

Bootstrapping giant graviton correlators

Song He (何颂),^{1,2,*} Canxin Shi (施灿欣)^{1,†} Yichao Tang (唐一朝)^{1,3,‡} and Congkao Wen (温从烤)^{4,§}

¹New Cornerstone Laboratory, *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*

²School of Fundamental Physics and Mathematical Sciences, *Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China*

³School of Physical Sciences, *University of Chinese Academy of Sciences, Beijing 100049, China*

⁴Centre for Theoretical Physics and Astronomy, *Department of Physics and Astronomy, Queen Mary University of London, London, E1 4NS, United Kingdom*



(Received 4 June 2026; accepted 3 August 2026; published 26 August 2026)

We develop bootstrap methods for mixed heavy-light four-point correlators $\langle GGOO \rangle$ in $\mathcal{N} = 4$ super-Yang-Mills theory at large N , where $O \equiv \mathcal{O}_2$ is the chiral primary operator in the stress-tensor multiplet and G are (dual) giant graviton operators with dimension of order N , including the maximal determinant case. The loop integrand is expanded in a basis of labeled f -graphs—necessarily including nonplanar topologies due to the dimension- N nature of the giant gravitons—and the coefficients are fixed by various bootstrap conditions, including double-triangle and triangle rules in the cusp and operator product expansion limits, integrated correlators from supersymmetric localization, and a ten-dimensional hidden symmetry, the latter also allowing extension to correlators involving generic chiral primaries $\mathcal{O}_k$. Together, these inputs uniquely determine the correlator through three loops, passing further nontrivial consistency checks. For the maximal determinant operator, we reproduce the known results through two loops and obtain the full three-loop correction.

DOI: [10.1103/9nr9-5y4f](https://doi.org/10.1103/9nr9-5y4f)

I. INTRODUCTION

Giant gravitons are a distinguished class of half-Bogomol'nyi-Prasad-Sommerfield (BPS) operators in $\mathcal{N} = 4$ super-Yang-Mills theory (SYM) whose conformal dimensions are of order N , placing them far outside the familiar single-trace sector. In the AdS/CFT correspondence they are dual to D3-branes wrapping cycles in $AdS_5 \times S^5$ [1–3], and their correlation functions probe a regime where large- N counting and perturbative dynamics interplay in ways not accessible to standard planar methods. Extending bootstrap techniques to these heavy operators presents qualitatively new challenges: the reduced symmetry enlarges the space of admissible integrands, nonplanar topologies contribute even at leading large N , and the relevant operator product expansion (OPE) data involving giant gravitons are less known. Recent developments—including powerful graphical

bootstrap methods, all-order results for integrated correlators from supersymmetric localization, and a hidden ten-dimensional symmetry—have nevertheless brought such correlators within reach of systematic perturbative analysis.

We study mixed heavy-light four-point correlators $\langle GGOO \rangle$, with two (dual) giant graviton operators G and two chiral primary operators of fixed dimensions. The giant gravitons are conveniently parametrized by α : for $0 < \alpha \leq 1$ they are the sphere giant gravitons, with $\alpha = 1$ the full determinant operator, while $\alpha < 0$ gives dual giant gravitons. For the light operators we focus on the dimension-two chiral primary $\mathcal{O}_2$ in the stress-tensor multiplet, and a recently proposed ten-dimensional hidden symmetry [4,5] then allows the results to be promoted to chiral primaries of generic dimension.

To implement the bootstrap, we expand the loop integrand in a basis of labeled f -graphs [6], crucially including nonplanar topologies. The coefficients in this ansatz are then fixed by several complementary inputs: double-triangle relations from lightlike cusp limits [7], triangle rules in the GG , OO , and GO OPE channels, integrated-correlator constraints [8–10] from supersymmetric localization, and a conjectured extension of the ten-dimensional hidden symmetry to generic giant graviton operators. We further exploit the fact that OPE coefficients in $\mathcal{N} = 4$

*Contact author: songhe@itp.ac.cn

†Contact author: shicanxin@itp.ac.cn

‡Contact author: tangyichao@itp.ac.cn

§Contact author: c.wen@qmul.ac.uk

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SYM admit representations in terms of harmonic sums [11], providing additional constraints.

Together, these (overconstrained) conditions uniquely determine the correlator through three loops. For $\alpha = 1$ we reproduce the known answer through two loops [12] and provide the complete three-loop result. We extract new OPE data throughout: in particular, the OPE coefficients d_S of two giant gravitons and a twist-two operator of spin S generically grow as $(\log S)^\ell$ at ℓ loops, but remarkably reduce to $\log S$ for the maximal determinant operator with $\alpha = 1$, confirming and extending an observation made at two loops [12]. The results exhibit further characteristic structure: enhanced permutation symmetry in the planar f -graphs at $\alpha = -1$, and in the very-heavy dual giant graviton limit $\alpha \rightarrow -\infty$, the correlator is dominated by a special class of f -graphs, in agreement with the expected semiclassical description.

II. BOOTSTRAP SETUP

The sphere and AdS giant gravitons are extracted from the generating functions as

$$G_{S;\alpha}(x, y) := \oint \frac{dz}{2\pi i} \frac{1}{z^{\alpha N+1}} \det(\mathbb{1} + zy \cdot \Phi(x)), \quad (1)$$

$$G_{\text{AdS};\beta}(x, y) := \oint \frac{dz}{2\pi i} \frac{1}{z^{\beta N+1}} \frac{1}{\det(\mathbb{1} - zy \cdot \Phi(x))}, \quad (2)$$

where $\Phi^I(x)$ with $I = 1, \dots, 6$ denotes the six scalars of $\mathcal{N} = 4$ SYM, and y_I is a null $\text{SO}(6)$ polarization vector. The parameters satisfy $0 < \alpha \leq 1$ and $\beta > 0$. Inspired by the symmetry $\alpha \leftrightarrow -\beta$ of the integrated correlators observed in [9], we use the unified notation

$$G_\alpha := \begin{cases} G_{S;\alpha} & \text{for } 0 < \alpha \leq 1, \\ G_{\text{AdS};-\alpha} & \text{for } \alpha < 0, \end{cases} \quad (3)$$

for the two types of giant gravitons. The four-point correlator we will consider is defined as

$$\mathcal{T}_\alpha := \frac{\langle G_\alpha(x_1, y_1) G_\alpha(x_2, y_2) \mathcal{O}_2(x_3, y_3) \mathcal{O}_2(x_4, y_4) \rangle}{\langle G_\alpha(x_1, y_1) G_\alpha(x_2, y_2) \rangle \langle \mathcal{O}_2(x_3, y_3) \mathcal{O}_2(x_4, y_4) \rangle}, \quad (4)$$

where $\mathcal{O}_2(x, y) = \text{Tr}[(y \cdot \Phi(x))^2]$ is the chiral primary operator. In the weak-coupling expansion, they can be expressed as [11,13]

$$\mathcal{T}_\alpha = \mathcal{T}_\alpha^{\text{free}} + \frac{4|\alpha|R_{1234}}{Nd_{12}^2 d_{34}^2 x_{13}^2 x_{24}^2} \sum_{\ell=1}^{\infty} a^\ell \mathcal{T}_\alpha^{(\ell)}, \quad (5)$$

where we have defined

$$a := \frac{g_{\text{YM}}^2 N}{4\pi^2}, \quad x_{ij}^2 := (x_i - x_j)^2, \quad d_{ij} := \frac{2y_i \cdot y_j}{x_{ij}^2}, \quad (6)$$

and the prefactor R_{1234} is fixed by the superconformal symmetry. We will now implement the bootstrap construction of the correlator.

Ansatz. We begin by introducing the basis for the loop integrands of the correlator. The ℓ -loop contribution can be expressed as

$$\mathcal{T}_\alpha^{(\ell)} = \frac{\xi_4}{\ell!} \int \frac{d^4 x_5 \cdots d^4 x_{4+\ell}}{(-4\pi^2)^\ell} F_{4+\ell}, \quad (7)$$

where $\xi_4 = x_{13}^4 x_{24}^4 x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2$. The integrand $F_{4+\ell}$ can be constructed through the chiral Lagrangian insertion procedure, as has been done for $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ [6,14]. In this way, $F_{4+\ell}$ can be expressed in terms of f -graphs, however, unlike the case of $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$, due to the insertion of two giant gravitons, $F_{4+\ell}$ has reduced permutation symmetry $S_2 \times S_{2+\ell}$. We will denote such more general f -graphs as *labeled* f -graphs [15]. In conclusion, we have

$$F_{4+\ell} = \sum_n c_n^{(\ell)} f_n^{(\ell)}, \quad (8)$$

$$f_n^{(\ell)} = \frac{p_n^{(\ell)}(x_i)}{\prod_{1 \leq i < j \leq 4+\ell} x_{ij}^2} + P_{12;34 \dots (4+\ell)}, \quad (9)$$

where $P_{12;34 \dots (4+\ell)}$ denotes permutations of $\{1, 2\}$ and $\{3, 4, \dots, 4+\ell\}$, and the numerator $p_n^{(\ell)}(x_i)$ is a monomial in x_{ij}^2 of degree $(\ell - 1)$ at each point x_i . For convenience, we normalize it so that after summing over the permutations, the coefficient of each term is 1. Here, $f_n^{(\ell)}$ can be represented by $(4+\ell)$ -point graphs: vertices denote the points x_i , with x_1 and x_2 distinguished to reflect the permutation structure, while solid and dashed lines denote factors of $1/x_{ij}^2$ and x_{ij}^2 , respectively. One- and two-loop examples are shown in Fig. 1—see Supplemental Material [16] for the explicit three-loop graph basis and related technical details. Importantly, the topology of the f -graphs (defined by the subgraph containing only solid lines) is not necessarily planar: the intricate structure and dimension- N nature of giant graviton operators allow nonplanar graphs to contribute even in the large- N limit. The coefficients $c_n^{(\ell)}$ are fixed by imposing several constraints, each of which yields linear relations among them. We now briefly summarize these constraints.

Double-triangle rule. The double-triangle rule originates from the behavior of the correlator $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ in the “cusp” limit, where three points become consecutively lightlike separated, i.e., $x_{ij}^2, x_{jk}^2 \rightarrow 0$ [7,24]. At the level of the integrand, the double-triangle rule takes the form

$$\lim_{\substack{x_{ij}^2, x_{jk}^2 \rightarrow 0 \\ x_n \rightarrow x_j}} \frac{x_{ij}^2 x_{jk}^2 x_{i,n}^2 x_{j,n}^2 x_{k,n}^2}{x_{ik}^2} F_n - 2x_{ij}^2 x_{jk}^2 F_{n-1} = 0. \quad (10)$$

It is straightforward to generalize this rule to giant graviton correlators. There are different kinds of cusp singularities depending on which operators participate in the cusp. We consider two kinds of double-triangle rules:

- (i) *GOO* cusp, with $x_{23}^2, x_{34}^2 \rightarrow 0$;
- (ii) *LOO* cusp, with $x_{53}^2, x_{34}^2 \rightarrow 0$.

Here, L denotes a Lagrangian insertion point, and in the example above this is the inserted point x_5 .

Triangle rule. The triangle rule is a graphical representation of the constraints imposed by the OPE limit [25]. In the case of the limit $x_i \rightarrow x_j$, the triangle rule for the integrand can be expressed as

$$\lim_{x_j, x_n \rightarrow x_i} x_{ij}^2 x_{i,n}^2 x_{j,n}^2 F_n - 2\gamma_O x_{ij}^2 F_{n-1} = 0, \quad (11)$$

where γ_O is the one-loop anomalous dimension of the exchanged operator that dominates the OPE limit. Because we have two kinds of operators, there are three different OPE channels:

- (i) *GG* channel, with $x_2 \rightarrow x_1$: the dominant contribution is from the Konishi operator $\mathcal{K}$, with $\gamma_{\mathcal{K}} = 3$ [26];
- (ii) *OO* channel, with $x_4 \rightarrow x_3$: the dominant contribution is also from the Konishi operator;
- (iii) *GO* channel, with $x_4 \rightarrow x_1$: the dominant contribution is from an unknown operator G' , whose anomalous dimension can be fixed using the triangle rule at two loops, yielding $\gamma_{G'} = 2 - \alpha$ [27].

Integrated-correlator constraints. As shown in [28,29], the integrated correlators [30,31], when applied to the perturbative expansion of the correlator, can be expressed as periods of f -graphs,

$$\mathcal{I}_\alpha(a) = - \sum_{\ell=1}^{\infty} \frac{4|\alpha|}{\ell!} \left(-\frac{a}{4}\right)^\ell \sum_n c_n^{(\ell)} \mathcal{P}(f_n^{(\ell)}), \quad (12)$$

where the periods of f -graphs are defined as

$$\mathcal{P}(f_n^{(\ell)}) = \frac{1}{(\pi^2)^{\ell+1}} \int \frac{d^4 x_1 \dots d^4 x_{\ell+4}}{\text{vol}(\text{SO}(1,5))} f_n^{(\ell)}(x_i). \quad (13)$$

Their explicit values through three loops are given as Supplemental Material. For convenience, we quote the giant graviton integrated correlators through three loops, as obtained from the supersymmetric localization [9],

$$\begin{aligned} \mathcal{I}_\alpha(a) = & 6\alpha\zeta(3)a + \frac{15}{2}\alpha(\alpha-4)\zeta(5)a^2 \\ & + \frac{35}{8}\alpha(2\alpha^2 - 15\alpha + 30)\zeta(7)a^3 + \dots \end{aligned} \quad (14)$$

Given the periods of the f -graph integrals, we can compute the integrated correlators from our ansatz, and matching

with the results from supersymmetric localization then imposes another set of constraints.

Extraction of twist-two OPE data. A further source of constraints comes from extracting the OPE data of the twist-two trajectory $\mathcal{O}_S$ with arbitrary spin S in the $\mathbf{20}'$ R -symmetry channel. In the double OPE limit $x_2 \rightarrow x_1, x_4 \rightarrow x_3$, its contribution takes the schematic form [32,33]

$$\mathcal{T}_\alpha|_{\text{twist-2}} = \sum_S d_S c_S g_{\Delta_S, S}(z, \bar{z}) + \dots, \quad (15)$$

where $g_{\Delta_S, S}(z, \bar{z})$ is the conformal block with $\Delta_S = 2 + S + \gamma_S$, and d_S and c_S denote the OPE coefficients of $\mathcal{O}_S$ with two giant gravitons and two chiral primary operators, respectively. Starting from the correlator ansatz with arbitrary f -graph coefficients, one can evaluate the corresponding conformal integrals and match the OPE expansion order by order. This allows us to extract the anomalous dimensions γ_S and structure constants d_S as functions of those coefficients, while also imposing non-trivial constraints on the f -graph coefficients.

Harmonic-sum representability. Once the twist-two OPE data have been extracted for sufficiently many spins, one can further constrain the correlator by demanding that they admit a closed-form representation in terms of nested harmonic sums [denoted as $S_{\vec{a}}(S)$],

$$\gamma_S = \sum_{\vec{a}} c_{\vec{a}} S_{\vec{a}}(S), \quad d_S = \sum_{\vec{a}} b_{\vec{a}} S_{\vec{a}}(S), \quad (16)$$

with the sums taken over a basis of harmonic sums of the appropriate transcendental weight [11]. In practice, we use the package `HarmonicSums` [34] to construct this basis, and then fix the coefficients by matching to the OPE data extracted at finite spin [12,15,33]. Requiring that the data fit a single harmonic-sum expression for arbitrary spin then gives additional constraints on the remaining f -graph coefficients. Therefore, harmonic-sum representability is an extra bootstrap input, rather than merely a compact way of presenting the final answer.

Hidden symmetry. It was recently proposed that the heavy-light correlators with maximal-determinant operators enjoy a ten-dimensional hidden symmetry [4,5,35] by extending what was known for four-point correlators of single-trace operators [36,37]. This was achieved by viewing the heavy-light correlators as defect systems, a perspective that naturally extends to more generic (dual) giant gravitons. We conjecture the hidden symmetry applies to heavy-light correlators with more general giant graviton operators considered here, supported by an explicit calculation at two loops [38]. We further impose the constraint, motivated by [29] for four-point correlators of single-trace operators, that after summing over R -symmetry channels at ℓ loops, all transcendental constants cancel except

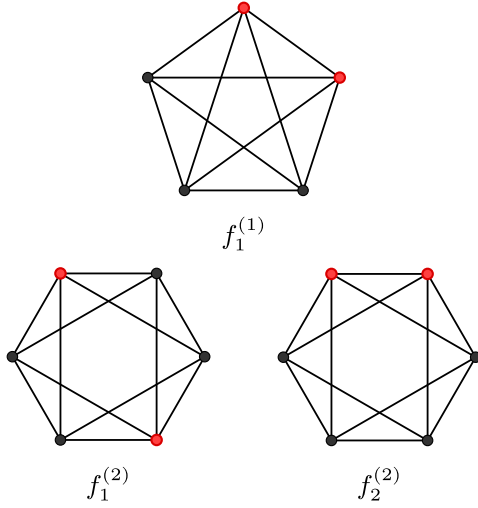


FIG. 1. Labeled f -graphs at one and two loops, where each solid line represents a propagator $1/x_{ij}^2$. The highlighted vertices indicate the distinguished points x_1 and x_2 .

$\zeta(2\ell + 1)$ —we assume this pattern extends to giant graviton correlators.

III. GIANT GRAVITON CORRELATORS UP TO TWO LOOPS

At the tree level, the correlator reads

$$T_\alpha^{\text{tree}} = \frac{2|\alpha|}{Nd_{12}^2 d_{34}^2} ((|\alpha|N - 1)d_{13}d_{14}d_{23}d_{24} + d_{12}d_{14}d_{23}d_{34} + d_{12}d_{13}d_{24}d_{34}) + O\left(\frac{1}{N^2}\right), \quad (17)$$

which is obtained from explicit Wick contractions, and we note the result contains an $O(N^0)$ contribution.

At one loop, there is a single f -graph, namely, the one-loop box integral, see also Fig. 1. We, therefore, only need to determine its overall coefficient, which is fixed by the integrated-correlator result (14). Using the period of the one-loop f -graph given as Supplemental Material, we find

$$F_5 = f^{(1)}(x_i). \quad (18)$$

At two loops, there are two independent f -graphs, whose definitions are given as Supplemental Material. The integrated-correlator constraint alone is not enough to determine them. Combining it with an OPE limit, for example the OO -channel OPE [39], we find

$$F_6 = f_2^{(2)}(x_i) - \alpha f_1^{(2)}(x_i). \quad (19)$$

We have also checked that this result is consistent with the double-triangle rules in the GOO and LOO cusp limits, and for $\alpha = 1$ it reproduces the known

maximal-determinant result of [12,15]. A few further comments are in order. First, when $\alpha = -1$ (i.e., for a dual giant graviton operator of conformal dimension N), the $S_2 \times S_4$ permutation symmetry of the two-loop correlator is enhanced to S_6 . Second, in the limit $\alpha \rightarrow -\infty$, the term $f_1^{(2)}(x_i)$ dominates, in agreement with expectations for the very-heavy dual giant graviton limit [9]. Third, the one- and two-loop results determine the anomalous dimension in the GO -channel OPE to be $\gamma_{G'} = 2 - \alpha$, as noted above. This result will be used in the three-loop bootstrap. Furthermore, assuming the ten-dimensional hidden symmetry [4,5,35], our result provides the generating function for the correlator $\langle GG\mathcal{O}_p \mathcal{O}_{p+2k} \rangle$ for generic p and k [40].

IV. GIANT GRAVITON CORRELATORS AT THREE LOOPS

We now consider the correlators at three loops. There are 15 labeled f -graphs at this order, which we denote by $f_{i,j}^{(3)}$ following the organization of Supplemental Material, where their explicit definitions are given. One Gram-determinant relation among the 15 integrals leaves 14 independent ones and we choose a basis in which the coefficient of $f_{1,2}^{(3)}$ is set to 1.

The number of independent coefficients fixed by each input is summarized in Table I. We find two minimal sets of constraints that determine all but one of the independent coefficients. In both sets, the GO -channel OPE and integrated-correlator constraints are indispensable. By contrast, the cusp constraints $GOO + LOO$ and the twist-two constraints $\text{Tw2} + \text{HS}$ are interchangeable: either pair, combined with GO and Int , leaves only a single parameter unfixed before imposing hidden symmetry. The agreement between these two routes provides a nontrivial consistency check, although the routes play rather different practical roles. The $GOO + LOO$ cusp constraints can be imposed directly at the integrand level, whereas the $\text{Tw2} + \text{HS}$ route

TABLE I. Number of independent three-loop coefficients fixed by each individual constraint and by two minimal constraint sets, each leaving one parameter unfixed.

Constraint	Coeffs. fixed
GOO cusp	7
LOO cusp	6
GG -channel OPE	1
OO -channel OPE	3
GO -channel OPE	2
Integrated correlator (Int)	2
Extraction of twist-two OPE data (Tw2)	7
Harmonic-sum representability (HS)	4
Minimal sets	Coeffs. fixed
$GO + \text{Int} + GOO + LOO$	13
$GO + \text{Int} + \text{Tw2} + \text{HS}$	13

is tied more directly to the extracted CFT data and its analytic dependence on spin, but requires evaluating the conformal integrals to obtain finite-spin OPE data. Additionally, the GG - and OO -channel OPE provide overconstraints of the system, and thus furnish further independent checks.

We denote the unfixed coefficient by c , and the correlator takes the following form:

$$F_7(c) = (2c + \alpha^2 + \alpha - 1)f_{1,4}^{(3)}(x_i) + cf_{1,1}^{(3)}(x_i) + f_{1,2}^{(3)}(x_i) + f_{1,3}^{(3)}(x_i) - 2(c - 2\alpha + 2)[f_{2,3}^{(3)}(x_i) - f_{3,3}^{(3)}(x_i)] + (c + \alpha)[2f_{3,1}^{(3)}(x_i) - f_{2,1}^{(3)}(x_i) - f_{4,1}^{(3)}(x_i)]. \quad (20)$$

We finally apply the ten-dimensional hidden symmetry to obtain the correlator $\langle GG\mathcal{O}_{k_1}\mathcal{O}_{k_2} \rangle$ at three loops. Unlike the simplest case $k_1 = k_2 = 2$, such generic four-point functions have nontrivial R -symmetry dependence, and their integrated correlators contain $\zeta(3)^2$ in addition to $\zeta(7)$, due to the nonplanar f -graphs. As shown as Supplemental Material, requiring the $\zeta(3)^2$ terms to cancel fixes $c = -\alpha$ [41]. The three-loop correlator is then given by

$$F_7 = -\alpha f_{1,1}^{(3)} + f_{1,2}^{(3)} + f_{1,3}^{(3)} + (\alpha^2 - \alpha - 1)f_{1,4}^{(3)} + (6\alpha - 4)[f_{2,3}^{(3)} - f_{3,3}^{(3)}]. \quad (21)$$

We note that when $\alpha = 1$, the first line in the above equation, which contains all the planar f -graphs, agrees with the result of [15], while the second line supplies the nonplanar contributions. As we will see shortly, these nonplanar f -graphs are required not only to match the integrated correlator but also to ensure the correct large-spin behavior of the OPE coefficients. When $\alpha = -1$, all the planar f -graphs have coefficient 1, so the planar sector acquires enhanced full permutation symmetry, as at two loops. Furthermore, the term that dominates in the limit $\alpha \rightarrow -\infty$ is $f_{1,4}^{(3)}(x_i)$, in agreement with the effective description of the very-heavy dual giant graviton limit [9]. It is intriguing that these three properties together, namely, those at $\alpha = 1$, $\alpha = -1$, and $\alpha \rightarrow -\infty$, would also fix the coefficient c in (20) to be $c = -\alpha$.

Large-spin limit of OPE data. Our result allows one to extract new OPE data. Here, we focus on the OPE coefficient d_S of two giant gravitons and a twist-two operator of spin S . We defer the explicit result of d_S at finite spin to Supplemental Material and concentrate on its large- S limit, which can be obtained from the harmonic-sum representation using the package `HarmonicSums` [34]. The one-parameter family $F_7(c)$ at three loops, together with the one- and two-loop results, implies the following large-spin expansion:

$$\begin{aligned} \ln(d_S/d_{S,\text{tree}})^2 = & -a(\zeta(2) + 2 \ln 2 \ln s) + \frac{a^2}{20} [(23 - 7\alpha)\zeta(2)^2 + (20 \ln 2 \zeta(2) - 30\alpha\zeta(3)) \ln s \\ & + 30 \ln 2 \zeta(3) + 10(1 - \alpha)\zeta(2)(\ln s)^2] - \frac{a^3}{840} [420 \ln 2 \zeta(2)\zeta(3) + 2100 \ln 2 \zeta(5) \\ & + 2(56c - 604\alpha + 78\alpha^2 + 915)\zeta(2)^3 + 35(18c + \alpha^2 - 88\alpha + 57)\zeta(3)^2 \\ & + 21(44 \ln 2 \zeta(2)^2 + 5(12c - \alpha^2 - 24\alpha + 21)\zeta(2)\zeta(3) - 50(3c - \alpha - \alpha^2 + 5)\zeta(5)) \ln s \\ & + 168(1 - \alpha)(9 - \alpha)\zeta(2)^2(\ln s)^2 + 70(1 - \alpha)(15 - \alpha)\zeta(3)(\ln s)^3] + O(1/s), \end{aligned} \quad (22)$$

where $\ln s := \ln S + \gamma_E$, with γ_E being Euler's constant. We note that remarkably, when $\alpha = 1$ (i.e., for the full determinant operator), all the terms proportional to $(\log s)^2$ and $(\log s)^3$ vanish. This was first observed at two loops in [12]. Our result shows that the cancellation of $(\log s)^k$ terms with $1 < k \leq \ell$ at ℓ loops occurs only for $\alpha = 1$ and persists to three loops [43]. At three loops, this cancellation relies crucially on the nonplanar f -graphs in (21).

V. CONCLUSION AND OUTLOOK

In this work we developed a bootstrap framework for mixed giant graviton correlators in $\mathcal{N} = 4$ super-Yang-Mills theory. Focusing on $\langle GGO \rangle$ in the leading large- N limit, we showed that a basis of labeled f -graphs, together

with cusp and OPE constraints, localization data, and hidden-symmetry input, is sufficient to determine the correlator through three loops. For the sphere giant graviton correlator, our construction reproduces the known results through two loops and yields the three-loop correction, which passes nontrivial checks from integrated correlators and from the large-spin limit of the extracted OPE data.

Our analysis suggests several natural directions for future work. It would be important to place the hidden symmetry for generic giant graviton sectors on firmer ground and to extend the bootstrap to more general $\langle GGO_{p_1}O_{p_2} \rangle$ correlators with nontrivial R -symmetry dependence. Another interesting direction is to move beyond the leading large- N limit, where finite- N effects should further illuminate the interplay between determinant

operators, nonplanar dynamics, and defectlike descriptions of giant gravitons. More broadly, the present results indicate that heavy BPS sectors admit a surprisingly rigid perturbative organization and may provide a useful bridge between localization, OPE bootstrap, and the emergent higher-dimensional structures of AdS/CFT.

It is also natural to proceed to higher-loop correlators. In this direction, the integrand-level nature of the cusp constraints should be especially useful, since extracting finite-spin OPE data from conformal integrals becomes substantially more demanding at higher loops. Another possible simplification comes from the observed correlation between the α dependence of an f -graph coefficient and the power of x_{12}^2 in the corresponding labeled f -graph. In the examples studied here, if a graph carries a factor $(x_{12}^2)^n$, with $n \geq -1$, then its coefficient is a polynomial in α of degree at most $n + 1$. If this pattern persists at higher loops, it would allow one to start from a smaller ansatz,

$$c_n^{(\ell)}(\alpha) = \sum_{i=0}^{n+1} c_{n,i}^{(\ell)} \alpha^i, \quad (23)$$

where the $c_{n,i}^{(\ell)}$ are rational numbers. For determining higher-loop correlators, the integrated-correlator results will again be important, and they are known to all orders [9]. However, the periods of the relevant f -graphs have not been computed exhaustively beyond three loops [44]. Higher-loop results would be especially interesting for clarifying some of the remarkable structures observed here, for instance, the

large-spin behavior at $\alpha = 1$ and the enhanced full permutation symmetry of the planar sector at $\alpha = -1$.

ACKNOWLEDGMENTS

We would like to thank Augustus Brown, Frank Coronado, Alessandro Georgoudis, Xuhang Jiang, Yunfeng Jiang, Yu Wu, Shun-Qing Zhang, Yang Zhang, and Xinan Zhou for insightful discussions. We also would like to thank Frank Coronado, Yunfeng Jiang, Yu Wu, and Yang Zhang for their comments on the draft. We are especially grateful to Xuhang Jiang for his help with the evaluation of the three-loop integrals. S. H. is supported by the National Natural Science Foundation of China under Grant No. 12225510 and No. 12447101, and by the New Cornerstone Science Foundation. C. S. is supported by the China Postdoctoral Science Foundation under Grant No. 2022TQ0346 and the National Natural Science Foundation of China under Grant No. 12347146. C. W. is supported by a Royal Society University Research Fellowship, No. URF\R\221015, and a STFC Consolidated Grant, No. ST/T000686/1 “Amplitudes, strings and duality.”

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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