

Nonunitary solar constraint for long-baseline neutrino experiments

Andrés López Moreno^{*}

*King's College London, Strand, London WC2R 2LS, United Kingdom and
Laboratoire d'Annecy De Physique Des Particules, 9 Chemin de Bellevue, 74940 Annecy, France*



(Received 3 April 2026; accepted 3 June 2026; published 25 June 2026)

Long-baseline neutrino oscillation experiments require external constraints on $\sin^2 \theta_{12}$ and Δm_{21}^2 to make precision measurements of the leptonic mixing matrix. These constraints come from measurements of the Mikheyev-Smirnov-Wolfenstein (MSW) mixing in solar neutrinos. Here we develop an adiabatic MSW approximation in the presence of heavy neutral leptons; it adds, to leading order, a single new parameter (α_{11}) representing the magnitude of the mixing between the ν_e state and the heavy sector. We use data from the Borexino, SNO, and KamLAND Collaborations to find a solar constraint appropriate for heavy neutral lepton searches in long-baseline oscillation experiments. Solar data yields a strongly correlated constraint on the solar mass splitting and the magnitude of ν_e nonunitary mixing, limiting the magnitude of the nonunitary parameter to $(1 - \alpha_{11}) < 0.046$ at the 99% credible region.

DOI: [10.1103/99t8-tl5w](https://doi.org/10.1103/99t8-tl5w)

I. INTRODUCTION

The next generation of long-baseline (LBL) neutrino experiments (DUNE [1], HK [2]) will bring an era of unprecedented precision measurements to neutrino oscillation physics. This will enable further testing of the current 3-flavor paradigm [3] against nonstandard neutrino interaction theories and sterile neutrino hypotheses.

LBL experiments compare the $\nu_\mu(\bar{\nu}_\mu)$ and $\nu_e(\bar{\nu}_e)$ fluxes of a neutrino beam at a near and far detector and thus have, at most, four oscillation channels; $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_\mu(\bar{\nu}_\mu)$, $\nu_e(\bar{\nu}_e) \rightarrow \nu_e(\bar{\nu}_e)$, $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$. Having few channels presents a difficulty when trying to produce constraints on neutrino propagation models with a large number of parameters. Indeed, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) oscillation framework has six free parameters (two mass square differences, three mixing angles, and one complex phase) and LBL experiments typically fix or impose external constraints on Δm_{21}^2 and θ_{12} to measure the remaining four parameters; θ_{13} , θ_{23} , Δm_{32}^2 , and δ_{CP} [4,5].

Other than agnostic probes of nonunitarity via goodness of fit comparisons [6], the search for neutrino oscillations with heavy neutral leptons (HNLs) is a natural target for LBL physics because it introduces few

new free parameters. These HNLs are well-motivated by low-scale type-I seesaw models [7] that give a satisfying explanation to the origin and smallness of neutrino masses—allowing for deviations from unitarity by introducing mixing into new electroweak-scale leptons. Due to their large mass, we expect seesaw-scale HNLs not to be kinematically available, thus not taking part in oscillations. This should manifest as a deficit in the unnormalized neutrino flux for all flavors and at all baselines due to a portion of the active states immediately shifting into the sterile sector [8]. In practice, near-detector normalization hides the deficit in all but the appearance channels [9].

The need for external solar constraints in long-baseline experiments becomes even greater when trying to set limits on nonunitary mixing models with additional degrees of freedom. In such setups, the usual solar constraint (which assumes unitarity) must be updated to be consistent with the nonunitary formalism. Currently, the lack of a nonunitary solar constraint is a hard wall for LBL analyzers hoping to search for HNL mixing in oscillation data. This paper aims to produce one such constraint by finding a nonunitary expression for the adiabatic Mikheyev-Smirnov-Wolfenstein (MSW) effect in the presence of HNLs.

In Sec. II we discuss the relevance of the solar constraint on the PMNS parameters for LBL experiments. In Sec. III we review the nonunitary neutrino mixing formalism and use it in deriving a nonunitary large mixing angle (LMA) MSW solution [10]. Finally, in Sec. IV we use the previous result to extract a new nonunitary solar constraint from Borexino [11], SNO [12], and KamLAND [13] data to be used in LBL nonunitary fits.

^{*}Contact author: andres.lopezmoreno@lapp.in2p3.fr

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

II. THE SOLAR CONSTRAINT IN LBL EXPERIMENTS

A. The need for a solar constraint

LBL experiments can produce ν_μ and $\bar{\nu}_\mu$ dominated beams by choosing the charge of decaying hadrons with a magnetic horn. These beams contain non-negligible ν_e and $\bar{\nu}_e$ fluxes which contribute to the oscillation signals in the far detector. In the 3-flavor (3ν) PMNS model, the oscillations follow the familiar formula,

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i>j}^3 \Re[U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}] \sin^2 \frac{\Delta \hat{m}_{ij}^2 L}{4E} \pm 2 \sum_{i>j}^3 \Im[U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}] \sin \frac{\Delta \hat{m}_{ij}^2 L}{2E}, \quad (1)$$

where $(U)_{\alpha j}$ is a change of basis matrix between the flavor states and the propagation states, and $\Delta \hat{m}_{ij}^2$ are the eigenvalue square differences (in vacuum, these correspond to the entries of the PMNS matrix and the mass square differences Δm_{ij}^2). The sign of the final term differentiates between ν and $\bar{\nu}$.

At the first oscillation maximum, which is the target of LBL experiments, we require that the term governing the oscillation be close to 1. That is, we need $\sin^2(\Delta m_{32}^2 L/4E) \approx \sin^2(\Delta m_{31}^2 L/4E) \approx 1$. This yields $L/E \approx 500$ km/GeV; since $\Delta m_{21}^2 \sim \mathcal{O}(10^{-5})$ eV², we find that at this baseline $\sin^2(\Delta m_{21}^2 L/4E) \sim \mathcal{O}(10^{-4})$. It follows that Δm_{21}^2 does not have a leading-order contribution in disappearance channels around the first oscillation maximum and so, using $\Delta m_{31}^2 \approx \Delta m_{32}^2$ and disregarding terms $< \mathcal{O}(10^{-3})$ we arrive at the following expressions for long-baseline oscillation probabilities:

$$P_{\nu_e \rightarrow \nu_e}^{\text{LBL}} \approx 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{32}^2 L}{4E}, \quad (2a)$$

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{LBL}} \approx 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{32}^2 L}{4E}, \quad (2b)$$

$$P_{\nu_\mu \rightarrow \nu_e}^{\text{LBL}} \approx \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{32}^2 L}{4E} \pm \frac{\Delta m_{21}^2 L}{4E} 8J_{CP} \sin^2 \frac{\Delta m_{32}^2 L}{4E}, \quad (2c)$$

where $J_{CP} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin \delta_{CP}$ is the Jarlskog invariant. Here we see that the leading contribution of the solar parameters $\Delta m_{21}^2, \theta_{12}$ is in the appearance channels, and not separable from $\sin \delta_{CP}$. Indeed, LBL experiments have limited sensitivity to the solar parameters, and need an external constraint to make precision measurements of δ_{CP} . A more detailed description of this effect and its consequences can be found in [14].

B. Current bounds on solar neutrino parameters

In the T2K [15] and NO ν A [16] experiments, the external constraint on the solar parameters comes from the 2019 Particle Data Group (PDG) report, which is derived from a combination of the KamLAND reactor data and various solar experiments [17]. KamLAND is a ≈ 1 kton liquid scintillator detector which measures the $\bar{\nu}_e$ spectrum from 55 surrounding nuclear reactors which act as isotropic $\bar{\nu}_e$ sources in the 1–10 MeV range. It provides constraints on Δm_{21}^2 and θ_{12} by fitting the 2-flavor $\bar{\nu}_e$ survival probability at a range of baselines where the frequency of the oscillations is dominated by Δm_{21}^2 [13,18,19]. To a very good approximation, the 3-flavor survival probability at KamLAND is given by

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \cong \cos^4 \theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2 L}{4E_\nu} \right], \quad (3)$$

where θ_{13} is small and well-constrained. In this expression, θ_{12} gives the depth of the oscillation and Δm_{21}^2 the L/E dependence; thus, KamLAND's Δm_{21}^2 and θ_{12} constraints are largely uncorrelated—this will prove helpful when redefining the angles in a nonunitary fit. Due to larger uncertainties in event rate predictions than in energy reconstruction [20], KamLAND produces a strong constraint on the mass difference but a weaker constraint on the angle. In contrast, solar neutrino experiments produce comparatively weaker Δm_{21}^2 measurement but have better θ_{12} precision. In this paper, we consider solar neutrino data taken by the Borexino and SNO Collaborations coming from p-p, pep, ^7Be , and ^8B processes [21].

Solar neutrino experiments measure ν_e survival probabilities by comparing predictions of the neutrino flux produced by nuclear processes in the sun [22,23] with the measured flux on Earth. In the adiabatic regime ($\Delta m_{21}^2 > 10^{-5}$ eV²), the survival probability can be approximated by a two-level transition between vacuum decoherent mixing and the MSW resonance [24]. Explicitly, for the standard 3-flavor scenario,

$$\cos 2\hat{\theta}_{12} = \frac{\cos 2\theta_{12} - \beta}{\sqrt{(\cos 2\theta_{12} - \beta)^2 + \sin^2(2\theta_{12})}},$$

$$P_{\nu_e \rightarrow \nu_e} = \frac{1}{2} \cos^4 \theta_{13} [1 + \cos 2\hat{\theta}_{12} \cos 2\theta_{12}] + \sin^4 \theta_{13}, \quad (4)$$

where $\hat{\theta}_{12}$ is the effective mixing angle near the transition point and β is the ratio of matter to vacuum effect near the MSW eigenvalue crossing. β can be written as

$$\beta = \frac{2\sqrt{2}G_f \cos^2 \theta_{13} n_e E}{\Delta m_{21}^2} \quad (5)$$

for n_e the electron density of the medium at production and E the neutrino energy.

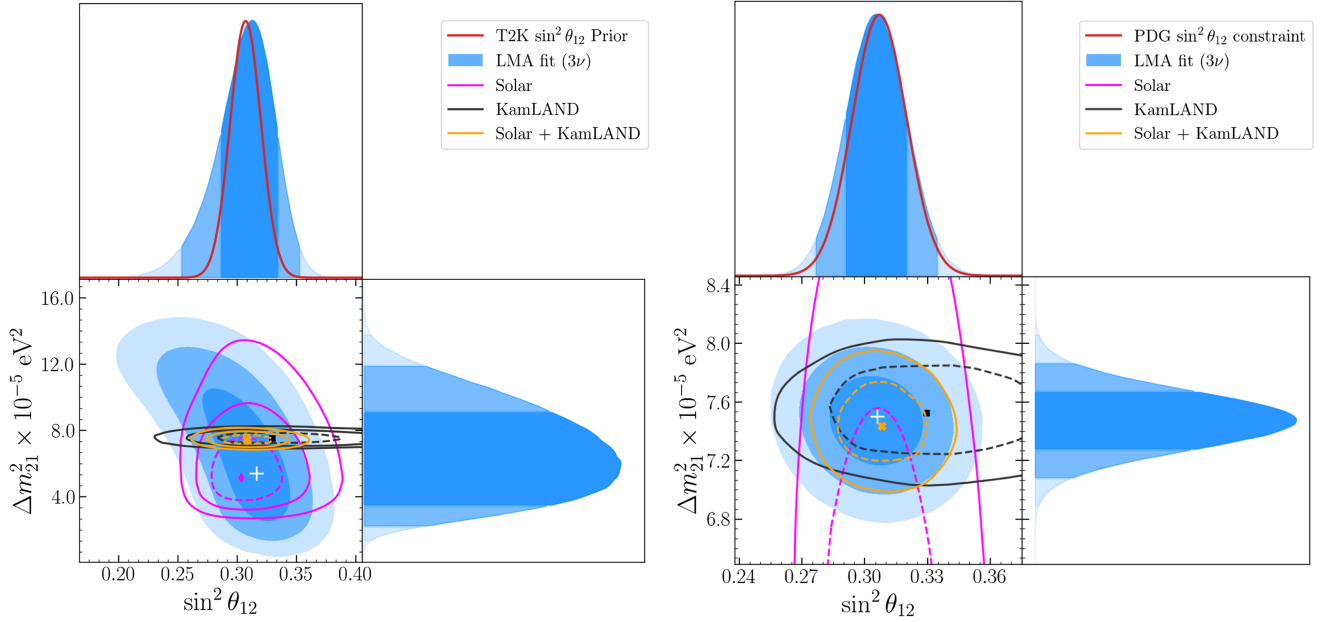


FIG. 1. 1σ , 2σ , and 3σ contours of the posterior distributions for our Bayesian fit of the LMA solution with a uniform prior in both parameters (left) and a Gaussian prior on Δm_{21}^2 (right); with 1σ , 2σ (right), and 3σ (left) contours for KamLAND's result (black), and the combined result from solar experiments (magenta). The red line corresponds to the $\sin^2 \theta_{12}$ constraint recommended by the Particle Data Group and the highest posterior density point of our analysis is shown as a white cross.

From Eqs. (4) and (5) we can see that a precise determination of θ_{12} can be made by measuring the survival probabilities in the limiting regimes of $\beta \rightarrow 0$ and $\beta > 1$, where $P_{\nu_e \rightarrow \nu_e}$ approaches

$$P_{\text{Vac}} = \cos^4 \theta_{13} \left[1 - \frac{1}{2} \sin^2 2\theta_{12} \right] + \sin^4 \theta_{13} \quad (6)$$

(vacuum-averaged mixing), and

$$P_{\text{MSW}} = \cos^4 \theta_{13} \sin^2 \theta_{12} + \sin^4 \theta_{13} \quad (7)$$

(MSW resonance), respectively.

This analysis uses publicly available data from solar neutrino experiments to fit the LMA curve to the reported survival probabilities for experiments at different energies. Survival probabilities for p-p, pep, ^7Be , and ^8B from the Borexino Collaboration are reported in [25], and a polynomial fit of the survival probability of ^8B at energies near 10 MeV from the SNO Collaboration is reported in [26]. While Borexino uses the high-metallicity Standard Solar Model (SSM) [27], SNO uses the BS05 SSM [26]; these models show strong agreement in the neutrino production zones so the impact in the reported probabilities is small [28]. First, we show that this analysis is robust enough to recover the current solar constraints; this justifies using it to arrive at a sensible bound on nonunitary parameters.

To this end, we perform two Bayesian fits of the survival probabilities described by equation (4); both with a Gaussian prior on $\sin^2 \theta_{13}$ ($\mu = 0.022$, $\sigma = 7 \times 10^{-4}$)

coming from reactor experiments [29,30] and a uniform prior on $\sin^2 \theta_{12}$. This is the prior recommended by the Particle Data Group [17], and is consistent with the flavor-anarchic Haar prior on $\text{U}(3)$ [31].

The fit is built via the (uncorrelated) sum of a Gaussian likelihood ratios of SNO's quadratic fit to the daytime probability, and Borexino's pp, ^7Be , pep, and ^8B measurements. Thus, for P^{SNO} , P_i^{Borex} the maximum likelihood points of the P_{ee} fits for the SNO and Borexino experiments (the index representing each solar process measured by Borexino), and $P(\theta)$ the MSW prediction,

$$-2 \ln \mathcal{L} \propto \int \int \Delta P(E; \theta) K^{-1}(E, E') \Delta P(E'; \theta) dE dE' + \sum_i \frac{(P_i^{\text{Borex}} - P_i(\theta))^2}{\sigma_i^2}, \quad (8)$$

where K is the Gaussian covariance kernel from SNO's polynomial P_{ee} fit [26] and $\Delta P(E; \theta) \equiv P^{\text{SNO}}(E) - P(E; \theta)$.

The first fit uses a uniform prior on Δm_{21}^2 and the second fit uses the KamLAND constraint as a Gaussian prior on Δm_{21}^2 ($\mu = 7.49 \times 10^{-5} \text{ eV}^2$, $\sigma = 0.2 \times 10^{-5} \text{ eV}^2$) [32]. Both fits assume the same solar model used in SNO's analysis. Figure 1 shows the posterior distributions of the fits with various overlaid constraints; the red curve corresponding to the PDG constraint is the prior used by the T2K experiment and is presented for reference. We find our fits to be in good agreement with the solar contours, but with

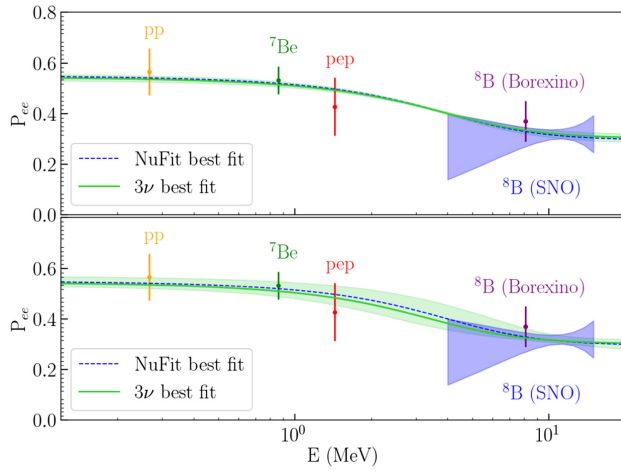


FIG. 2. Solar survival probabilities from Borexino and SNO. The highest posterior density point and 1σ contours for our Bayesian fit with (above) and without (below) a Gaussian prior on Δm^2_{21} are shown in green. The LMA solution for NuFit’s global best-fit values is shown in blue for reference.

weaker constraining power; without the KamLAND prior, the fit produces wider Δm^2_{21} contours. This is not surprising because we have no access to Borexino’s binned data, which would give a more precise constraint on the position of the MSW transition point. Despite this, the resulting distribution is consistent with the $\sin^2 \theta_{12}$ constraint recommended by the Particle Data Group, particularly after applying the KamLAND prior.

Figure 2 shows the fit results in terms of the ν_e survival probability, as well as the data used for the analysis.

III. NONUNITARITY IN NEUTRINO OSCILLATIONS

This analysis considers nonunitary effects due to charged-current transitions from the usual neutrinos to heavy isosinglet leptons (HNLs) which arise naturally from type-I seesaw models [33]. We adopt the formalism described by Escribuela *et al.* in [8], where the heavy states separate from the oscillation at production and we observe a constant flux deficit in the active sector. Under this assumption, the effective 3×3 mixing matrix between the active states is no longer unitary and can be parametrized by a lower-triangular matrix as

$$N = AU = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U, \quad (9)$$

where U is the usual PMNS matrix [34,35] and A is the matrix encoding the nonunitary contribution to the oscillations. Only the off-diagonal elements of A are allowed to have complex components. In this model,

the matter potential has to be altered too so that, as shown in [36], the effective 3-flavor propagation Hamiltonian becomes

$$\mathcal{H}_{\text{prop}} = NMN^\dagger + (NN^\dagger) \begin{pmatrix} v_{cc} - v_{nc} & 0 & 0 \\ 0 & -v_{nc} & 0 \\ 0 & 0 & -v_{nc} \end{pmatrix} (NN^\dagger), \quad (10)$$

where M is the matrix of mass eigenstates, $v_{cc} = \pm\sqrt{2}G_f N_e$ is the usual charged-current contribution and $v_{nc} = \pm(\sqrt{2}/2)G_f N_n$ is the neutral-current contribution, which can no longer be ignored because mixing with the sterile states may give flavor sensitivity to the neutral-current potential. The choice of sign differentiates between ν and $\bar{\nu}$.

A. Explicit form of the nonunitary propagation Hamiltonian

In the PDG parametrization, the unitary lepton mixing matrix is given by $U = e^{i\theta_{23}\lambda_7} e^{i\theta_{13}\lambda_5} e^{i\theta_{12}\lambda_2}$ with a complex phase appended to the off-diagonal components of $e^{i\theta_{13}\lambda_5}$, where λ_j are Gell-Mann matrices. Since we are only concerned with a survival probability $P(\nu_e \rightarrow \nu_e)$, we can use a similar argument to [37] to eliminate this complex phase δ_{CP} .

Inspired by the 3-flavor MSW derivation ([10]), we can further simplify the calculations by considering the propagation Hamiltonian in an alternate basis where we rotate by $e^{-i\theta_{23}\lambda_7}$ and $e^{-i\theta_{13}\lambda_5}$ so that the unitary part of the mixing matrix represents mixing between the mass states and some new nonflavor eigenstates. Usually, this reduces the unitary PMNS matrix to $U = e^{i\theta_{12}\lambda_2}$. Here, to leading order on the nonunitary corrections, we can commute the rotations with the lower-triangular matrix A while only accruing small errors [38] on the electron row of the vacuum hamiltonian $\mathcal{H}_{\text{vac}(1i)}$. Thus,

$$\mathcal{H}_{\text{vac}} \approx A e^{i\theta_{12}\lambda_2} M e^{-i\theta_{12}\lambda_2} A^\dagger. \quad (11)$$

Similarly, the relevant entries of the matter contribution are largely invariant to commuting the nonunitary matrix $NN^\dagger (= AA^\dagger)$ with $e^{i\theta_{23}\lambda_7}$. This allows us to absorb the rotation into the unitary potential $V = -\text{Diag}(v_{cc} - v_{nc}, v_{nc}, v_{nc})$ for free, resulting in

$$\mathcal{H}_{\text{mat}} \approx e^{-i\theta_{13}\lambda_5} (AA^\dagger) V (AA^\dagger) e^{i\theta_{13}\lambda_5}. \quad (12)$$

Factoring out the energy dependence, we find that the relevant entries of the traceless vacuum component become, to first order in the small parameters,

$$\begin{aligned}
\mathcal{H}_{\text{vac}(11)} &\approx -\alpha_{11}^2 \Delta m_{21}^2 \cos 2\theta_{12}, \\
\mathcal{H}_{\text{vac}(12)} &\approx \alpha_{11} \Delta m_{21}^2 (\alpha_{22} \sin 2\theta_{12} - \overline{\alpha_{21}} \cos 2\theta_{12}), \\
\mathcal{H}_{\text{vac}(13)} &\approx \alpha_{11} \Delta m_{21}^2 (\overline{\alpha_{32}} \sin 2\theta_{12} - \overline{\alpha_{31}} \cos 2\theta_{12}), \\
\mathcal{H}_{\text{vac}(22)} &\approx \alpha_{22} \Delta m_{21}^2 (\alpha_{22} \cos 2\theta_{12} + \mathcal{O}(|\alpha_{21}|)),
\end{aligned} \tag{13}$$

and, in the matter component,

$$\begin{aligned}
\mathcal{H}_{\text{mat}(11)} &\approx \alpha_{11}^4 c_{13}^2 (v_{cc} - v_{nc}) \\
&\quad - \alpha_{11}^3 \sin 2\theta_{13} \Re\{\alpha_{31}\} (v_{cc} - 2v_{nc}), \\
\mathcal{H}_{\text{mat}(12)} &\approx \alpha_{11}^3 (c_{13} \overline{\alpha_{21}} (v_{cc} - 2v_{nc}) - 2s_{13} \overline{\alpha_{32}} v_{nc}), \\
\mathcal{H}_{\text{mat}(13)} &\approx s_{13} c_{13} (\alpha_{11}^4 - \alpha_{33}^4) (v_{cc} - v_{nc}) \\
&\quad + c_{13}^2 \alpha_{11}^3 \overline{\alpha_{31}} (v_{cc} - 2v_{nc}), \\
\mathcal{H}_{\text{mat}(22)} &\approx -\alpha_{22}^4 v_{nc}.
\end{aligned} \tag{14}$$

B. Nonunitary adiabatic approximation

To proceed with an analysis we need to confirm that, at the energy and matter density scales in the sun, nonunitary neutrino evolution remains adiabatic.

The resonance takes place at $\mathcal{H}_{11} \approx \mathcal{H}_{22}$. Using the nonunitary Hamiltonian shifts the resonance from the usual condition $E v_{cc} = \Delta m_{21}^2 \cos 2\theta_{12}$ to

$$\begin{aligned}
&\frac{\alpha_{11}^4}{\alpha_{11}^2 + \alpha_{22}^2} 2E v_{cc} - (\alpha_{11}^2 - \alpha_{22}^2) 2E v_{nc} \\
&= \Delta m_{21}^2 (\cos 2\theta_{12} + \mathcal{O}(|\alpha_{21}|) \sin 2\theta_{12}).
\end{aligned} \tag{15}$$

Notice that, as one should expect, the usual relation is recovered in the unitary limit. Then, even assuming nonunitarity on the scale of the reactor angle θ_{13} , and taking (inside the Sun) $v_{nc} \approx v_{cc}/4$, the resonant density increases by no more than $\sim 20\%$ —only a few percent closer to the Sun's core [23].

The adiabatic condition requires the rate of change of the effective mixing angle θ_m to be much smaller than the instantaneous mass splitting at resonance ΔH_{res} . Due to the scale of neutrino masses, we assume a decoupling of the 1–2 and 1–3 resonance. Then, projecting the solar sector into a 2×2 space yields,

$$\tan 2\theta_m \simeq \frac{2|\mathcal{H}_{12}|}{\mathcal{H}_{22} - \mathcal{H}_{11}} \quad \text{and} \quad \Delta H_{\text{res}} \simeq 2|\mathcal{H}_{12}|. \tag{16}$$

Thus, adiabaticity holds if

$$\frac{|\mathcal{H}_{22} - \mathcal{H}_{11}|'_{\text{res}}}{8|\mathcal{H}_{12}|^2} \ll 1, \tag{17}$$

which, assuming unitarity, recovers the familiar

$$\gamma_{\odot} \equiv \frac{E \cos(2\theta_{12})}{\Delta m_{21}^2 \sin^2(2\theta_{12}) h_e} \ll 1 \tag{18}$$

for h_e the coefficient of the exponential electron density profile in the sun. Substituting for 8^B neutrinos one finds $\gamma_{\odot} \sim 10^{-4}$, indicating a strongly adiabatic transition.

Setting $h_n \approx h_e$, the nonunitary expressions (13) and (14) yield,

$$\hat{\gamma}_{\odot} \equiv \frac{(\alpha_{11}^2 + \alpha_{22}^2) E \cos(2\theta_{12}) + \mathcal{O}(|\alpha_{21}|)}{2h_e \alpha_{11}^2 \alpha_{22}^2 \Delta m_{21}^2 \sin^2 \theta_{12}} \ll 1. \tag{19}$$

Once again assuming a deviation from unitarity as large as the reactor angle, we find that $\hat{\gamma}_{\odot}$ is only about 4% larger than $\gamma_{\odot}$. The problem is still very strongly adiabatic.

C. Nonunitary survival probability

In order to find an approximation analogous to that of the unitary formalism, we want to write $\mathcal{H}_{\text{prop}}$ at some energy and matter density as $\hat{N} M \hat{N}^\dagger$, where $\hat{N}$ is an effective mixing matrix of the form $A \hat{U}$ with $A(\alpha_{ij})$ the same lower-triangular matrix and $\hat{U}(\hat{\theta}_{ij})$ still unitary. In this case, the expression for $\cos(2\hat{\theta}_{12})$ in Eq. (4) is still a valid approximation, but the ratio β will reflect the new matter effect and the nonunitary contribution to the vacuum Hamiltonian. Then, comparing expressions (13) and (14), the new matter-to-vacuum ratio is

$$\beta_{NU} = \frac{2E}{\Delta m_{21}^2} \left(\alpha_{11}^2 c_{13}^2 v_{cc} + (\alpha_{22}^2 - c_{13}^2 \alpha_{11}^2) v_{nc} + \mathcal{O}(|\alpha_{21}|) v_{nc} \right). \tag{20}$$

Then, we follow the original 3-flavor eigenvalue crossing derivation in [10], where the survival probability averaged over the oscillation period can be written as

$$\begin{aligned}
\langle P_{ee} \rangle &= |a_1 N_{e1}(L_0) N_{e1}(L_f)|^2 + |a_2 N_{e2}(L_0) N_{e1}(L_f)|^2 \\
&\quad + |a_1 N_{e2}(L_0) N_{e2}(L_f)|^2 + |a_2 N_{e1}(L_0) N_{e2}(L_f)|^2 \\
&\quad + |a_3 N_{e3}(L_0) N_{e3}(L_f)|^2,
\end{aligned} \tag{21}$$

for $N_{ai}(L_0)$, $N_{ai}(L_f)$ elements of the effective mixing matrices at production and detection respectively, and a_i the resonance couplings. Since the ν_1/ν_2 resonance appears at much lower energies than the ν_2/ν_3 resonance, we assume they are fully decoupled so that $|a_1|^2 = \cos^2 \theta_{13} P_{\text{jump}}$, $|a_2|^2 = 1 - \cos^2 \theta_{13} P_{\text{jump}}$ and $|a_3|^2 = 1$, where P_{jump} is probability for a neutrino to jump between

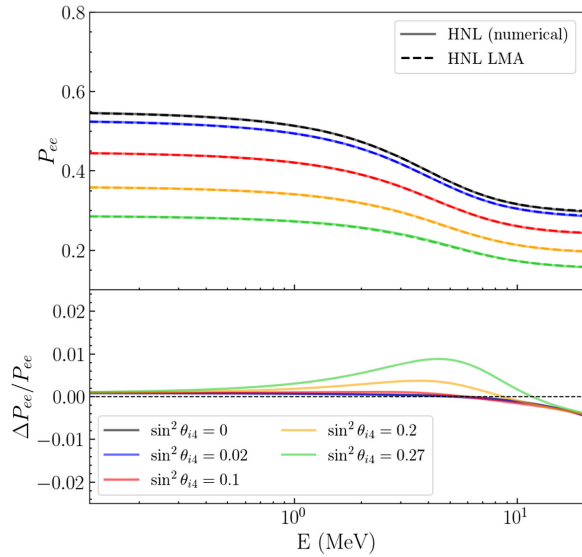


FIG. 3. Comparison of the usual (black) and nonunitary (colored) LMA approximations against numerical solutions in the presence of a single GeV scale HNL, assuming the electron and neutron densities of the sun's core. The magnitude of the mixing into the sterile sector was set to the same value for all three active neutrinos. In this case, the nonunitary parameter α_{11} corresponds to $\cos \theta_{14}$.

mass states at the eigenvalue crossing. Using $\hat{N}$ and N for the effective mixing matrices in matter and vacuum, the survival probability becomes

$$\begin{aligned} \langle P_{ee} \rangle = & |a_1 \hat{N}_{e1} N_{e1}|^2 + |a_2 \hat{N}_{e2} N_{e1}|^2 \\ & + |a_1 \hat{N}_{e2} N_{e2}|^2 + |a_2 \hat{N}_{e1} N_{e2}|^2 \\ & + |a_3 \hat{N}_{e3} N_{e3}|^2. \end{aligned} \quad (22)$$

Once again exploiting the energy scale of the ν_1/ν_2 resonance, ν_3 is strongly decoupled [10] so that $\hat{\theta}_{13} \approx \theta_{13}$ and $\hat{\theta}_{23} \approx \theta_{23}$ and Eq. (22) simplifies to

$$\begin{aligned} \langle P_{ee} \rangle = & \alpha_{11}^4 \left(c_{13}^4 \left[\frac{1}{2} + \left(\frac{1}{2} - P_{\text{jump}} \cos^2 \theta_{13} \right) \right. \right. \\ & \left. \left. \times \cos 2\theta_{12} \cos 2\hat{\theta}_{12} \right] + s_{13}^4 \right). \end{aligned} \quad (23)$$

For the Sun's density profile, the transition is adiabatic: $P_{\text{jump}} \approx 0$ [10,39], and so we end up with

$$\langle P_{ee} \rangle = \alpha_{11}^4 \left(\frac{1}{2} \cos^4 \theta_{13} [1 + \cos 2\hat{\theta}_{12} \cos 2\theta_{12}] + \sin^4 \theta_{13} \right), \quad (24)$$

which resembles the usual LMA solution, with an additional α_{11}^4 factor. Here $\cos 2\hat{\theta}_{12}$ must be calculated using the β_{NU} from Eq. (20).

Finally, notice that the dominant term in Eq. (20) is $\alpha_{11}^2 c_{13}^2 v_{cc}$, as the neutral current contribution $\alpha_{22}^2 - c_{13}^2 \alpha_{11}^2$ is of the scale of the off-diagonal elements of A , and further suppressed by v_{cc} being around a factor of 4 smaller than v_{nc} in the Sun. This implies that the nonunitary survival probability for ^8B neutrinos is only very weakly dependent on any new parameters other than α_{11} . We can approximate the behaviour of the subleading neutral current contribution while only incurring a $\mathcal{O}(|\alpha_{21}|)$ error as

$$\beta \approx \frac{\sqrt{2} G_f E}{\Delta m_{21}^2} \left(2\alpha_{11}^2 c_{13}^2 N_e + (1 - c_{13}^2 \alpha_{11}^2) N_n \right). \quad (25)$$

Figure 3 shows the unitary and nonunitary LMA approximations against numerical solutions for increasingly large nonunitary mixing at the energy and matter density ranges of solar neutrinos. Even for large unitarity violation ($1 - \alpha_{ii} < 0.1$), the error in our approximation remains subpercent.

IV. NONUNITARY SOLAR OSCILLATION FITS

The MSW approximation in the presence of HNLs was fitted to solar and reactor data. Taking N_e and N_n as given in the neutrino production regions of the sun by the BS05 SSM, we performed a Bayesian fit to the same solar data used for the unitary fit in Sec. II. We note that KamLAND's experimental setup is such that matter effects play no role in its oscillation measurement, so any nonunitary effects in its Δm_{21}^2 constraint come from nonunitary effects in the vacuum Hamiltonian. Since HNLs only appear as a normalization factor in vacuum oscillations, their presence does not affect the period of the oscillations; it is simple to see that the (already angle-uncorrelated) Δm_{21}^2 measurement at KamLAND is not affected under this relaxation of the unitarity condition. Taking this into account, we use the KamLAND Δm_{21}^2 constraint as a prior for our nonunitary analysis.

Figure 4 shows the result of the fits with and without KamLAND's Δm_{21}^2 constraint. The nonunitary parameter is written as $1 - \alpha_{11}^2$ because in a $3 + 1$ scenario this quantity corresponds to the $\sin^2 \theta_{14}$ angle. Without the KamLAND constraint, the nonunitary parameter is strongly correlated with the solar mass splitting; this is an expected feature because a larger Δm_{21}^2 will move the transition point towards higher energies, thus increasing the survival probability for ^8B

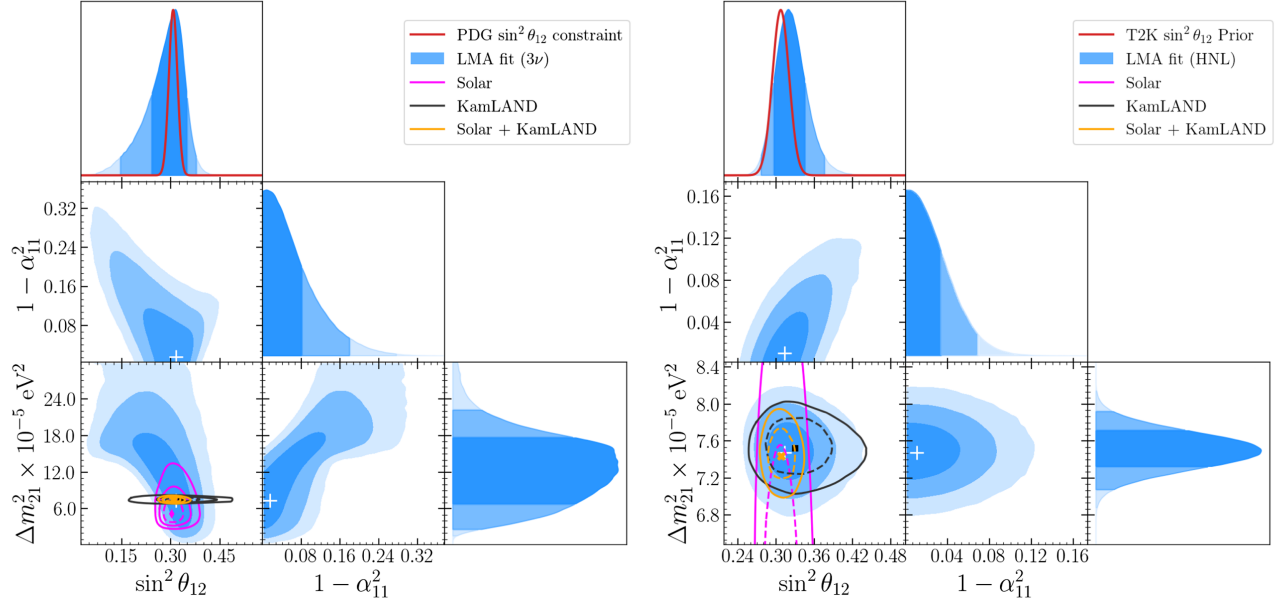


FIG. 4. 1σ , 2σ , and 3σ contours of the posterior distributions for our Bayesian fit of the nonunitary LMA solution with a uniform prior in all parameters (left) and a Gaussian prior on Δm_{12}^2 (right); with 1σ , 2σ (right), and 3σ (left) contours for KamLAND's result (black) and the combined result from solar experiments (magenta). The red line corresponds to the $\sin^2 \theta_{12}$ constraint recommended by the Particle Data Group and the best-fit point of our analysis is shown as a white cross.

neutrinos—an excess that can be balanced by $\alpha_{11} < 1$ acting as a normalization factor decreasing the expected flux.

The results are consistent with no HNL mixing, and using the KamLAND mass difference we achieve constraints comparable to the current strongest limits (coming from joint-fits of reactor and short-baseline data [9,36,40]), constraining $(1 - \alpha_{11}) < 0.046$ at 99% credibility. Table I compares these results with current constraints coming from reactor, accelerator, and lepton universality data; the solar limit can compete with other

oscillation-only constraints and may contribute to strengthening the global limit. Unsurprisingly, introducing nonunitary parameters comes at the cost of reducing our sensitivity to θ_{12} ; Table II shows the relaxation of the measurement and drift to larger mixing angles to compensate for the nonunitary disappearance. Moreover, the posteriors for α_{11} and $\sin^2 \theta_{12}$ are highly correlated—this is further indication that the usual solar constraint used by long-baseline experiments is not adequate for HNL studies.

Mirroring the presentation of the unitary fits, Fig. 5 shows the HNL fits in ν_e survival probability space. As expected, the contours are similar to those in Fig. 2; differing most at the MSW transition in the ≈ 3 MeV range. Enhanced measurements of ^8B and pep neutrinos in this energy range may allow for stronger constraints on nonunitary ν_e mixing.

TABLE I. Current constraints for $1 - \alpha_{11}$ at the 90% and 99% confidence (credible) levels. The global seesaw fit (second row) uses lepton flavor violation (LFT) data and is therefore not an oscillation constraint. The global oscillation fit uses a combination of short-baseline (SBL), long-baseline, and reactor data. The top-row measurement considers SBL accelerator data only.

Dataset	90% CL	99% CL
NOMAD + NuTeV [8]	< 0.031	< 0.056
Global seesaw fit (+LFT) [36,41]	$< 2.6 \times 10^{-3}$	$< 3.78 \times 10^{-3}$
Global oscillation fit (SBL + LBL + reactor) [42]	< 0.02	< 0.05
<i>This work</i>	< 0.028	< 0.046

TABLE II. Comparison of this work's unitary and nonunitary constraints marginalized onto $\sin^2 \theta_{12}$ presented in this work. The nonunitary framework doubles the uncertainty on the mixing angle and shifts the best-fit (highest posterior density) point to larger values by $\approx 4\%$.

Model	Best-fit	σ
Unitary (PDG)	$\sin^2 \theta_{12} = 0.307$	0.013
Nonunitary	$\sin^2 \theta_{12} = 0.318$	0.025

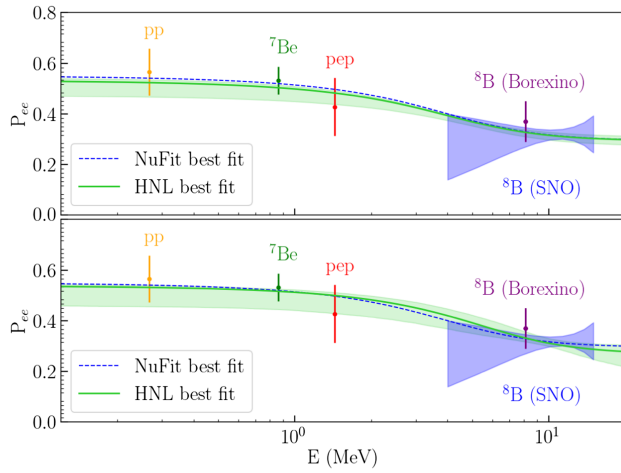


FIG. 5. Solar survival probabilities from Borexino and SNO. The highest posterior density point and 1σ contours for our nonunitary Bayesian fit with (above) and without (below) a Gaussian prior on Δm^2_{21} are shown in green. The LMA solution for NuFit’s global best-fit values is shown in blue for reference. Note that the best-fit line is not centered around the 1σ contours because the posteriors are non-Gaussian.

V. CONCLUSION

In this work we have derived an approximation for solar survival probabilities under the HNL formalism discussed in [8]. The approximation introduces one new parameter and is accurate for small deviations from unitarity. We have discussed the importance of an external constraint on θ_{12} for LBL oscillation analysis and have reproduced current constraints using solar and reactor data. We have used the newly derived nonunitary approximation to constrain the α_{11} parameter and provided a correlated θ_{12} constraint, which is necessary for nonunitary LBL analyses.

The result has been a weakening of the current θ_{12} measurement but a competitive constraint on α_{11} when compared against short-baseline and global oscillation fits. Due to the role of the θ_{12} constraint on δ_{CP} measurements in LBL experiments, we can expect a decline in sensitivity to CP -violation when performing HNL fits using this work as the external solar constraint.

The fits presented in this work used publicly available data and lacked the energy-dependent information necessary to reject the low- θ_{12} /high- Δm^2_{21} region of the 3-flavor LMA phase space without the help of KamLAND’s Δm^2_{21} constraint, but we have shown that solar data can produce competitive constraints on the mixing between ν_e and the HNL sector and have provided a tentative constraint for unitarity violation in LBL oscillations. The nonunitary solar constraint presents strong correlations between the solar mixing angle and the nonunitary parameter α_{11} , which reinforces the need for a special solar constraint when performing HNL searches using LBL oscillation data.

Finally, we note that the presence of nonunitarity relaxes the solar-KamLAND Δm^2_{21} tension by allowing a larger mass splitting in solar measurements. Indeed, the highest posterior density of our solar-data-only HNL fit agrees with the KamLAND Δm^2_{21} best-fit point, which should be blind to HNL effects.

The solar sector has the power to set strong limits on nonunitary neutrino mixing that have wider implications in explaining nuclear anomalies [43] and is vital for accessing the off-diagonal entries of the nonunitary matrix in LBL experiments. We look forward to in-depth solar HNL analyses, particularly with the inclusion of data from the Super-Kamiokande experiment [44], anticipating leading constraints on the magnitude of ν_e nonunitary mixing and a possible resolution of the KamLAND Δm^2_{21} tension.

ACKNOWLEDGMENTS

I would like to thank Lukas Berns and Mariam Tórtola for providing helpful conversations and advice on the nuances of the nonunitary oscillation formalism, Jeanne Wilson and Daniel Cookman for insight into solar neutrino physics at SNO, and Asher Kaboth, Nikolaos Kouvasos, Teppei Katori, and Francesca di Lodovico for useful comments on drafts for this work.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] B. Abi *et al.* (DUNE Collaboration), Deep underground neutrino experiment (DUNE), far detector technical design report, volume II: DUNE physics, [arXiv:2002.03005](#).
- [2] K. Abe *et al.* (Hyper-Kamiokande Proto- Collaboration), Physics potential of a long-baseline neutrino oscillation experiment using a J-PARC neutrino beam and Hyper-Kamiokande, *Prog. Theor. Exp. Phys.* **2015**, 53C02 (2015).

- [3] C. Giganti, S. Lavignac, and M. Zito, Neutrino oscillations: The rise of the PMNS paradigm, *Prog. Part. Nucl. Phys.* **98**, 1 (2018).
- [4] M. A. Acero *et al.* (NOvA Collaboration), First measurement of neutrino oscillation parameters using neutrinos and antineutrinos by NOvA, *Phys. Rev. Lett.* **123**, 151803 (2019).

- [5] K. Abe *et al.* (T2K Collaboration), Constraint on the matter–antimatter symmetry-violating phase in neutrino oscillations, *Nature (London)* **580**, 339 (2020); **583**, E16 (2020).
- [6] S. A. R. Ellis, K. J. Kelly, and S. W. Li, Leptonic unitarity triangles, *Phys. Rev. D* **102**, 115027 (2020).
- [7] A. E. Cárcamo Hernández, M. González, and N. A. Neill, Low scale type I seesaw model for lepton masses and mixings, *Phys. Rev. D* **101**, 035005 (2020).
- [8] F. J. Escrivuela, D. V. Forero, O. G. Miranda, M. Tortola, and J. W. F. Valle, On the description of nonunitary neutrino mixing, *Phys. Rev. D* **92**, 053009 (2015); **93**, 119905(E) (2016).
- [9] D. V. Forero, C. Giunti, C. A. Ternes, and M. Tortola, Nonunitary neutrino mixing in short and long-baseline experiments, *Phys. Rev. D* **104**, 075030 (2021).
- [10] X. Shi and D. N. Schramm, Solar neutrinos and the MSW effect for three neutrino mixing, *Phys. Lett. B* **283**, 305 (1992).
- [11] G. Alimonti *et al.* (Borexino Collaboration), The Borexino detector at the Laboratori Nazionali del Gran Sasso, *Nucl. Instrum. Methods Phys. Res., Sect. A* **600**, 568 (2009).
- [12] J. Boger *et al.* (SNO Collaboration), The Sudbury neutrino observatory, *Nucl. Instrum. Methods Phys. Res., Sect. A* **449**, 172 (2000).
- [13] A. Piepke (KamLAND Collaboration), KamLAND: A reactor neutrino experiment testing the solar neutrino anomaly, *Nucl. Phys. B, Proc. Suppl.* **91**, 99 (2001).
- [14] P. B. Denton and J. Gehrlein, Solar parameters in long-baseline accelerator neutrino oscillations, *J. High Energy Phys.* **06** (2023) 090.
- [15] K. Abe *et al.* (T2K Collaboration), The T2K experiment, *Nucl. Instrum. Methods Phys. Res., Sect. A* **659**, 106 (2011).
- [16] M. A. Acero *et al.* (NOvA Collaboration), Improved measurement of neutrino oscillation parameters by the NOvA experiment, *Phys. Rev. D* **106**, 032004 (2022).
- [17] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **98**, 030001 (2018).
- [18] K. Eguchi *et al.* (KamLAND Collaboration), First results from KamLAND: Evidence for reactor anti-neutrino disappearance, *Phys. Rev. Lett.* **90**, 021802 (2003).
- [19] S. Parke, What is Δm_{ee}^2 ?, *Phys. Rev. D* **93**, 053008 (2016).
- [20] S. Abe *et al.* (KamLAND Collaboration), Precision measurement of neutrino oscillation parameters with KamLAND, *Phys. Rev. Lett.* **100**, 221803 (2008).
- [21] G. D. O. Gann, K. Zuber, D. Bemmerer, and A. Serenelli, The future of solar neutrinos, *Annu. Rev. Nucl. Part. Sci.* **71**, 491 (2021).
- [22] M. Asplund, N. Grevesse, A. J. Sauval, and P. Scott, The chemical composition of the Sun, *Annu. Rev. Astron. Astrophys.* **47**, 481 (2009).
- [23] J. N. Bahcall and C. Pena-Garay, Solar models and solar neutrino oscillations, *New J. Phys.* **6**, 63 (2004).
- [24] T.-K. Kuo and J. T. Pantaleone, The solar neutrino problem and three neutrino oscillations, *Phys. Rev. Lett.* **57**, 1805 (1986).
- [25] M. Agostini *et al.* (BOREXINO Collaboration), Comprehensive measurement of pp -chain solar neutrinos, *Nature (London)* **562**, 505 (2018).
- [26] B. Aharmim *et al.* (SNO Collaboration), Combined analysis of all three phases of solar neutrino data from the Sudbury Neutrino Observatory, *Phys. Rev. C* **88**, 025501 (2013).
- [27] J. N. Bahcall, A. M. Serenelli, and S. Basu, 10,000 standard solar models: A Monte Carlo simulation, *Astrophys. J. Suppl. Ser.* **165**, 400 (2006).
- [28] N. F. Fiúza de Barros, Precision measurement of neutrino oscillation parameters: Combined three-phase results of the Sudbury Neutrino Observatory, Ph. D. thesis, Lisbon University, 2011.
- [29] M. C. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, NuFIT: Three-flavour global analyses of neutrino oscillation experiments, *Universe* **7**, 459 (2021).
- [30] D. Adey *et al.* (Daya Bay Collaboration), Measurement of the electron antineutrino oscillation with 1958 days of operation at Daya Bay, *Phys. Rev. Lett.* **121**, 241805 (2018).
- [31] M. L. Eaton and W. D. Sudderth, Group invariant inference and right haar measure, *J. Stat. Plann. Infer.* **103**, 87 (2002).
- [32] A. Gando *et al.* (KamLAND Collaboration), Constraints on θ_{13} from a three-flavor oscillation analysis of reactor antineutrinos at KamLAND, *Phys. Rev. D* **83**, 052002 (2011).
- [33] A. M. Abdullahi *et al.*, The present and future status of heavy neutral leptons, *J. Phys. G* **50**, 020501 (2023).
- [34] With (possibly) different mixing angles and CP -phase than in the unitary case.
- [35] S. K. Agarwalla, S. Das, A. Giarnetti, and D. Meloni, Model-independent constraints on non-unitary neutrino mixing from high-precision long-baseline experiments, *J. High Energy Phys.* **07** (2022) 121.
- [36] F. J. Escrivuela, D. V. Forero, O. G. Miranda, M. Tortola, and J. W. F. Valle, Probing CP violation with non-unitary mixing in long-baseline neutrino oscillation experiments: DUNE as a case study, *New J. Phys.* **19**, 093005 (2017).
- [37] H. Yokomakura, K. Kimura, and A. Takamura, Overall feature of CP dependence for neutrino oscillation probability in arbitrary matter profile, *Phys. Lett. B* **544**, 286 (2002).
- [38] Linear on the deviation from unitarity but suppressed by the smallness of θ_{13} .
- [39] M. C. Gonzalez-Garcia and Y. Nir, Neutrino masses and mixing: Evidence and implications, *Rev. Mod. Phys.* **75**, 345 (2003).
- [40] M. Blennow, P. Coloma, E. Fernandez-Martinez, J. Hernandez-Garcia, and J. Lopez-Pavon, Non-unitarity, sterile neutrinos, and non-standard neutrino interactions, *J. High Energy Phys.* **04** (2017) 153.

- [41] E. Fernandez-Martinez, J. Hernandez-Garcia, and J. Lopez-Pavon, Global constraints on heavy neutrino mixing, [J. High Energy Phys.](#) **08** (2016) 033.
- [42] S. Parke and M. Ross-Lonergan, Unitarity and the three flavor neutrino mixing matrix, [Phys. Rev. D](#) **93**, 113009 (2016).
- [43] P. B. Denton and J. Gehrlein, Neutrino constraints and the ATOMKI X17 anomaly, [Phys. Rev. D](#) **108**, 015009 (2023).
- [44] K. Abe *et al.* (Super-Kamiokande Collaboration), Solar neutrino measurements in Super-Kamiokande-IV, [Phys. Rev. D](#) **94**, 052010 (2016).