_i \Gamma\left(s_i + \frac{3-(n_s-1)}{4}\right) \Gamma\left(s_i + \frac{1-(n_s-1)}{4}\right) \delta\left(\frac{3}{2}-s_3-s_4-s_5-s_6\right) \\
& \rightarrow \frac{1}{3-3(n_s-1)-4(s_2+s_4+s_6)} \delta(s_1+s_2) \\
& \times \prod_i \Gamma\left(s_i + \frac{3-(n_s-1)}{4}\right) \Gamma\left(s_i + \frac{1-(n_s-1)}{4}\right) \delta\left(\frac{3}{2}-s_3-s_4-s_5-s_6\right). \tag{B16}
\end{aligned}$$

Notice that in the last step, because of the loop momentum, a nonvanishing result as $\tau \rightarrow 0$ requires $s_1 + s_2 = 0$. We will focus on the poles of this expression, rather than the full one-loop result, as this is what is relevant for determining the divergences in the final result.

In Mellin space, divergences in these loop calculations manifest themselves as collisions between two or more poles. A properly regulated theory should only have simple poles, and the collision of the poles when the regulator is removed gives rise to logarithmic divergences (similar to dimensional regularization). We will show that this does not arise, as expected from the EFT description. To see this, we can first integrate over s_1 using the δ function to set $s_1 = -s_2$. We can also integrate out s_5 and set $s_5 = 3/2 - s_3 - s_4 - s_6$. Then we have

$$\begin{aligned}
& \frac{1}{3-3(n_s-1)-4(s_2+s_4+s_6)} \Gamma\left(s_2 + \frac{3-(n_s-1)}{4}\right) \Gamma\left(s_2 + \frac{1-(n_s-1)}{4}\right) \\
& \cdot \Gamma\left(-s_2 + \frac{3-(n_s-1)}{4}\right) \Gamma\left(-s_2 + \frac{1-(n_s-1)}{4}\right) \cdot \Gamma_{s_3, s_4, s_6} \\
& \cdot \Gamma\left(-s_3 - s_4 - s_6 + \frac{9-(n_s-1)}{4}\right) \Gamma\left(-s_3 - s_4 - s_6 + \frac{7-(n_s-1)}{4}\right). \tag{B17}
\end{aligned}$$

Now, we have some poles on the negative part of the number line and some poles on the positive part of the number line due to the Γ function. It is clear from inspection that the left and right poles are well separated, and a line going through $s_i^C = 0$ is all that is required. The key point was that due to the time derivative, $\dot{\zeta}^3$ has no poles on the positive number line. This ensures that none of the s_2, s_3, s_4, s_6 poles can give rise to double poles in the expression above.

We now want to calculate the finite part of the sum. We take $n_s - 1 = 0$ to simplify the calculation. The pole structure we care about is

$$\begin{aligned}
& \frac{-1)^{-2s_4-2s_6}}{3-3(n_s-1)-4(s_2+s_4+s_6)} \Gamma\left(s_2 + \frac{3-(n_s-1)}{4}\right) \Gamma\left(s_2 + \frac{1-(n_s-1)}{4}\right) \\
& \cdot \Gamma\left(-s_2 + \frac{3-(n_s-1)}{4}\right) \Gamma\left(-s_2 + \frac{1-(n_s-1)}{4}\right) \cdot \Gamma_{s_3, s_4, s_6} \\
& \cdot \Gamma\left(-s_3 - s_4 - s_6 + \frac{9-(n_s-1)}{4}\right) \Gamma\left(-s_3 - s_4 - s_6 + \frac{7-(n_s-1)}{4}\right). \tag{B18}
\end{aligned}$$

We first sum the s_3 poles, since it only has Γ matrices,

$$\int \frac{ds_3}{(2\pi i)} \Gamma\left(s_3 + \frac{3}{4}\right) \Gamma\left(s_3 + \frac{1}{4}\right) \Gamma\left(-s_3 - s_4 - s_6 + \frac{9}{4}\right) \Gamma\left(-s_3 - s_4 - s_6 + \frac{7}{4}\right) = 2^{-7+4(s_4+s_6)} \pi \Gamma(4-2(s_4+s_6)). \tag{B19}$$

Let us turn to s_2 poles next, which give us,

$$\begin{aligned}
& \frac{1}{3-4(s_2+s_4+s_6)} \Gamma\left(s_2+\frac{3}{4}\right) \Gamma\left(s_2+\frac{1}{4}\right) \Gamma\left(-s_2+\frac{3}{4}\right) \Gamma\left(-s_2+\frac{1}{4}\right) \\
&= 2\pi \frac{{}_pF_q\left[\left(1, \frac{3}{2}-(s_4+s_6), 1-(s_4+s_6)\right), \left(2-(s_4+s_6), \frac{5}{2}-(s_4+s_6)\right), 1\right]}{(4-4(s_4+s_6))(6-4(s_4+s_6))} \\
&= \frac{\pi}{4} \left[H_n\left(\frac{3}{4}-(s_4+s_6)\right) - H_n(-(s_4+s_6)) \right],
\end{aligned} \tag{B20}$$

where H_n is the Harmonic numbers, defined as

$$H_n(x) = \sum_{i=0}^x \frac{1}{i}. \tag{B21}$$

Putting these together, we get that the loop integral gives us

$$\begin{aligned}
\frac{\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle}{P_\zeta} &= -\frac{\lambda^2}{2^{10} \pi^3} \int \frac{ds_4 ds_6}{(2\pi i)^2} (-1)^{-2s_4-2s_6} 2^{4(s_4+s_6)} \Gamma(4-2(s_4+s_6)) \left[H_n\left(\frac{3}{4}-(s_4+s_6)\right) - H_n(-(s_4+s_6)) \right] \\
&\times \Gamma\left(s_4+\frac{3}{4}\right) \Gamma\left(s_4+\frac{1}{4}\right) \Gamma\left(s_6+\frac{3}{4}\right) \Gamma\left(s_6+\frac{1}{4}\right).
\end{aligned} \tag{B22}$$

While we are unable to find an analytic sum over the poles of s_4, s_6 , we can perform a numerical sum. Concretely, we can define

$$\begin{aligned}
\xi &= \int \frac{ds_4 ds_6}{(2\pi i)^2} (-1)^{-2s_4-2s_6} 2^{4(s_4+s_6)} \left[H_n\left(\frac{3}{4}-(s_4+s_6)\right) - H_n(-(s_4+s_6)) \right] \\
&\times \Gamma(4-2(s_4+s_6)) \Gamma\left(s_4+\frac{3}{4}\right) \Gamma\left(s_4+\frac{1}{4}\right) \Gamma\left(s_6+\frac{3}{4}\right) \Gamma\left(s_6+\frac{1}{4}\right).
\end{aligned} \tag{B23}$$

First, we compute the first n_1 poles of s_4 coming from $\Gamma(s_4+3/4)$. We call this function $\xi_1(n_1)$. The sum of residues of these poles gives us

$$\begin{aligned}
\xi_1(n_1) &= \sum_{m=1}^{n_1} \int \frac{ds_6}{2\pi i} (-1)^{-2s_6} 2^{4s_6} \Gamma\left(s_6+\frac{3}{4}\right) \Gamma\left(s_6+\frac{1}{4}\right) \\
&\times \left[(-1)^{3/2+2m} \frac{(-1)^m}{m!} 2^{-3-4m} \left(H_n\left(\frac{6}{4}+m-s_6\right) - H_n\left(\frac{3}{4}+m-s_6\right) \right) \cdot \Gamma\left(-m-\frac{1}{2}\right) \Gamma\left(\frac{11}{2}+2m-2s_6\right) \right] \\
&= \int \frac{ds_6}{2\pi i} (-1)^{-2s_6} 2^{4s_6} \Gamma\left(s_6+\frac{3}{4}\right) \Gamma\left(s_6+\frac{1}{4}\right) \sum_{m=1}^{n_1} f_1(m).
\end{aligned} \tag{B24}$$

Now, we sum the first n_1 poles of s_4 coming from $\Gamma(s_4+1/4)$. We call this function $\xi_2(n_1)$. We get that

$$\begin{aligned}
\xi_2(n_1) &= \sum_{m=1}^{n_1} \int \frac{ds_6}{2\pi i} (-1)^{-2s_6} 2^{4s_6} \Gamma\left(s_6+\frac{3}{4}\right) \Gamma\left(s_6+\frac{1}{4}\right) \left[(-1)^{1/2+2m} \frac{(-1)^m}{m!} 2^{-1-4m} \left(H_n(1+m-s_6) - H_n\left(\frac{1}{4}+m-s_6\right) \right) \right. \\
&\quad \left. \cdot \Gamma\left(-m+\frac{1}{2}\right) \Gamma\left(\frac{9}{2}+2m-2s_6\right) \right] \\
&= \int \frac{ds_6}{2\pi i} (-1)^{-2s_6} 2^{4s_6} \Gamma\left(s_6+\frac{3}{4}\right) \Gamma\left(s_6+\frac{1}{4}\right) \sum_{m=1}^{n_1} f_2(m).
\end{aligned} \tag{B25}$$

We of course have that $\xi = \lim_{n_1 \rightarrow \infty} [\xi_1(n_1) + \xi_2(n_1)]$. Now, we want to simplify ξ_1 and ξ_2 further, by also summing over the s_6 poles. Because s_4, s_6 were symmetric, let us also sum over the first n_1 poles for s_6 . We first sum over the $\Gamma(s_6+3/4)$ poles, and then $\Gamma(s_6+1/4)$ poles. Let us define the new functions as

$$\begin{aligned}\xi'_1(n_1) &= \sum_{l=1}^{n_1} (-1)^{3/2+2l} 2^{-3-4l} \frac{(-1)^l}{l!} \Gamma\left(-l - \frac{1}{2}\right) \left(\sum_{m=1}^{n_1} f_1(m) + f_2(m)\right), \\ \xi'_2(n_1) &= \sum_{l=1}^{n_1} (-1)^{1/2+2l} 2^{-1-4l} \frac{(-1)^l}{l!} \Gamma\left(-l + \frac{1}{2}\right) \left(\sum_{m=1}^{n_1} f_1(m) + f_2(m)\right).\end{aligned}\quad (\text{B26})$$

Finally, then, we have that

$$\xi = \lim_{n \rightarrow \infty} (\xi'_1(n) + \xi'_2(n)). \quad (\text{B27})$$

Now, this form makes it conducive to numerically evaluate it. We comment that we see the sum start to asymptote at around $n_1 = 120$, to a value of

$$\xi \approx -2.972 \times 10^9. \quad (\text{B28})$$

Then, the loop result is given by

$$\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle'_{1\text{-loop}} = P_\zeta(k) \left[-\frac{\lambda^2}{2^{10} \pi^3} \xi \right] = -9.36 \times 10^4 \lambda^2 P_\zeta. \quad (\text{B29})$$

We just comment that this is a scheme-dependent finite contribution. It is important to point out that this is not equivalent to the result of [88], which points $\xi \propto 1/(n_s - 1)$, which would be divergent in this case. While we would still like to have a better understanding of why this constant is large, it does not appear to signal a deeper issue with perturbation theory.

As we have seen, we chose contours such that $s_1 = -s_2$, and we got a finite scheme-dependent result.

This counterterm can be absorbed into the six derivative operators. Choosing contours, however, is equivalent to choosing schemes. We could have just recognized, when we encountered $(k\tau)^{-2s_1-2s_2}$, that since all the Γ function poles lie on the negative axis, we can never get a function that behaves like τ^0 . Thus, if we do not enforce the $s_1 = -s_2$ constraint coming from the time integral (which we are not forced to do anyways), we will immediately see that no contribution survives in the $\tau \rightarrow 0$ limit, and we get that the loop integral is 0, as we would expect from a dim reg calculation to regulate power law divergences. Regardless of whether we choose a scheme where we get a finite contribution or 0, we see that no log will be generated, and hence, Mellin representation shows that the loop is regulated without any need for RG.

b. (2, 0) leg

The (2, 0) leg will be given by the complex conjugate of the (0, 2) leg. Since we have already established that the (0, 2) leg has no poles, the (2, 0) leg will have no poles either. Moreover, we will again get the same finite scheme-dependent answer from the loop integral.

c. (1, 1) leg

The (1, 1) leg is given by

$$\begin{aligned}\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle' &= - \int_{-\infty}^{\tau} d\tau_1 \int_{-\infty}^{\tau} d\tau_2 \langle H_{\text{int}}(\tau_1) \zeta(\vec{k}) \zeta(\vec{k}') H_{\text{int}}(\tau_2) \rangle' \\ &= -\lambda^2 \int_{-\infty}^{\tau} d\tau_1 a^4(\tau_1) \int_{-\infty}^{\tau} d\tau_2 a^4(\tau_2) \int \Pi_i \frac{d^3 p_i}{(2\pi)^3} \delta^3(p_{123}) \delta^3(p_{456}) \langle \dot{\zeta}_{123} \zeta(\vec{k}) \zeta(\vec{k}') \dot{\zeta}_{456} \rangle.\end{aligned}\quad (\text{B30})$$

After Wick contractions, we will get

$$\begin{aligned}\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle &= -\lambda^2 \int_{-\infty}^{\tau} d\tau_1 a^4(\tau_1) \int_{-\infty}^{\tau} d\tau_2 a^4(\tau_2) \\ &\quad \times \int \frac{d^3 p}{(2\pi)^3} \dot{\zeta}_c(\vec{k}, \tau_1) \zeta^*(\vec{k}, \tau) \zeta(\vec{k}, \tau) \dot{\zeta}_c^*(\vec{k}, \tau_2) \dot{\zeta}_c(\vec{p}, \tau_1) \dot{\zeta}_c^*(\vec{p}, \tau_2) \dot{\zeta}_c(\vec{k} - \vec{p}, \tau_1) \dot{\zeta}_c^*(\vec{k} - \vec{p}, \tau_2) \\ &= -\lambda^2 \zeta(\vec{k}, \tau) \zeta^*(\vec{k}', \tau) \int_{-\infty}^{\tau} d\tau_1 a^4(\tau_1) \int_{-\infty}^{\tau} d\tau_2 a^4(\tau_2) \int \frac{d^3 p}{(2\pi)^3} \\ &\quad \times \prod_{i=1}^6 \int \frac{ds_i}{(2\pi i)} \Gamma\left(s_i + \frac{3 - (n_s - 1)}{4}\right) \Gamma\left(s_i + \frac{1 - (n_s - 1)}{4}\right) \left(-\frac{ip_i \tau_i}{2}\right)^{-2s_i} \\ &\quad \times (-1)^{-2s_2-2s_4-2s_6} \frac{1}{\pi^3} (-\tau_1)^{\frac{9-3(n_s-1)}{2}} (-\tau_2)^{\frac{9-3(n_s-1)}{2}}.\end{aligned}\quad (\text{B31})$$

Compared to the $(2, 0)$ leg, the major difference is in the time integral. Instead of a nested time integral, we get two time integrals running from $-\infty \rightarrow \tau$. Thus we have

$$\int_{-\infty}^{\tau} d\tau_2 a^4(\tau_2) (\tau_2)^{\frac{9-3(n_s-1)}{2}} (\tau_2)^{-2(s_2+s_4+s_6)} = \frac{2(\tau)^{\frac{3-3(n_s-1)}{2}-2(s_2+s_4+s_6)}}{3-3(n_s-1)-4(s_2+s_4+s_6)}, \quad (\text{B32})$$

and the same for the τ_1 integral. The momentum integral remains the same,

$$\int \frac{d^3 p}{(2\pi)^3} p^{-2(s_3+s_4+s_5+s_6)} \rightarrow -\frac{i}{2\pi} \delta\left(\frac{3}{2} - (s_3 + s_4 + s_5 + s_6)\right). \quad (\text{B33})$$

Now, we see that since we have all the poles on the negative axis, we cannot get any time dependence in the $\tau \rightarrow 0$ limit. Thus, Mellin representation sets this term to 0.

3. One loop from $\dot{\xi}(\partial\dot{\xi})^2$

Now we would like to repeat the above analysis of the one-loop power spectrum for the $\bar{\lambda} a \dot{\xi}_c (\partial_i \dot{\xi}_c)^2$ interaction. We will focus only on showing the absence of divergences. We focus on the $(2,0)$ one-loop term, written schematically as

$$\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle = -\bar{\lambda}^2 \int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^2(\tau_2) \int \Pi_i \frac{d^3 p_i}{(2\pi)^3} \delta^3(k_{123}) \delta^3(k_{456}) \langle \zeta(p) \zeta(p') \dot{\xi}_1 (\partial_i \dot{\xi})_{23}^2 \dot{\xi}_4 (\partial_i \dot{\xi})_{56}^2 \rangle. \quad (\text{B34})$$

As we saw in the previous example, the other (m, n) terms have the same pole structure, and hence showing that the poles are well separated in this example will take care of all the poles. Of course, we have multiple Wick contractions that are possible. We explain in detail the pole structure of the two possible Wick contractions.

a. First Wick contraction

The first contraction will focus on terms where both the $\dot{\xi}$ contract with the external $\zeta(\vec{k}), \zeta(\vec{k}')$. Then, we get

$$\begin{aligned} \langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle &= -\bar{\lambda}^2 \int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^2(\tau_2) (k \cdot (k-p))^2 \\ &\quad \times \int \frac{d^3 p}{(2\pi)^3} \zeta(\vec{k}, \tau) \zeta(\vec{k}', \tau) \dot{\xi}_c^*(\vec{k}, \tau_1) \dot{\xi}_c^*(\vec{k}', \tau_2) \zeta_c(\vec{p}, \tau_1) \zeta_c^*(\vec{p}, \tau_2) \zeta_c(\vec{k}-\vec{p}, \tau_1) \zeta_c^*(\vec{k}-\vec{p}, \tau_2) \\ &= -\bar{\lambda}^2 \zeta(\vec{k}, \tau) \zeta(\vec{k}', \tau) \int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^2(\tau_2) \int \frac{d^3 p}{(2\pi)^3} \\ &\quad \times \left(\prod_{i=1}^6 \int \frac{ds_i}{(2\pi i)} \Gamma\left(s_i + \frac{3-(n_s-1)}{4}\right) \Gamma\left(s_i - \frac{3-(n_s-1)}{4}\right) \left(-\frac{ip_i \tau_i}{2}\right)^{-2s_i} \right) \\ &\quad \times (-1)^{-2s_1-2s_2-2s_4-2s_6} (-\tau_1)^{\frac{9-3(n_s-1)}{2}} (-\tau_2)^{\frac{9-3(n_s-1)}{2}} (k \cdot (k-p))^2. \end{aligned} \quad (\text{B35})$$

As before, we can evaluate the loop integral assuming the internal momentum is hard ($p \gg k$), so that

$$\int \frac{d^3 p}{(2\pi)^3} p^{4-2(s_3+s_4+s_5+s_6)} \rightarrow -\frac{i}{2\pi} \delta\left(\frac{7}{2} - s_3 + s_4 + s_5 + s_6\right). \quad (\text{B36})$$

We also calculate the integrals as before, to find

$$\int_{-\infty}^{\tau_1} d\tau_2 a^2(\tau_2) (\tau_2)^{\frac{9-3(n_s-1)}{2}} (\tau_2)^{-2(s_2+s_4+s_6)} = \frac{2(\tau_1)^{\frac{7-3(n_s-1)}{2}-2(s_2+s_4+s_6)}}{7-3(n_s-1)-4(s_2+s_4+s_6)}, \quad (\text{B37})$$

and

$$\int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) (\tau_1)^{\frac{9-3(n_s-1)}{2}} (\tau_1)^{-2(s_1+s_3+s_5)} = \frac{\tau^{7-3(n_s-1)-2(s_1+s_2+\dots+s_6)}}{7-3(n_s-1)-2(s_1+s_2+\dots+s_6)}. \quad (\text{B38})$$

Using the δ function from the momenta integral, we again reduce to a $\tau^{-2(s_1+s_2)}$ dependence. Since s_1, s_2 are the poles associated with the ζ term, these poles are negative, and the integral is set to 0.

b. Other Wick contraction

To convince the reader that the same pattern holds, let us look at the term where the $\partial_i \zeta$ contracts with the external ζ . We get

$$\begin{aligned} \langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle &= -\bar{\lambda}^2 \int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^2(\tau_2) (p \cdot k)^2 \\ &\times \int \frac{d^3 k}{(2\pi)^3} \zeta(\vec{k}, \tau) \zeta(\vec{k}', \tau) \zeta_c(\vec{k} - \vec{p}, \tau_1) \zeta_c^*(\vec{k} - \vec{p}, \tau_2) \zeta_c^*(\vec{p}, \tau_1) \zeta_c(\vec{p}, \tau_2) \zeta_c(\vec{k}, \tau_1) \zeta_c^*(\vec{k}, \tau_2) \\ &= -\bar{\lambda}^2 \zeta(\vec{p}, \tau) \zeta(\vec{p}, \tau) \int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 a^2(\tau_2) \int \frac{d^3 k}{(2\pi)^3} (-\tau_1)^{\frac{9-3(n_s-1)}{2}} (-\tau_2)^{\frac{9-3(n_s-1)}{2}} (p \cdot k)^2 \\ &\times \prod_{i=1}^6 (-1)^{-2s_2-2s_3-2s_4-2s_6} \int \frac{ds_i}{(2\pi i)} \Gamma\left(s_i + \frac{3-(n_s-1)}{4}\right) \\ &\times \Gamma\left(s_i - \frac{3-(n_s-1)}{4} \middle| s_i + \frac{1-(n_s-1)}{4}\right) \left(-\frac{ik_i \tau_i}{2}\right)^{-2s_i}, \end{aligned} \quad (\text{B39})$$

where $\Gamma(a|b)$ picks the appropriate Γ function $\Gamma(a)$ or $\Gamma(b)$ for the given s_i , depending on whether it is a ζ or ζ_c . Now, it is time to do the integrals in various fashion. We will first start with the momentum integral, which reads

$$\int \frac{d^3 p}{(2\pi)^3} p^{2-2(s_3+s_4)} (k-p)^{-2(s_5+s_6)}. \quad (\text{B40})$$

We take the hard limit, ($p \gg k$), giving us

$$\int \frac{d^3 p}{(2\pi)^3} p^{2-2(s_3+s_4+s_5+s_6)} \rightarrow -\frac{i}{2\pi} \delta\left(\frac{5}{2} - (s_3 + s_4 + s_5 + s_6)\right). \quad (\text{B41})$$

Now, we proceed to do the time integral. The τ_2 integral gives us

$$\int_{-\infty}^{\tau_1} d\tau_2 a^2(\tau_2) (\tau_2)^{\frac{9-3(n_s-1)}{2}} (\tau_2)^{-2(s_2+s_4+s_6)} = \frac{2(\tau_1)^{\frac{7-3(n_s-1)}{2}-2(s_2+s_4+s_6)}}{7-3(n_s-1)-4(s_2+s_4+s_6)} \quad (\text{B42})$$

as from before. The τ_1 integral then gives us

$$\int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) (\tau_1)^{\frac{9-3(n_s-1)}{2}} (\tau_1)^{-2(s_1+s_3+s_5)} = \frac{\tau^{7-3(n_s-1)-2(s_1+s_2+\dots+s_6)}}{7-3(n_s-1)-2(s_1+s_2+\dots+s_6)}. \quad (\text{B43})$$

Again, we have that enforcing the momentum integral δ function, we get $(k\tau)^{1-2(s_1+s_2)}$, which is set to 0 by our Mellin prescription in the $\tau \rightarrow 0$ limit.

APPENDIX C: FLAT SPACETIME EXAMPLE

Let us take a flat spacetime example to understand the renormalization to the propagator due to a ϕ^3 interaction term. Modes deep inside the de Sitter horizon effectively

behave as if they are in flat spacetime, so understanding the field in flat spacetime will guide our de Sitter result as well. The Lagrangian we consider is

$$\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} (\partial_i \phi)^2 + \frac{g}{\Lambda^2} \dot{\phi}^3, \quad (\text{C1})$$

where Λ is the UV scale where the EFT description breaks down. Now, we calculate the corrections to the in-in

propagator. The interaction picture operator in flat space is given by

$$\phi(\vec{k}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{k}} (a_{\vec{k}} e^{ikx-ikt} + a_{\vec{k}}^\dagger e^{-ikx+ikt}). \quad (C2)$$

At lowest order, the correction to $\langle \phi(\vec{k})\phi(\vec{k}') \rangle$ is given by

$$\begin{aligned} \langle \phi(\vec{k})\phi(\vec{k}') \rangle &= i^2 \int d^3x_1 dt_1 \int d^3x_2 dt_2 \\ &\times [H_{\text{int}}(x_2, t), [H_{\text{int}}(x_1, t), \phi(\vec{k})\phi(\vec{k}')]], \end{aligned} \quad (C3)$$

where $H_{\text{int}} = \frac{g}{\Lambda^2} \dot{\phi}^3$. Considering the (0, 2) contraction, we have

$$\begin{aligned} \langle \phi(\vec{k})\phi(\vec{k}') \rangle &= - \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 \\ &\times \langle \phi(\vec{k})\phi(\vec{k}') H_{\text{int}}(t_1) H_{\text{int}}(t_2) \rangle \\ &= - \frac{g^2}{\Lambda^4} e^{-2ikt} \int \frac{d^3p}{(2\pi)^3} \frac{p^2 q^2 k^2}{k^2 q p} \int_{-\infty}^t dt_1 e^{i(k-q-p)t_1} \\ &\times \int_{-\infty}^{t_1} dt_2 e^{i(k+q+p)t_2}, \end{aligned} \quad (C4)$$

where $q = |\vec{p} - \vec{k}|$. Doing the time integrals, we get

$$\langle \phi(\vec{k})\phi(\vec{k}') \rangle = \frac{g^2}{\Lambda^4} \int \frac{d^3p}{(2\pi)^3} \frac{qk}{2k(k+q+p)}. \quad (C5)$$

The free theory correlation function is given by

$$\langle \phi(k)\phi(k') \rangle = \frac{1}{k}. \quad (C6)$$

Now, just from dimensional analysis, since the only scale in the p integral is k , we will have that, up to scaleless

integrals, the term that will be produced, when expanded in p/k is (large loop momenta)

$$\langle \phi(\vec{k})\phi(\vec{k}') \rangle \sim \left(\alpha \frac{g^2 k^3}{\Lambda^4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^3} \right), \quad (C7)$$

where α is some number. Similarly, if we look at the (1,1) leg, we will get that the loop correction is given by

$$\begin{aligned} \langle \phi(\vec{k})\phi(\vec{k}') \rangle &= - \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 \\ &\times \langle \phi(\vec{k})\phi(\vec{k}') H_{\text{int}}(t_1) H_{\text{int}}(t_2) \rangle \\ &= \frac{g^2}{\Lambda^4} \int \frac{d^3p}{(2\pi)^3} \frac{qk}{(k+q+p)^2}. \end{aligned} \quad (C8)$$

Similarly, for large internal momenta ($p \gg k$), we can extract a term

$$\langle \phi(\vec{k})\phi(\vec{k}') \rangle \sim \left(\beta \frac{g^2 k^3}{\Lambda^4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^3} \right). \quad (C9)$$

We can guess the local operator that absorbs these counterterms. Let us take the interaction term

$$S_{\text{int}} = \int d^4x \frac{g^2}{\Lambda^4} \phi \partial_i^6 \phi. \quad (C10)$$

This produces a correction of the form

$$\langle \phi(\vec{k})\phi(\vec{k}') \rangle = i \frac{g^2}{\Lambda^4} \frac{e^{-2ikt}}{k^2} \cdot k^6 \cdot \int dt e^{2ikt} = \frac{g^2}{\Lambda^4} k^3, \quad (C11)$$

which is exactly the structure required to absorb the divergence from the loop integral. Note that similarly, we can also have $\phi \partial_i^6 \phi$ as a counterterm (these are the same operators, as one can see using the EOM). This shows that the loops generated by our interaction Lagrangian are absorbed by six derivative counterterms.

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