

Probing the dark axion portal via J/ψ decays at BESIII and STCF

Zeren Simon Wang,¹ Dazhuang He,^{2,*} and Yu Zhang^{1,†}

¹*School of Physics, Hefei University of Technology, Hefei 230601, People's Republic of China*

²*College of Physics and Electronic Engineering, Heze University, No.2269 University Road, Heze, Shandong, 274015, People's Republic of China*



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Large numbers of J/ψ mesons can be resonantly produced at BESIII and STCF at the center-of-mass energy $\sqrt{s} = 3.097$ GeV. Such J/ψ mesons may undergo rare decays into an axionlike particle (ALP) a and a dark photon γ' through the dark axion portal. We investigate the exclusion reach of the existing BESIII dataset and the projected sensitivity of STCF, focusing on the monophoton signature. We perform Monte Carlo simulations of the signal and estimate both the irreducible $J/\psi \rightarrow \gamma\nu\bar{\nu}$ background and the continuum and resonant three-photon reducible backgrounds. Detector-induced migration of three-photon events into the mono-photon signal region is parametrized by the event-level leakage probability $\epsilon_{3\gamma}^{\text{leak}} = 0, 10^{-6}, 10^{-4}, 10^{-3}$. In the zero- and sufficiently small-leakage benchmarks, the existing BESIII dataset could probe previously unexplored parameter space, while STCF can further improve the coupling reach. Increasing leakage probabilities progressively weaken these projections, highlighting the importance of detector-level three-photon rejection in realizing the projected reach.

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I. INTRODUCTION

In recent years, models of portal physics are under scrutiny, as to whether they serve as a bridge between the Standard Model (SM), visible sector and a dark, invisible sector. Traditionally, such models include the fermion portal with heavy neutral leptons [1–3], the scalar portal with a new scalar particle [4–9], the vector portal with a dark photon γ' [10–15], and the pseudoscalar portal with an axion or axionlike particle (ALP) a [16–22]. The predicted new particles listed above couple to the SM particles either directly or via mixing with them. An additional portal, called the dark axion portal (DAP), was introduced more recently in Ref. [23]. As an independent portal, the DAP provides a novel link between the dark photon and the ALP, featured with the coupling $G_{a\gamma\gamma'}$, instead of being a naive combination of the vector and pseudoscalar portals. The DAP can be constructed from a triangle anomaly, in the underlying UV-complete models such as the dark KSVZ axion model discussed in Ref. [23].

The DAP scenario is well motivated, for solving critical issues in particle physics and cosmology, including the

strong CP problem [23–25] and the hierarchy problem with a cosmological relaxation solution [26–28], as well as providing a dark-matter candidate [23,29–31]. For further past studies on the dark axion portal, we refer to Refs. [32–41]. Such studies cover contexts of cosmological observations, colliders including B factories, beam-dump experiments, astrophysics, and neutrino experiments.

In this work, we investigate the sensitivity of the existing Beijing Spectrometer III (BESIII) experiment at the Beijing Electron Positron Collider II (BEPC II) [42] in Beijing, China, as well as that of the proposed Super Tau–Charm Facility (STCF) [43,44] in Hefei, China, to the dark axion portal. BESIII and STCF are intensity-frontier electron-positron collider experiments, scanning the center-of-mass (COM) energy over about 2–5 GeV and 2–7 GeV, respectively, in the tau-charm regime. In particular, both experiments can operate at the COM energy of 3.097 GeV on the $J/\psi(1S)$ resonance, yielding $\mathcal{O}(10^{10})$ and $\mathcal{O}(10^{12})$ J/ψ mesons, respectively. The J/ψ mesons can undergo two-body decays via an off shell SM photon γ into a dark photon and an ALP in the DAP scenario, i.e., $J/\psi \rightarrow a\gamma'$, as studied in the context of the LHC far detectors [38] and the beam-dump experiments NA64e, NA64 μ , LDMX, and M³ [30,31].

We focus on the phenomenology with the single coupling $G_{a\gamma\gamma'}$ in this work. With this coupling present, an additional coupling $G_{aZ\gamma'}$ arises from the SM gauge invariance, with the relation $|G_{aZ\gamma'}/G_{a\gamma\gamma'}| = \tan\theta_W$ where θ_W is the weak mixing angle [39]. Since we consider

*Contact author: dzhe1998@gmail.com

†Contact author: dayu@hfut.edu.cn

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BESIII and STCF on the $J/\psi(1S)$ resonance, the Z-boson contributions to the signal process are negligible, and we will therefore ignore the coupling $G_{aZ\gamma'}$ in our phenomenological analysis.

The sensitivity projections presented in this work are based on simulations and analytic estimates of the signal and background contributions. Although we make every effort to model the relevant BESIII detector response and the anticipated STCF performance, our results remain phenomenological estimates. In particular, the background estimates and the resulting sensitivity projections may be optimistic. They will ultimately require validation and refinement by the BESIII and future STCF Collaborations using full detector simulations and data-driven studies based on control samples.

This paper is organized as follows. In Sec. II, we present the model setup, computing the decay rates of the J/ψ meson, dark photon, and ALP. Then in Sec. III, we introduce the BESIII and STCF experiments, detail our simulation procedures, as well as elaborate on background estimates. We show the numerical results in Sec. IV and conclude the paper in Sec. V. Additionally, Appendix A provides the analytic differential decay rates for the signal and irreducible background, while Appendix B details our benchmark estimate of the reducible three-photon backgrounds.

II. THEORETICAL SETUP

We focus on the following interaction Lagrangian with a single term proportional to the coupling $G_{a\gamma\gamma'}$ [23,45]:

$$\mathcal{L}_{\text{DAP}} \supset \frac{G_{a\gamma\gamma'}}{2} a F_{\mu\nu} \tilde{Z}'^{\mu\nu}, \quad (1)$$

where $G_{a\gamma\gamma'}$ has mass dimension $[-1]$, and $F_{\mu\nu}$ is the electromagnetic field-strength tensor and $Z'^{\mu\nu}$ is the field-strength tensor of the dark $U(1)_D$ gauge field. $\tilde{Z}'^{\mu\nu}$ is the dual of $Z'^{\mu\nu}$.

As discussed above, the interaction in Eq. (1) may induce J/ψ decays into a dark photon and an ALP, and we compute the corresponding decay width to be

$$\Gamma(J/\psi \rightarrow a\gamma') = \frac{e^2 Q_c^2 f_{J/\psi}^2 G_{a\gamma\gamma'}^2}{96\pi m_{J/\psi}^5} \lambda^{3/2}(m_{J/\psi}^2, m_a^2, m_{\gamma'}^2), \quad (2)$$

where e denotes the electric charge coupling, $Q_c = 2/3$ is the electric charge in units of e , $f_{J/\psi}$ and $m_{J/\psi}$ are the decay constant and mass of J/ψ , respectively, and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ labels the Källén function. m_a and $m_{\gamma'}$ are the masses of the ALP and the dark photon, respectively.

We notice, on the other hand, that the decay width of the SM process $J/\psi \rightarrow e^+e^-$ [46] is

$$\Gamma(J/\psi \rightarrow e^+e^-) = \frac{4\pi\alpha^2 Q_c^2 f_{J/\psi}^2}{3m_{J/\psi}}, \quad (3)$$

where $\alpha = e^2/4\pi$ is the fine structure constant. We thus express $\Gamma(J/\psi \rightarrow a\gamma')$ in terms of $\Gamma(J/\psi \rightarrow e^+e^-)$,

$$\Gamma(J/\psi \rightarrow a\gamma') = \frac{G_{a\gamma\gamma'}^2 \Gamma(J/\psi \rightarrow e^+e^-)}{32\pi\alpha m_{J/\psi}^4} \times \lambda^{3/2}(m_{J/\psi}^2, m_a^2, m_{\gamma'}^2), \quad (4)$$

where $\Gamma(J/\psi \rightarrow e^+e^-) = \Gamma_{J/\psi} \cdot \text{BR}(J/\psi \rightarrow e^+e^-) = 5.53 \text{ keV}$ with the measured J/ψ total decay width $\Gamma_{J/\psi} = 92.6 \text{ keV}$ and $\text{BR}(J/\psi \rightarrow e^+e^-) = 5.971\%$ [47].

In Fig. 1, we show the Feynman diagram of the decay $J/\psi \rightarrow a\gamma'$ via an off shell SM photon. The vertex associated with the coupling $G_{a\gamma\gamma'}$ is labeled with a blue blob.

Further, we display in Fig. 2 a plot of $\text{BR}(J/\psi \rightarrow a\gamma') = \Gamma(J/\psi \rightarrow a\gamma')/\Gamma_{J/\psi}$ as a function of $m_{\gamma'}$, for $m_a = 0$ and $G_{a\gamma\gamma'} = 10^{-5} \text{ GeV}^{-1}$. For this benchmark, the branching ratio is of order 10^{-11} . $\Gamma(J/\psi \rightarrow a\gamma')$ [see Eq. (4)] is symmetric under $m_a \leftrightarrow m_{\gamma'}$ and scales as $G_{a\gamma\gamma'}^2$.

The leading two-body decays induced by the $G_{a\gamma\gamma'}$ coupling and their corresponding partial widths are

$$\Gamma(\gamma' \rightarrow a\gamma) = \frac{G_{a\gamma\gamma'}^2}{96\pi} m_{\gamma'}^3 \left(1 - \frac{m_a^2}{m_{\gamma'}^2}\right)^3, \quad (5)$$

$$\Gamma(a \rightarrow \gamma'\gamma) = \frac{G_{a\gamma\gamma'}^2}{32\pi} m_a^3 \left(1 - \frac{m_{\gamma'}^2}{m_a^2}\right)^3. \quad (6)$$

The two modes are kinematically allowed for $m_{\gamma'} > m_a$ and $m_a > m_{\gamma'}$, respectively.

We work in a simplified benchmark in which $G_{a\gamma\gamma'}$ is the only new-physics coupling relevant at the J/ψ energy scale. In particular, dark-photon kinetic mixing and additional ALP couplings to SM particles are assumed to be negligible. If appreciable, such interactions would open decays such as $\gamma' \rightarrow \ell^+\ell^-$, $a \rightarrow \ell^+\ell^-$, and $a \rightarrow \gamma\gamma$, and

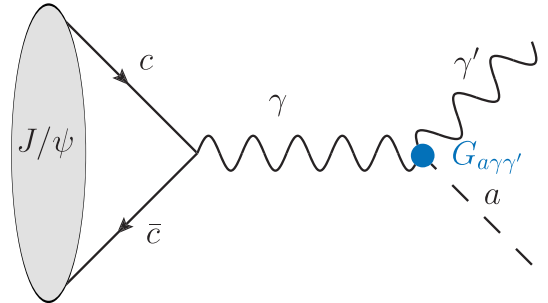


FIG. 1. The Feynman diagram depicting the J/ψ decay into a dark photon and an ALP via an s -channel SM photon.

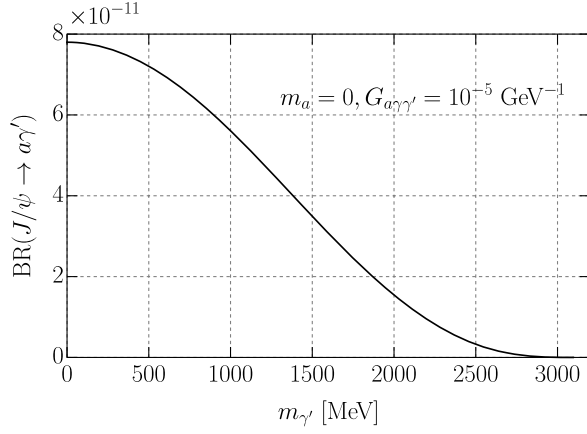


FIG. 2. The decay branching ratio of $J/\psi \rightarrow a\gamma'$ as a function of $m_{\gamma'}$ for $m_a = 0$ and $G_{a\gamma\gamma'} = 10^{-5} \text{ GeV}^{-1}$.

could lead to experimentally cleaner final states containing charged-lepton pairs, possibly accompanied by a photon and/or missing energy. These signatures depend on additional independent couplings and lie outside the scope of our single-coupling mono-photon analysis.

Even within this single-portal-coupling benchmark, the photon in the radiative transition can be off shell, giving rise to the Dalitz-type three-body decays $\gamma' \rightarrow a\gamma^* \rightarrow af\bar{f}$ and $a \rightarrow \gamma'\gamma^* \rightarrow \gamma'f\bar{f}$, when kinematically allowed. These modes are suppressed by the additional electromagnetic coupling and the three-body phase space, and typically contribute only at the percent level to the total decay width [38,39]. We therefore neglect their effects on the lifetimes and mono-photon sensitivities considered here.

The proper decay lengths $c\tau_a$ and $c\tau_{\gamma'}$ are computed as $c\hbar/\Gamma_a$ and $c\hbar/\Gamma_{\gamma'}$, respectively, where Γ_a and $\Gamma_{\gamma'}$ are the total decay widths of the ALP and the dark photon. In our benchmark, the relevant total width is approximated by the dominant two-body partial width in Eq. (5) or Eq. (6), depending on the mass hierarchy. The percent-level three-body contributions discussed above are neglected.

We show in Fig. 3 a plot of $c\tau_{a/\gamma'} \cdot G_{a\gamma\gamma'}^2$ as a function of $m_{a/\gamma'}$. The red (blue) curve is for the decay of γ' (a) with $m_a \ll m_{\gamma'}$ ($m_{\gamma'} \ll m_a$). The two curves differ by a factor of 3 vertically, as can be inferred from Eq. (5) and Eq. (6). It is easy to see that if the value of $G_{a\gamma\gamma'}$ decreases or the mass of the heavier particle is reduced, its lifetime is enhanced, potentially making it long-lived.

In this work, we propose a search for a single-photon signature arising when the heavier of γ' and a decays into the lighter state and a photon. We require the decay to occur within a three-dimensional lab-frame distance $L_{\text{dec}} < L_{\text{prompt}} \equiv 1 \text{ cm}$ from the interaction point (IP) and therefore, treat the resulting photon as prompt in our analysis. In the next section, we will discuss our search strategy in detail.

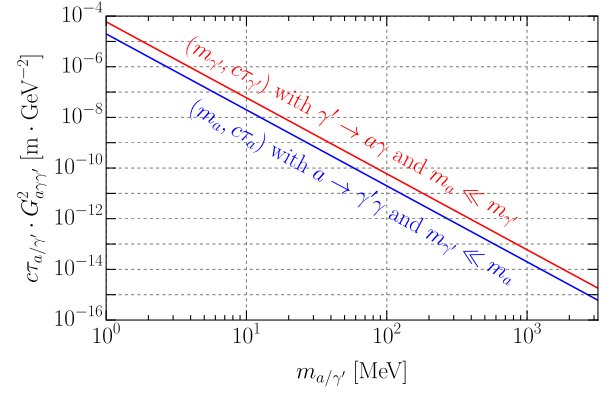


FIG. 3. The proper decay length $c\tau_{a/\gamma'}$ of the heavier new particle (γ' or a) scaled with $G_{a\gamma\gamma'}^2$ as a function of its mass, with the other particle being much lighter or massless.

III. EXPERIMENTS AND SIMULATION

We consider the low-energy e^+e^- collider experiments BESIII and STCF, focusing on the COM energy $\sqrt{s} = 3.097 \text{ GeV}$ at the $J/\psi(1S)$ resonance. The enhancement in the signal-event yields brought by the resonant production of the J/ψ mesons, compared to the nonresonant continuum process, is expected to result in excellent sensitivities to the DAP.

Ignoring potential, minor effects of initial-state radiation of the incoming electron and positron beams, we perform truth-level Monte Carlo (MC) simulations of J/ψ meson decays at rest into an ALP and a dark photon. The J/ψ mesons considered here are produced through $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi$ at BESIII and STCF. In the limit of negligible electron mass, current conservation implies that the longitudinal component of the intermediate virtual photon does not contribute, so that the produced J/ψ mesons are transversely polarized. Averaging over the two transverse helicity states ($\lambda = \pm 1$), the corresponding differential decay rate is

$$\frac{d\Gamma_T(J/\psi \rightarrow a\gamma')}{d\cos\theta_a} = \Gamma(J/\psi \rightarrow a\gamma') \frac{3}{8} (1 + \cos^2\theta_a), \quad (7)$$

where θ_a denotes the polar angle of the ALP (equivalently, for the dark photon, since the two particles are produced back-to-back) with respect to the beam axis, and the subscript T indicates the average over the two transverse polarization states of the J/ψ meson. This angular distribution is taken into account in the simulation. We require the heavier of the ALP and dark photon to decay into an SM photon and the lighter particle. For the ALP and dark-photon decays, we assume isotropic angular distributions, which is exact for the spin-0 ALP but approximate for the spin-1 dark photon.¹ We

¹We have checked numerically that the sensitivity results are negligibly affected by the precise choice of the polar-angle distributions.

provide in Appendix A the analytic differential decay widths of the J/ψ mesons in both signal and irreducible background events.

In the J/ψ rest frame, the photon recoil mass satisfies $m_{\text{rec}}^2 = m_{J/\psi}^2 - 2m_{J/\psi}E_\gamma$ and is therefore in one-to-one correspondence with the photon energy, rather than constituting an independent observable. The recoil system consists of two identical invisible particles, implying $m_{\text{rec}} \geq 2m_L$, where m_L denotes the mass of the lighter of γ' and a . The recoil-mass distribution is continuous, and both its kinematic end points and shape depend on the masses of the two new particles. With only one observed photon, only the total recoil four-momentum can be reconstructed, while the individual four-momenta of the two invisible particles cannot be determined.

We then impose the following kinematic-acceptance requirements for the final-state photons, in reference to the cuts used by the BESIII Collaboration [48]: $|\cos\theta_\gamma| < 0.8$ and $E_\gamma > 25$ MeV, where we consider only the barrel region. For the photons satisfying these conditions, an efficiency of 99% is applied. For the STCF experiment, we take the same event-selection cuts as described here.

For each simulated signal event, we apply the probability that the heavier of a and γ' decays within a three-dimensional lab-frame distance $L_{\text{dec}} < L_{\text{prompt}} \equiv 1$ cm from the IP,

$$\epsilon_{\text{prompt}} = 1 - e^{-L_{\text{prompt}}/(\beta\gamma_L c\tau)}, \quad (8)$$

where $\beta = p/E$ is the speed of the decaying particle with p and E labeling the three-momentum magnitude and energy of the heavier particle, $\gamma_L = E/m$ denotes its Lorentz boost factor with mass m , and $c\tau$ is its proper decay length. For both BESIII and STCF, L_{prompt} is within the beam-pipe vacuum region [42,44]. For a photon reaching the barrel electromagnetic calorimeter (EMC) of inner radius R_{EMC} , the maximum additional nonpointing angle relative to the corresponding IP-pointing direction is bounded by $\delta_{\text{np}}^{\text{max}} \leq \arcsin(L_{\text{prompt}}/R_{\text{EMC}})$. Using the EMC radii of 94 cm for BESIII [42] and 105 cm for STCF [43], this additional angle is smaller than 0.61° and 0.55° , respectively. At the level of our phenomenological treatment, these subdegree deviations are negligible; we therefore apply the same flat 99% efficiency benchmark as for photons originating from the IP. A dedicated analysis of decays beyond L_{prompt} , for which non-pointing-photon information may become relevant, is left for future work.

For the proposed mono-photon search, both reducible and irreducible backgrounds exist. The reducible backgrounds stem from $e^+e^- \rightarrow \gamma + X$, where the photon γ is the only particle detected and X denotes anything undetected by the detector. Such processes include mainly those with X being a pair of charged fermions, a photon pair, or a single photon. For the Born two-photon final state, the recoil photon is back-to-back with the tagged photon and

therefore, lies within the nominal detector acceptance whenever the tagged photon does, in the idealized point-symmetric limit. Its rejection nevertheless relies on successful reconstruction of the recoil photon. Geometric gaps, inactive regions, and reconstruction inefficiencies can therefore produce residual two-photon leakage into the mono-photon sample. The other reducible processes can likewise contribute when the additional final-state particles escape detection.

For reducible configurations in which the undetected particles escape through the nominal forward or backward blind regions, transverse-momentum and energy conservation gives the following upper bound on the energy of the visible photon as a function of its polar angle θ_γ [49,50]:

$$E_b^{\text{max}}(\theta_\gamma) = \frac{\sqrt{s}}{1 + \sin\theta_\gamma/\sin\theta_b}, \quad (9)$$

where, following Ref. [51], we have $\sin\theta_b = \sqrt{1 - \cos^2\theta_b}$ with $|\cos\theta_b| = 0.95$ corresponding to the boundary of all the subdetectors at BESIII [52]. We take the same value of $\cos\theta_b$ for STCF. The requirement $E_\gamma > E_b^{\text{max}}(\theta_\gamma)$ provides a powerful kinematic veto under the idealized assumption of complete detector coverage and perfect veto efficiency down to θ_b . In a realistic detector, however, geometric gaps, inactive regions, and reconstruction inefficiencies can cause a small fraction of multiparticle events to violate this idealized rejection. We therefore do not assume that this requirement completely removes the reducible backgrounds. In the present work, we explicitly estimate only the three-photon contribution and parametrize its migration into the mono-photon signal region by the event-level probability $\epsilon_{3\gamma}^{\text{leak}}$, as detailed in Appendix B. The Born process $e^+e^- \rightarrow 2\gamma$, unresolved $2\gamma(+\gamma)$ configurations, radiative-Bhabha backgrounds, and other detector-dependent reducible sources are not included in this benchmark estimate; their migration into the mono-photon signal region requires separate detector-level estimates. Note that the cut $E_\gamma > E_b^{\text{max}}(\theta_\gamma)$ is always stronger than the $E_\gamma > 25$ MeV one in the present study.

The dominant irreducible background stems from $J/\psi \rightarrow \gamma\nu\bar{\nu}$ decays, with a branching ratio recently estimated with lattice QCD to be $\text{BR}(J/\psi \rightarrow \gamma\nu\bar{\nu}) = 1.04(7)(8) \times 10^{-10}$ in the SM [53]. We use the lattice result to normalize the total decay rate of $J/\psi \rightarrow \gamma\nu\bar{\nu}$, while the kinematic acceptance is evaluated with the leading-order analytic spectrum given below. Uncertainties associated with this spectrum modeling are not included. Instead of performing a detailed simulation, here we provide an analytic estimate of the irreducible-background levels at BESIII and STCF.

For BESIII, the total number of resonant J/ψ events collected is determined to be $N_{J/\psi}^{\text{BESIII}} = (10087 \pm 44) \times 10^6$ [54]. For STCF, we have $N_{J/\psi}^{\text{STCF}} = 3.4 \times 10^{12}$ J/ψ

mesons per year with an integrated luminosity $\mathcal{L}_{\text{STCF}} = 1 \text{ ab}^{-1}$ [43]. Since STCF is planned to operate at $\sqrt{s} = 3.097 \text{ GeV}$ for one year, this would lead to non-negligible irreducible-background levels. We compute analytically the double differential decay rate of $J/\psi \rightarrow \gamma\nu\bar{\nu}$ for the transversely polarized J/ψ mesons, and obtain

$$\frac{d^2\Gamma_T(J/\psi \rightarrow \gamma\nu\bar{\nu})}{dE_\gamma d\cos\theta_\gamma} \propto E_\gamma(3m_{J/\psi} - 4E_\gamma + (4E_\gamma - m_{J/\psi})\cos^2\theta_\gamma), \quad (10)$$

with $\cos\theta_\gamma$ ranging between -1 and 1 and E_γ between 0 and $m_{J/\psi}/2$. The subscript T has the same meaning as that in Eq. (7).

We thus estimate the cut efficiency of $|\cos\theta_\gamma| < 0.8$ and $E_\gamma > E_b^{\text{max}}(\theta_\gamma)$ with the following formula:

$$\epsilon_{\text{irr}} = \frac{\int_{-0.8}^{0.8} d\cos\theta_\gamma \int_{E_b^{\text{max}}(\theta_\gamma)}^{m_{J/\psi}/2} dE_\gamma \frac{d^2\Gamma_T(J/\psi \rightarrow \gamma\nu\bar{\nu})}{dE_\gamma d\cos\theta_\gamma}}{\int_{-1}^1 d\cos\theta_\gamma \int_0^{m_{J/\psi}/2} dE_\gamma \frac{d^2\Gamma_T(J/\psi \rightarrow \gamma\nu\bar{\nu})}{dE_\gamma d\cos\theta_\gamma}} \approx 49.8\%. \quad (11)$$

The quantity $\epsilon_{\text{irr}} = 49.8\%$ in Eq. (11) is the kinematic acceptance. Including the flat single-photon reconstruction efficiency $\epsilon_\gamma \approx 0.99$, the total efficiency is

$$\epsilon_{\text{irr}}^{\text{total}} = \epsilon_{\text{irr}}\epsilon_\gamma \approx 49.3\%. \quad (12)$$

Using the central value of the SM branching ratio, we therefore obtain

$$N_{\text{irr}}^{\text{BESIII}} = 0.518, \quad N_{\text{irr}}^{\text{STCF}} = 174.5. \quad (13)$$

We then derive the signal-event numbers with the following expression:

$$N_S = N_{J/\psi} \cdot \text{BR}(J/\psi \rightarrow a\gamma') \cdot \epsilon, \quad (14)$$

where ϵ is a selection-efficiency factor, defined as

$$\epsilon = \frac{1}{N_{\text{MC}}} \sum_{i=1}^{N_{\text{MC}}} \epsilon_{\text{acc}}^i \cdot \epsilon_{\text{prompt}}^i \cdot \epsilon_{E_\gamma}^i, \quad (15)$$

where N_{MC} is the total number of MC simulation events, ϵ_{acc}^i is the acceptance efficiency (including $|\cos\theta_\gamma| < 0.8$, $E_\gamma > 25 \text{ MeV}$, and the photon reconstruction efficiency of 99%) of the SM photon in the i th simulated signal event, $\epsilon_{\text{prompt}}^i$ is the prompt-decay probability defined in Eq. (8), and $\epsilon_{E_\gamma}^i = 1$ if $E_\gamma^i > E_b^{\text{max}}(\theta_\gamma^i)$ and is zero otherwise.

The binary signal-selection factor $\epsilon_{E_\gamma}^i$ should not be confused with $\epsilon_{3\gamma}^{\text{leak}}$. The former is applied to each simulated signal event and implements the requirement

$E_\gamma > E_b^{\text{max}}(\theta_\gamma)$, whereas the latter is a net event-level migration probability applied only to the reducible three-photon background. Since $\epsilon_{3\gamma}^{\text{leak}}$ includes reconstruction of the surviving tag photon, no additional factor of 0.99 is applied to the three-photon yields.

We have also compared the normalized photon-energy and polar-angle distributions of the signal and the irreducible background after the kinematic selections above are imposed. The distributions substantially overlap over most of the parameter space, with the angular distribution providing little discrimination; some separation in E_γ appears only when the heavier new state approaches the kinematic limit, owing to the mass-dependent lower end point of the signal spectrum. We therefore retain an inclusive event-counting analysis for the sensitivity projections, while noting that a dedicated detector-level binned analysis could exploit this high-energy region and sidebands to reduce the dependence on the absolute background normalization.

In the following section, we present the estimated exclusion reach for BESIII and the projected sensitivity for STCF. For an experiment $X = \text{BESIII, STCF}$, let $N_{3\gamma,X}^{\text{tag}}$ denote the number of truth-level resolved three-photon events for which at least one photon satisfies the full tag requirements given in Appendix B, before detector-induced leakage. We then define the included benchmark background as

$$B_X = N_{\text{irr}}^X + \epsilon_{3\gamma}^{\text{leak}} N_{3\gamma,X}^{\text{tag}}. \quad (16)$$

For each leakage benchmark, we adopt the following approximate phenomenological 95% confidence-level (CL) signal-yield criterion,

$$N_{S,X}^{95} = \max(3, 2\sqrt{B_X}). \quad (17)$$

The $2\sqrt{B_X}$ term accounts only for statistical counting fluctuations in the Gaussian approximation, while the three-event floor is retained in the low-background regime. At each fixed value of $\epsilon_{3\gamma}^{\text{leak}}$, B_X is treated as exactly known. Background-normalization uncertainties associated with $\mathcal{B}(J/\psi \rightarrow \gamma\nu\bar{\nu})$, the three-photon production rates, and the detector-dependent leakage probability are not included or profiled. The resulting curves should therefore be interpreted as optimistic phenomenological benchmarks rather than full detector-level expected limits. The corresponding values are listed in Appendix B.

IV. NUMERICAL RESULTS

In Fig. 4, we present the estimated exclusion reach in the coupling $G_{a\gamma\gamma'}$ as a function of $m_{\gamma'}$ for $m_a \ll m_{\gamma'}$ (the left plot) and of m_a for $m_{\gamma'} \ll m_a$ (the right plot), for the full BESIII dataset already collected (red) and the projected STCF sample (blue). The results of the BESIII experiment

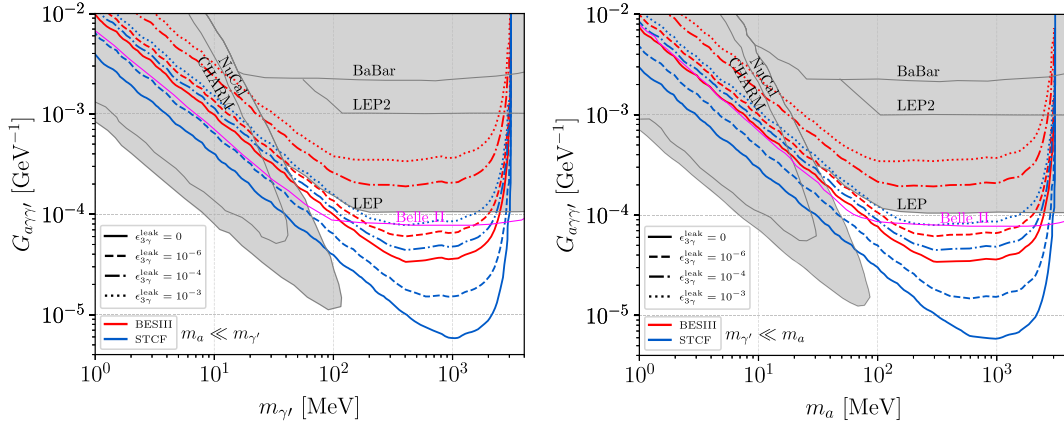


FIG. 4. Exclusion reach from the existing BESIII dataset and projected sensitivity of STCF to $G_{a\gamma\gamma'}$ as a function of $m_{\gamma'}$ for $m_a \ll m_{\gamma'}$ (the left panel) and of m_a for $m_{\gamma'} \ll m_a$ (the right panel). The present upper bounds are displayed in the gray area, obtained from the beam-dump experiments CHARM [32,38,55,56] and NuCal [38,57,58], as well as the e^+e^- colliders BABAR [32,59], LEP [39,60,61], and LEP2 [39,62]. Projected future limits at Belle II with 50 ab^{-1} [32,39] are also shown. All BESIII and STCF curves impose the prompt-decay requirement $L_{\text{dec}} < L_{\text{prompt}} \equiv 1 \text{ cm}$. All the accelerator constraints and projections shown here are based on single-photon signatures. Red and blue curves correspond to BESIII and STCF, respectively. For each experiment, the solid, dashed, dash-dotted, and dotted curves denote $\epsilon_{3\gamma}^{\text{leak}} = 0, 10^{-6}, 10^{-4}, 10^{-3}$, respectively.

correspond to the $\sim 10^{10} J/\psi$ events that have been collected, and those of STCF are for $N_{J/\psi}^{\text{STCF}} = 3.4 \times 10^{12}$ expected for an integrated luminosity of 1 ab^{-1} that can be accumulated with only one year's operation. Future projections at Belle II with an integrated luminosity of 50 ab^{-1} are also shown (the magenta curve) [32,39].

The two mass hierarchies exhibit similar qualitative behavior. With increasing mass, the sensitivities of BESIII and STCF are enhanced initially, and then enter a plateau phase, before deteriorating rapidly near the kinematic end point. At small masses, the heavier particle tends to have a relatively long lifetime, so that its probability of decaying within $L_{\text{prompt}} = 1 \text{ cm}$ of the interaction point is suppressed, weakening the sensitivity. As its mass increases, the lab-frame decay length decreases and the prompt-decay probability approaches unity, producing the plateau behavior shown in the figure. Finally, for even larger masses approaching $m_{J/\psi}$, the phase-space suppression effects begin to dominate, strongly impairing the sensitivities.

In the zero- and small-leakage benchmarks, BESIII and STCF can probe portal couplings beyond the existing bounds in part of the parameter space. The precise improvement over current constraints and over the Belle II projection becomes progressively weaker for larger leakage probabilities and therefore depends on the detector performance. This advantage is primarily driven by the resonant production and the large samples of J/ψ mesons available at BESIII and STCF, whereas the Belle II study relies on the nonresonant continuum process $e^+e^- \rightarrow a\gamma'$ [32].

The zero-leakage curves should be interpreted as idealized best-case sensitivities. As $\epsilon_{3\gamma}^{\text{leak}}$ increases, the large underlying three-photon rate raises the required signal yield and progressively weakens the coupling reach.

In Fig. 4, we have assumed either $m_a \ll m_{\gamma'}$ or $m_{\gamma'} \ll m_a$. If the mass of the lighter particle approaches that of the heavier one, the sensitivities weaken as a result of the reduced phase space in the decay of the heavier particle.

V. CONCLUSIONS

In this work, we have investigated the sensitivity of the existing BESIII dataset and the future STCF collider to the dark axion portal, via J/ψ decays into an axionlike particle and a dark photon. The resonant production of the J/ψ mesons at the COM energy of $\sqrt{s} = 3.097 \text{ GeV}$ leads to enhanced production yields of the signal events and hence to strong sensitivity reach. Concretely, for BESIII, we consider $N_{J/\psi}^{\text{BESIII}} = 10087 \times 10^6$ corresponding to the J/ψ meson events that have been collected, while for STCF, we take $N_{J/\psi}^{\text{STCF}} = 3.4 \times 10^{12}$ corresponding to an integrated luminosity of 1 ab^{-1} .

We have performed Monte Carlo simulations to estimate the signal acceptance and have included both the irreducible $J/\psi \rightarrow \gamma\nu\bar{\nu}$ background and a benchmark estimate of the three-photon reducible backgrounds.

The kinematic requirement based on Eq. (9) provides an idealized veto but does not guarantee complete rejection in a realistic detector. We therefore parametrize migration of taggable three-photon events into the mono-photon signal region by the net event-level probability $\epsilon_{3\gamma}^{\text{leak}}$. For $0 \leq \epsilon_{3\gamma}^{\text{leak}} \leq 10^{-3}$, the included irreducible-plus-three-photon background ranges from approximately 0.52 to 2.48×10^4 events at BESIII and from 174.5 to 8.11×10^6 events at STCF. The zero-leakage curves represent the

idealized best-case benchmarks, while the finite-leakage curves quantify their dependence on the detector performance.

A reliable determination of the event-level three-photon leakage probability $\epsilon_{3\gamma}^{\text{leak}}$, together with separate estimates of the Born-level two-photon, unresolved $2\gamma(+\gamma)$, radiative-Bhabha, and other detector-dependent backgrounds, will require collaboration-level full simulations and data-driven control samples.

We have presented the exclusion reach associated with the existing BESIII dataset, together with the projected sensitivity of STCF, at 95% CL, considering either $m_a \ll m_{\gamma'}$ or $m_{\gamma'} \ll m_a$. Our numerical results indicate that within the zero- and sufficiently small three-photon-leakage benchmarks, the existing BESIII dataset could probe parameter space beyond current bounds, while STCF could substantially extend this reach. The comparison with Belle II becomes progressively less favorable as the leakage probability increases. The favorable reach in the small-leakage regime is primarily driven by resonant production and the large samples of J/ψ mesons available at BESIII and STCF. Thus, this work illustrates the potential constraining power of the tau-charm factories on new-physics scenarios predicting new particles arising from rare decays of the J/ψ meson.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: ANALYTIC EXPRESSIONS OF DIFFERENTIAL DECAY RATES OF J/ψ IN BOTH SIGNAL AND BACKGROUND PROCESSES

We consider both unpolarized and transversely polarized J/ψ mesons.

For the signal process $J/\psi \rightarrow a\gamma'$, the differential decay rate of a transversely polarized J/ψ meson, $\frac{d\Gamma_T(J/\psi \rightarrow a\gamma')}{d\cos\theta_a}$, is given by²

$$\begin{aligned} \frac{d\Gamma_T(J/\psi \rightarrow a\gamma')}{d\cos\theta_a} &= \frac{\alpha f_{J/\psi}^2 G_{a\gamma'}^2}{144 m_{J/\psi}^5} (1 + \cos^2\theta_a) \\ &\times \lambda^{3/2}(m_{J/\psi}^2, m_a^2, m_{\gamma'}^2) \\ &= \Gamma(J/\psi \rightarrow a\gamma') \frac{3}{8} (1 + \cos^2\theta_a), \end{aligned} \quad (\text{A1})$$

where we have averaged over the two transverse helicity states by taking a factor 1/2 and used Eq. (2).

For an unpolarized J/ψ meson, the differential decay rate $\frac{d\Gamma(J/\psi \rightarrow a\gamma')}{d\cos\theta_a}$ is independent of $\cos\theta_a$ and is simply,

$$\frac{d\Gamma(J/\psi \rightarrow a\gamma')}{d\cos\theta_a} = \frac{\Gamma(J/\psi \rightarrow a\gamma')}{2}. \quad (\text{A2})$$

For the background process $J/\psi \rightarrow \gamma\nu\bar{\nu}$, the double differential decay rate $\frac{d^2\Gamma_T(J/\psi \rightarrow \gamma\nu\bar{\nu})}{dE_\gamma d\cos\theta_\gamma}$ for one generation of the neutrinos reads

$$\begin{aligned} \frac{d^2\Gamma_T(J/\psi \rightarrow \gamma\nu\bar{\nu})}{dE_\gamma d\cos\theta_\gamma} &= \frac{\alpha G_F^2 |\psi_{J/\psi}(0)|^2 Q_c^2}{4\pi^2 m_{J/\psi}} E_\gamma (3m_{J/\psi} - 4E_\gamma \\ &+ (4E_\gamma - m_{J/\psi}) \cos^2\theta_\gamma), \end{aligned} \quad (\text{A3})$$

where G_F is the Fermi constant and the nonperturbative parameter $\psi_{J/\psi}(0)$ is related to the $f_{J/\psi}$ by the following relation, which we have derived from Eq. (13) of Ref. [63]:

$$f_{J/\psi} = -2\sqrt{\frac{N_c}{m_{J/\psi}}} \psi_{J/\psi}(0), \quad (\text{A4})$$

where $N_c = 3$ is the color factor of quarks. Similar to Eq. (A1), we have averaged over the two transverse helicity states in Eq. (A3).

For an unpolarized J/ψ meson, we find

$$\frac{d\Gamma(J/\psi \rightarrow \gamma\nu\bar{\nu})}{dE_\gamma} = \frac{4\alpha G_F^2 |\psi_{J/\psi}(0)|^2 Q_c^2}{3\pi^2 m_{J/\psi}} E_\gamma (m_{J/\psi} - E_\gamma), \quad (\text{A5})$$

for one generation of the neutrinos. Equation (A5) is in agreement with Eq. (10) of Ref. [64].³

APPENDIX B: BENCHMARK ESTIMATE OF REDUCIBLE THREE-PHOTON BACKGROUNDS

We consider three sources of reducible three-photon background to the mono-photon search: (i) the continuum QED process $e^+e^- \rightarrow 3\gamma$; (ii) the resonant cascade processes $e^+e^- \rightarrow J/\psi \rightarrow \gamma P$, followed by $P \rightarrow \gamma\gamma$, with

²As mentioned in Sec. I, we ignore the Z-boson contributions to the signal process in this work since we study low-energy e^+e^- collider experiments BESIII and STCF.

³We note, however, in Ref. [64], a factor of 4 is missing in its Eq. (7) for the expression of the matrix element of $J/\psi \rightarrow \gamma\nu\bar{\nu}$.

$P = \pi^0, \eta, \eta', \eta_c$; and (iii) the direct resonant process $e^+e^- \rightarrow J/\psi \rightarrow 3\gamma$. A truth-level three-photon event from any of these sources is classified as taggable if at least one of its photons satisfies

$$|\cos\theta_\gamma| < 0.8, \quad E_\gamma > 25 \text{ MeV}, \quad E_\gamma > E_b^{\max}(\theta_\gamma). \quad (\text{B1})$$

This is an event-level selection: each event is counted at most once.

For the continuum contribution, we generate $e^+e^- \rightarrow 3\gamma$ at leading order at $\sqrt{s} = 3.097 \text{ GeV}$ with MadGraph5_aMC@NLO [65,66], using the full SM implementation and the physical electron mass $m_e = 0.511 \text{ MeV}$. Each of the three generated photon is required at generation level to satisfy $E_\gamma > 25 \text{ MeV}$. No generator-level angular cut is imposed; in particular, photons with $|\cos\theta_\gamma| > 0.95$ are thus retained. The physical electron mass regulates the initial-state collinear enhancement. We obtain

$$\sigma_{\text{cont}}^{\text{gen}} = 63.6 \text{ nb}, \quad A_{\text{cont}} = 0.1028, \quad (\text{B2})$$

and therefore,

$$\sigma_{\text{cont}}^{\text{tag}} = \sigma_{\text{cont}}^{\text{gen}} A_{\text{cont}} = 6.54 \text{ nb}. \quad (\text{B3})$$

Here, A_{cont} denotes the event-level taggable acceptance under the truth-level selections defined above.

For the continuum contribution only, we further classify the selected events according to N_{fwd} , defined as the number of the two non-tag photons satisfying $|\cos\theta_\gamma| \geq 0.95$. If several photons satisfy the full tag selection, the highest-energy one is chosen as the tag. We obtain

N_{fwd}	0	1	2
fraction of tagged events	23.38%	76.62%	0
$\sigma_{\text{cont}}^{\text{tag}}(N_{\text{fwd}})[\text{nb}]$	1.53	5.01	0

The $N_{\text{fwd}} = 1$ category matches the topology with one additional photon lying outside the detector boundary and the other evading the detector veto. For $N_{\text{fwd}} = 0$, both additional photons must evade the veto. No $N_{\text{fwd}} = 2$ event survives the ideal kinematic requirement.

For the resonant cascade modes $J/\psi \rightarrow \gamma P$, $P \rightarrow \gamma\gamma$, with $P = \pi^0, \eta, \eta', \eta_c$, we use the transverse J/ψ angular distribution proportional to $1 + \cos^2\theta$ and generate the pseudoscalar decay isotropically in the P rest frame. The direct $J/\psi \rightarrow 3\gamma$ contribution is modeled with a transversely polarized version of the lowest-order Ore-Powell matrix element [67,68], analogous to the ortho-positronium-based signal model used in the BESIII analysis [69]. The resulting acceptances and effective branching fractions are summarized in Table I.

TABLE I. Kinematic acceptances and branching-ratio-weighted acceptances for the resonant three-photon backgrounds. The branching ratios are extracted from Ref. [47].

Channel	A	$\text{BR}_{\text{chain}} A$
$\gamma\pi^0$ with $\pi^0 \rightarrow \gamma\gamma$	0.73	2.46×10^{-5}
$\gamma\eta$ with $\eta \rightarrow \gamma\gamma$	0.76	3.26×10^{-4}
$\gamma\eta'$ with $\eta' \rightarrow \gamma\gamma$	0.80	9.80×10^{-5}
$\gamma\eta_c$ with $\eta_c \rightarrow \gamma\gamma$	0.81	3.14×10^{-6}
direct 3γ	0.86	9.97×10^{-6}

The cascade sum is

$$R_{\gamma P} = \sum_P \text{BR}(J/\psi \rightarrow \gamma P) \text{BR}(P \rightarrow \gamma\gamma) A_P = 4.52 \times 10^{-4}, \quad (\text{B4})$$

while the direct contribution is

$$R_{\text{direct}} = \text{BR}(J/\psi \rightarrow 3\gamma) A_{3\gamma} = 9.97 \times 10^{-6}. \quad (\text{B5})$$

Here, A_P and $A_{3\gamma}$ denote the corresponding event-level taggable acceptances for the cascade and direct three-photon samples, respectively.

For experiment X (BESIII/STCF), the number of taggable three-photon events before detector-induced leakage is

$$N_{3\gamma, X}^{\text{tag}} = \mathcal{L}_X \sigma_{\text{cont}}^{\text{tag}} + N_{J/\psi, X} (R_{\gamma P} + R_{\text{direct}}). \quad (\text{B6})$$

Using $\mathcal{L}_{\text{BESIII}} = 3.083 \text{ fb}^{-1}$ [54], $\mathcal{L}_{\text{STCF}} = 1 \text{ ab}^{-1}$ [43], $N_{J/\psi}^{\text{BESIII}} = 1.0087 \times 10^{10}$, and $N_{J/\psi}^{\text{STCF}} = 3.4 \times 10^{12}$, we obtain

$$N_{3\gamma, \text{BESIII}}^{\text{tag}} = 2.48 \times 10^7, \quad N_{3\gamma, \text{STCF}}^{\text{tag}} = 8.11 \times 10^9. \quad (\text{B7})$$

We define $\epsilon_{3\gamma}^{\text{leak}}$ as the net event-level probability for a taggable three-photon event to be reconstructed in the exactly-one-photon signal region. It is applied once to each event,

$$N_{3\gamma, X}^{\text{red}} = \epsilon_{3\gamma}^{\text{leak}} \cdot N_{3\gamma, X}^{\text{tag}}. \quad (\text{B8})$$

Because this probability includes reconstruction of the surviving tag photon, no additional factor of $\epsilon_\gamma = 0.99$ is applied to $N_{3\gamma, X}^{\text{red}}$.

The signal-event thresholds in Table II are evaluated using Eq. (17) and account only for statistical counting fluctuations. Background-normalization uncertainties are not included.

The continuum and resonant contributions are added incoherently, and interference effects are neglected. The continuum result corresponds to a generator-level

TABLE II. Included benchmark background yields and signal-event thresholds for the four event-level three-photon leakage benchmarks.

Experiment	$\epsilon_{3\gamma}^{\text{leak}}$	B	N_S^{95}
BESIII	0	0.518	3
	10^{-6}	25.3	10.1
	10^{-4}	2.48×10^3	99.6
	10^{-3}	2.48×10^4	315.0
STCF	0	174.5	26.4
	10^{-6}	8.28×10^3	182.0
	10^{-4}	8.11×10^5	1800.8
	10^{-3}	8.11×10^6	5694.0

three-photon sample defined by the requirement $E_\gamma > 25$ MeV for each photon. The Born process $e^+e^- \rightarrow 2\gamma$ is not included. Configurations containing an additional photon below this generation-level energy cutoff, referred to here as unresolved $2\gamma(+\gamma)$ backgrounds, are likewise not included. Their detector-induced migration is not described by $\epsilon_{3\gamma}^{\text{leak}}$ and would require a separate detector-level leakage estimate. We also do not include radiative-Bhabha backgrounds, higher-order QED corrections, detector-response or background-normalization systematic uncertainties, or correlations among inefficiencies. Accordingly, the values of $\epsilon_{3\gamma}^{\text{leak}}$ should be regarded as detector-performance benchmarks rather than predictions. Their determination requires full detector simulations and data-driven control samples.

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- [1] R. E. Shrock, *Phys. Lett.* **96B**, 159 (1980).
[2] R. E. Shrock, *Phys. Rev. D* **24**, 1232 (1981).
[3] R. E. Shrock, *Phys. Rev. D* **24**, 1275 (1981).
[4] D. O’Connell, M. J. Ramsey-Musolf, and M. B. Wise, *Phys. Rev. D* **75**, 037701 (2007).
[5] J. D. Wells, in *Perspectives on LHC Physics*, edited by G. Kane and A. Pierce (World Scientific, Singapore, 2008), pp. 283–298.
[6] C. Bird, P. Jackson, R. V. Kowalewski, and M. Pospelov, *Phys. Rev. Lett.* **93**, 201803 (2004).
[7] M. Pospelov, A. Ritz, and M. B. Voloshin, *Phys. Lett. B* **662**, 53 (2008).
[8] G. Krnjaic, *Phys. Rev. D* **94**, 073009 (2016).
[9] I. Boiarska, K. Bondarenko, A. Boyarsky, V. Gorkavenko, M. Ovchinnikov, and A. Sokolenko, *J. High Energy Phys.* **11** (2019) 162.
[10] L. B. Okun, *Sov. Phys. JETP* **56**, 502 (1982), http://jetp.ras.ru/cgi-bin/dn/e_056_03_0502.pdf.
[11] P. Galison and A. Manohar, *Phys. Lett.* **136B**, 279 (1984).
[12] B. Holdom, *Phys. Lett.* **166B**, 196 (1986).
[13] C. Boehm and P. Fayet, *Nucl. Phys.* **B683**, 219 (2004).
[14] M. Pospelov, *Phys. Rev. D* **80**, 095002 (2009).
[15] D. Curtin, R. Essig, S. Gori, and J. Shelton, *J. High Energy Phys.* **02** (2015) 157.
[16] R. D. Peccei and H. R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977).
[17] R. D. Peccei and H. R. Quinn, *Phys. Rev. D* **16**, 1791 (1977).
[18] E. Witten, *Phys. Lett.* **149B**, 351 (1984).
[19] J. P. Conlon, *J. High Energy Phys.* **05** (2006) 078.
[20] N. Arkani-Hamed, L. Motl, A. Nicolis, and C. Vafa, *J. High Energy Phys.* **06** (2007) 060.
[21] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, and J. March-Russell, *Phys. Rev. D* **81**, 123530 (2010).
[22] M. Cicoli, M. Goodsell, and A. Ringwald, *J. High Energy Phys.* **10** (2012) 146.
[23] K. Kaneta, H.-S. Lee, and S. Yun, *Phys. Rev. Lett.* **118**, 101802 (2017).
[24] K. Kaneta, H.-S. Lee, and S. Yun, *Phys. Rev. D* **95**, 115032 (2017).
[25] E. Broadberry, S. Das, A. Hook, and G. Marques Tavares, *J. High Energy Phys.* **03** (2025) 215.
[26] P. W. Graham, D. E. Kaplan, and S. Rajendran, *Phys. Rev. Lett.* **115**, 221801 (2015).
[27] K. Choi, H. Kim, and T. Sekiguchi, *Phys. Rev. D* **95**, 075008 (2017).
[28] V. Domcke, K. Schmitz, and T. You, *J. High Energy Phys.* **07** (2022) 126.
[29] P. deNiverville and H.-S. Lee, *Phys. Rev. D* **100**, 055017 (2019).
[30] A. S. Zhevlakov, D. V. Kirpichnikov, and V. E. Lyubovitskij, *Phys. Rev. D* **106**, 035018 (2022).
[31] S. N. Gninenko, N. V. Krasnikov, V. E. Lyubovitskij, S. Kuleshov, A. S. Zhevlakov, I. V. Voronchikhin, and D. V. Kirpichnikov, *Phys. Rev. D* **113**, 095028 (2026).
[32] P. deNiverville, H.-S. Lee, and M.-S. Seo, *Phys. Rev. D* **98**, 115011 (2018).
[33] O. E. Kalashev, A. Kusenko, and E. Vitagliano, *Phys. Rev. D* **99**, 023002 (2019).
[34] P. Deniverville, H.-S. Lee, and Y.-M. Lee, *Phys. Rev. D* **103**, 075006 (2021).
[35] A. Hook, G. Marques-Tavares, and C. Ristow, *J. High Energy Phys.* **06** (2021) 167.
[36] A. Hook, G. Marques-Tavares, and C. Ristow, *J. High Energy Phys.* **05** (2024) 086.
[37] H. Hong, U. Min, M. Son, and T. You, *J. High Energy Phys.* **03** (2024) 155.
[38] K. Jodłowski, *Phys. Rev. D* **108**, 115017 (2023).
[39] K. Jodłowski, *J. High Energy Phys.* **08** (2025) 022.
[40] N. Ness and B. Cimring, *Phys. Rev. D* **114**, 035021 (2026).
[41] Y. Shen, J. Tang, L. Wang, Y. Wu, and L. Yang, *arXiv:* 2603.24050.

- [42] M. Ablikim *et al.* (BESIII Collaboration), *Nucl. Instrum. Methods Phys. Res., Sect. A* **614**, 345 (2010).
- [43] M. Achasov *et al.*, *Front. Phys. (Beijing)* **19**, 14701 (2024).
- [44] X.-C. Ai *et al.*, *Nucl. Sci. Tech.* **36**, 242 (2025).
- [45] D. Ejlli, *Eur. Phys. J. C* **78**, 63 (2018).
- [46] N. Brambilla *et al.* (Quarkonium Working Group Collaboration) (2004), [10.5170/CERN-2005-005](https://arxiv.org/abs/10.5170/CERN-2005-005).
- [47] S. Navas *et al.* (Particle Data Group Collaboration), *Phys. Rev. D* **110**, 030001 (2024).
- [48] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **96**, 112008 (2017).
- [49] Z. Liu, Y.-H. Xu, and Y. Zhang, *J. High Energy Phys.* **06** (2019) 009.
- [50] Y. Zhang, W.-T. Zhang, M. Song, X.-A. Pan, Z.-M. Niu, and G. Li, *Phys. Rev. D* **100**, 115016 (2019).
- [51] Z. Liu and Y. Zhang, *Phys. Rev. D* **99**, 015004 (2019).
- [52] D. M. Asner *et al.*, *Int. J. Mod. Phys. A* **24**, S1 (2009), <https://www.worldscientific.com/toc/ijmpa/24/supp01?srsId=AfmBOopJIT28upvvbMGo2nrVyPmNR1SYyoX1aMDiDiNChL7hUIz42LQV>.
- [53] Y. Meng, N. Li, C. Liu, H. Yan, K.-L. Zhang, and X.-Z. Zhang, *Phys. Rev. D* **114**, 014524 (2026).
- [54] M. Ablikim *et al.* (BESIII Collaboration), *Chin. Phys. C* **46**, 074001 (2022).
- [55] S. N. Gninenko, *Phys. Rev. D* **85**, 055027 (2012).
- [56] S. N. Gninenko, *Phys. Lett. B* **713**, 244 (2012).
- [57] J. Blumlein *et al.*, *Z. Phys. C* **51**, 341 (1991).
- [58] J. Blumlein and J. Brunner, *Phys. Lett. B* **701**, 155 (2011).
- [59] B. Aubert *et al.* (BABAR Collaboration), in 34th International Conference on High Energy Physics (2008), [arXiv:0808.0017](https://arxiv.org/abs/0808.0017).
- [60] R. Akers *et al.* (OPAL Collaboration), *Z. Phys. C* **65**, 47 (1995).
- [61] P. Abreu *et al.* (DELPHI Collaboration), *Z. Phys. C* **74**, 577 (1997).
- [62] J. Abdallah *et al.* (DELPHI Collaboration), *Eur. Phys. J. C* **38**, 395 (2005).
- [63] G. Hao, Y. Jia, C.-F. Qiao, and P. Sun, *J. High Energy Phys.* **02** (2007) 057.
- [64] D.-N. Gao, *Phys. Rev. D* **90**, 077501 (2014).
- [65] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, *J. High Energy Phys.* **07** (2014) 079.
- [66] R. Frederix, S. Frixione, V. Hirschi, D. Pagani, H. S. Shao, and M. Zaro, *J. High Energy Phys.* **07** (2018) 185; **11** (2021) 85.
- [67] A. Ore and J. L. Powell, *Phys. Rev.* **75**, 1696 (1949).
- [68] G. S. Adkins, *Phys. Rev. Lett.* **76**, 4903 (1996).
- [69] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **87**, 032003 (2013).