

Constraints on the odderon amplitude in the color glass condensate framework

M. Roa^{1,2,*}, M. Siddikov^{1,3,†}, Y. Gentile^{2,‡} and I. Zemlyakov^{1,4,§}

¹*Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

²*Facultad de Ingeniería, Laboratorio DataScience, Universidad de Playa Ancha, Leopoldo Carvallo 270, Valparaíso, Chile*

³*Centro Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile*

⁴*Instituto de Física, Pontificia Universidad Católica de Valparaíso, Avenida Brasil 2950, Valparaíso, Chile*



(Received 6 March 2026; accepted 8 July 2026; published 10 August 2026)

In this manuscript, we analyze the elastic pp and $p\bar{p}$ cross section in the color glass condensate framework, treating them as a dilute-dense system, and derive the phenomenological constraints (upper bounds) on the odderon-mediated part of the dipole scattering amplitude $\mathcal{O}(Y, \mathbf{r}, \mathbf{b})$. For our analysis, we used the experimental data available from TOTEM-D0 and ISR Collaborations and constructed a combination of the cross sections which is directly proportional to the odderon-mediated part of the amplitude. This allows one to suppress possible uncertainties associated with charge-parity even part of the amplitude. We analyze two phenomenological parametrizations of odderons and demonstrate that, after minor adjustments to the global normalization, they can describe the experimental data reasonably well, although due to large experimental uncertainties the odderon amplitude remains loosely constrained.

DOI: 10.1103/9kg5-3t22

I. INTRODUCTION

The odderon, a charge-parity-odd (C -odd) counterpart of the well-established Pomeron exchange, remains one of the most elusive objects in the theory of strong interactions. Originally predicted in the 1970s within the framework of Regge theory [1,2], it is interpreted in the language of quantum chromodynamics (QCD) as the exchange of an odd number of Reggeized gluons in a color singlet configuration in the t channel [3–5]. In the dilute (perturbative) systems, the odderon arises as a coherent exchange of three interacting Reggeized gluons in the t channel, and its amplitude should satisfy the Bartels-Kwiecinski-Praszalowicz equation [6,7], written for the special case of three Reggeized gluons. Beyond that limit, the odderon has been explored in nonperturbative approaches such as the stochastic vacuum model [8] and holographic QCD [9], where it emerges from the exchange of specific string modes in the dual gravitational background. In the

color glass condensate (CGC) framework, the odderon admits a natural description in terms of correlators of Wilson lines with definite symmetry under charge conjugation, and its evolution can be studied via extensions of the Balitsky-Kovchegov (BK) Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner equations [10–12].

Since the odderon exchange is an essentially nonperturbative phenomenon, at present the amplitude of the odderon-mediated scattering cannot be predicted unambiguously from first principles and should be extracted from the experimental data. Phenomenologically, the odderon can manifest itself in observables that are sensitive to the difference between particle and antiparticle scattering [13–15] or in exclusive processes which require exchange of C parity in the t channel [16]. However, the experimental identification of the odderons remains challenging due to significant contributions from various Pomeron-mediated mechanisms that can mimic a weak odderon signal or provide sizable backgrounds. At present, the most compelling evidence in favor of odderons comes from a recent comparison of the differential cross sections for proton-proton and proton-antiproton elastic scattering at the LHC and Tevatron [17–20]. However, the estimates of the statistical significance of this signal vary significantly due to experimental and theoretical ambiguities (see a short overview of recent developments in [21,22]). For odderons, these uncertainties are especially pronounced because in the cross section it contributes commingled with a large C -even contribution, so the small errors of the latter can drastically change the

*Contact author: michael.roa@upla.cl

†Contact author: marat.siddikov@usm.cl

‡Contact author: yanal.garrido@alumnos.upla.cl

§Contact author: izemlyakov@usm.cl

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

estimates of the odderon amplitude and even cast doubt on its existence. This motivates the construction of the observables designed to be exclusively sensitive to odderon exchange, free from contamination by C -even backgrounds. On the other hand, the estimates of the odderon signal at future experiments are based on phenomenological parametrizations [16,23] that include an arbitrary parameter which controls the magnitude of odderon. In this manuscript, we try to find the upper limit on the odderon amplitude in the CGC framework and also propose a new observable (combination of cross sections) that could be more sensitive to the odderon contributions.

The paper is structured as follows. In the following Sec. II, we briefly summarize the main theoretical tools used for description of the proton structure in high-energy collisions. In Sec. III, we analyze in detail the elastic proton-proton collision scattering, discuss the role of odderons, and introduce the observable that allows one to extract the contributions of pure odderons. Later, in Sec. IV, we perform phenomenological analysis and compare theoretical predictions for this observable with existing experimental data. We also analyze upper limits for the odderon amplitude which are compatible with data. Finally, in Sec. V, we draw conclusions.

II. THEORETICAL FRAMEWORK

A. Structure of the proton and its description in high-energy processes

In general, the elementary particle is a sophisticated ensemble of Fock states with different number of quarks and gluons. However, we expect that the dominant contributions to many processes comes from the valence three-quark component of the proton. The light-cone wave function of the latter is given by [24–26]

$$|P\rangle \approx \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{\sqrt{x_1 x_2 x_3}} \delta(1 - x_1 - x_2 - x_3) \times \int \frac{d^2 k_1 d^2 k_2 d^2 k_3}{(16\pi^3)^3} 16\pi^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times \psi_3(x_1, \mathbf{k}_1; x_2, \mathbf{k}_2; x_3, \mathbf{k}_3) \times \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} |p_1, i_1, f_1; p_2, i_2, f_2; p_3, i_3, f_3\rangle, \quad (1)$$

where $P^\mu = (P^+, m_N^2/2P^+, \mathbf{0}_\perp)$ is the 4-momentum of the proton and $\psi_3(x_i, \mathbf{k}_i)$ is the universal (process-independent) nonperturbative light-front wave function of the three-quark Fock state. In Eq. (1), the variables x_i represent

the longitudinal momentum carried by each quark, and $\mathbf{k}_i$ characterize the transverse momenta of the internal motion of the quarks in the transverse plane. The Levi-Civita tensor $\epsilon_{i_1 i_2 i_3}$ reflects overall antisymmetry with respect to the color indices of individual quarks in the color singlet proton. Finally, the indices f_1, f_2 , and f_3 correspond to flavor labels of individual quarks. In our approach, we use this valence Fock state as the starting point for modeling the exclusive observables. The wave function is normalized such that the proton state satisfies the usual light-front normalization condition

$$\langle K|P\rangle = 16\pi^3 P^+ \delta(P^+ - K^+) \delta(\mathbf{P}_\perp - \mathbf{K}_\perp), \quad (2)$$

which for the three-quark system translates into

$$\int dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \times \int \frac{d^2 k_1 d^2 k_2 d^2 k_3}{(16\pi^3)^3} (16\pi^3) \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) |\psi_3|^2 = 1. \quad (3)$$

For phenomenological analysis, frequently the wave function is chosen in the Gaussian form [27]:

$$\psi_3^{(G)}(x_1, \mathbf{k}_1; x_2, \mathbf{k}_2; x_3, \mathbf{k}_3) = N \exp(-\mathcal{M}^2/2\beta^2), \quad (4)$$

$$\mathcal{M}^2 = \sum_{i=1}^3 \frac{\mathbf{k}_{\perp i}^2 + m^2}{x_i},$$

where N is the normalization constant. The mass of the constituent quark m and the parameter β are treated as free parameters fitted to the electroweak properties of the baryon octet as explained in Refs. [28,29]. The main advantage of the parametrization (4) is its simplicity, which allows one to get analytic results for the configuration space wave function. At very high-energy processes, only one of the three quarks participates (interacts with other partons), so the three-quark system may be reduced to a simpler quark-diquark system, which is characterized by the light-cone fraction x carried by the active quark and the effective separation $\mathbf{r}$ between the quark and the center of mass of two spectators. This approximation for the three-quark system was proposed in the early papers on the baryon structure in QCD [30–32] and since that time has been constantly used as a basis for various phenomenological models of the proton structure. The corresponding (squared of) wave function of the quark-diquark system is given by

$$|\psi_2(x, \mathbf{r})|^2 = \int dx_2 \int \frac{d^2 k_1 d^2 k_2}{(16\pi^3)^2} e^{i\mathbf{k}_1 \cdot \mathbf{r}} |\psi_3(x, \mathbf{k}_1; x_2, \mathbf{k}_2; x_3 = 1 - x - x_2, \mathbf{k}_3 = -\mathbf{k}_1 - \mathbf{k}_2)|^2. \quad (5)$$

For the Gaussian parametrization (4), we may obtain

$$|\psi_2^{(G)}(x, \mathbf{r})|^2 = \frac{N^2 \beta^4 x}{256 \pi^4} \int_0^{1-x_1} dx_2 x_2 (1-x-x_2) \exp \left(-\frac{\beta^2 \mathbf{r}^2 x(1-x)}{4} - \frac{m^2((x+x_2)(1-x_2)-x^2)}{\beta^2 x x_2 (1-x-x_2)} \right). \quad (6)$$

For further numerical estimates, we used the values of parameters $m = 0.26$ GeV and $\beta = 1.1$ GeV provided in [28,29], though we had to double the value of the parameter β in order to match the experimental root-mean-square (rms) radius of the proton [33]:

$$\langle r_p^2 \rangle = \frac{\int_0^1 dx \int d^2 r r^2 |\psi_2^{(G)}(x, \mathbf{r})|^2}{\int_0^1 dx \int d^2 r |\psi_2^{(G)}(x, \mathbf{r})|^2} \approx (0.83 \text{ fm})^2. \quad (7)$$

While the function (6) provides a reasonable estimate for the variables dominated by the valence quarks, it is not universal. In order to assess the model dependence introduced by the choice of the wave function, we will also use in parallel the AdS/QCD-based parametrization suggested in Ref. [34]:

$$\psi_2^{(\text{AdS/QCD})}(x, \mathbf{r}) = \frac{1}{2\sqrt{\pi}} \sqrt{\frac{q_\tau(x)}{f(x)}} (1-x) \exp \left[-\frac{(1-x)^2}{8f(x)} \mathbf{r}^2 \right], \quad (8)$$

where the parameter τ denotes the so-called twist of the Fock component. The functions $q_\tau(x)$ and $f(x)$ that control the longitudinal and transverse shape, respectively, are given by [35,36]

$$q_\tau(x) = \frac{1}{N_\tau} (1-w(x))^{\tau-2} w(x)^{-1/2} w'(x), \quad f(x) = \frac{1}{4\Lambda} \log \left(\frac{1}{w(x)} \right), \quad (9)$$

$$w(x) = x^{1-x} e^{-a(1-x)^2}, \quad a = 0.531 \pm 0.037, \quad \sqrt{\Lambda} = 0.548 \text{ GeV}. \quad (10)$$

Similar to (6), the parametrization (8) exhibits a Gaussian falloff at large distances, reflecting the confinement, although with a different x -dependent slope. We checked that this parametrization gives acceptable value of the rms radius $\sqrt{\langle r^2 \rangle} \approx 0.7$ fm, which is approximately 20% smaller than (7). In Fig. 1, we have shown the wave functions (6) and (8), together with their first moments $\int dx |\psi_2(x, \mathbf{r})|^2$ which will be needed later. In what follows, we will use for our estimates both wave functions, considering the difference between them as a tentative estimate of the size of nonperturbative effects.

B. Dipole scattering in the CGC framework

The CGC framework describes the high-energy scattering of the dilute-dense system in the eikonal approximation. In this picture, the interaction of each parton of the (dilute) projectile with the target is encoded in the universal Wilson lines $U(\mathbf{x}_\perp)$ defined as

$$U(\mathbf{x}_\perp) = P \exp \left(ig \int dx^- A_a^+(x^-, \mathbf{x}_\perp) t^a \right), \quad (11)$$

where we use notation $\mathbf{x}_\perp$ for the impact parameter of the parton, A_a^μ for the gluonic field of the target, and t_a for the

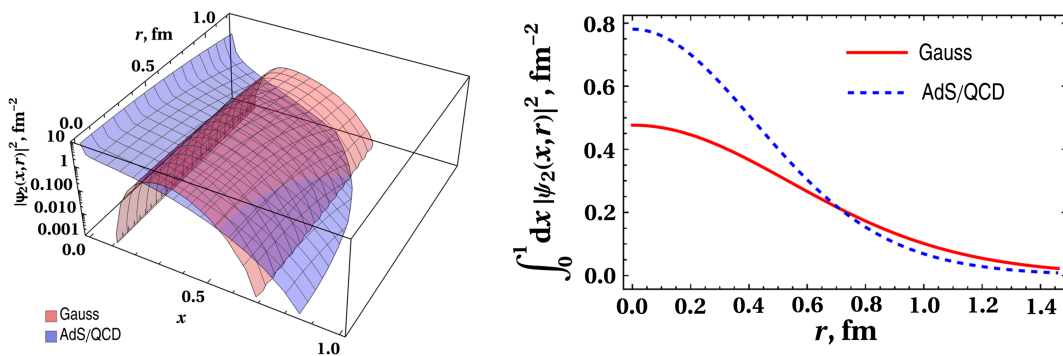


FIG. 1. Left: comparison of the wave functions (6) and (8). Right: comparison of the x -integrated wave functions (6) and (8). See the text for more details.

usual color group generators in the irreducible representation which corresponds to the parton ($\mathbf{3}$, $\bar{\mathbf{3}}$, or $\mathbf{8}$ for quark, antiquark, or gluon, respectively). The interaction of the quark-antiquark (or quark-diquark) system in this approach is given by the two-point correlator [11,12]

$$\frac{1}{N_c} \langle \text{tr}(U(\mathbf{x}_q)U^\dagger(\mathbf{x}_{\bar{q}})) \rangle_Y = 1 - \mathcal{N}(Y, \mathbf{r}, \mathbf{b}) + i\mathcal{O}(Y, \mathbf{r}, \mathbf{b}),$$

$$\mathbf{r} = \mathbf{x}_q - \mathbf{x}_{\bar{q}}, \quad \mathbf{b} = z\mathbf{x}_q + (1-z)\mathbf{x}_{\bar{q}} \quad (12)$$

where the angular bracket $\langle \dots \rangle_Y$ in the left-hand side of (12) stands for averaging over all the gluonic field configurations of the target, Y is the rapidity of the dipole,

$\mathbf{r}$ is the separation between the quarks, and $\mathbf{b}$ is the impact parameter. The functions $\mathcal{N}$ and $\mathcal{O}$ correspond to C -even and C -odd parts of the dipole scattering amplitude, respectively, and are frequently referred to as “Pomeron” and “odderon” contributions. These functions have different parity under permutation of the quark and antiquark coordinates; namely, the function $\mathcal{O}$ changes its sign under $\mathbf{r} \rightarrow -\mathbf{r}$, whereas the function $\mathcal{N}$ does not change under this transformation. This implies that the function $\mathcal{O}$ should have nontrivial dependence on the orientation of the vector $\mathbf{r}$ with respect to some external vector (e.g., impact parameter $\mathbf{b}$ or spin S). The rapidity dependence of the functions $\mathcal{N}$ and $\mathcal{O}$ should satisfy the Balitsky-Kovchegov evolution equations [10–12]

$$\frac{\partial \mathcal{N}(Y, \mathbf{r}, \mathbf{b})}{\partial Y} = \int_{\mathbf{r}_1} \mathcal{K}(\mathbf{r}, \mathbf{r}_1) [\mathcal{N}(Y, \mathbf{r}_1, \mathbf{b}) + \mathcal{N}(Y, \mathbf{r}_2, \mathbf{b}) - \mathcal{N}(Y, \mathbf{r}, \mathbf{b}) - \mathcal{N}(Y, \mathbf{r}_1, \mathbf{b})\mathcal{N}(Y, \mathbf{r}_2, \mathbf{b}) + \mathcal{O}(Y, \mathbf{r}_1, \mathbf{b})\mathcal{O}(Y, \mathbf{r}_2, \mathbf{b})], \quad (13)$$

$$\frac{\partial \mathcal{O}(Y, \mathbf{r}, \mathbf{b})}{\partial Y} = \int_{\mathbf{r}_1} \mathcal{K}(\mathbf{r}, \mathbf{r}_1) [\mathcal{O}(Y, \mathbf{r}_1, \mathbf{b}) + \mathcal{O}(Y, \mathbf{r}_2, \mathbf{b}) - \mathcal{O}(Y, \mathbf{r}, \mathbf{b}) - \mathcal{N}(Y, \mathbf{r}_1, \mathbf{b})\mathcal{O}(Y, \mathbf{r}_2, \mathbf{b}) - \mathcal{O}(Y, \mathbf{r}_1, \mathbf{b})\mathcal{N}(Y, \mathbf{r}_2, \mathbf{b})], \quad (14)$$

where $\mathbf{r}_2 = \mathbf{r} - \mathbf{r}_1$ and $\mathcal{K}(\mathbf{r}, \mathbf{r}_1)$ in (13) and (14) is the standard Balitsky-Fadin-Kuraev-Lipatov kernel with running coupling corrections:

$$\mathcal{K}(\mathbf{r}, \mathbf{r}_1) = \frac{\alpha_s(\mathbf{r})N_c}{2\pi^2} \left[\frac{\mathbf{r}^2}{\mathbf{r}_1^2 \mathbf{r}_2^2} + \frac{1}{\mathbf{r}_1^2} \left(\frac{\alpha_s(\mathbf{r}_1)}{\alpha_s(\mathbf{r}_2)} - 1 \right) + \frac{1}{\mathbf{r}_2^2} \left(\frac{\alpha_s(\mathbf{r}_2)}{\alpha_s(\mathbf{r}_1)} - 1 \right) \right]. \quad (15)$$

The dependence on the impact parameter $\mathbf{b}$ in (13) and (14) appears only parametrically, which implies that the evolution can be done independently for each $\mathbf{b}$. Because of the above-mentioned different symmetry properties of the Pomerons and odderons, the dipole amplitude $\mathcal{N}(Y, \mathbf{r}, \mathbf{b})$ saturates (cannot exceed unity) at very high energy, whereas odderons tend to decrease with energy [37]. While the function $\mathcal{N}(Y, \mathbf{r}, \mathbf{b})$ is reasonably well understood from the phenomenological fits of the deep inelastic scattering (DIS) (at least for small size dipoles), the function $\mathcal{O}(Y, \mathbf{r}, \mathbf{b})$ at present is poorly constrained, although it remains significantly smaller than $\mathcal{N}$ in all the available parametrizations. From the left panel in Fig. 2, we can see that the ratio of the C -odd to C -even components of the dipole scattering amplitude $|\mathcal{O}|^2/|\mathcal{N}|^2$ does not exceed a few percent. For this reason, the last term in the second line in Eq. (13) may be safely disregarded, so Eq. (13) decouples from odderons. This justifies the odderon-independent analysis of the C -even dipole amplitude and allows one to fix the latter with very good precision using available precise data from HERA [38]. The remaining equation (14) at fixed $\mathcal{N}(Y, \mathbf{r}, \mathbf{b})$ turns into a linear

homogeneous integro-differential equation with respect to $\mathcal{O}$. In order to understand how much the uncertainty of the dipole amplitude $\mathcal{N}(Y, \mathbf{r}_1, \mathbf{b})$ affects the odderon evolution, we evaluated numerically the evolution of the odderon amplitude from [16] over a large interval $\Delta Y = \ln(10^3)$ using three different parametrizations of $\mathcal{N}$ from [16,39,40] (see Sec. IV for more details). As we can see from the right panel in Fig. 2, the uncertainty of the dipole amplitude $\mathcal{N}(Y, \mathbf{r}, \mathbf{b})$ has only minor influence on the result of evolution and does not exceed a few percent in the region $r \lesssim 1$ fm which gives the dominant contribution. Since Eq. (14) is a linear homogeneous integro-differential equation with respect to $\mathcal{O}$, formally its general solution is defined up to rescaling transformation

$$\mathcal{O}(Y, \mathbf{r}, \mathbf{b}) \rightarrow \lambda \mathcal{O}(Y, \mathbf{r}, \mathbf{b}), \quad \lambda = \text{const}, \quad (16)$$

where λ should be fixed from the experimental data. Because of the lack of sufficient experimental evidence, in many parametrizations, which are formulated as initial conditions for evolution equations (13) and (14), the parameter λ remains largely unconstrained (provided λ is not very large,

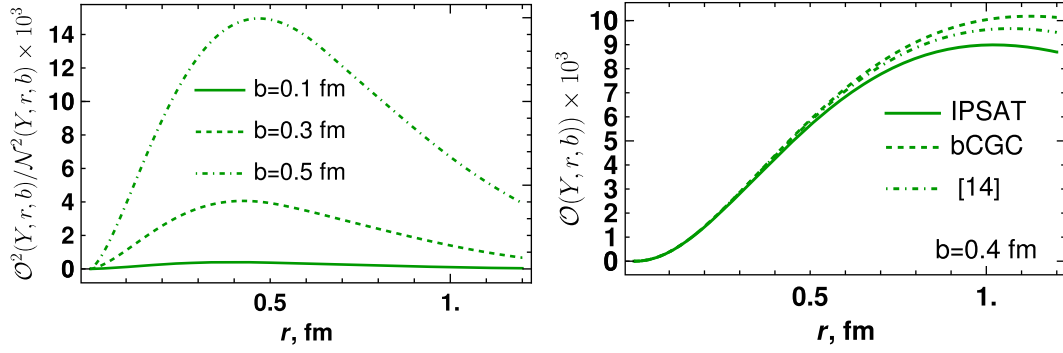


FIG. 2. Left: the ratio of the C -odd and C -even terms $\sim \mathcal{O}^2, \mathcal{N}^2$ that appear in the right-hand side of (13). For the sake of definiteness, we used the parametrization of odderons and Pomerons from [16], but we checked that the parametrization from [12,23] also gives small numbers which do not exceed a few percent. This plot clearly demonstrates that the odderons are negligible for the evolution of the C -even dipole amplitude. Right: the dependence of the odderon evolution on the parametrization of the dipole amplitude. At the initial scale, we use parametrization from [16] for the odderon amplitude and parametrizations from [16,39,40] for the C -even amplitude $\mathcal{N}(Y, r, b)$ (the latter three are fitted to the experimental data from HERA [38]). The plot represents the amplitude $\mathcal{O}(Y, r, b)$ at the final scale [rapidity difference $\Delta Y = Y_{\text{fin}} - Y_{\text{ini}} = \ln(10^3)$]. In the region $r \lesssim 1$ fm, which gives the dominant contribution to the cross section, the uncertainty due to choice of the dipole amplitude does not exceed a few percent.

so that the identity $\mathcal{O} \ll \mathcal{N}$ remains valid). For this reason, the predictions done with such parametrizations are given up to global normalization factor. In this work, we explore two popular parametrizations of the odderon amplitude available from [16] and [12,23] described below.

1. The odderon parametrization from Ref. [16]

The parametrization proposed in Ref. [16] incorporates the transverse geometry of the proton through a Gaussian profile and introduces a local saturation scale that depends on both the dipole size and the impact parameter. At the initial scale Y_0 , which corresponds to $x \approx 10^{-2}$, the odderon amplitude is chosen as

$$\mathcal{O}(Y_0, \mathbf{r}, \mathbf{b}) = \frac{\Lambda_{\text{JV}}}{8} \left[R_A \frac{dT_A(b)}{db} A^{2/3} \frac{\sigma_0}{2} \right] Q_{S,0}^3 A^{1/2} r^3 (\hat{\mathbf{r}} \cdot \hat{\mathbf{b}}) \times \log \left(\frac{1}{r\Lambda_{\text{QCD}}} + e_c e \right) \exp \left[-\frac{1}{4} r^2 Q_{0,p}^2(\mathbf{r}, \mathbf{b}) \right], \quad (17)$$

where the exponential suppression of large dipoles is controlled by the local saturation scale $Q_{0,p}^2(\mathbf{r}, \mathbf{b})$:

$$Q_{0,p}^2(\mathbf{r}, \mathbf{b}) \equiv AT_p(\mathbf{b}) \frac{\sigma_0}{2} Q_{S,0}^2 \log \left(\frac{1}{r\Lambda_{\text{QCD}}} + e_c e \right), \quad T_p(\mathbf{b}) = \frac{1}{\pi R_p^2} \exp \left(-\frac{b^2}{R_p^2} \right), \quad (18)$$

$$Q_{S,0}^2 = 0.06 \text{ GeV}^2, \quad e_c = 18.9, \quad \frac{\sigma_0}{2} = 16.36 \text{ mb}. \quad (19)$$

The proton radius R_p is fixed using the relation $\pi R_p^2 = \sigma_0/2 = 4\pi B_p$, and the value of the constant Λ_{JV}

for the sake of definiteness was chosen as in the Jeon-Venugopalan model [16,41]:

$$\lambda = \Lambda_{\text{JV}} = -\frac{3}{16} \frac{N_c^2 - 4}{(N_c^2 - 1)^2} \frac{Q_{S,0}^3 A^{1/2} R_A^3}{\alpha_s^3 A^2}. \quad (20)$$

In view of the above-mentioned normalization uncertainty, we will replace Λ_{JV} with $\lambda \Lambda_{\text{JV}}$ and will treat λ as a free parameter whose values can change in a certain range dictated by consistency with experimental pp and $p\bar{p}$ data.

2. The odderon parametrization from Refs. [12,23]

The parametrization proposed in [12,23] does not take into account the impact parameter dependence of the dipole and considers that the odderon amplitude $\mathcal{O}$ has a nontrivial dependence on the angle between the dipole separation $\mathbf{r}$ and the transverse spin $\mathbf{S}$ of the target. This parametrization is better suited for description of the observables which include explicitly the dependence on the spin of the target, such as the gluon Sivers function. Explicitly, the parametrization is given by

$$\mathcal{O}(Y_0, \mathbf{r}) = 2\kappa Q_{s,0}^3 r^2 [\mathbf{S} \times \mathbf{r}]_z \exp(-r^2 Q_{s,0}^2) = \kappa Q_{s,0}^3 r^3 \sin \theta \exp(-r^2 Q_{s,0}^2), \quad (21)$$

where θ is the angle between the vectors $\mathbf{S}$ and $\mathbf{r}$, the variable $Q_{s,0} \approx 1$ GeV is the initial saturation scale, and κ is a dimensionless parameter controlling the normalization and the sign of the odderon amplitude. While in [23] the value of $\kappa = 1/3$ was used for definiteness, as we discussed earlier, we will assume that it may be multiplied by an arbitrary constant λ . The lack of explicit b dependence in the parametrization (21) is not relevant for inclusive observables. However, for exclusive processes, the b

TABLE I. Impact-parameter profile functions $S(b)$ used in this work. In what follows, we will refer to them as $S_1(b)$, $S_2(b)$, and $S_3(b)$, respectively. The effective parameter m determines suppression of the odderons at large values of the impact parameter b .

Ref.	Profile function $S(b)$	m
[42]	$mbK_1(mb)$	0.5412 GeV
[43]	$\text{Exp}(-mb)$	0.840 GeV
[39]	$\text{Exp}(-mb^2)$	0.070 GeV ²

dependence plays a crucial role and determines the dependence of the cross section on the invariant momentum transfer t . For this reason, we introduce a multiplicative impact-parameter-dependent profile $S(b)$ into (21), namely, replace (21) with

$$\mathcal{O}(Y_0, \mathbf{r}, \mathbf{b}) = \lambda \kappa Q_{s0}^3 r^3 (\hat{\mathbf{r}} \cdot \hat{\mathbf{b}}) \exp(-r^2 Q_{s0}^2) S(b). \quad (22)$$

For the profile $S(b)$, we analyze different parametrizations of $S(b)$ suggested in Refs. [39,42,43] in the context of modeling the dipole scattering amplitude and summarized in Table I. The parameter λ (its upper possible value) will be fixed later from the experimental fit of pp and $p\bar{p}$ data. Since in what follows we will consider only unpolarized observables, in (22) we also slightly modified the dependence on the orientation of $\mathbf{r}$ and replaced the spin-dependent factor $[\mathbf{S} \times \hat{\mathbf{r}}]_z$ with $(\hat{\mathbf{r}} \cdot \hat{\mathbf{b}})$. We can see that the amplitude (22) satisfies the desired antisymmetry property $\mathcal{O}(-\mathbf{r}, \mathbf{b}) = -\mathcal{O}(\mathbf{r}, \mathbf{b})$ of the odderon amplitude.

III. ELASTIC pp SCATTERING

As we mentioned in the Introduction, odderons have been predicted many years ago; however, for a long time, any attempts to detect them experimentally yielded only upper limits, and at present the only compelling indication in favor of odderons is the mismatch of the pp and $p\bar{p}$ elastic cross-sections measured by the TOTEM [20] and D0 [19] Collaborations. For this reason, it is not surprising that the estimates for the odderon amplitude vary significantly and include unknown constants. To the best of our knowledge, most of the previous attempts to describe these data were based on the effective models that encode C -odd and C -even parts of the elastic pp amplitude [44–47]. However, the main limitation of those studies is the lack of universality: Its extension to other odderon-mediated processes is not straightforward, whereas extrapolation to different energies relies on additional assumptions that should be fixed from experimental data rather than from the QCD evolution equations.

In our phenomenological analysis, we largely follow the approach outlined in [48] and compute rigorously the differential cross section $d\sigma/dt$ for the elastic proton-proton and proton-antiproton scattering using the color glass condensate framework. We will consider the pp and

$p\bar{p}$ as a dilute-dense system, treating one of the hadrons (projectile) as an ensemble of partons (quark-diquark system), whose scattering in the gluonic field of the other hadron is described by the Pomeron and odderon scattering amplitudes. According to canonical CGC rules, the interaction of the color singlet three-quark system with the target should be described by the correlator

$$S(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \frac{1}{3!} \epsilon_{b_1 b_2 b_3} \epsilon^{a_1 a_2 a_3} \langle U_{a_1}^{b_1}(\mathbf{x}_1) U_{a_2}^{b_2}(\mathbf{x}_2) U_{a_3}^{b_3}(\mathbf{x}_3) \rangle, \quad (23)$$

where the Levi-Civita symbol $\epsilon_{a_1 a_2 a_3}$ reflects antisymmetrization with respect to color singlet indices and the angular brackets denote averaging over all color sources. The color averaging leads to a strong suppression of the color charges separated by distances $|\mathbf{x}_i - \mathbf{x}_j| \gtrsim Q_s^{-1}$, where Q_s is the saturation scale (inverse of the color correlation length). As was discussed in [12,49], in general, this correlator does not reduce to the “dipolar” amplitudes $\mathcal{N}$ and $\mathcal{O}$ but may also include additional quadrupolar contributions, which mix up nontrivially in the process of evolution. Since the early days of QCD, it has been understood that, for many high-energy processes, the hard interaction involves a single active quark within the proton, while the remaining two act as a joint source of the color field with nearly coinciding coordinates. In view of the $SU(3)$ identity $\epsilon_{a_1 a_2 a_3} U_{a_1}^{b_1}(\mathbf{x}) U_{a_2}^{b_2}(\mathbf{x}) \approx \epsilon^{b_1 b_2 b_3} [U^\dagger(\mathbf{x})]_{b_3}^{a_3}$, a pair of overlapping quarks in the three-quark ensemble is effectively replaced by a single diquark, so the proton is treated as a bound quark-diquark state [30–32]. For such configurations, the correlator (23) effectively reduces to a combination of the “dipolar” amplitudes $\mathcal{N}$ and $\mathcal{O}$, and the cross sections of the elastic pp and $p\bar{p}$ scattering may be represented as [50–52]

$$\frac{d\sigma^{(\pm)}}{dt} = \frac{1}{16\pi} |\mathcal{A}^{(\pm)}(s, t)|^2, \quad (24)$$

where the amplitude is given by

$$\begin{aligned} \mathcal{A}^{(\pm)}(t) = & i \int d^2 \mathbf{r} d^2 \mathbf{b} d\mathbf{x} |\psi_2(\mathbf{x}, \mathbf{r})|^2 \\ & \times e^{-i(\mathbf{b} + \mathbf{x}\mathbf{r}) \cdot \mathbf{\Delta}} [\mathcal{N}(Y, \mathbf{r}, \mathbf{b}) \mp i\mathcal{O}(Y, \mathbf{r}, \mathbf{b})] \pm \mathcal{A}_\gamma(t) \end{aligned} \quad (25)$$

and the choice of upper or lower sign in $\pm, \mp$ corresponds to pp or $p\bar{p}$, respectively. The variable $\mathbf{\Delta}$ in (25) is the transverse momentum transfer between the two hadrons (it is related to the Mandelstam variable t by $t = -\mathbf{\Delta}^2$). The last term in (25) corresponds to the electromagnetic contribution with photon exchange in the t channel, which potentially can also contribute to odderon-mediated processes, as was discussed long ago in [53]. Its evaluations is

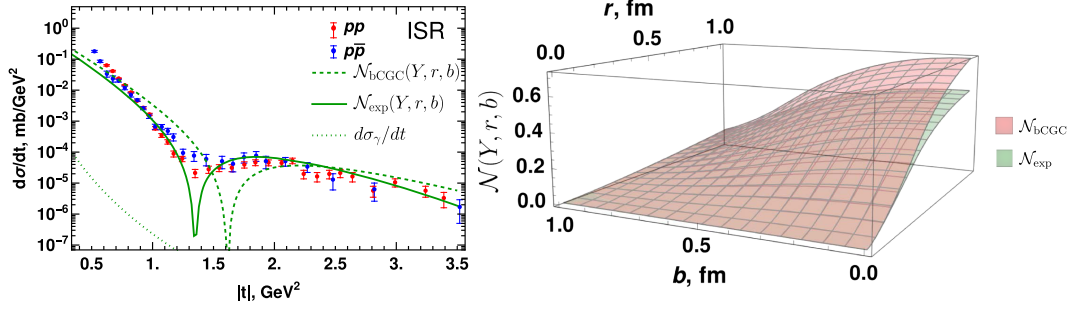


FIG. 3. Left: contribution of the dominant (C -even) part of the dipole amplitude to the elastic cross section $d\sigma/dt$. The dotted curve $d\sigma_\gamma/dt$ is added for reference and corresponds to the photon-only contribution. We use notations $\mathcal{N}_{\text{bCGC}}$ for the bCGC parametrization of the dipole amplitude from [39] and $\mathcal{N}_{\text{exp}} = 1 - \exp(-\mathcal{N}_{\text{bCGC}})$ for the exponentiated form suggested in [50,51] (it reduces to $\mathcal{N}_{\text{bCGC}}$ for small-size dipoles that dominate in DIS kinematics). The evaluation was performed using the wave functions (8). A qualitatively similar behavior was observed for the TOTEM-D0 kinematics. Right plot: comparison of the two parametrizations $\mathcal{N}_{\text{bCGC}}$ and $\mathcal{N}_{\text{exp}}$ in configuration space. The plot illustrates that the parametrizations nearly coincide for small-size dipoles. See the text for more details.

straightforward in helicity basis using the light-cone rules from [24,25]. As was demonstrated in [48], the helicity flip of each nucleon decreases the amplitude $\mathcal{A}_\gamma(t)$ by the factor $\sim \sqrt{|t|/4M^2} F_2(t)/F_1(t)$, which is small in the kinematics of interest [33,54]. For the sake of simplicity, in what follows we will focus on the helicity nonflip part, which is given explicitly in the single-photon approximation by¹ [48]

$$\mathcal{A}_\gamma(t) = 4\pi\alpha_{\text{em}} \frac{4}{t} F_1^2(t), \quad (26)$$

where α_{em} is the fine structure constant and $F_1(t)$ stands for the Dirac form factor of the proton. Since we do not consider very small $|t|$ in our analysis, we do not take into account the so-called Coulomb phase factor $e^{i\alpha_{\text{em}}\phi_c}$ which was evaluated in [53,55,56] and takes into account certain higher-order corrections. In order to demonstrate the validity of the quark-diquark approximation, in Fig. 3 we have shown the corresponding contributions of the dominant (C -even) amplitude $\mathcal{N}$ to the cross sections $d\sigma/dt$. We may observe that this approach provides a reasonable qualitative description of the data. The diffractive minimum (dip) occurs due to cancellation of contributions of different kinematic regions in the integral (25) and is very sensitive to the b dependence implemented in dipole amplitude for large-size dipoles. The region near the dip obtains the dominant contribution from various subleading mechanisms and can have enhanced sensitivity to various C -odd contributions. We also need to mention that the above-mentioned uncertainty in b dependence implemented in dipole parametrization for large dipoles has only minimal influence for analysis of the C -odd sector, because the last two terms in (14) in the saturated regime give a vanishingly small contribution. This happens because the

C -even part $\mathcal{N}$ is approximately constant in the saturated regime (so it can be taken out of the integrand), and the remaining integral vanishes in view of the antisymmetry property of the odderon amplitude, $\mathcal{O}(Y, -\mathbf{r}, \mathbf{b}) = -\mathcal{O}(Y, \mathbf{r}, \mathbf{b})$.

A. Observable for extracting the C -odd contribution

In the elastic cross section (24) and (25), the contribution of the odderon is commingled with that of Pomeron and photon, which complicates the analysis. At present, the strongest evidence in favor of odderons is the mismatch of TOTEM [20] and D0 [18,19] data near the so-called diffractive minimum of the elastic cross section, at relatively large $|t| \in (0.5-1)$ GeV². However, even in this kinematics, the odderons still remain a small correction and reveal themselves only from mismatch of pp and $\bar{p}p$ data. We will not consider in this manuscript the contribution of odderons to the ratio of the real to imaginary parts of the forward scattering amplitude, because the latter may be strongly influenced by other mechanisms (e.g., contributions of the so-called secondary odderons [57]). Since in (25) the odderon and photon contribute with the same sign, for simplicity we will split the amplitude (25) into the C -odd and C -even parts:

$$\mathcal{A}_0(t) = - \int d^2\mathbf{r} d^2\mathbf{b} dx |\psi_2(x, \mathbf{r})|^2 \times e^{-i(\mathbf{b}+\mathbf{xr})\cdot\Delta} \mathcal{O}(Y, \mathbf{r}, \mathbf{b}) + \mathcal{A}_\gamma(t), \quad (27)$$

$$\mathcal{A}_N(t) = (\rho + i) \int d^2\mathbf{r} d^2\mathbf{b} dx |\psi_2(x, \mathbf{r})|^2 \times e^{-i(\mathbf{b}+\mathbf{xr})\cdot\Delta} \mathcal{N}_{\text{DIS}}(Y, \mathbf{r}, \mathbf{b}), \quad (28)$$

tacitly assuming that the contribution of the “pure” odderon can be obtained from $\mathcal{A}_0$ subtracting the well-known photon amplitude. In the C -even contribution, we replaced the dipole amplitude $\mathcal{N}$ with its imaginary part $\mathcal{N}_{\text{DIS}}(Y, \mathbf{r}, \mathbf{b})$ measurable in deep inelastic scattering,

¹Since we use nonrelativistic normalization of the bra and ket states, the amplitude differs from [48] by factor $2/s$, where the Mandelstam variable s is the invariant energy of the collision.

multiplied by additional prefactor

$$\mathcal{N}(Y, \mathbf{r}, \mathbf{b}) = (\rho + i)\mathcal{N}_{\text{DIS}}(Y, \mathbf{r}, \mathbf{b}), \quad \rho \equiv \frac{\text{Re}\mathcal{A}}{\text{Im}\mathcal{A}}. \quad (29)$$

The elastic cross section (24) in terms of these notations can be rewritten as

$$\frac{d\sigma^{(\pm)}}{dt} = \frac{1}{16\pi} [|\mathcal{A}_{\mathbb{N}}|^2 + |\mathcal{A}_{\mathbb{O}}|^2 \pm 2\text{Re}(\mathcal{A}_{\mathbb{N}}^* \mathcal{A}_{\mathbb{O}})]. \quad (30)$$

If we take into account the smallness of the amplitude $\mathcal{A}_{\mathbb{O}}$ and disregard the term $|\mathcal{A}_{\mathbb{O}}|^2$ in (30), we may obtain for the sum and the difference of the pp and $p\bar{p}$ cross sections:

$$\begin{aligned} \frac{d\sigma^{(+)}}{dt} + \frac{d\sigma^{(-)}}{dt} &\approx \frac{1}{8\pi} |\mathcal{A}_{\mathbb{N}}|^2, \\ \frac{d\sigma^{(+)}}{dt} - \frac{d\sigma^{(-)}}{dt} &= \frac{1}{4\pi} \text{Re}(\mathcal{A}_{\mathbb{N}}^* \mathcal{A}_{\mathbb{O}}). \end{aligned} \quad (31)$$

Using symmetries of $\mathcal{O}$, it is possible to show that the amplitude $\mathcal{A}_{\mathbb{O}}$ is a real-valued function, so we may rewrite (31) as

$$\Sigma = \frac{1}{\sqrt{2\pi}} \frac{\rho}{\sqrt{1+\rho^2}} \mathcal{A}_{\mathbb{O}} = \frac{d\sigma^{(pp)}/dt - d\sigma^{(p\bar{p})}/dt}{\sqrt{d\sigma^{(pp)}/dt + d\sigma^{(p\bar{p})}/dt}}, \quad (32)$$

which allows one to relate directly the effective odderon amplitude $\mathcal{A}_{\mathbb{O}}$ with physically measurable cross sections. Since this observable does not include the contribution of Pomerons, we believe that it is better suited for phenomenological studies, since it allows one to single out a feeble odderon signal. We may observe that the coefficient in front of $\mathcal{A}_{\mathbb{O}}$ depends on the ratio ρ of real to imaginary parts of the dipole amplitudes defined in (29). At very high energies, it is expected that the amplitude is dominated by the imaginary part, so the parameter $\rho \ll 1$, thus leading to a strong suppression of the odderon signal. While it is possible to estimate the ratio of the real and imaginary parts ρ using dispersion relations [58,59], this would require additional model assumptions and introduce dependence on the parametrization of the dipole amplitude $\mathcal{N}_{\text{DIS}}$. For this reason, in what follows, we will use the experimentally measured value $\rho \approx 0.1$ [22,60].

IV. COMPARISON OF PHENOMENOLOGICAL MODELS WITH EXPERIMENTAL DATA

The main goal of this phenomenological analysis is to compare the available phenomenological models with experimental data and find upper limits for the odderon amplitude. For this reason, we will focus on the mismatch of pp and $p\bar{p}$ cross section observed at ISR [61] and a similar mismatch of D0 and TOTEM experimental data reported in [17–20]. For the TOTEM-D0 odderon

measurements, we extracted the numerical values of the extrapolated pp and measured $p\bar{p}$ cross sections directly from Fig. 4 in Ref. [18]. This dataset consists of 16 points in total: eight experimental points corresponding to the D0 $p\bar{p}$ measurement and eight additional points corresponding to the extrapolated pp cross section from TOTEM. For each data point, we included both the central value and the experimental uncertainty. We need to mention that at present the TOTEM data have been questioned by ATLAS Collaboration, and a comparison of their data with D0 suggests a significantly smaller signal (see [21,22] for details). While the latter is important for the global analysis and estimate of statistical significance, in our case this set simply decreases the upper values of the odderon signal. For the ISR, we used the experimental points in the range $0.6 < |t| < 2 \text{ GeV}^2$ (20 points in total), because in this range the pp and $p\bar{p}$ cross sections are measured in the same narrow bins in $|t|$. For larger $|t|$, the bins used for measurement of pp and $p\bar{p}$ cross sections do not coincide, and for this reason their comparison would involve the additional model assumptions that can be detrimental for studies of the feeble odderon signal. The inclusion of the ISR data requires due caution. The data from [60] do not present interest for odderon searches, since in the small- t kinematics the photon exchange contribution $\mathcal{A}_\gamma(t)$ is strongly enhanced and constitutes the dominant mechanism that explains the difference of pp and $p\bar{p}$ cross sections. The data from [61] present more interest, since their kinematic domain partially overlaps with that of TOTEM-D0. While these data have large systematic uncertainty (20%–30%), the latter is largely due to the global normalization (scale) uncertainty and can be directly related to the (systematic) normalization uncertainty of the odderon signal (32). In what follows, we sum the systematic and statistic errors in quadrature. Despite the large *relative* error, the ISR data impose useful upper limits for the magnitude of the odderon amplitude, especially in the kinematics of large $|t| > 1 \text{ GeV}^2$, where there are no other experimental constraints.

As we explained earlier, for each pair of experimental points $(d\sigma_{pp}/dt, d\sigma_{p\bar{p}}/dt)$, we construct the observable (32) and compare it with theoretical predictions obtained using different parametrizations of the odderon amplitudes and wave functions. The relative experimental error of the derived observable (32) increased drastically due subtraction of very close numbers, in agreement with well-known results from elementary statistics. From Fig. 4, we may observe that TOTEM-D0 and ISR data are marginally consistent with each other in the kinematics $|t| < 0.7 \text{ GeV}^2$: It may be nearly impossible to find a parametrization of the odderon amplitude $\mathcal{O}(\mathbf{r}, \mathbf{b})$ that satisfies the BK equation and fits simultaneously a homogeneous t dependence at ISR and node in the same variable in the kinematics of D0. In the same Fig. 4, we also have shown the results for the variable Σ found with

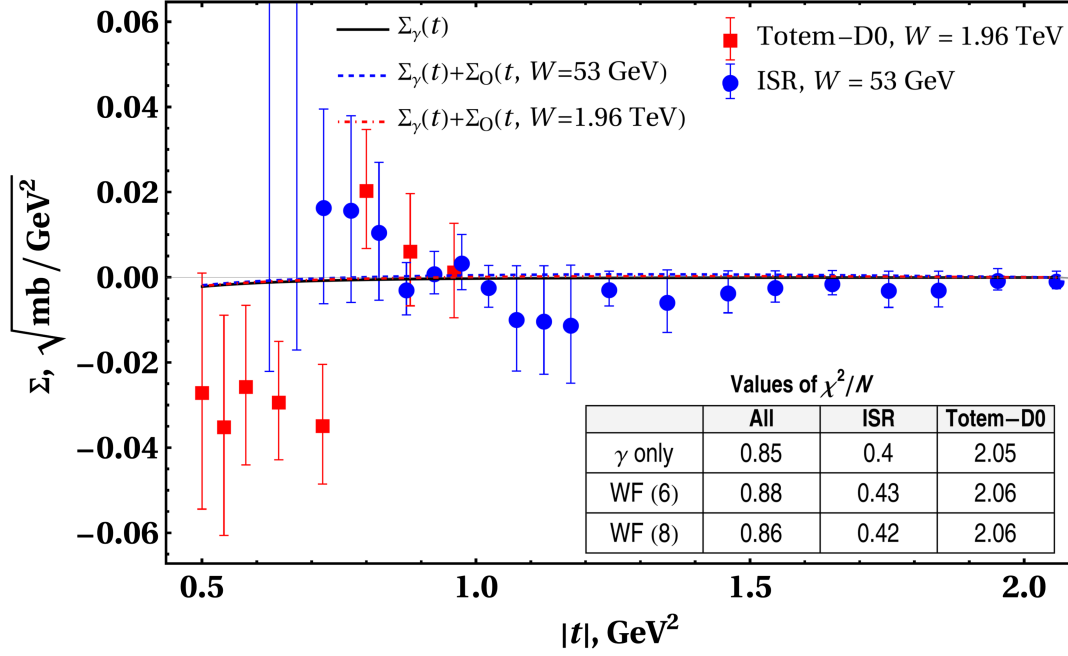


FIG. 4. The predictions for the odderon-mediated observable Σ with parametrization (17) and the wave functions (6) and (8), respectively (all curves nearly coincide with the photon-only contribution Σ_γ). The points with error bars (“experimental data” for the observable Σ) were derived from ISR [61] and D0-TOTEM experimental data in [17–20] as explained in the text. For ISR, the systematic and statistic errors were added in quadrature. The solid black curve $\Sigma_\gamma(t)$ corresponds to the energy-independent contribution of the t -channel photon exchange, whereas $\Sigma_O(t)$ is the energy-dependent contribution of the odderon. The quoted values of χ^2/N in the column “all” in the table correspond to the global analysis (TOTEM-D0 and ISR), whereas columns “ISR” and “Totem-D0” take into account only the data from these collaborations. The first row in the table gives values for the photon-only contribution, whereas the last two rows take into account odderons using the wave functions (6) and (8), respectively.

parametrization (17) of the odderon amplitude and different wave functions (6) and (8). We solved the BK evolution equations (13) and (14) numerically, using a three-step third-order Adams-Bashforth method, as described in [16]. For each impact parameter b , we used a lattice with $N_r \times N_{\phi_r} = 400 \times 100$ nodes (with logarithmic step in the r direction). Since the initial scale of the parametrization (22) was not defined in the paper, we assumed that evolution is done from the initial scale $Y_0 = \ln(1/x_0)$, where $x_0 = 10^{-3}$, and up to a scale $Y_{\text{fin}} = \ln(s/m_N^2)$, where $\sqrt{s}$ corresponds to 53 GeV for ISR and 1.96 TeV for D0. Since the right-hand side of (13) and (14) does not depend explicitly on Y , the effects of evolution depend only on the difference of rapidities (and initial conditions). In view of this invariance, the effects of evolution remain invariant under any redefinition (rapidity shift) $Y \rightarrow Y + \text{const}$. For the dipole amplitude $\mathcal{N}_{\text{DIS}}(\mathbf{r}, \mathbf{b})$ at the initial scale we assume the b-CGC parametrization [39], which yields the constants $\alpha \approx 0.15$ and $\rho \approx 0.25$. We found that the evolution suppresses the odderon signal, in agreement with earlier findings of [23,48]. We may see that the choice of the wave function has only mild influence on the shape of the t dependence and mostly affects the magnitude of the odderon-mediated contribution. This happens because the wave functions do not depend directly

on the impact parameter b (Fourier conjugate of the variable $\Delta_\perp \approx \sqrt{|t|}$), and the average dipole size is determined by the shape of the odderon amplitude rather than by the wave function alone. In both cases, the goodness-of-fit metric χ^2/N is close to one, which signals that both parametrizations describe the experimental data reasonably well, which might be partially due to large error bars of the experimental data. For comparison, for the photon-only contribution (Σ_γ), we obtained $\chi^2/N = 0.85$. If we omit the ISR data with large error bars and consider only TOTEM-D0 data, then the χ^2/N increases up to ≈ 2.05 (p value 0.02), in agreement with [62]. This signals tension of the photon-only model and suggests that the statistical significance of the odderon is $\approx 2.3\sigma$.

We also tried to modify the parametrization (17), adding a free parameter λ (normalization constant) in front of it, as explained in (16), and fixing its value from the global minimization of $\chi^2/\text{d.o.f.}$ In Fig. 5, we have shown the results of the fit, together with obtained value of λ and goodness-of-fit metric $\chi^2/\text{d.o.f.}$, for different choices of the wave functions and subsets of experimental data. As we mentioned earlier, the choice of the wave function mostly affects the magnitude of the odderon-mediated contribution, which can be absorbed by redefinition of the parameter λ that minimizes the $\chi^2/\text{d.o.f.}$ For this reason, the plots

in the left and the right columns have similar shapes. In the first row in Fig. 5, we have shown the results of the global fit that treats the ISR and TOTEM-D0 data on equal footing. We may see that minimization does not decrease significantly the values of $\chi^2/\text{d.o.f.}$ and sets very loose constraints on possible values of the normalization λ , although it indicates that the data may be described better

with negative and relatively large values of λ . Partially such large values are due to the fact that in the difference of the pp and $p\bar{p}$ cross sections (32) the odderon amplitude is multiplied by the small parameter ρ . Despite large errors, the extracted values of the parameter λ are in agreement with each other, which corroborates consistency of our approach and robustness of our results to the choice of the

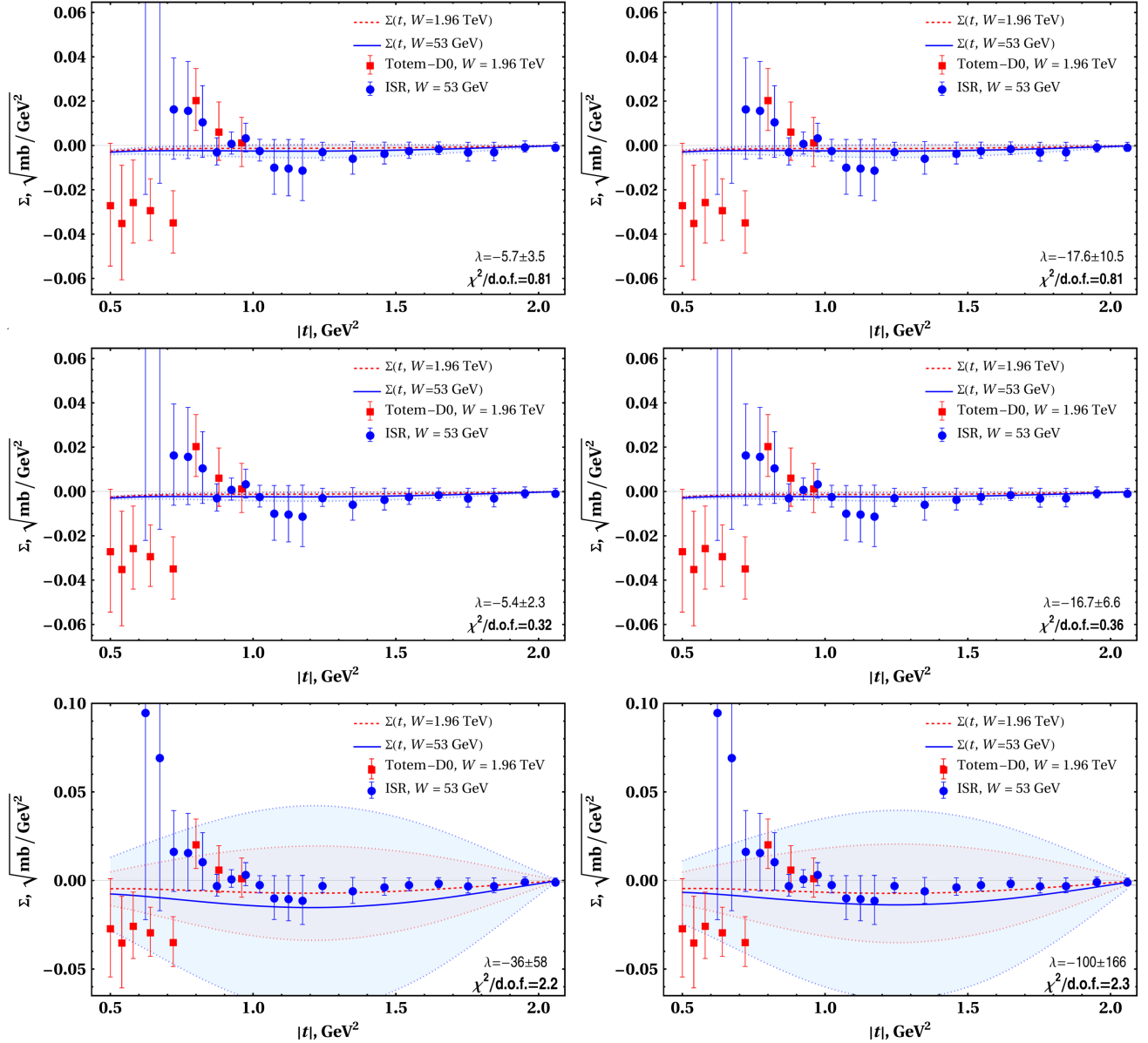


FIG. 5. The predictions for the odderon-mediated observable Σ with parametrization (17). The upper row corresponds to the global fit, the central row corresponds to the fit of ISR data only, and the last row includes only TOTEM-D0 data into the fit (see the text for details). The left and right columns correspond to the wave functions (6) and (8), respectively. The points with error bars (experimental data for the observable Σ) were derived from ISR [61] and D0-TOTEM experimental data in [17–20]. The values of λ (together with errors) and the corresponding $\chi^2/\text{d.o.f.}$ are shown in the lower right corner. The quoted values of $\chi^2/\text{d.o.f.}$ correspond only to the points that were taken into account during the fit and should not be confused with global goodness-of-fit metrics (see the text for more details). The values of $\chi^2/\text{d.o.f.}$ shown in the last two plots exceed χ^2/N shown in the last column of the inline table in Fig. 4 only due to the difference of divisors; the corresponding $\chi^2/N = 1.92$ for the TOTEM-D0 fit. The colored bands show the uncertainty of the fit (95% confidence level).

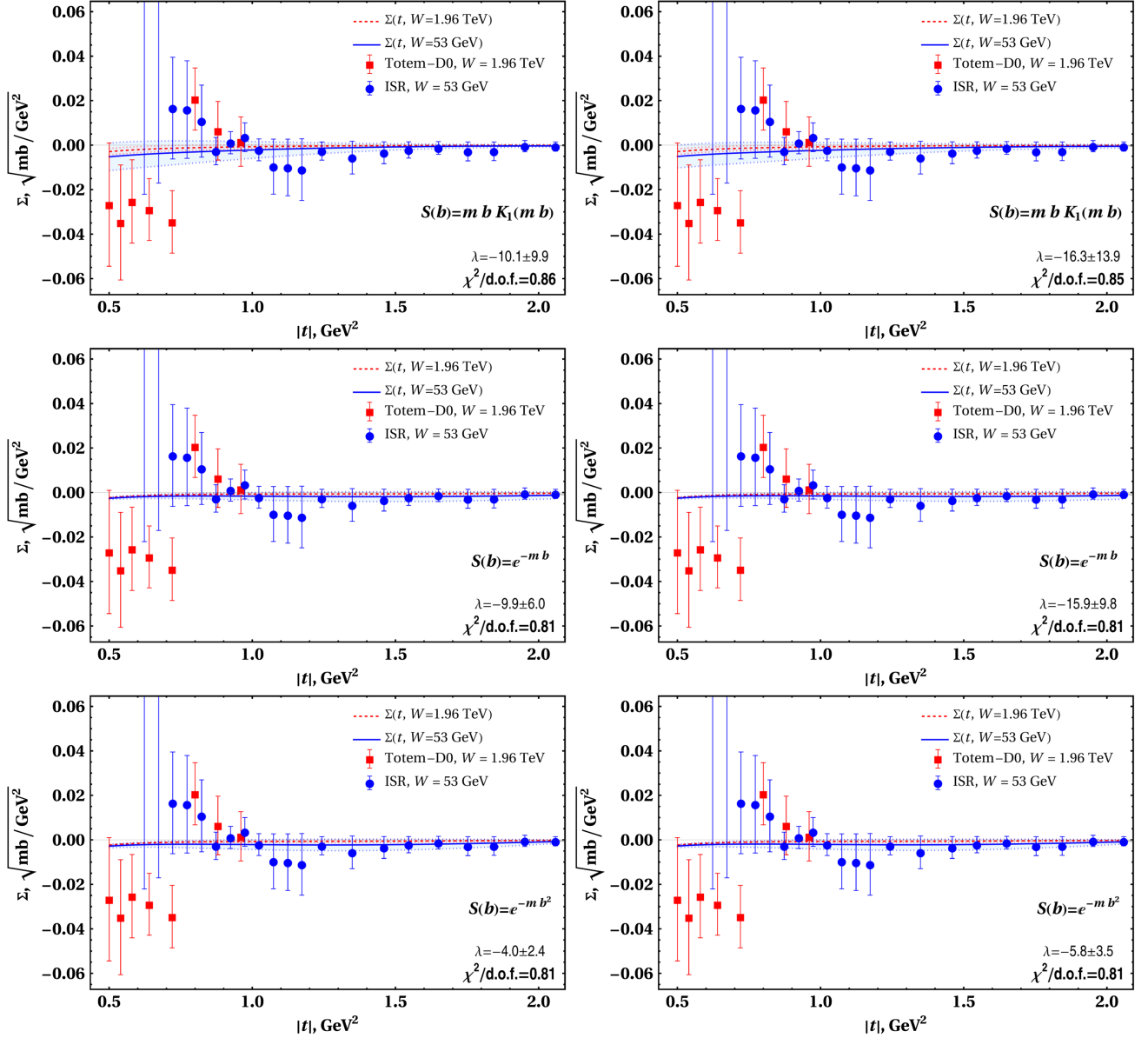


FIG. 6. The predictions for the odderon-mediated observable Σ with parametrization (22) and the profiles $S_i(b)$ from Table I (m is a profile-dependent constant). The left and right columns correspond to the wave functions (6) and (8), respectively. The quoted values of $\chi^2/\text{d.o.f.}$ correspond to the global fit; the fit of ISR-only data decreases $\chi^2/\text{d.o.f.}$ —however, it yields nearly the same values of λ . The points with error bars (experimental data for the observable Σ) were derived from ISR [61] and D0-TOTEM experimental data in [17–20].

proton wave function. In order to understand better which of the two datasets constrains stronger the odderon amplitude, we also performed similar fits using separately ISR or TOTEM-D0 data and present the results in the second and the third rows in Fig. 5, respectively. The fit of ISR data yields approximately the same confidence interval for λ , although the values of $\chi^2/\text{d.o.f.}$ are smaller than in the global fit. At the same time, the fit of the TOTEM-D0 data alone (see the last row) provides significantly worse constraints on the parameter λ and overall goodness-of-fit metrics. Such large values of λ (of the order of $\lambda \sim 100$) clearly would lead to an unphysically large cross section for

the exclusive π^0 , η_c , and χ_c production² in contradiction to experimental upper bounds at HERA [64] and would break severely the known theoretical constraints on the odderon amplitude [65,66]

²For our estimates of the cross sections for the exclusive production of π^0 , η_c , and χ_c , we took predictions from [16,63] and scaled them using the factor λ^2 . The pion size (rms radius) is approximately larger than that of η_c and χ_c ; for this reason, for our rough estimates we used the wave function of η_c with properly adjusted radius and normalization.

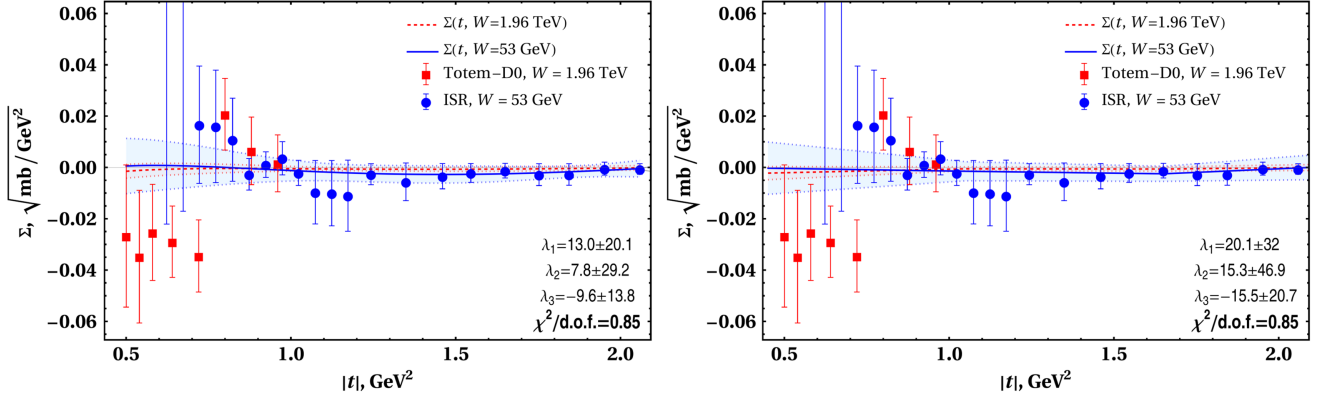


FIG. 7. The predictions for the odderon-mediated observable Σ with parametrization (22) and profile (34). The left and right columns correspond to the wave functions (6) and (8), respectively. The points with error bars (experimental data) for the observable Σ were derived from ISR [61] and D0-TOTEM experimental data in [17–20]. The quoted values of $\chi^2/\text{d.o.f.}$ correspond to the global fit; the fit of ISR-only data decreases $\chi^2/\text{d.o.f.}$ —however, it yields nearly the same values of λ_n .

$$(4 - 3\mathcal{N}(\mathbf{r}, \mathbf{b}))\mathcal{N}^3(\mathbf{r}, \mathbf{b}) - 6(6 - 6\mathcal{N}(\mathbf{r}, \mathbf{b}) + \mathcal{N}^2(\mathbf{r}, \mathbf{b})) \times \mathcal{O}^2(\mathbf{r}, \mathbf{b}) - 3\mathcal{O}^4(\mathbf{r}, \mathbf{b}) \geq 0. \quad (33)$$

For this reason, we believe that the observed TOTEM-D0 mismatch hardly can be attributed to the odderon signal. Our findings indicate that, in the framework of chosen parametrizations, the large- t data from ISR impose more stringent upper limits for odderon amplitude than the TOTEM-D0 data. While in all fits we found reasonable values of $\chi^2/\text{d.o.f.}$, we may observe that the errors of the parameter λ remain elevated, which signals that existing experimental data constrain poorly the amplitude of odderons. This is not surprising, given the low statistical significance of the odderons.

In Fig. 6, we provided result of a similar analysis for the parametrization (22), assuming that the profile $S(b)$ is given by one of the functions $S_1 \dots S_3$ defined in Table I. For all possible choices of $S_i(b)$, the t dependence of the parameter Σ is very similar to what we obtained earlier using the parametrization (17). As explained earlier, the uncertainty in the choice of the wave function has only minor effect on the shape of the t dependence and largely reduces to normalization uncertainty; for this reason, the fits with different wave functions (left and right columns) differ only by the values of constant λ that minimizes $\chi^2/\text{d.o.f.}$

Finally, in Fig. 7, we present the results of analysis in which we assumed that the profile $S(b)$ in the parametrization (22) may be represented as a linear superposition of the functions $S_1 \dots S_3$ defined in Table I, namely,

$$S(b) = \sum_{n=1}^3 \lambda_n S_n(b), \quad (34)$$

and treated λ_n as free parameters. This parametrization is sufficiently flexible to represent any profile that has a

maximum at the center and decreases on the proton's periphery. In view of the above-mentioned linearity of the BK evolution for odderons in the limit $\mathcal{O} \ll \mathcal{N}$, each term in the right-hand side of (34) can be evolved independently, thus drastically simplifying the numerical procedure. Because of similarity of the shapes of the functions $S_n(b)$, this approach suffers from sizable correlation between different parameters (the corresponding nondiagonal elements of the correlation matrix are between 0.5 and 1). For this reason, the fitted curves have enhanced error bands compared to previous fits. At TOTEM-D0 energy range, the contribution of odderon remain small due to suppression of its amplitude by BK evolution; for this reason, the inclusion of TOTEM-D0 data increases the $\chi^2/\text{d.o.f.}$ yet has a very mild influence on the fit parameters.

Although the extracted constraints suggest that the normalization factor λ may be large, we checked that the values of $\lambda \lesssim 10$ neither undermine our assumption that $|\mathcal{A}_0|^2$ term in (30) can be neglected nor lead to an unphysically large cross section for exclusive π^0 production at HERA [64].

V. CONCLUSIONS

In summary, in this manuscript, we developed a formalism which can be used to extract the C -odd contributions to the elastic nucleon-nucleon scattering and constrain phenomenologically the odderon amplitude $\mathcal{O}(\mathbf{r}, \mathbf{b})$ in the color glass condensate framework. In contrast to previous studies, we constructed the combination of the pp and $p\bar{p}$ cross sections defined in (32) which is directly proportional to the amplitude mediated by C -odd exchanges in the t channel. We also demonstrated that the smallness of odderon amplitude ($\mathcal{O} \ll \mathcal{N}$) allows one to completely decouple the equation for the C -even amplitude $\mathcal{N}$, and, thus, a global simultaneous fit of the C -odd and C -even

amplitudes reduces to their sequential extraction from the experimental data. Since the experimental datasets used to fix the amplitudes $\mathcal{N}$ and $\mathcal{O}$ have a significantly different number of events and precision, such a sequential procedure allows one to measure more accurately the feeble odderon signal. We demonstrated that variation of the dipole amplitude within a range allowed by the experimental data from HERA has only minor influence on the odderon evolution.

Using popular parametrizations of the wave functions, we found upper constraints on the magnitude of the odderon amplitude. While our estimates depend on the choice of the parameter ρ defined in (29) and the wave function of the proton, we found that these nonperturbative factors affect only the global normalization (at most a factor of 2 uncertainty) but almost do not change the *shape* of the t dependence of the cross section, which is controlled by the impact parameter of the odderon. In our phenomenological studies, we focused on the TOTEM-D0 and ISR data, although these datasets are only marginally consistent with each other in the kinematics $|t| < 0.7 \text{ GeV}^2$. We analyzed two widely used parametrizations of odderons and found that adding them at face value almost does not change χ^2/N compared to the photon-only contribution. We also tried to vary their magnitudes (normalizations), treating them as free parameters; however, this only marginally improved description of the experimental data. In these global fits, the most stringent constraints on odderon amplitude come from the ISR data at large $|t| > 1 \text{ GeV}^2$. While the obtained values of $\chi^2/\text{d.o.f.}$ are small, due to large experimental uncertainties, the constraints on the odderon magnitude are very loose and suggest that the normalization constant λ is negative. The inclusion of the TOTEM-D0 data increases the global $\chi^2/\text{d.o.f.}$ yet has only mild influence on the parameters, because odderon amplitude is suppressed at D0 kinematics by BK evolution. The attempts to interpret the TOTEM-D0 data as a manifestation of odderon exchanges would inevitably require a significantly larger odderon amplitude, violating the known theoretical constraint (33). Furthermore, this would lead to a tension with ISR and HERA data. Our findings agree with earlier claims in [44] that TOTEM-D0 signal is too large for odderons and may underestimate certain systematic errors that affect global normalization of data measured by different collaborations. Precisely, the authors of [44] had to introduce the normalization factors N_i to reconcile discrepancies of different experiments and get a consistent global fit.

Through a detailed comparison of multiple saturation scale profiles and proton wave function parametrizations, we have shown that our analysis remains robust across a

variety of modeling choices and, thus, captures the essential physics of C -odd exchange. Unfortunately, the large errors of ISR and TOTEM-D0 data for $|t| \in (0.5, 0.7) \text{ GeV}^2$, as well as lack of other experimental evidence in favor of odderons, does not allow us to make more precise fits.³ However, the approach developed here lays the groundwork for future developments and allows one to consider on equal footing the odderon contribution to elastic nucleon-nucleon scattering together with odderon signal on other channels. Potentially, such constraints can be obtained from spin-sensitive polarized observables [48] and odderon-mediated quarkonia photoproduction [16] that can be measured at the future Electron-Ion Collider.

The main results of this manuscript are the theoretical expressions for amplitudes. The phenomenological estimates for the cross sections presented in this manuscript are also provided as tabulated plain text data files in Supplemental Material [67].

ACKNOWLEDGMENTS

We thank our colleagues at UTFSM university and Eugene Levin for encouraging discussions. This research was partially supported by ANID CCTVal CIA250027 (Chile) and ANID grants Fondecyt Regular No. 1251975 and Fondecyt Postdoctoral No. 3230699. I. Z. acknowledges funding from the Fellowship ANID Beca de Doctorado Nacional No. 21250067 and comprehensive support from the Institute of Physics of PUCV. Y. G. acknowledges financial support provided by the Universidad de Playa Ancha de Ciencias de la Educación through the 2024 Research Seed Grants Program (Concurso Semilleros de Investigación 2024), D.E. 0295/2024, Project Code No. SEIV 24-24, which significantly contributed to the development of this research. Powered@NLHPC: This research was partially supported by the supercomputing infrastructure of the NLHPC (ECM-02).

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

³While there are plenty of high-precision pp elastic scattering data from LHC, to the best of our knowledge, at present there are no plans to study experimentally the $p\bar{p}$ elastic scattering in the foreseeable future at the same energies. For this reason, all the attempts to extract the odderon signal from pp scattering at LHC would require extrapolation to lower energies of older $p\bar{p}$ experiments and inevitably would suffer from the same problem as the above-mentioned TOTEM-D0 joint analysis.

- [1] L. Lukaszuk and B. Nicolescu, *Lett. Nuovo Cimento* **8**, 405 (1973).
- [2] P. Gauron, L. Lukaszuk, and B. Nicolescu, *Phys. Lett. B* **294**, 298 (1992).
- [3] Y. V. Kovchegov and E. Levin, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. **33**, 1 (2012).
- [4] C. Ewerz, [arXiv:hep-ph/0306137](https://arxiv.org/abs/hep-ph/0306137).
- [5] J. Bartels, L. N. Lipatov, and G. P. Vacca, *Phys. Lett. B* **477**, 178 (2000).
- [6] J. Bartels, *Nucl. Phys.* **B175**, 365 (1980).
- [7] J. Kwiecinski and M. Praszalowicz, *Phys. Lett.* **94B**, 413 (1980).
- [8] H. G. Dosch and Y. A. Simonov, *Phys. Lett. B* **205**, 339 (1988).
- [9] R. C. Brower, M. Djuric, and C. I. Tan, *J. High Energy Phys.* **07** (2009) 063.
- [10] A. Kovner and M. Lublinsky, *J. High Energy Phys.* **02** (2007) 058.
- [11] Y. V. Kovchegov, L. Szymanowski, and S. Wallon, *Phys. Lett. B* **586**, 267 (2004).
- [12] Y. Hatta, E. Iancu, K. Itakura, and L. McLerran, *Nucl. Phys.* **A760**, 172 (2005).
- [13] G. Antchev *et al.* (TOTEM Collaboration), *Eur. Phys. J. C* **79**, 103 (2019).
- [14] G. Antchev *et al.* (TOTEM Collaboration), *Eur. Phys. J. C* **79**, 785 (2019).
- [15] G. Antchev *et al.* (TOTEM Collaboration), *Eur. Phys. J. C* **79**, 861 (2019).
- [16] S. Benić, D. Horvatić, A. Kaushik, and E. A. Vivoda, *Phys. Rev. D* **108**, 074005 (2023).
- [17] E. Martynov and B. Nicolescu, *Phys. Lett. B* **778**, 414 (2018).
- [18] V. M. Abazov *et al.* (D0 and TOTEM Collaborations), *Phys. Rev. Lett.* **127**, 062003 (2021).
- [19] V. M. Abazov *et al.* (D0 Collaboration), *Phys. Rev. D* **86**, 012009 (2012).
- [20] G. Antchev *et al.* (TOTEM Collaboration), *Eur. Phys. J. C* **80**, 91 (2020).
- [21] T. Csörgő, T. Novik, R. Pasechnik, A. Ster, and I. Szanyi, *Universe* **10**, 264 (2024).
- [22] G. Aad *et al.* (ATLAS Collaboration), *Eur. Phys. J. C* **83**, 441 (2023).
- [23] X. Yao, Y. Hagiwara, and Y. Hatta, *Phys. Lett. B* **790**, 361 (2019).
- [24] G. P. Lepage and S. J. Brodsky, *Phys. Rev. D* **22**, 2157 (1980).
- [25] S. J. Brodsky, H. C. Pauli, and S. S. Pinsky, *Phys. Rep.* **301**, 299 (1998).
- [26] A. Dumitru and T. Stebel, *Phys. Rev. D* **99**, 094038 (2019).
- [27] S. J. Brodsky and F. Schlumpf, *Phys. Lett. B* **329**, 111 (1994).
- [28] F. Schlumpf, *Phys. Rev. D* **48**, 4478 (1993).
- [29] F. Schlumpf, *Phys. Rev. D* **47**, 4114 (1993); **49**, 6246(E) (1994).
- [30] M. Gell-Mann, *Phys. Lett.* **8**, 214 (1964).
- [31] M. Ida and R. Kobayashi, *Prog. Theor. Phys.* **36**, 846 (1966).
- [32] D. B. Lichtenberg and L. J. Tassie, *Phys. Rev.* **155**, 1601 (1967).
- [33] S. Navas *et al.* (Particle Data Group Collaboration), *Phys. Rev. D* **110**, 030001 (2024).
- [34] G. F. de Teramond *et al.* (HLFHS Collaboration), *Phys. Rev. Lett.* **120**, 182001 (2018).
- [35] S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and J. Erlich, *Phys. Rep.* **584**, 1 (2015).
- [36] S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and C. Lorc, *Phys. Lett. B* **759**, 171 (2016).
- [37] C. Contreras, E. Levin, R. Meneses, and M. Sanhueza, *Phys. Rev. D* **101**, 096019 (2020).
- [38] F. D. Aaron *et al.* (H1 and ZEUS Collaborations), *J. High Energy Phys.* **01** (2010) 109.
- [39] A. H. Rezaeian and I. Schmidt, *Phys. Rev. D* **88**, 074016 (2013).
- [40] A. H. Rezaeian, M. Siddikov, M. Van de Klundert, and R. Venugopalan, *Phys. Rev. D* **87**, 034002 (2013).
- [41] S. Jeon and R. Venugopalan, *Phys. Rev. D* **71**, 125003 (2005).
- [42] M. Roa, J. Garrido, and M. Guevara, *Phys. Rev. D* **110**, 074006 (2024).
- [43] C. Contreras, E. Levin, and M. Sanhueza, *Phys. Rev. D* **104**, 116020 (2021).
- [44] E. G. S. Luna, M. G. Ryskin, and V. A. Khoze, *Phys. Rev. D* **110**, 014002 (2024).
- [45] V. A. Khoze, A. D. Martin, and M. G. Ryskin, *Phys. Lett. B* **784**, 192 (2018).
- [46] M. Broilo, D. A. Fagundes, E. G. S. Luna, and M. Peláez, *Phys. Rev. D* **103**, 014019 (2021).
- [47] E. Gotsman, E. Levin, and I. Potashnikova, *Phys. Rev. D* **101**, 094021 (2020).
- [48] Y. Hagiwara, Y. Hatta, R. Pasechnik, and J. Zhou, *Eur. Phys. J. C* **80**, 427 (2020).
- [49] I. Balitsky and A. V. Grabovsky, *J. High Energy Phys.* **01** (2015) 009.
- [50] A. Kormilitzin and E. Levin, [arXiv:0809.3886](https://arxiv.org/abs/0809.3886).
- [51] C. Contreras, E. Levin, and M. Sanhueza, *Phys. Rev. D* **106**, 034011 (2022).
- [52] C. Flensburg, G. Gustafson, and L. Lonnblad, *Eur. Phys. J. C* **60**, 233 (2009).
- [53] N. H. Buttmore, E. Gotsman, and E. Leader, *Phys. Rev. D* **18**, 694 (1978); **35**, 407(E) (1987).
- [54] M. K. Jones *et al.* (Jefferson Lab Hall A Collaboration), *Phys. Rev. Lett.* **84**, 1398 (2000).
- [55] H. A. Bethe, *Ann. Phys. (N.Y.)* **3**, 190 (1958).
- [56] G. B. West and D. R. Yennie, *Phys. Rev.* **172**, 1413 (1968).
- [57] E. Gotsman, E. Levin, and I. Potashnikova, *Phys. Lett. B* **786**, 472 (2018).
- [58] J. B. Bronzan, G. L. Kane, and U. P. Sukhatme, *Phys. Lett.* **49B**, 272 (1974).
- [59] E. Gotsman, E. M. Levin, and U. Maor, *Z. Phys. C* **57**, 677 (1993).
- [60] N. Amos *et al.*, *Nucl. Phys.* **B262**, 689 (1985).
- [61] A. Breakston *et al.*, *Phys. Rev. Lett.* **54**, 2180 (1985).
- [62] Z. F. Cui, D. Binosi, C. D. Roberts, S. M. Schmidt, and D. N. Triantafyllopoulos, *Phys. Lett. B* **839**, 137826 (2023).

- [63] S. Benić, A. Dumitru, A. Kaushik, L. Motyka, and T. Stebel, *Phys. Rev. D* **110**, 014025 (2024).
- [64] C. Adloff *et al.* (H1 Collaboration), *Phys. Lett. B* **544**, 35 (2002).
- [65] T. Lappi, A. Ramnath, K. Rummukainen, and H. Weigert, *Phys. Rev. D* **94**, 054014 (2016).
- [66] J. Berntsen, T. O. Espelid, and A. Genz, *ACM Trans. Math. Softw.* **17**, 437 (1991).
- [67] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/9kg5-3t22> for tabulated data and the file README.txt in that section for more details.