

Scale invariant radiative neutrino mass model

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(Received 15 February 2026; revised 29 June 2026; accepted 7 July 2026; published 20 July 2026)

We propose a scale invariant radiative neutrino mass model with custodial symmetry by introducing two real singlet scalars to the scotogenic model. Masses of an inert doublet scalar and right-handed neutrinos are induced by a vacuum expectation value (VEV) of a singlet scalar caused through the Coleman-Weinberg mechanism. It violates spontaneously both the custodial symmetry and the scale invariance. The weak scale can take a suppressed value compared with the singlet scalar VEV because of the custodial symmetry. In this framework we study phenomenological consequences for neutrino mass, dark matter, and baryon number asymmetry by assuming a texture for neutrino Yukawa couplings. Since the required dark matter abundance cannot be explained by a neutral component of the inert doublet scalar in that case, the lightest right-handed neutrino should be dark matter. Mass of the dark matter is predicted to be less than $O(1)$ MeV and baryon number asymmetry could be explained through resonant leptogenesis.

DOI: [10.1103/9r8v-12cn](https://doi.org/10.1103/9r8v-12cn)

I. INTRODUCTION

The standard model (SM) is a successful theoretical framework that can explain experimental results known by now although some unsatisfactory points remain in it. The fatal ones among them are that it cannot explain both existence of neutrino masses and dark matter (DM) and also the asymmetry of matter and antimatter in the Universe. These are confirmed through various experiments and observations [1–3]. It suggests that the SM should be extended so as to explain them at least. Since the SM has a robust structure, its extension seems to be desirable to give a simultaneous explanation for them without causing any contradictions.

As such an example, we have the scotogenic model [4], which is a simple extension of the SM with an inert doublet scalar η and three right-handed neutrinos N_j . If these new contents are assumed to have a Z_2 odd charge and the SM contents have its even charge, these three problems can be simultaneously solved under the assumption that the Z_2 symmetry is exact [5]. Unfortunately, the model cannot give any explanation for mass scales contained in it. The model is characterized by three mass scales, that is, the ordinary weak scale whose origin is a vacuum expectation value (VEV) of the Higgs scalar, the mass of the inert doublet scalar, and the one of the right-handed neutrinos.

A typical feature of the model is that the latter two scales could be taken at a TeV region. However, these are fixed by hand and the well-known hierarchy problem is left as an untouchable problem in the model.

Recently, an interesting solution for the hierarchy problem of the SM has been proposed as an extended model based on both the classical scale invariance and the custodial symmetry [6,7]. In the model given in [7], the scalar sector is extended by two real singlet scalars, and the Higgs doublet scalar is assumed to compose a 5-plet of custodial symmetry $SO(5)$ with one of these singlet scalars. The Higgs scalar behaves as Nambu-Goldstone bosons due to the spontaneous breaking of this custodial symmetry at an intermediate scale. The breaking is caused by a VEV of the singlet scalar due to the Coleman-Weinberg mechanism [8]. Since negative squared Higgs mass can be induced radiatively through the explicit breaking of the custodial symmetry, the weak scale could be much smaller than the intermediate scale thanks to the custodial symmetry.

In this paper, we consider to apply this idea to the scotogenic model by introducing two Z_2 even singlet scalars. This extended model may be considered as a UV model, which can give a solution to the problem in the scotogenic model. The above mentioned two mass scales, that is, the inert doublet scalar mass and the right-handed neutrino mass, are considered to be generated through the couplings with the singlet scalar, which causes the spontaneous breaking of the custodial symmetry. Since these couplings violate the custodial symmetry explicitly, they have to be constrained so that the weak scale is generated as an appropriately suppressed one. Quartic couplings of the inert doublet scalar in the scalar potential are also constrained for the same reason although they play a

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crucial role to reduce the DM abundance if DM is a neutral component of the inert doublet scalar. These suggest that phenomenological consequences could be largely changed from the usual ones of the scotogenic model. As such examples, we discuss DM and leptogenesis by assuming a texture of neutrino Yukawa couplings. We find that DM should be a lightest right-handed neutrino. Mass degeneracy among the inert doublet scalar and the right-handed neutrinos, which are required for the suitable generation of the weak scale makes the resonant leptogenesis work well.

The remaining parts are organized as follows. In Sec. II, we define the model studied in this paper. We address how the weak scale can be induced appropriately and what constraints should be imposed on the custodial symmetry violating couplings. In Sec. III, we discuss consequences caused by such constraints for the neutrino mass generation, the DM abundance, and the baryon number asymmetry through leptogenesis. We summarize the paper in Sec. IV.

II. SCALE INVARIANT SCOTOGENIC MODEL

A model considered in this paper is an extension of the scotogenic model with two real singlet scalars ϕ and S . It is assumed to have Z_2 invariance for which the inert doublet scalar η and the right-handed neutrinos N_j are assumed to have its odd charge but two new singlet scalars have its even charge as the SM contents. We also impose on the model both the classical scale invariance and the custodial symmetry $SO(5)$. The Higgs doublet scalar H and ϕ are assumed to transform as a 5-plet of this $SO(5)$.

The model is defined by renormalizable scalar potential V and Yukawa couplings of N_j in the leptonic sector as

$$\begin{aligned}
 V = & \lambda_{c1} \left[(H^\dagger H) + \frac{1}{2} \phi^2 \right]^2 + \frac{1}{2} \lambda_{c2} \left[(H^\dagger H) + \frac{1}{2} \phi^2 \right] S^2 \\
 & + \frac{1}{4!} \lambda_S S^4 + \lambda_\eta (\eta^\dagger \eta)^2 \\
 & + \frac{1}{2} \lambda_{\eta\phi} \phi^2 (\eta^\dagger \eta) + \frac{\tilde{\lambda}_5}{2} [(H^\dagger \eta)^2 + (\eta^\dagger H)^2], \\
 -\mathcal{L}_Y = & \sum_{j=1}^3 \left(\sum_{\alpha=e,\mu,\tau} h_{\alpha j} \bar{\ell}_\alpha \eta N_j + \frac{1}{2} y_{N_j} \phi \bar{N}_j N_j^c \right) + \text{H.c.}, \quad (1)
 \end{aligned}$$

where ℓ_α is a left-handed doublet lepton. Quartic scalar couplings $S^3\phi$ and $S\phi^3$ are not included in the scalar potential since they violate the custodial symmetry. Both quartic couplings $\tilde{\lambda}_3(H^\dagger H)(\eta^\dagger \eta)$ and $\tilde{\lambda}_4(H^\dagger \eta)(\eta^\dagger H)$ in the ordinary scotogenic model are not included since they also violate it. On the other hand, the custodial symmetry is explicitly violated by gauge interactions and Yukawa couplings in the SM. Although quartic couplings in the second line of V and Yukawa couplings $y_{N_j} \phi \bar{N}_j N_j^c$ of the right-handed neutrinos violate it, they are included as important terms. These couplings not only determine the mass of η and N_j through the VEV of ϕ but also play crucial

roles for the generation of the weak scale as shown later. If we rewrite the potential V as

$$\begin{aligned}
 V_0 = & \lambda_H (H^\dagger H)^2 + \lambda_{H\phi} \phi^2 (H^\dagger H) + \frac{1}{4} \lambda_\phi \phi^4 \\
 & + \frac{1}{2} \lambda_{HS} S^2 (H^\dagger H) + \frac{1}{4} \lambda_{\phi S} \phi^2 S^2 \\
 & + \frac{1}{4!} \lambda_S S^4 + \lambda_\eta (\eta^\dagger \eta)^2 + \frac{1}{2} \lambda_{\eta\phi} \phi^2 (\eta^\dagger \eta) \\
 & + \frac{\tilde{\lambda}_5}{2} [(H^\dagger \eta)^2 + (\eta^\dagger H)^2], \quad (2)
 \end{aligned}$$

the custodial symmetry imposes the conditions

$$\lambda_H = \lambda_{H\phi} = \lambda_\phi = \lambda_{c1}, \quad \lambda_{HS} = \lambda_{\phi S} = \lambda_{c2} \quad (3)$$

at a cutoff scale which we take as the Planck scale in the present study. Conditions $\lambda_H, \lambda_\phi, \lambda_S, \lambda_\eta > 0$ should be satisfied for the stability of the potential at least.

If the spontaneous symmetry breaking of the custodial symmetry is caused by a nonzero VEV of ϕ due to quantum effects at some intermediate scale, λ_ϕ should be small enough at that scale. Such a situation is expected to happen in a case where V_0 has a flat direction determined mainly by ϕ [9]. Here, we consider background fields of the scalars, which are represented as S_b, η_b, ϕ_b , and H_b . Components of the last one are expressed as $\frac{1}{\sqrt{2}} H_{bi} (i = 1-4)$. In the case $S_b = \eta_b = 0$, V_0 has a flat direction determined by ϕ_b and H_b under a condition

$$2\lambda_H (H_b^\dagger H_b) + \lambda_{H\phi} \phi_b^2 = 0, \quad \lambda_H = \frac{\lambda_{H\phi}^2}{\lambda_\phi}. \quad (4)$$

This condition requires $\lambda_{H\phi} < 0$ and the first one may be represented as

$$\begin{aligned}
 H_{b1} = H_{b2} = H_{b3} = 0, \quad H_{b4} &= \sqrt{\frac{-\lambda_{H\phi}}{\lambda_H - \lambda_{H\phi}}} v, \\
 \phi_b &= \sqrt{\frac{\lambda_H}{\lambda_H - \lambda_{H\phi}}} v, \quad (5)
 \end{aligned}$$

where v is defined as $v = \sqrt{2H_b^\dagger H_b + \phi_b^2}$. If $\lambda_H \gg |\lambda_{H\phi}|$ is satisfied, H_{b4} can be much smaller than ϕ_b .

We focus our study on this flat direction defined by nonzero background fields ϕ_b and H_{b4} . The fields on this background can get mass depending on them. If we define x as $x \equiv H_{b4}/\phi_b$, mass eigenvalues of six scalar contents are found to be

$$(\lambda_{H\phi} + \lambda_H x^2) \phi_b^2, \quad \frac{1}{2} (\lambda_{\phi S} + \lambda_{HS} x^2) \phi_b^2 \quad (6)$$

for $H_{1,2,3}$ and S , and also the ones of the mass matrix

$$\begin{pmatrix} 3\lambda_\phi + \lambda_{H\phi}x^2 & 2\lambda_{H\phi}x \\ 2\lambda_{H\phi}x & \lambda_{H\phi} + 3\lambda_Hx^2 \end{pmatrix} \phi_b^2 \quad (7)$$

for (ϕ, H_4) . Mass of the inert doublet η and the right-handed neutrino N_j is, respectively, given as

$$m_\eta^2 = \frac{1}{2}\lambda_{\eta\phi}\phi_b^2, \quad M_j = y_{N_j}\phi_b, \quad (8)$$

where a contribution from the $\tilde{\lambda}_5$ term is not explicitly written in the formula of m_η^2 . Using these field dependent mass eigenvalues, the one-loop effective potential can be expressed as

$$V_{\text{eff}} = V_0 + V_1, \quad V_1 = \sum_i \frac{n_i(-1)^{2s_i}}{64\pi^2} m_i^4 \left(\ln \frac{m_i^2}{\mu^2} - \frac{3}{2} \right), \quad (9)$$

where n_i and s_i are a degree of freedom and a spin of the field i , respectively.

We are interested in a vacuum of the model which satisfies $x \ll 1$. Such a vacuum can be determined as a minimum of the effective potential V_{eff} , which can be considered as a function of H_b and $\phi_b(H_b)$ [6,7]. It can be found by solving the potential minimum condition

$$\frac{dV_{\text{eff}}}{dH_b} = \frac{\partial V_{\text{eff}}}{\partial H_b} + \frac{d\phi_b}{dH_b} \frac{\partial V_{\text{eff}}}{\partial \phi_b} = 0. \quad (10)$$

By solving this equation for ϕ_b at $H_b = 0$, we find a solution ϕ_0 under an assumption $\lambda_{1c} \ll \lambda_{2c}$ as

$$\begin{aligned} \ln \frac{\phi_0^2}{\mu^2} &= \frac{-64\pi^2\lambda_\phi - \lambda_{\phi S}^2 \left(\ln \frac{\lambda_{\phi S}}{2} - 1 \right) + \mathcal{A}}{\lambda_{\phi S}^2 + 16\lambda_{H\phi}^2 + 36\lambda_\phi^2 + 4\lambda_{\eta\phi}^2 - 8\sum_j y_{N_j}^4} \\ &\simeq -64\pi^2 \frac{\lambda_\phi}{\lambda_{\phi S}^2} - \ln \frac{\lambda_{\phi S}}{2} + 1, \end{aligned} \quad (11)$$

where $\mathcal{A}$ is represented as¹

$$\begin{aligned} \mathcal{A} &= -16\lambda_{H\phi}^2 (\ln \lambda_{H\phi} - 1) - 36\lambda_\phi^2 (\ln 3\lambda_\phi - 1) \\ &\quad - 4\lambda_{\eta\phi}^2 \left(\ln \frac{\lambda_{\eta\phi}}{2} - 1 \right) + 8\sum_j y_{N_j}^4 (\ln y_{N_j}^2 - 1). \end{aligned}$$

If η and $N_{2,3}$ have nearly degenerate mass, Eq. (8) suggests $\lambda_{\eta\phi} \sim 2y_{N_{2,3}}^2$ and then the last two terms in both $\mathcal{A}$ and a denominator of Eq. (11) cancel each other in the case $y_{N_1} \ll y_{N_{2,3}}$.² In that case, their contribution could be suppressed to the same order or less compared with the one from $\lambda_{\phi S}$ when $\lambda_\phi, |\lambda_{H\phi}| \ll \lambda_{\phi S}$ is satisfied. We can expect that the VEV v_ϕ of ϕ described by ϕ_0 could take a value in TeV regions if couplings λ_ϕ and $\lambda_{\phi S}$ are fixed appropriately.

In order to determine a VEV of H , we consider a perturbative expansion of the effective potential V_{eff} with respect to x around the minimum ϕ_0 given in Eq. (11). For this purpose, we consider expanding it by ϵ , which is formally introduced by replacing of x and $\lambda_{H\phi}$ with ϵx and $\epsilon^2 \lambda_{H\phi}$, respectively [6,7]. This expansion guarantees the condition (4) since this replacement keeps it unchanged. We pick up higher order terms of x by applying this expansion method up to ϵ^4 terms. As a result of this procedure, we can derive the effective scalar potential $\mathcal{V}$ at μ , which includes the scalar potential of the ordinary scotogenic model as its part. Such scalar potential $\mathcal{V}$ is found to be written as

$$\begin{aligned} \mathcal{V} &= \left(\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_\phi \right) v_\phi^2 (H^\dagger H) + \left(\lambda_H - \frac{\lambda_{HS}^2}{\lambda_{\phi S}^2} \lambda_\phi + \frac{\lambda_{HS}^2}{64\pi^2} \right) (H^\dagger H)^2 + \lambda_\eta (\eta^\dagger \eta)^2 + \frac{\tilde{\lambda}_5}{2} [(H^\dagger \eta)^2 + (\eta^\dagger H)^2] + \frac{\lambda_{\eta\phi}}{2} v_\phi^2 (\eta^\dagger \eta) \\ &\quad + \sum_k \frac{n_k(-1)^{2s_k}}{64\pi^2} m_k^4 \left(\ln \frac{m_k^2}{\mu_0^2} - C_k \right) - \frac{\lambda_{\phi S}^2}{512\pi^2} v_\phi^4 + \frac{3\lambda_{\phi S}^2}{512\pi^2} \phi^4 + \frac{1}{2} \tilde{\beta}_{\lambda_\phi} v_\phi^2 \phi^2 + 2 \left(\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_\phi + \frac{1}{64\pi^2} \lambda_{\phi S} \lambda_{HS} \right) v_\phi \phi (H^\dagger H) \\ &\quad + \left(\lambda_{H\phi} + \frac{7}{64\pi^2} \lambda_{HS} \lambda_{\phi S} \right) \phi^2 (H^\dagger H) + \frac{1}{4!} \lambda_S S^4 + \frac{1}{2} \lambda_{\phi S} (v_\phi + \phi)^2 S^2 + \frac{1}{2} \lambda_{HS} (H^\dagger H) S^2, \end{aligned} \quad (12)$$

where ϕ is redefined as $v_\phi + \phi$ and couplings between ϕ and η are abbreviated. $\tilde{\beta}_{\lambda_\phi}$ equals the β function of the quartic coupling λ_ϕ given in the Appendix except for the contribution from the anomalous dimension of ϕ . The summation for k should be done for the SM contents and $C_k = \frac{3}{2}(\frac{5}{6})$ for scalars and fermions (vector bosons). Since the first two lines of $\mathcal{V}$ are considered to correspond to the

¹Since this expression becomes complex for $\lambda_{H\phi} < 0$, one might suspect the validity of Eq. (11) in that case. However, if μ is fixed as a value where $\lambda_{H\phi}$ changes its sign from positive to negative, it can give a consistent and stable result. In fact, since the corresponding term is smaller than others by several orders of magnitude in $\mathcal{A}$ around such a μ , this can be a good prescription for the problem.

²We discuss the assumption $M_1 \ll M_{2,3}$ in the part for the mass generation of active neutrinos later.

scalar potential of the ordinary scotogenic model given as³

$$V_{\text{scot}} = m_H^2 H^\dagger H + \tilde{\lambda}_1 (H^\dagger H)^2 + \tilde{\lambda}_2 (\eta^\dagger \eta)^2 + \tilde{\lambda}_3 (H^\dagger H)(\eta^\dagger \eta) + \tilde{\lambda}_4 (H^\dagger \eta)(\eta^\dagger H) + \frac{\tilde{\lambda}_5}{2} [(H^\dagger \eta)^2 + (\eta^\dagger H)^2] + m_\eta^2 \eta^\dagger \eta + \sum_k \frac{n_k (-1)^{2s_k}}{64\pi^2} m_k^4 \left(\ln \frac{m_k^2}{\mu_0^2} - C_k \right), \quad (13)$$

we find the parameters in Eq. (13) can be expressed by using the ones in Eq. (2) at μ as follows,

$$m_H^2 \simeq \left(\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_\phi \right) v_\phi^2, \quad \tilde{\lambda}_1 \simeq \lambda_H - \frac{\lambda_{HS}^2}{\lambda_{\phi S}^2} \lambda_\phi + \frac{\lambda_{HS}^2}{64\pi^2},$$

$$m_\eta^2 = \frac{1}{2} \lambda_{\eta\phi} v_\phi^2, \quad \tilde{\lambda}_2 = \lambda_\eta, \quad \tilde{\lambda}_3 \simeq 0, \quad \tilde{\lambda}_4 \simeq 0. \quad (14)$$

Nonzero $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ could be induced radiatively at this scale even if they are assumed to be zero at the cutoff scale. However, their values are found to be small enough and then we can suppose them to be zero safely. Mass of the right-handed neutrino N_j is given in Eq. (8).

If both $\lambda_\phi > 0$ and $\lambda_{H\phi} < \lambda_\phi$ are satisfied at some critical scale, the first condition in (14) shows that $m_H^2 < 0$ could be realized to induce the spontaneous electroweak symmetry breaking. The difference $|\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_\phi|$ could be substantially suppressed to result in $|m_H^2| \ll v_\phi^2$ because of the custodial symmetry, which guarantees $\lambda_{H\phi} \simeq \lambda_\phi$ and $\lambda_{HS} \simeq \lambda_{\phi S}$. The Higgs scalar in this model is not a pure SM Higgs scalar. As shown in Eqs. (7) and (12), it has a small mixing with ϕ . If we write the mass eigenstates as h for a Higgs-like scalar and h_ϕ , their mass eigenvalues are found through $\partial^2 \mathcal{V} / \partial f_i \partial f_j$. They are written as

$$m_h^2 \simeq -2 \left(\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_\phi \right) v_\phi^2, \quad m_{h_\phi}^2 \simeq \frac{\lambda_{\phi S}^2}{32\pi^2} v_\phi^2, \quad (15)$$

and their mixing angle θ is expressed as

$$\tan \theta \simeq \frac{2 \left(\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_\phi + \frac{\lambda_{HS} \lambda_{\phi S}}{64\pi^2} \right) v_\phi v_H}{m_{h_\phi}^2 - m_h^2}. \quad (16)$$

Since $m_{h_\phi}^2$ is determined by the β function of the quartic coupling λ_ϕ , h_ϕ can be identified with a dilaton, a pseudo

³It should be noted that the present low energy effective model is not the ordinary scotogenic model but it is extended by the additional couplings of ϕ and S from the ordinary scotogenic model as found in $\mathcal{V}$. However, they do not affect the important formulas (21) and (23) for the inert doublet scalar mass and the neutrino mass in the scotogenic model.

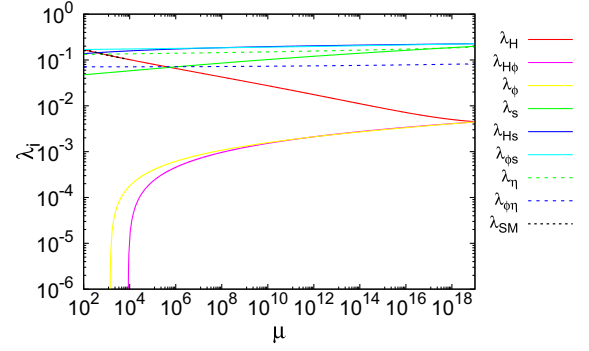


FIG. 1. Solutions of the RGEs for the quartic scalar couplings in the model. The parameters given in Eq. (17) are used as the initial values at the Planck scale. $y_{N_{2,3}}$ and $\lambda_{\eta\phi}$ fixed at the Planck scale in Eq. (17) induce the nearly degenerate mass for η and $N_{2,3}$ at $\mu \simeq M_2$. The result is confirmed to be stable for y_{N_1} if its initial value is taken less than 10^{-4} .

Nambu-Goldstone boson caused by the spontaneous violation of the classical scale invariance. The Higgs VEV $\langle H \rangle (\equiv v_H / \sqrt{2})$ can be estimated through a relation $v_H = m_h / \sqrt{2\tilde{\lambda}_1}$ by using $\tilde{\lambda}_1$ and m_h .

Now, we examine this scenario numerically by fixing the parameters in V_0 at the Planck scale. In this paper, we pick up benchmark parameters as an example and present physical results for them to describe features of the model. As such benchmark parameters, we take them at the Planck scale as follows,

$$\lambda_{c1} = 4.48 \times 10^{-3}, \quad \lambda_{c2} = 2.27 \times 10^{-1}, \quad \lambda_S = 0.2,$$

$$\lambda_\eta = 0.19, \quad \tilde{\lambda}_5 = -3 \times 10^{-3}, \quad 2\lambda_{\eta\phi} = (y_{N_2} + y_{N_3})^2,$$

$$y_{N_2} = y_{N_3} = 2.03 \times 10^{-1}, \quad (17)$$

where $y_{N_1} \ll y_{N_2}$ is assumed. Couplings which violate the custodial symmetry are listed in the second line.⁴ Our observations addressed above can be confirmed by solving the renormalization group equations (RGEs) of the relevant couplings for these initial values. RGEs used in this analysis are presented in the Appendix.

In Fig. 1, we plot solutions of the RGEs of the quartic scalar couplings in Eq. (2). In this plot, we use Eq. (26) for the neutrino Yukawa couplings h_{aj} that are fixed through the neutrino oscillation data as discussed in the next part. Their values at the Planck scale are determined from them. The figure shows that λ_ϕ takes a small positive value at a critical scale $\mu \simeq 8800$ GeV where $\lambda_{H\phi}$ changes its sign. On the other hand, λ_H takes a much larger value than $|\lambda_{H\phi}|$ thanks to the top Yukawa coupling, which is favored as

⁴A simple relation is assumed between $\lambda_{\eta\phi}$ and $y_{N_{2,3}}$ so that m_η and $M_{2,3}$ take a degenerate value at a low energy scale. λ_S and λ_η should be fixed so as to guarantee their positivity at the intermediate scale.

addressed below Eq. (5). Since $\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_{\phi} < 0$ is realized and its absolute value is sufficiently suppressed, we find from Eq. (14) that a desirable m_H^2 could be generated to cause a VEV of the Higgs scalar such as $v_H \ll v_{\phi}$. Using the solutions in Fig. 1, we find $v_{\phi} \simeq 7880$ GeV from Eq. (11) and rather good numerical values are obtained for the Higgs mass, the Higgs VEV, the top mass, and the dilaton mass such that

$$\begin{aligned} m_h &\simeq 137 \text{ GeV}, & v_H &\simeq 242 \text{ GeV}, \\ m_t &\simeq 175 \text{ GeV}, & m_{h_{\phi}} &\simeq 77 \text{ GeV}. \end{aligned} \quad (18)$$

The mixing between the Higgs-like boson and the dilaton is estimated as $\tan \theta \simeq 0.04$. The masses of η , $N_{2,3}$, and S are found to be

$$m_{\eta} \simeq 1480 \text{ GeV}, \quad M_{2,3} \simeq 1500 \text{ GeV}, \quad m_S \simeq 2320 \text{ GeV}. \quad (19)$$

Small mass difference of N_2 and N_3 is induced from the neutrino Yukawa couplings $h_{\alpha j}$ through the RGE evolution of y_{N_j} . It is estimated as $(M_2 - M_3)/M_2 = 8.8 \times 10^{-7}$ at their mass scale.

It may be instructive to examine how $|m_H^2| \ll v_{\phi}^2$ can be realized more closely. The difference $\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_{\phi}$ at a critical scale is determined by the difference of their β functions. If we take account of the constraint imposed by the custodial symmetry, it could be estimated as

$$\begin{aligned} \beta_{\lambda_{H\phi}} - \beta_{\lambda_{\phi}} &\simeq \frac{1}{16\pi^2} \left[\lambda_{H\phi} \left(6y_t^2 + 2 \sum_j y_{N_j}^2 + 12\lambda_H - \frac{3}{2}g_Y^2 \right. \right. \\ &\quad \left. \left. - \frac{9}{2}g_2^2 \right) - 4\lambda_{\phi} \sum_j y_{N_j}^2 + \frac{1}{2}\lambda_{\phi S}(\lambda_{HS} - \lambda_{\phi S}) \right. \\ &\quad \left. + 4 \sum_j y_{N_j}^4 - 2\lambda_{\eta\phi}^2 \right] \\ &\equiv \beta_1 + \beta_2 + \beta_3, \end{aligned} \quad (20)$$

where β_1 , β_2 , and β_3 correspond to the first line, the first term in the second line, and the last two terms in the second line, respectively. We should note that β_2 cannot be neglected even under the constraint of the custodial symmetry since it could give the same order contribution as the one from β_1 under the initial condition $\lambda_{c2} \gg \lambda_{c1}$ assumed in Eq. (17). On the other hand, since β_3 comes from the terms that break the custodial symmetry, it is not controlled by the symmetry so that there is no reason why its contribution can be sufficiently suppressed. Here, we should note that two terms in β_3 are relevant to the masses of N_j and η that are generated through the VEV of ϕ , respectively. If the mass of $N_{2,3}$ is nearly degenerate with

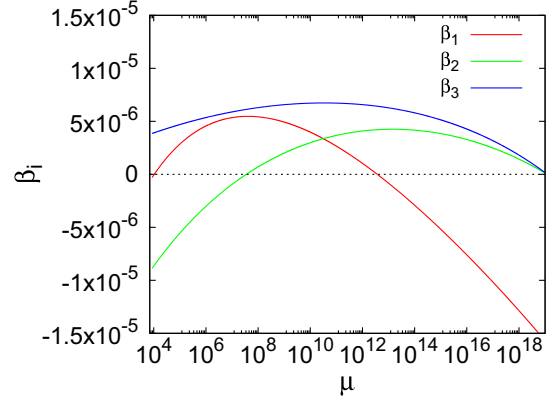


FIG. 2. Evolution of β_1, β_2 , and β_3 in Eq. (20), whose definition is given in the text. The initial condition $\lambda_{c1} \ll \lambda_{c2}$ allows β_2 to play an important role in spite of the custodial symmetry. The behavior of the β_1 is considered to be caused by the top Yukawa coupling.

the one of η and then $2y_{N_{2,3}}^2 \simeq \lambda_{\eta\phi}$ is satisfied at low energy regions,⁵ its contribution could be constrained.

Assuming this mass degeneracy and using the parameters in Eq. (17), we plot the evolution of β_1, β_2 , and β_3 in Fig. 2. The figure shows that the magnitude of β_3 is comparable to the one of β_1 and β_2 throughout the relevant energy scale. Both β_2 and β_3 are found to play a crucial role to cancel the negative contribution of β_1 at high energy regions. It makes $\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_{\phi}$ possible to take a negative and appropriate value at low energy regions.

This analysis shows that the present model can induce three mass scales of the scotogenic model in a desirable way as long as suitable terms for explicit breaking of the custodial symmetry are introduced. In the next Section, we study phenomenological consequences caused by the constraints on the parameters of the scotogenic model given in Eq. (14). We focus our attention on their influence to neutrino mass, dark matter, and leptogenesis for the baryon number asymmetry. Numerical study in the following part is based on the benchmark parameters presented in Eq. (17) and the results of the RGEs analysis in this Section.

III. PHENOMENOLOGICAL CONSEQUENCES

A. Neutrino mass

The inert doublet scalar η is a key ingredient of the neutrino mass generation in the scotogenic model. Mass of its components is expressed as

$$M_{\eta^\pm}^2 = m_\eta^2 + \frac{1}{2}\tilde{\lambda}_3 v_H^2, \quad M_{\eta_R}^2 = m_\eta^2 + \frac{1}{2}(\tilde{\lambda}_3 + \tilde{\lambda}_4 \pm \tilde{\lambda}_5) v_H^2, \quad (21)$$

⁵This condition is found to play a crucial role in the study of dark matter and leptogenesis as seen later.

where $\eta^\pm$ are charged components and $\eta_{R(I)}$ is a real (imaginary) part of the neutral one, respectively. In the present extended model, $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ are assumed to be zero at the Planck scale. Although they take nonzero values radiatively through RGE effects of the small custodial symmetry breaking such as the $\tilde{\lambda}_5$ coupling assumed in Eq. (17), they are found to be negligibly small. Taking account of this point and the expression of m_η^2 given in Eq. (14), the mass of each component of η is found to be represented as

$$M_{\eta^\pm}^2 \simeq \frac{1}{2} \lambda_{\eta\phi} v_\phi^2, \quad M_{\eta_{R/I}}^2 \simeq \frac{1}{2} \lambda_{\eta\phi} v_\phi^2 \pm \frac{1}{2} \tilde{\lambda}_5 v_H^2. \quad (22)$$

Neutrino mass in the scotogenic model is generated through one-loop diagrams that have a neutral component of η and a right-handed neutrino N_j in the internal lines. A mass matrix caused by them can be expressed as

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_{j=1}^3 h_{\alpha j} h_{\beta j} \Lambda_j, \quad \Lambda_j = \frac{\tilde{\lambda}_5 v_H^2}{32\pi^2 M_j} \frac{M_j^2}{M_\eta^2 - M_j^2} \left(1 + \frac{M_j^2}{M_\eta^2 - M_j^2} \ln \frac{M_j^2}{M_\eta^2} \right), \quad (23)$$

where $M_\eta^2 = \frac{1}{2}(M_{\eta_R}^2 + M_{\eta_I}^2)$. As found from this formula, $\tilde{\lambda}_5 \neq 0$ is a necessary condition of the neutrino mass generation in this scheme. Since a small $\tilde{\lambda}_5$ term is included in Eq. (1) as one of the custodial symmetry violating terms,⁶ active neutrino masses can be generated through this process also in the present model. As is well known, they can be consistent with the neutrino oscillation data for such a small $\tilde{\lambda}_5$ even if M_j and M_η take TeV scale values.

For simplicity, we assume that neutrino Yukawa couplings $h_{\alpha j}$ realize the tribimaximal mixing [10] in the neutrino sector,⁷ and we take them in the weak interaction basis as [14,15]

$$h_{ej} = h_{\mu j} = -h_{\tau j} = \frac{1}{\sqrt{3}} h_j e^{i\gamma_j} (j = 1, 2); \quad h_{e3} = 0, \quad h_{\mu 3} = h_{\tau 3} = \frac{1}{\sqrt{2}} h_3 e^{i\gamma_3}. \quad (24)$$

In that case, neutrino mass eigenvalues are found to be expressed as

⁶It should be noted that $\tilde{\lambda}_5$ is irrelevant to both $\beta_{\lambda_{H\phi}}$ and β_{λ_ϕ} in Eq. (20) and then the difference $\lambda_{H\phi} - \frac{\lambda_{HS}}{\lambda_{\phi S}} \lambda_\phi$ is not affected by it.

⁷Although it cannot cause a nonzero mixing angle θ_{13} in the neutrino sector, a favorable value of θ_{13} could be obtained in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [11] through the help of mixing in the charged lepton sector [12,13].

$$m_1 = 0, \quad m_2 = h_1^2 \Lambda_1 e^{2i\gamma_1} + h_2^2 \Lambda_2 e^{2i\gamma_2}, \quad m_3 = h_3^2 \Lambda_3 e^{2i\gamma_3}. \quad (25)$$

This formula suggests that N_1 could be irrelevant to the neutrino mass eigenvalues for a sufficiently small h_1 . Data for the neutrino mass and mixing obtained from experiments and cosmological observations suggest that m_1 is sufficiently small and the tribimaximal mixing is rather good. If the normal ordering, the hierarchical mass, and the tribimaximal mixing are satisfied in the neutrino sector, Eq. (24) can be derived automatically and then the qualitative conclusions obtained on the basis of this assumption could be robust although details might be dependent on the numerical assumption. However, we should note that the conclusions given here could be changed if these conditions are not satisfied in the neutrino sector.

If we apply the RGE results obtained for the benchmark parameters given in Eq. (17) to this formula, the neutrino oscillation data [16] are found to be explained by taking the neutrino Yukawa couplings h_2 and h_3 as

$$h_2 = 4.24 \times 10^{-4}, \quad h_3 = 1.58 \times 10^{-3}. \quad (26)$$

The RGE evolution of $y_{N_{2,3}}$ caused by $h_{2,3}$ is found to resolve the mass degeneracy of N_2 and N_3 at low energy regions as addressed in the previous part.

N_1 can be irrelevant to the result given in Eq. (26) when h_1 is small enough. We should also note that h_1 can take a larger value than h_2 without affecting it if $M_1 \ll M_\eta$ is satisfied. In that case, Λ_1 can be approximated as $\Lambda_1 \simeq \frac{\tilde{\lambda}_5 v_H^2}{32\pi^2} \frac{M_1}{M_\eta^2}$, and then it takes a heavily suppressed value compared with Λ_2 . As a result, if the condition

$$h_1^2 \left(\frac{M_1}{1 \text{ GeV}} \right) < 10^{-6} \quad (27)$$

is satisfied, h_2 in Eq. (26) is not changed. It suggests that $h_1 > h_2$ is possible for $M_1 < O(1) \text{ GeV}$ in the present case.

It may also be useful to give a remark on the relation of this condition to a constraint imposed by the lepton flavor violating process (LFV) $\mu \rightarrow e\gamma$, which gives an upper bound for h_1 [17]. Its branching ratio can be expressed in the present model as [18]

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{3\alpha}{64\pi(G_F M_{\eta^\pm})^2} \left| \sum_{j=1,2} \frac{h_j^2 e^{2i\gamma_j}}{3} F_2 \left(\frac{M_j^2}{M_{\eta^\pm}^2} \right) \right|^2, \quad F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1-x)^4}. \quad (28)$$

If we apply it to the model with $M_1 \ll M_\eta$, the present experimental bound gives a constraint $h_1 < 0.12$

independently of M_1 since $F_2(x)$ converges to $\frac{1}{6}$ at $x \rightarrow 0$. Thus, this constraint should be taken care in the case $M_1 \lesssim 100$ keV since it could be stronger than the condition (27). In the next part, we discuss dark matter and leptogenesis taking account of these points.

B. Dark matter

The existence of a stable particle is guaranteed by the exact Z_2 symmetry. A possible DM candidate is either the lightest neutral component of η or the lightest right-handed neutrino N_1 which has its odd parity. Since M_{η_R} is smaller than $M_{\eta^\pm}$ and M_{η_i} in the case $\tilde{\lambda}_4, \tilde{\lambda}_5 < 0$ as found from Eq. (22),⁸ η_R is considered to be a DM candidate. If reheating temperature is higher than the mass of the DM candidate and it could be in the thermal equilibrium, the DM abundance might be realized as their thermal relics through the freeze-out scenario [19]. In fact, inflation is expected to be caused by the singlet scalar S if the Planck mass is induced in some way and S has a nonminimal coupling $\frac{\xi}{2}S^2R$ with the Ricci scalar R [20]. It can be consistent with the CMB data of the Planck satellite [21] if ξ is fixed appropriately for the quartic coupling λ_S . Moreover, preheating following it can be expected to realize reheating temperature higher than $O(10^4)$ GeV [20,22]. It is high enough to make η and $N_{2,3}$ in the thermal bath.

A DM candidate η_R is considered to be in the thermal equilibrium by the SM interactions. On the other hand, $N_{2,3}$ could be in the thermal equilibrium through the 2-2 scatterings $\eta\eta^\dagger \rightarrow N_i N_j^c$ and $\ell_\alpha \ell_\beta^\dagger \rightarrow N_i N_j^c$ caused by the Yukawa couplings h_2 and h_3 given in Eq. (26), which explain the neutrino oscillation data. The production of N_1 is expected to be proceeded through the scattering $N_j N_j \rightarrow N_1 N_1$ ($j = 2, 3$) caused by y_{N_j} [23] and also both the above scatterings and the decay of η caused by h_1 . Here, we consider the Boltzmann equations for the number density of η_R and N_1 based on these processes. We express the number density of particle species i in the comoving volume as $Y_i = \frac{n_i}{s}$ where n_i and s are the number density of the particle i and the entropy density of the Universe, respectively. If we define a dimensionless parameter z for a temperature T as $z \equiv \frac{M_2}{T}$ and represent an equilibrium value of Y_i as Y_i^{eq} , the Boltzmann equations for Y_{η_R} and Y_{N_1} can be written as

$$\frac{dY_{\eta_R}}{dz} = -\frac{z}{H(M_2)s} \left[\sum_{a,b=\eta_R,\eta_L,\eta^\pm} \gamma^S(ab) \left(\frac{Y_{\eta_R}^2}{Y_{\eta_R}^{\text{eq}2}} - 1 \right) + \gamma^D(\eta_R) \left(\frac{Y_{\eta_R}}{Y_{\eta_R}^{\text{eq}}} - 1 \right) \right], \quad (29)$$

⁸For the assumed parameters in Eq. (17), we find that $\tilde{\lambda}_4 < 0$ is satisfied at the relevant scale.

$$\frac{dY_{N_1}}{dz} = -\frac{z}{H(M_2)s} \left[\left\{ \sum_{j=2,3} \gamma_1^S(N_j N_j) + \sum_{a=\eta,\ell} \gamma_1^S(aa^\dagger) \right\} \times \left(\frac{Y_{N_1}^2}{Y_{N_1}^{\text{eq}2}} - 1 \right) - \gamma^D(\eta_R) \left(\frac{Y_{\eta_R}}{Y_{\eta_R}^{\text{eq}}} - 1 \right) \right], \quad (30)$$

where $H(M_2)$ is the Hubble parameter at $T = M_2$. Reaction density of the decay $\eta_R \rightarrow \nu N_2$ is represented by $\gamma^D(\eta_R)$, and reaction densities of the scatterings $\ell \ell^\dagger \leftrightarrow N_j^c N_j$, $\eta \eta^\dagger \leftrightarrow N_j^c N_j$, and $N_i N_i \leftrightarrow N_j N_j$ are represented by $\gamma_j^S(\ell \ell^\dagger)$, $\gamma_j^S(\eta \eta^\dagger)$, and $\gamma_j^S(N_i N_i)$. They are, respectively, related to the thermally averaged decay width $\langle \Gamma_a \rangle$ for $a \rightarrow ij$ and the thermally averaged cross section $\langle \sigma_{ab} |v| \rangle$ for the scattering $ab \leftrightarrow ij$ as [24]

$$\gamma^D(a) = \langle \Gamma_a \rangle n_a^{\text{eq}}, \quad \gamma^S(ab) = \langle \sigma_{ab} |v| \rangle n_a^{\text{eq}} n_b^{\text{eq}}, \quad (31)$$

where n_a^{eq} is the equilibrium number density of a particle a .

In the η_R mass region considered in this study, which is the so called high mass region ($M_{\eta_R} \gtrsim 500$ GeV) in the inert doublet model, the couplings $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ play a crucial role to reduce its relic abundance through the coannihilation among the components of η [25]. It is taken into account in $\gamma^S(ab)$ of Eq. (29). Unfortunately, since the absolute values of $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ in this model are much smaller than the required ones, the relic η_R is difficult to be reduced to the observed DM abundance.⁹ As a result, we have to consider the lightest right-handed neutrino N_1 as DM. In that case, $M_1 < M_{\eta_R}$ has to be satisfied. It does not bring any contradiction between the present scheme for the neutrino mass generation and the neutrino oscillation data as found from the discussion on the neutrino mass. The lightest right-handed neutrino has been studied as a DM candidate from various view points [13–15,17,26,27].

We define $Y_{N_1}^\infty$ as the N_1 relic density in the present Universe that satisfies $\Omega_{N_1} h^2 = 0.12$. We also define $Y_{\eta_R}^f$ as a value of Y_{η_R} at a temperature where the η_R coannihilation by the electroweak gauge interaction freezes out. The fact that η_R finally decays to N_1 requires that they satisfy $Y_{\eta_R}^f \leq Y_{N_1}^\infty$. Since the solution of the above Boltzmann equations shows that $Y_{\eta_R}^f \simeq Y_{N_1}^\infty$ is realized for $M_1 \simeq 45$ GeV, we find that $M_1 \lesssim 45$ GeV should be satisfied as an upper bound of M_1 . Within this range of M_1 , N_1 could be produced to reach the thermal equilibrium through the processes caused by h_1 and y_{N_j} . In that case,

⁹Moreover, we find that the couplings in the scalar potential V_0 reach the Landau pole at a scale like $O(10^8)$ GeV much lower than the Planck scale if $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ have magnitude, which can reduce the η_R abundance sufficiently. It means that the present framework cannot be consistent with the η_R DM in this mass region.

however, its annihilation described by $\gamma_1^S(\ell\ell^\dagger)$ and $\gamma_1^S(\eta\eta^\dagger)$ is suppressed due to $M_1 \ll M_\eta$ and then the N_1 abundance is difficult to be reduced to the required level for the bound of h_1 discussed in the last part of the previous Section.¹⁰ Although the freeze-out scenario is difficult to be applied to N_1 in this model, the freeze-in scenario [28] could work well for N_1 to realize the suitable relic abundance.

Freeze-in of N_1 is caused through the processes addressed above, which are caused by the couplings y_{N_1} and h_1 . Here, we should note that $M_1 (\equiv y_{N_1} v_\phi)$ determines both the N_1 production due to the scattering $N_j N_j \rightarrow N_1 N_1$ represented by $\gamma_1^S(N_j N_j)$ in Eq. (30) and a value of $Y_{N_1}^\infty$. The former is proportional to $y_{N_1}^2$ and the latter is proportional to $y_{N_1}^{-1}$. If this scattering process for a certain M_1 value results in $Y_{N_1} > Y_{N_1}^\infty$, the N_1 abundance cannot be reduced to $Y_{N_1}^\infty$ since the N_1 annihilation is ineffective just as in the freeze-out case. This means that there is an upper bound for M_1 , which is fixed by $Y_{N_1} = Y_{N_1}^\infty$. It is much smaller than the one obtained from the condition $Y_{\eta_R}^f \sim Y_{N_1}^\infty$. On the other hand, although this scattering process for a smaller M_1 results in $Y_{N_1} < Y_{N_1}^\infty$, the decay of the η_R in the thermal bath could help Y_{N_1} to reach $Y_{N_1}^\infty$ if h_1 takes a suitable value. For much smaller M_1 , the $N_j N_j$ scattering becomes ineffective to generate N_1 . However, in that case, both the scattering processes and the η_R decay caused by h_1 could make Y_{N_1} reach $Y_{N_1}^\infty$ since h_1 can take a larger value within the constraints derived from Eq. (27) and the LFV. These are considered to be general features found for the N_1 DM in this model independently of details of parameters assumed here.

In Fig. 3, we plot Y_{N_1} obtained as a solution of the Boltzmann equations for three typical cases defined by (M_1, h_1) . They correspond to three possibilities described above, respectively. In this calculation, $M_{2,3}$ and M_{η_R} are fixed using the values given in Eq. (19). The upper bound of M_1 is found to be 2.1 MeV for which the $N_j N_j$ scattering realizes $Y_{N_1}^\infty$ by itself. In the $M_1 = 1.0$ MeV case, the $N_j N_j$ scattering produces N_1 but its abundance cannot reach $Y_{N_1}^\infty$. However, the N_1 produced through the decay of η_R in the thermal bath supplies the deficit abundance of N_1 to make it reach $Y_{N_1}^\infty$ if h_1 is fixed to a suitable value within the allowed range. For much smaller values of M_1 , the processes caused by a larger value of h_1 becomes dominant ones. We can find it from the Y_{N_1} evolution in the case 10 keV. From these results, we can conclude that the model predicts a right-handed neutrino DM in the mass range less than $O(1)$ MeV. It is interesting that this range includes the

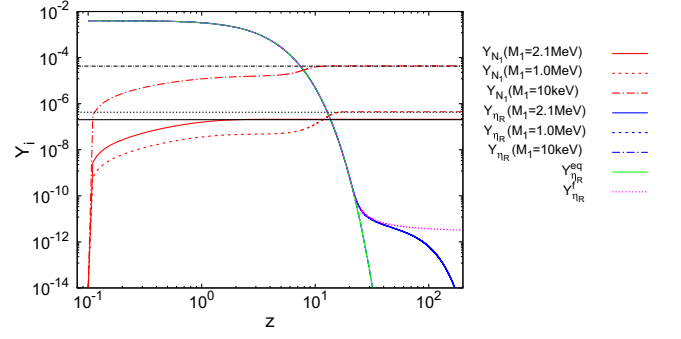


FIG. 3. Evolution of the abundance of N_1 and η_R in the comoving volume is drawn as a function of $z (\equiv M_2/T)$. As initial values at $z = 0.1$, $Y_{\eta_R} = Y_{\eta_R}^{\text{eq}}$ and $Y_{N_1} = 0$ are assumed. Y_{N_1} is plotted by red lines for three cases of (M_1, h_1) ; (2.1 MeV, 10^{-8}) (solid), (1.0 MeV, 1.3×10^{-5}) (dashed), and (10 keV, 2.8×10^{-4}) (dash dotted). Horizontal black lines represent $Y_{N_1}^\infty$, which is required to realize $\Omega_{N_1} h^2 = 0.12$ for each mass. The same type of lines is used as the ones of Y_{N_1} for the corresponding mass. Y_{η_R} is plotted by blue lines for each M_1 value. In latter two cases, $Y_{\eta_R} = Y_{\eta_R}^{\text{eq}}$ is realized through the processes caused by h_1 . Only in the $M_1 = 2.1$ MeV case, Y_{η_R} is found to leave the equilibrium due to a small h_1 value.

mass of the sterile neutrino for which it could give an explanation for the small scale structure problem [29]. It should be noted that this sterile neutrino has no mixing with active neutrinos. Although numerical values like the upper bound of M_1 depend on the values of parameters given in Eq. (17), the features found here are expected to be rather general for this framework and the right-handed neutrino could be light DM with mass less than a GeV scale.

Finally, we give a brief comment on a possibility such that the lightest neutral component of η is DM and $M_\eta < M_1$ is satisfied in this framework that is featured by the small couplings $\tilde{\lambda}_{3,4,5}$. In the present work, we confine our study to the case where η_R mass is in the high mass region. However, if its mass is in the low mass region ($\lesssim 80$ GeV) or in the intermediate mass region (80–200 GeV), its relic abundance is known to be realized through the SM interaction [25] although their allowed parameter space is severely constrained through various experiments like the direct detection [30] by now. Since $M_\eta < M_j$ breaks the assumption adopted here, we need to study whether the weak scale can be generated successfully in such cases. It is an interesting subject in this model but it is beyond the scope of the present study.

C. Baryon number asymmetry

In this model, the mass ordering $M_1 < m_\eta < M_2 \simeq M_3$ is satisfied for the Z_2 odd contents as shown in Eq. (19) and the decay width Γ_j of N_j is found to be $\Gamma_2 < \Gamma_3$. It suggests that baryon number asymmetry is expected to be produced through leptogenesis caused by the out-of-equilibrium

¹⁰Coannihilation among the right-handed neutrinos might solve this difficulty [15]. However, N_1 does not have a degenerate mass with $N_{2,3}$ in the present model.

decay of N_2 to η .¹¹ Almost degenerate masses of N_2 and N_3 realized in the model are expected to enhance the CP asymmetry ε in the N_2 decay so that a value of ε could be large enough for the generation of the required lepton number asymmetry. It can be expressed by using Eq. (24) as [32]

$$\varepsilon = \frac{\text{Im}[(\sum_a h_{a2} h_{a3}^*)^2]}{(\sum_a h_{a2} h_{a2}^*)(\sum_a h_{a3} h_{a3}^*)} \frac{(M_2^2 - M_3^2) M_2 \Gamma_{N_2}}{(M_2^2 - M_3^2)^2 + M_2^2 \Gamma_{N_2}^2} \\ = \frac{2}{3} \frac{(M_2^2 - M_3^2) M_2 \Gamma_{N_2}}{(M_2^2 - M_3^2)^2 + M_2^2 \Gamma_{N_2}^2}, \quad (32)$$

where we assume $\sin 2(\gamma_2 - \gamma_3) = 1$ in the second line. If we define $\delta \equiv (M_2 - M_3)/M_2$ as a parameter to represent the mass degeneracy between N_2 and N_3 , the parameters given in Eq. (17) realize $\delta = O(10^{-6})$ at the scale M_2 and then ε takes a value of $O(10^{-5})$.

In the study of leptogenesis in the present model with $M_2 \simeq M_\eta$, we should note that the lepton number asymmetry and the N_1 abundance could have a close relation since η produced in the N_2 decay can contribute to the N_1 abundance through the η decay. If we take account of this point, we have to solve the Boltzmann equations for η_R, N_1, N_2, N_3 , and the lepton number asymmetry Y_L as coupled equations. As found from Fig. 3, the freeze-out of the coannihilation of η occurs around $z \simeq 25$, and then the η produced through the N_2 decay at $z \lesssim 25$ is expected to be in the thermal equilibrium. This situation is the same in the present case except that the additional η is supplied in the thermal bath. Thus, when we take account of the effect of N_2 decay, we have to assume a smaller value for h_1 in order to realize the required N_1 relic abundance compared

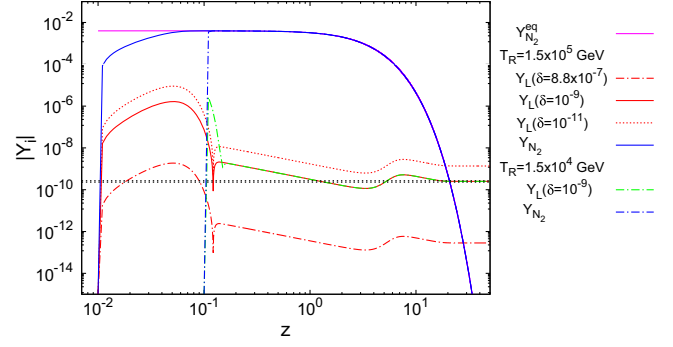


FIG. 4. Evolution of Y_{N_2} and Y_L generated by the N_2 decay. Y_L is plotted for three values of δ assuming $T_R = 1.5 \times 10^5$ GeV. Horizontal dashed lines represent a region of Y_L required for the explanation of the baryon number asymmetry in the Universe. In the case $T_R = 1.5 \times 10^4$ GeV, Y_L is also plotted for the case $\delta = 10^{-9}$, which coincides with the one obtained in the case $T_R = 1.5 \times 10^5$ GeV. The behavior of Y_L around $z \simeq 10$ is considered to be caused by the N_2 generation due to $\gamma_2^S(N_3 N_3)$ and the Y_L washout due to $\gamma^S(\ell \ell)$ whose magnitudes are reversed there.

with the one used in Fig. 3. In this point, $M_1 = 2.1$ MeV is an exceptional case since N_1 is generated only through the scatterings caused by y_{N_j} . We can confirm it by solving the Boltzmann equations. As a result, when we consider leptogenesis in this model, the equations for $Y_{N_{2,3}}$ and Y_L can be treated separately from the ones for Y_{η_R} and Y_{N_1} . Three cases given in Fig. 3 can be treated as the same case in the study of leptogenesis since the relevant equations are independent of M_1 and h_1 .

The Boltzmann equations for them are given as

$$\frac{dY_L}{dz} = \frac{-z}{H(M_2)s} \left[-\varepsilon \gamma^D(N_2) \left(\frac{Y_{N_2}}{Y_{N_2}^{\text{eq}}} - 1 \right) + \left\{ \sum_{k=2,3} \gamma^D(N_k) + 2(\gamma^S(\ell \eta^\dagger) + \gamma^S(\ell \ell)) \right\} \frac{Y_L}{Y_{\ell}^{\text{eq}}} \right], \\ \frac{dY_{N_3}}{dz} = \frac{-z}{H(M_2)s} \left[\left\{ \sum_{a=\eta, \ell} \gamma_3^S(a a^\dagger) + \gamma_3^S(N_2 N_2) \right\} \left(\frac{Y_{N_3}^2}{Y_{N_3}^{\text{eq}2}} - 1 \right) + \gamma^D(N_3) \left(\frac{Y_{N_3}}{Y_{N_3}^{\text{eq}}} - 1 \right) \right], \\ \frac{dY_{N_2}}{dz} = \frac{-z}{H(M_2)s} \left[\left\{ \sum_{a=\eta, \ell} \gamma_2^S(a a^\dagger) + \gamma_2^S(N_3 N_3) \right\} \left(\frac{Y_{N_2}^2}{Y_{N_2}^{\text{eq}2}} - 1 \right) + \gamma^D(N_2) \left(\frac{Y_{N_2}}{Y_{N_2}^{\text{eq}}} - 1 \right) \right], \quad (33)$$

where we take account of $Y_{N_2}^{\text{eq}} \simeq Y_{N_3}^{\text{eq}}$, $\gamma^S(\ell \eta^\dagger)$ and $\gamma^S(\ell \ell)$ are reaction densities of the lepton number violating scatterings $\ell \eta^\dagger \leftrightarrow \ell^\dagger \eta$ and $\ell \ell \leftrightarrow \eta \eta$. We solve them for an initial condition such that $Y_{N_j} = Y_L = 0$ at the reheating temperature T_R .

¹¹We confine our study to the thermal leptogenesis and do not consider a possibility of nonthermal leptogenesis [31].

The solutions of these Boltzmann equations are shown in Fig. 4.¹² In this calculation, we apply the same parameters used in Fig. 3. We find that the lepton number asymmetry Y_L sufficient for the explanation of the baryon number asymmetry in the Universe cannot be generated for the

¹²The baryon number asymmetry is estimated by using the lepton number asymmetry at $z \simeq 15$ since the sphaleron is in the thermal equilibrium at $z \lesssim 15$ for the M_2 assumed here.

mass degeneracy δ derived by using $y_{N_{2,3}}$ assumed in Eq. (17). This δ is not small enough to make the CP asymmetry ε large enough since the washout processes are not sufficiently suppressed around $z \simeq 1$. To improve this result, we may consider the realization of stricter mass degeneracy between N_2 and N_3 . We modify the relation between y_{N_2} and y_{N_3} given in Eq. (17) to

$$y_{N_3} = (1 + \delta_0)y_{N_2}, \quad y_{N_2} = 0.203. \quad (34)$$

If δ_0 is tuned appropriately, δ at the scale M_2 can be smaller than $O(10^{-9})$. In that case, ε can take a value larger than $O(10^{-3})$ while other results are kept unchanged. In fact, the sufficient lepton number asymmetry can be generated as found in Fig. 4. Although initial values of parameters have to be tuned at the Planck scale, promising low energy results can be obtained in a consistent way. It is noticeable that the mass degeneracy among $N_{2,3}$ and η_R that is imposed for the realization of the appropriate electroweak symmetry breaking could play an important role also for the successful leptogenesis.

IV. SUMMARY

The scotogenic model is a promising simple extension of the SM at TeV scales. It can explain the neutrino mass, the DM existence, and the baryon number asymmetry in the Universe, which are unsolvable problems in the SM. This model has three mass scales such as the electroweak scale, the masses of the inert doublet scalar, and the right-handed neutrinos. The latter two are usually assumed to be in TeV regions. However, the model cannot explain both the origin of these mass scales and their stability against the quantum correction just as the SM.

In this paper, in order to solve this problem, the model is extended by imposing the classical scale invariance and the custodial symmetry $SO(5)$. By introducing the two real scalars, the scalar potential is modified to be invariant under these symmetries. An intermediate scale is induced through the spontaneous breaking of these symmetries caused by the Coleman-Weinberg mechanism. Four Nambu-Goldstone bosons are brought about as a result of the custodial symmetry breaking $SO(5) \rightarrow SO(4)$. Three of them are eaten by the weak gauge boson and a remaining one becomes the Higgs-like scalar. It has small mixing with a dilaton that appears along with the violation of the scale invariance. Since the negative squared Higgs mass

generated from the intermediate scale is suppressed by the custodial symmetry, the weak scale can be much smaller than the intermediate scale.

The imposed symmetries constrain the scalar potential of the ordinary scotogenic model, which is included as a part of this extended model. This constrained scalar potential and the existence of the singlet scalars could cause phenomenological consequences different from the ones of the ordinary scotogenic model. In this study, a certain texture of the neutrino Yukawa couplings is assumed for the analysis. Using such an assumption, we find that a DM candidate has to be not the lightest neutral component of the inert doublet scalar η but the lightest right-handed neutrino. Moreover, the freeze-out scenario cannot be applied for it to explain the DM abundance and then only the freeze-in scenario is applicable for it. Since the right-handed neutrino mass is induced through the coupling to a singlet scalar, the lightest right-handed neutrino as DM is produced through the scattering among right-handed neutrinos mediated by it. This nature severely constrains its mass in the range less than $O(1)$ MeV. It contains a mass region where the small scale structure problem could be resolved partially through a sterile neutrino. The baryon number asymmetry is found to be explained through the resonant leptogenesis. An interesting point of the model is that the mass degeneracy among $N_{2,3}$ and η assumed for the appropriate generation of the electroweak scale also plays a crucial role to make the model consistent with the successful leptogenesis through the resonance. Although further study is required to clarify the phenomenological consequences in the cases defined by more general neutrino Yukawa textures, the model with a suitable neutrino Yukawa texture may be considered as a candidate of the model that can solve the problems in the scotogenic model.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX

We use one-loop RGEs in this study for simplicity. RGEs for the scalar quartic coupling constants in Eq. (2) are given as

$$\begin{aligned} \beta_{\lambda_H} &= \frac{1}{16\pi^2} \left[24\lambda_H^2 + 2\lambda_{H\phi}^2 + \frac{1}{2}\lambda_{HS}^2 - 6y_t^4 + \frac{3}{8}(g_Y^2 + g_2^2)^2 + \frac{3}{4}g_2^4 + \lambda_H(12y_t^2 - 3g_Y^2 - 9g_2^2) \right], \\ \beta_{\lambda_{H\phi}} &= \frac{1}{16\pi^2} \left[6\lambda_{H\phi}\lambda_\phi + 12\lambda_H\lambda_{H\phi} + 8\lambda_{H\phi}^2 + \frac{1}{2}\lambda_{\phi S}\lambda_{HS} + \frac{1}{2}\lambda_{H\phi} \left(12y_t^2 + 4\sum_j y_{N_j}^2 - 3g_Y^2 - 9g_2^2 \right) \right], \end{aligned}$$

$$\begin{aligned}
\beta_{\lambda_\phi} &= \frac{1}{16\pi^2} \left(18\lambda_\phi^2 + 8\lambda_{H\phi}^2 + \frac{1}{2}\lambda_{\phi S}^2 + 2\lambda_{\eta\phi}^2 - 4\sum_j y_{N_j}^4 + 4\lambda_\phi \sum_j y_{N_j}^2 \right), \\
\beta_{\lambda_{HS}} &= \frac{1}{16\pi^2} \left[12\lambda_H\lambda_{HS} + 2\lambda_{H\phi}\lambda_{\phi S} + 2\lambda_S\lambda_{HS} + 4\lambda_{HS}^2 + \frac{1}{2}\lambda_{HS}(12h_t^2 - 3g_Y^2 - 9g_2^2) \right], \\
\beta_{\lambda_{\phi S}} &= \frac{1}{16\pi^2} \left(6\lambda_{\phi S}\lambda_\phi + 8\lambda_{H\phi}\lambda_{HS} + 2\lambda_S\lambda_{\phi S} + 4\lambda_{\phi S}^2 + 2\lambda_{\phi S}\sum_j y_{N_j}^2 \right), \\
\beta_{\lambda_S} &= \frac{1}{16\pi^2} (3\lambda_S^2 + 3\lambda_{\phi S}^2 + 12\lambda_{HS}^2), \\
\beta_{\lambda_\eta} &= \frac{1}{16\pi^2} \left[24\lambda_\eta^2 + \frac{1}{2}\lambda_{\eta\phi}^2 + \frac{3}{8}(g_Y^2 + g_2^2)^2 + \frac{3}{4}g_2^4 - \lambda_\eta(3g_Y^2 + 9g_2^2) \right], \\
\beta_{\lambda_{\eta\phi}} &= \frac{1}{16\pi^2} \lambda_{\eta\phi} \left[6\lambda_\phi + 12\lambda_\eta + 4\lambda_{\eta\phi} + \frac{1}{2} \left(4\sum_j y_{N_j}^2 - 3g_Y^2 - 9g_2^2 \right) \right], \\
\beta_{\tilde{\lambda}_5} &= \frac{1}{16\pi^2} \tilde{\lambda}_5 [4\lambda_H + 4\lambda_\eta + (6h_t^2 - 3g_Y^2 - 9g_2^2)].
\end{aligned} \tag{A1}$$

In these RGEs, we take account of the top Yukawa coupling y_t , the neutrino Yukawa couplings h_j , and y_{N_j} among the Yukawa couplings in the model. One-loop RGEs for h_j and y_{N_j} are given as

$$\begin{aligned}
\beta_{h_i} &= \frac{h_i}{16\pi^2} \left[\frac{5}{2}(h_1^2 + h_2^2) + h_3^2 + \frac{1}{2}y_{N_i}^2 - \frac{9}{4}g_2^2 + \frac{1}{4}g_Y^2 \right] \quad (i = 1, 2), \\
\beta_{h_3} &= \frac{h_3}{16\pi^2} \left[h_1^2 + h_2^2 + \frac{5}{2}h_3^2 + \frac{1}{2}y_{N_3}^2 - \frac{9}{4}g_2^2 + \frac{1}{4}g_Y^2 \right], \\
\beta_{y_{N_j}} &= \frac{y_{N_j}}{16\pi^2} \left(3y_{N_j}^2 + 2\sum_{k=1}^3 y_{N_k}^2 + 2h_j^2 \right) \quad (j = 1, 2, 3).
\end{aligned} \tag{A2}$$

It should be noted that the β functions of the scalar quartic couplings indirectly depend on the neutrino Yukawa couplings h_j through y_{N_j} . Thus, when N_1 is very light and then y_{N_1} is sufficiently small, the evolution of these quartic couplings is almost independent of h_1 as long as the condition (27) is satisfied.

Scalar quartic couplings

$$\frac{1}{2}\lambda_{\eta S}S^2(\eta^\dagger\eta) + \tilde{\lambda}_3(H^\dagger H)(\eta^\dagger\eta) + \tilde{\lambda}_4(H^\dagger\eta)(\eta^\dagger H) \tag{A3}$$

can be induced radiatively through their RGEs, which are given as

$$\beta_{\lambda_{\eta S}} = \frac{1}{16\pi^2} \lambda_{\phi S}\lambda_{\eta\phi}, \quad \beta_{\tilde{\lambda}_3} = \frac{1}{16\pi^2} (\lambda_{H\phi}\lambda_{\eta\phi} + \tilde{\lambda}_5^2), \quad \beta_{\tilde{\lambda}_4} = \frac{1}{16\pi^2} \tilde{\lambda}_5^2. \tag{A4}$$

In the β functions given in Eqs. (A1) and (A4), terms proportional to $\lambda_{\eta S}$, $\tilde{\lambda}_3$, and $\tilde{\lambda}_4$ are not explicitly written since they are assumed to be zero at the Planck scale. Although they give much smaller contributions than other couplings, they are taken into

account in the present numerical calculation. We confirm that absolute values of $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ are of $O(10^{-6})$ at TeV regions for the benchmark parameters. It justifies our treatment of these couplings in the present study.

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