

Optimal estimation of Higgs-gauge boson couplings at the future e^+e^- colliders

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The proposed e^+e^- collider offers an ideal environment for precise estimation of Higgs boson properties which are of utmost importance to validate the Standard Model of particle physics. We investigate hVV couplings, where $V \in \{Z, \gamma\}$ with single Higgs production associated with Z boson at the proposed e^+e^- machine with $\sqrt{s} = 250$ GeV, within the Standard Model effective field theory framework. We employ the recoil mass of the dilepton system, to select the signal phase space, i.e., $Zh \rightarrow l^+l^-b\bar{b}$ events. The constraints on the Wilson coefficients (WCs) are obtained using the optimal observable technique. On comparison with the current experimental limits at 68% CL with 138 fb^{-1} luminosity, our limits are tighter by a factor ranging from 1.5–10 for CP -even operators, while CP -odd WCs shows comparable limits.

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I. INTRODUCTION

The discovery of the Higgs boson at the Large Hadron Collider (LHC) by the ATLAS [1] and CMS [2,3] Collaborations unleashes a cornerstone achievement in particle physics, completing the particle spectrum of the Standard Model (SM) [4]. The Higgs field endows the $W^\pm$ and Z bosons, as well as the SM fermions with mass through electroweak symmetry breaking (EWSB) [5–7], while preserving the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance and renormalizability. Although the SM provides a remarkably successful theory up to the electroweak scale and has passed through numerous precision tests, it leaves several fundamental questions unresolved. Among these are the naturalness of the electroweak scale, the origin and structure of the Yukawa couplings responsible for fermion mass hierarchies, the mechanism of underlying CP violation, the origin of neutrino masses and mixing etc. In the SM framework, such features are often accommodated through *ad hoc* parameters without offering a dynamical explanations. Furthermore, the SM does not contain a viable dark matter candidate and fails to account for the observed baryon asymmetry of the Universe. Given that the Higgs boson is intimately connected to the mechanism of mass generation and the vacuum structure, it is widely

anticipated that it may serve as a portal to beyond-the-SM (BSM) physics.

A multitude of experimental investigations at LHC [8–22] have imposed stringent constraints on the spin-parity quantum numbers of the Higgs boson, as well as its interactions with gluons and electroweak (EW) gauge bosons. Current measurements are in agreement with the SM prediction of a scalar boson with quantum numbers $J^{PC} = 0^{++}$. Nonetheless, the presence of small deviations from the SM in the form of anomalous couplings remains experimentally permissible. In BSM theories, interactions with the Higgs boson may occur through several anomalous couplings, which lead to new tensor structures in the interaction terms that can be both CP -even or CP -odd. The hadronic environment of the LHC imposes limitations on measurement precision due to QCD backgrounds, pile-up, and uncertainties associated with parton distribution functions (PDFs). This has motivated the proposal of high-luminosity and high-precision lepton colliders, such as the ILC [23–27], FCC-ee [28], CEPC [29–31] and CLIC [32–35], which offer a complementary environment for Higgs boson studies. Projected experimental uncertainties on Higgs couplings at future e^+e^- colliders are expected to reach the per-mille level [26,36–39], significantly surpassing the capabilities of the LHC.

A particularly clean setting for precision Higgs physics is provided by an e^+e^- collider operating at a center-of-mass energy of $\sqrt{s} = 250$ GeV, which is the focus of current work. In this energy regime, the dominant Higgs production channel is the Higgs-strahlung process, $e^+e^- \rightarrow Zh$, which proceeds via s -channel exchange of a Z boson. The Higgs-strahlung production mechanism allows for model-independent measurements of Higgs boson properties by

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reconstructing the Higgs mass from the recoil mass spectrum of the Z boson, independent of the Higgs decay mode. Moreover, the ability to polarize the initial e^- and e^+ beams in the linear collider provides an additional handle to disentangle helicity structures of electroweak interactions and enhance sensitivity to specific interaction channels.

In the current work, we study the potential of ILC to probe the anomalous Higgs-gauge couplings in Zh production process within the SM effective field theory (SMEFT) framework. The absence of direct evidence for new states at the LHC suggests that new degrees of freedom may reside at energy scales higher than currently accessible, with their low-energy imprints manifesting as subtle modifications to SM couplings. In such scenario, SMEFT provides a powerful tool to parametrize deviations from the SM predictions in a systematic and gauge-invariant manner, wherein higher-dimensional operators are added to the SM Lagrangian. Our aim is to obtain the optimal limit on the operator coefficients contributing to hZZ vertex and Zh production at ILC.

The current work is organized as follows. In Sec. II, we discuss the SMEFT framework and the relevant operators. In Sec. III, we discuss the event selection using the dilepton recoil mass observable. We focus on $l^+l^-b\bar{b}$ final state owing to large Higgs branching fraction to $b\bar{b}$ channel. In Sec. IV, we discuss in brief the optimal observable technique. In Sec. V, we discuss the one parameter limits on Wilson coefficients obtained for unpolarized and polarized beams scenario. We conclude in Sec. VI.

II. SMEFT FRAMEWORK AND RELEVANT OPERATORS

The SMEFT Lagrangian is written as a perturbative expansion in terms of the inverse power of new physics (NP) scale Λ where decoupling occurs, accompanied with $\mathcal{O}_i$ operators of dimension- d constructed of the SM fields obeying SM gauge invariance [40],

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right), \quad (1)$$

where C_i are the Wilson coefficients. Since at each order of d , the amplitude becomes suppressed as $(E/\Lambda)^{(d-4)}$, the dominant SMEFT contribution comes from lowest dimension operator that contributes to the process. Also the SMEFT construction remains valid only when the scale of the reaction remains below the cut off scale Λ , requiring us to adhere to CM energy of the collider $\sqrt{s} < \Lambda$. Throughout the analysis, we set $\Lambda = 1$ TeV. Assuming the lepton and baryon number conservation, the lowest order of SMEFT begins at dimension-6. We consider three

CP -even and their dual CP -odd operators inducing anomalous contribution to Zh process. The relevant operators in Warsaw basis are [41]

$$\begin{aligned} \mathcal{O}_{HW} &= (H^\dagger H) W_{\mu\nu}^i W^{i\mu\nu}, & \mathcal{O}_{H\tilde{W}} &= (H^\dagger H) \tilde{W}_{\mu\nu}^i W^{i\mu\nu}, \\ \mathcal{O}_{HWB} &= (H^\dagger \tau^i H) W_{\mu\nu}^i B^{\mu\nu}, & \mathcal{O}_{H\tilde{W}B} &= (H^\dagger \tau^i H) \tilde{W}_{\mu\nu}^i B^{\mu\nu}, \\ \mathcal{O}_{HB} &= (H^\dagger H) B_{\mu\nu} B^{\mu\nu}, & \mathcal{O}_{H\tilde{B}} &= (H^\dagger H) \tilde{B}_{\mu\nu} B^{\mu\nu}. \end{aligned} \quad (2)$$

Here, the operators $\mathcal{O} \in \{\mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{HWB}\}$ are CP -even while the remaining subset of operators are CP -odd. The field tensor are defined as $W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon^{ijk}W_\mu^j W_\nu^k$, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and the dual field tensor is $\tilde{V}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} V^{\rho\sigma}$ ($V = W^i, B$), with the Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$; following standard notation $\epsilon_{0123} = 1$. H denotes the SM Higgs isodoublet. Upon spontaneous symmetry breaking it acquires nonzero vacuum expectation value v , which takes the form $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$ in unitary gauge, where h denotes SM Higgs field. These operators thereafter induces anomalous contribution to hZZ coupling of the form

$$\delta\Gamma_{hZZ} = \kappa_{hZZ} \left(\frac{h}{v} Z_{\mu\nu} Z^{\mu\nu} \right) + \kappa_{h\tilde{Z}Z} \left(\frac{h}{v} \tilde{Z}_{\mu\nu} Z^{\mu\nu} \right), \quad (3)$$

where

$$\begin{aligned} \kappa_{hZZ} &= \frac{2v^2}{\Lambda^2} [\cos^2 \theta_W C_{HW} + \cos \theta_W \sin \theta_W C_{HWB} \\ &\quad + \sin^2 \theta_W C_{HB}], \\ \kappa_{h\tilde{Z}Z} &= \frac{2v^2}{\Lambda^2} [\cos^2 \theta_W C_{H\tilde{W}} + \cos \theta_W \sin \theta_W C_{H\tilde{W}B} \\ &\quad + \sin^2 \theta_W C_{H\tilde{B}}]. \end{aligned} \quad (4)$$

Apart from anomalous hZZ coupling, the operators allows for tree-level s -channel Zh production mediated by massless photon, which otherwise is a loop process in the SM. The anomalous $hZ\gamma$ coupling is obtained as

$$\delta\Gamma_{hZ\gamma} = \kappa_{hZ\gamma} \left(\frac{h}{v} Z_{\mu\nu} A^{\mu\nu} \right) + \kappa_{h\tilde{Z}\gamma} \left(\frac{h}{v} \tilde{Z}_{\mu\nu} A^{\mu\nu} \right), \quad (5)$$

where

$$\begin{aligned} \kappa_{hZ\gamma} &= \frac{v^2}{\Lambda^2} [2 \cos \theta_W \sin \theta_W (C_{HW} - C_{HB}) \\ &\quad + (\sin^2 \theta_W - \cos^2 \theta_W) C_{HWB}], \\ \kappa_{h\tilde{Z}\gamma} &= \frac{v^2}{\Lambda^2} [2 \cos \theta_W \sin \theta_W (C_{H\tilde{W}} - C_{H\tilde{B}}) \\ &\quad + (\sin^2 \theta_W - \cos^2 \theta_W) C_{H\tilde{W}B}]. \end{aligned} \quad (6)$$

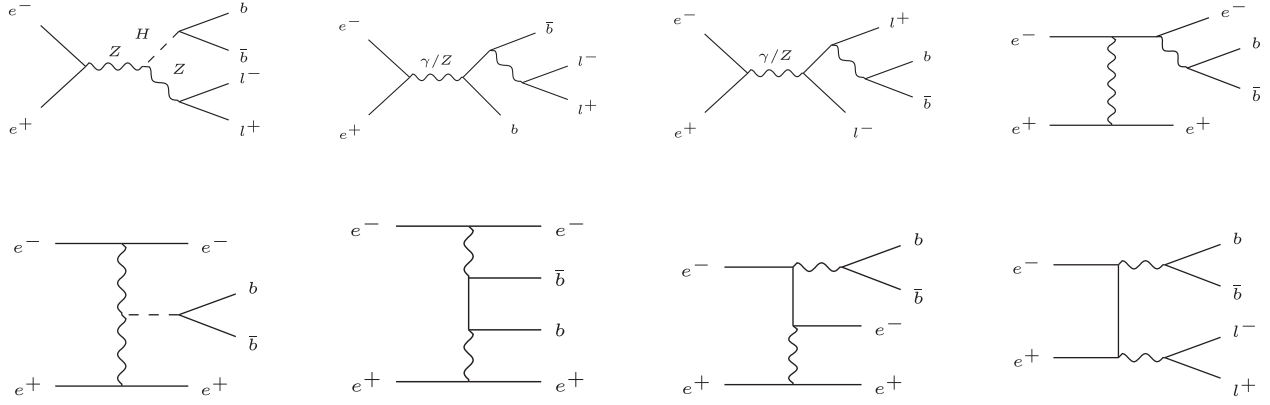


FIG. 1. Representative Feynman diagrams denoting the production of two leptons and two b quarks at the leading order in the SM.

We do not consider operators affecting $f\bar{f}V$ or $f\bar{f}VH$ couplings as they are tightly constrained from large electron-positron collider data. Various studies [42–74] has been done to probe the structure of anomalous Higgs-gauge couplings using the kinematic and angular distribution at LHC at e^+e^- colliders. Quantum tomography [75–79] has also been used to constrain anomalous hVV , $V \in \{Z, W\}$ coupling. On the experimental side, the current limits at 68% confidence level (CL) on the Wilson coefficients (WCs) of the above operators are provided by CMS [80] (assuming $\Lambda = 1$ TeV),

$$\begin{aligned}
 C_{HW} &= [-0.79, +0.51], & C_{H\tilde{W}} &= [-0.76, +0.41], \\
 C_{HWB} &= [-1.62, +1.50], & C_{H\tilde{W}B} &= [-1.57, +0.83], \\
 C_{HB} &= [-0.23, +0.16], & C_{H\tilde{B}} &= [-0.23, +0.12].
 \end{aligned} \tag{7}$$

As stated before, in this work we explore the extent to which optimal observable analysis of production cross section can constrain modifications to the Higgs-gauge couplings, and highlight the complementarity of this program with current LHC studies. Our analysis underscores the central role that an e^+e^- Higgs factory can play in elucidating the nature of electroweak symmetry breaking and probing the structure of physics beyond the SM.

III. COLLIDER ANALYSIS: EVENT SELECTION

The signal process considered here is the Higgs-strahlung process followed by leptonic decay of Z boson and Higgs boson decay to pair of b -quark at ILC, i.e., $e^+e^- \rightarrow Z(l^+l^-)h(b\bar{b})$ for the SM as well as dimension-6 effective operators listed in Eq. (2) that contribute to it. The dominant background to the $l^+l^-b\bar{b}$ final state arises from $e^+e^- \rightarrow Z(l^+l^-)Z(b\bar{b})$. A subdominant background from Z -boson fusion Higgs production, $e^+e^- \rightarrow e^+e^-h(b\bar{b})$, contributes negligibly at $\sqrt{s} = 250$ GeV and is therefore neglected. The $ZZ\gamma$ process could also dilute the analysis,

however the p_T cut on final photons reduces such contribution significantly. We also consider the effect of initial state radiation (ISR) on the overall selection kinematics of the processes. The schematic Feynman diagrams representing signal and some of the background processes are illustrated in Fig. 1.

Operationally, the dim-6 operators listed in Eq. (2) are implemented in FeynRules [81] to obtain an Universal FeynRules Output (UFO) model files. The corresponding UFO model [82] is exported to MG5_aMC [83] for Monte Carlo event generation. The matrix level events are then passed through Pythia8 [84] for showering of partons and hadronization of final state particles. Finally, the detector simulation are implemented using Delphes3 [85] with the ILC parameters [86]. For b -tagging of final jets, we employ the loose working point, corresponding to an average tagging efficiency of 80%. Event selection requires exactly two oppositely charged leptons and two b -tagged jets, with the additional condition that the final state contains no hard ($p_T > 10$ GeV) photons.

In order to select the phase space which maximizes the significance, a number of kinematic variables are constructed using the four momenta of final four objects. We train LightGBM boosted decision tree (BDT) models separately for the unpolarized and polarized beam configurations; $(P_{e^+}, P_{e^-}) = (+30\%, -80\%)$ and $(-30\%, +80\%)$, respectively. The input features for the BDTs consist of several kinematic observables, whose details are provided in Appendix A. Among these variables, the recoil mass of the dilepton system, m_{Recoil} , emerges as the most effective discriminator between signal and background. The distribution normalized to $\mathcal{L} = 1000 \text{ fb}^{-1}$ representing m_{Recoil} for signal and two major background processes (ZZ and $ZZ\gamma$) with and without ISR are shown in Fig. 2. The p_T cut in the final photons remove the $ZZ\gamma$ significantly and the corresponding distribution are rescaled by factor of 100 in integrated luminosity for visualization. The number of events for the case of signal (Zh) reduces due to ISR as the interaction energy lowers below 250 GeV leading to lower cross section. However, no significant change in

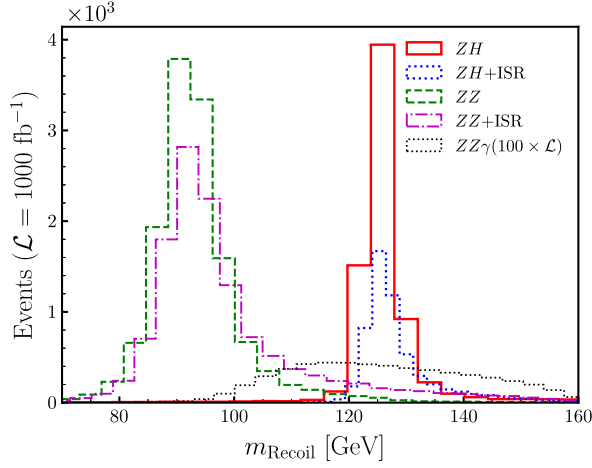


FIG. 2. Recoil mass distribution normalized to $\mathcal{L} = 1000 \text{ fb}^{-1}$ for signal and two background processes viz. ZZ and ZZ γ . The kinematic effect of ISR are highlighted for Zh and ZZ processes.

shape and count is observed for ZZ process. Nevertheless, the recoil mass stands out to nullify the effect of ISR on segregating the signal phase space from that of background. We apply the following selection cut on the recoil mass:

$$|m_{\text{Recoil}} - M_h| < 2 \text{ GeV}. \quad (8)$$

Events and signal significance before and after the M_{recoil} cut is furnished in Table I for both polarized and unpolarized beams. After applying the recoil mass cut, we find that the background becomes nearly negligible compared to the signal.

IV. OPTIMAL STATISTICAL SIGNIFICANCE

The optimal observable technique (OOT) [87–91] is a powerful method to estimate model parameters or couplings from experimental data with maximum statistical sensitivity. By constructing observables that are analytically derived from the dependence of the differential cross section on the parameters of interest, this approach ensures the most efficient use of the available kinematic

information, by minimizing the covariance matrix. It is particularly useful in precision measurements, where small deviations from the SM predictions are expected to be estimated.

The differential cross section for any given collider process can be written in the form,

$$\begin{aligned} \mathcal{O}(\phi) &= \left. \frac{d\sigma}{d\phi} \right|_{\text{observed}} = \epsilon_S \left. \frac{d\sigma}{d\phi} \right|_{S, \text{theory}} + \epsilon_B \left. \frac{d\sigma}{d\phi} \right|_{B, \text{theory}} \\ &= \sum_i g_i f_i(\phi), \end{aligned} \quad (9)$$

where ϕ is a phase space variable, g_i are the functions containing the NP couplings and f_i are the functions of phase space variable, ϕ . The subscripts S and B refers to signal and background, respectively. ϵ_S and ϵ_B are the signal and background detection efficiencies, following all the cuts and final state signal identification. The optimal covariance matrix upon minimization ($\partial V_{ij}/\partial g_j = 0$) is obtained as

$$V_{ij} = \frac{M_{ij}^{-1}}{\mathcal{L}_{\text{int}}} = \frac{1}{\mathcal{L}_{\text{int}}} \int \frac{f_i(\phi) f_j(\phi)}{\mathcal{O}(\phi)} d\phi. \quad (10)$$

Correspondingly optimal χ^2 is given by

$$\chi^2 = \sum_{i,j} (g_i - g_i^0) V_{ij}^{-1} (g_j - g_j^0) \Big|_{g=g^0}, \quad (11)$$

where g^0 correspond to the seed values of the coefficients g . The input seed values can come from different sources like available measurements, predictions from a different experiment, etc. In case of sensitivity prediction of an unobserved NP scenario, the seed value is usually determined by setting the seed values of NP couplings to zero.

For $e^+e^- \rightarrow Zh$ production at the ILC, we map the accessible phase space using the variable, $\cos \theta$, which is the emerging angle of the Z boson. The differential observable can be written in terms of polarization degrees as

TABLE I. Cutflow table for signal and backgrounds, at the ILC $\sqrt{s} = 250 \text{ GeV}$ and an integrated luminosity of 1000 fb^{-1} .

(P_{e^+}, P_{e^-})	Process	Before M_{recoil} cut		After M_{recoil} cut	
		Events	$\mathcal{Z} = S/B$	Events	$\mathcal{Z} = S/B$
Unpolarized	Signal (S)	1879	0.11	1610	36.9
	Background (B)	16652		44	
(+30%, -80%)	Signal (S)	2580	0.12	2189	31.2
	Background (B)	21252		70	
(-30%, +80%)	Signal (S)	2045	0.13	1746	61.0
	Background (B)	15722		29	

$$\begin{aligned}
\mathcal{O}(\cos \theta) = \epsilon_S \frac{d\sigma}{d\cos \theta} \Big|_{\text{S,theory}} &= \epsilon_S \{g_0^L(1 - \overline{P_{e^+}})(1 - P_{e^-}) + g_0^R(1 + \overline{P_{e^+}})(1 + P_{e^-})\} f_0 \\
&+ \epsilon_S \{g_1^L(1 - \overline{P_{e^+}})(1 - P_{e^-}) + g_1^R(1 + \overline{P_{e^+}})(1 + P_{e^-})\} f_1 \\
&+ \epsilon_S \{g_2^L(1 - \overline{P_{e^+}})(1 - P_{e^-}) + g_2^R(1 + \overline{P_{e^+}})(1 + P_{e^-})\} f_2.
\end{aligned} \tag{12}$$

The background contribution is small after the signal selection cuts are applied, as seen from Sec. III, and this has been neglected to estimate optimal sensitivity. The linearly independent phase space functions f_i are defined as follows:

$$f_0 = \beta, \quad f_1 = \beta \cos \theta, \quad f_2 = \beta \cos^2 \theta, \tag{13}$$

where

$$\beta = \frac{\sqrt{\lambda(s, m_Z^2, m_H^2)}}{32\pi s^2}. \tag{14}$$

Here, λ is the Kallen function defined as $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. With $\sqrt{s} = 250$ GeV, $m_Z = 91$ GeV and $m_h = 125$ GeV, we obtain: $\beta \approx 31$ pb. With all the SM constants replaced by their respective values, the dimensionless g_i as in Eq. (12), turns out to be functions of the WCs only. We list the expressions for g_i in Appendix B. Given that there is no hint of BSM, we choose the seed values of all the WCs to be zero to find an optimal range in which they can be extracted.

V. RESULTS

In this section, we present projected sensitivities to the dimension-6 operators in Eq. (2), derived using the optimal observable technique. Constraints on the corresponding WCs are obtained following the optimized signal selection strategy described earlier. We do the analysis for various beam polarization configurations, highlighting the improvement in sensitivity enabled by polarized e^+e^- collisions. For benchmarking, we also compare our projections with existing limits from the LHC and provide estimates relevant to future collider runs.

Table II summarizes the optimal 68% CL limits (corresponding to $\chi^2 = 1$) on the WCs at the ILC with $\sqrt{s} = 250$ GeV, in comparison with current CMS constraints at 13 TeV with $\mathcal{L}_{\text{int}} = 138 \text{ fb}^{-1}$. Two ILC scenarios are considered; (I) unpolarized beams with $\mathcal{L}_{\text{int}} = 138 \text{ fb}^{-1}$, and (II) a polarized setup combining (+30%, -80%) and (-30%, +80%) beam configurations, each contributing 69 fb^{-1} , summing to the same total integrated luminosity. The sensitivity to the CP -even operator C_{HW} improves by approximately a factor of 6 in the unpolarized ILC scenario compared to the current LHC constraints, with beam polarization providing an additional enhancement to about a factor of 7. For C_{HWB} , the improvement is even more

pronounced, reaching nearly a factor of 7 in the unpolarized case and exceeding a factor of 10 when polarized beams are employed. In the case of C_{HB} , the unpolarized ILC limit is slightly weaker than the LHC bound; however, the introduction of beam polarization yields an improvement of roughly 1.5 times. These results highlight the critical role of beam polarization in maximizing the precision reach of future lepton colliders.

For the CP -odd operators, the current LHC limits generally demonstrate stronger sensitivity compared to the ILC projections obtained via the optimal observable technique. This reduced sensitivity at the ILC arises because the optimal observable method probes CP -odd effects predominantly at the quadratic level in the differential cross section, which inherently suppresses the constraints. As reflected in Table II, the allowed parameter ranges for these operators remain significantly wider at the ILC, even with beam polarization. To enhance sensitivity to CP -odd operators, alternative strategies—such as leveraging spin-polarization asymmetries or dedicated CP -odd observables—could provide more powerful and direct probes, thus offering complementary avenues to tighten these bounds at future lepton colliders. A detailed study exploiting these observables to constrain CP -odd operators will be presented in future work.

We further study the projected sensitivities of the relevant SMEFT operator coefficients at the ILC with an integrated luminosity of 1000 fb^{-1} per polarization configuration, and a combined dataset yielding 2000 fb^{-1} luminosity. The one-parameter sensitivity projections are

TABLE II. 68% CL bounds on CP -even (*top half*) and CP -odd (*bottom half*) operator coefficients at the LHC vs from Zh production at the ILC 250 GeV. For polarized case, we choose $\mathfrak{L}_{\text{int}} = 69 \text{ fb}^{-1}$ for polarization setups (+30%, -80%) and (-30%, +80%) each.

WCs	LHC (13 TeV 138 fb ⁻¹)	ILC (250 GeV 138 fb ⁻¹)	
		Unpolarized	Polarized (±30%, ∓80%)
C_{HW}	[-0.79, +0.51]	[-0.11, +0.11]	[-0.09, +0.08]
C_{HWB}	[-1.62, +1.50]	[-0.24, +0.22]	[-0.16, +0.15]
C_{HB}	[-0.23, +0.16]	[-0.69, +0.41]	[-0.14, +0.14]
$C_{H\tilde{W}}$	[-0.76, +0.41]	[-1.36, +1.36]	[-1.16, +1.16]
$C_{H\tilde{W}B}$	[-1.57, +0.83]	[-2.50, +2.50]	[-1.98, +1.98]
$C_{H\tilde{B}}$	[-0.23, +0.12]	[-2.16, +2.16]	[-1.84, +1.84]

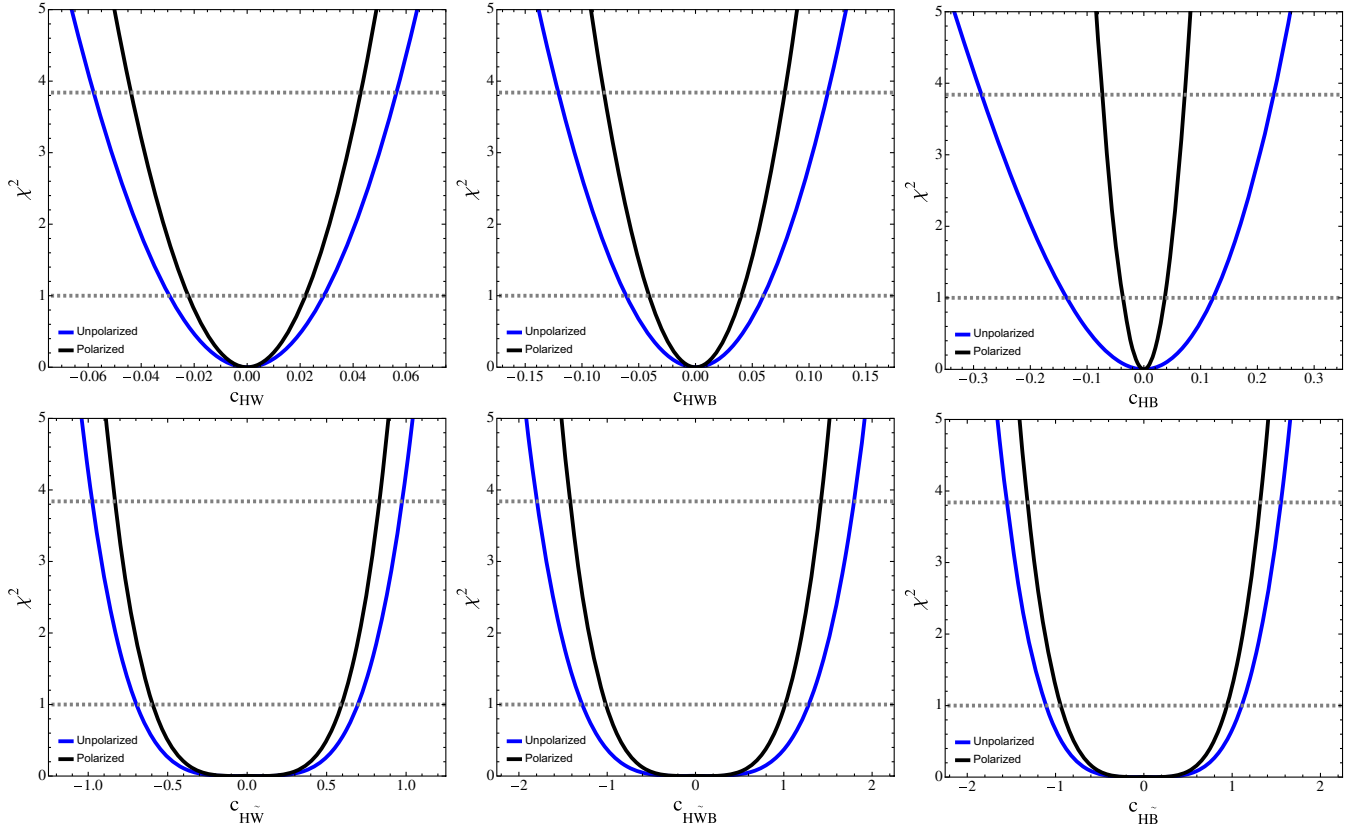


FIG. 3. One parameter optimal sensitivity plots from Zh production at the ILC 250 GeV for unpolarized and polarized setups. For the unpolarized case, $\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$, and for the polarized case each polarization setup i.e., $(+30\%, -80\%)$ and $(-30\%, +80\%)$ corresponds to $\mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1}$, hence giving a combined luminosity, $\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$.

shown in Fig. 3 for graphical representation. The gray dashed lines indicate the 68% CL i.e., $\chi^2 = 1$ and 95% CL ($\chi^2 = 3.84$) thresholds. The corresponding numerical limits are listed in Tab. III. For the CP -even operators, the results demonstrate notable variations in sensitivity across the different configurations. The C_{HW} operator benefits significantly from beam polarization; the $(+30\%, -80\%)$ setup yields the most stringent bounds at both 68% and 95% confidence levels, improving roughly by a factor of two compared to the unpolarized case. The $(-30\%, +80\%)$ configuration shows weaker sensitivity for this operator, illustrating the asymmetric impact of polarization states.

In the case of C_{HWB} , the situation is reversed with the $(-30\%, +80\%)$ polarization providing the strongest limits, surpassing both the unpolarized and the $(+30\%, -80\%)$ runs by a considerable margin. This complementarity between polarized states underscores the importance of running both configurations for a comprehensive probe. The combined analysis again improves the bounds further, achieving nearly a factor of two improvement over single configurations. For C_{HB} , the $(-30\%, +80\%)$ polarization scenario delivers the best sensitivity, with bounds tighter by roughly a factor of four compared to the unpolarized run.

Turning to the CP -odd operators, the sensitivities exhibit a more varied pattern. For $C_{H\bar{W}}$, the $(+30\%, -80\%)$ polarization setup outperforms the others, improving constraints by approximately 30% relative to unpolarized collisions. The flipped polarization configuration is less sensitive.

For $C_{H\bar{W}B}$ and $C_{H\bar{B}}$, the $(-30\%, +80\%)$ run provides stronger bounds than either unpolarized or flipped configuration individually. However, combining all data still leads to a modest but meaningful improvement, demonstrating the statistical benefit of aggregating diverse polarization states.

Overall, these projections emphasize the critical role of beam polarization in enhancing sensitivity to dimension-6 operators at the ILC, with different operators responding distinctly to polarization choices. The combined dataset invariably delivers the most stringent bounds, leveraging the complementarity and statistical gains across all configurations.

We further compare our results with Ref. [72], which analyzed $e^+e^- \rightarrow Zh$ at $\sqrt{s} = 250 \text{ GeV}$ under identical beam polarizations and an integrated luminosity of 2000 fb^{-1} . That study parametrized the anomalous hZZ vertex in terms of three real couplings, a_Z , b_Z , and $\tilde{b}_Z$,

TABLE III. Projected sensitivities of CP -even (*top half*) and CP -odd (*bottom half*) SMEFT operator coefficients from Zh production at the ILC 250 GeV. For the polarized setup, we combine polarization setups: $(+30\%, -80\%)$ and $(-30\%, +80\%)$, each with luminosities, $\mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1}$, hence the combined setup of $\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$.

WCs	CL	Projected sensitivities				
		Unpolarized		$(+30\%, -80\%)$ $\mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1}$	$(-30\%, +80\%)$ $\mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1}$	Polarized $\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$
		$\mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1}$	$\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$			
C_{HW}	68%	$[-0.04, +0.04]$	$[-0.03, +0.03]$	$[-0.02, +0.02]$	$[-0.17, +0.16]$	$[-0.02, +0.02]$
	95%	$[-0.08, +0.08]$	$[-0.06, +0.06]$	$[-0.04, +0.04]$	$[-0.34, +0.31]$	$[-0.04, +0.04]$
C_{HWB}	68%	$[-0.09, +0.08]$	$[-0.06, +0.06]$	$[-0.33, +0.31]$	$[-0.04, +0.04]$	$[-0.04, +0.04]$
	95%	$[-0.17, +0.16]$	$[-0.12, +0.12]$	$[-0.66, +0.60]$	$[-0.08, +0.08]$	$[-0.08, +0.08]$
C_{HB}	68%	$[-0.20, +0.17]$	$[-0.14, +0.12]$	$[-0.10, +0.10]$	$[-0.04, +0.04]$	$[-0.04, +0.04]$
	95%	$[-0.44, +0.31]$	$[-0.28, +0.23]$	$[-0.19, +0.20]$	$[-0.08, +0.08]$	$[-0.07, +0.07]$
$C_{H\tilde{W}}$	68%	$[-0.83, +0.83]$	$[-0.70, +0.70]$	$[-0.60, +0.60]$	$[-2.18, +2.18]$	$[-0.59, +0.59]$
	95%	$[-1.16, +1.16]$	$[-0.98, +0.98]$	$[-0.83, +0.83]$	$[-3.05, +3.05]$	$[-0.83, +0.83]$
$C_{H\tilde{W}B}$	68%	$[-1.52, +1.52]$	$[-1.28, +1.28]$	$[-4.28, +4.28]$	$[-1.02, +1.02]$	$[-1.02, +1.02]$
	95%	$[-2.14, +2.14]$	$[-1.80, +1.80]$	$[-5.99, +5.99]$	$[-1.42, +1.42]$	$[-1.42, +1.42]$
$C_{H\tilde{B}}$	68%	$[-1.32, +1.32]$	$[-1.11, +1.11]$	$[-2.04, +2.04]$	$[-0.95, +0.95]$	$[-0.94, +0.94]$
	95%	$[-1.85, +1.85]$	$[-1.55, +1.55]$	$[-2.86, +2.86]$	$[-1.33, +1.33]$	$[-1.32, +1.32]$

extracted from angular and cross section observables. In the SMEFT framework, our dimension-6 operators contribute only to b_Z and $\tilde{b}_Z$. Inverting the couplings, we obtain $b_Z \in [-0.0079, +0.0079]$ and $\tilde{b}_Z \in [-1.517, +1.517]$. The bound on the CP -even parameter b_Z agrees well with Ref. [72], while the much weaker limit on the CP -odd parameter $\tilde{b}_Z$, about two orders of magnitude larger, reflects the limited CP sensitivity of total cross section observables and underscores the need for CP -odd and spin-correlation analyses, as discussed above.

In addition, the two parameter sensitivity contours are illustrated in Fig. 4, providing a complementary view of the correlations and degeneracies between operator coefficients that are not captured in the one parameter analysis. Additional sensitivity contours are presented in Appendix D.

VI. CONCLUSION

The associated production of a single Higgs boson with a Z boson—commonly referred to as the Higgs-strahlung process—offers a clean and model-independent channel to probe Higgs-gauge couplings with high precision. In this study, we focus on the final state consisting of two oppositely charged, same-flavor leptons and two b -tagged jets in e^+e^- collisions at a center-of-mass energy of $\sqrt{s} = 250 \text{ GeV}$, incorporating scenarios with polarized initial beams. The deviations from the Standard Model (SM) expectations are parametrized by the SMEFT framework, allowing us to constrain the coefficients of higher-dimensional operators that modify the hZZ and $hZ\gamma$ interactions.

Event selection is performed via the recoil mass technique, targeting the Higgs boson through the invariant mass

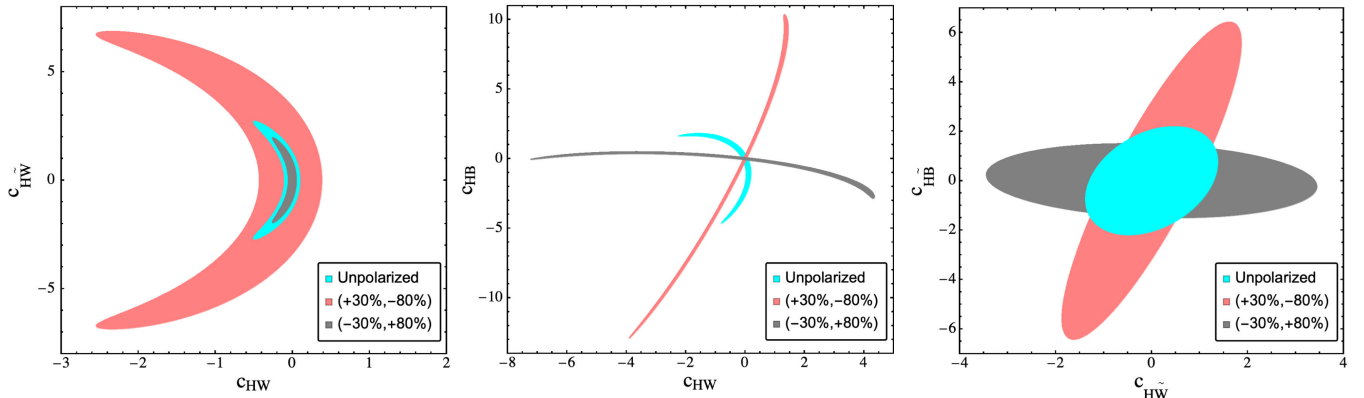


FIG. 4. Two parameter 95% CL optimal sensitivity plots from Zh production at the ILC 250 GeV for unpolarized and different polarization setups. For each setup, $\mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1}$.

spectrum of the Z -tagged dilepton system. The statistical extraction of the sensitivity to the WCs is carried out using the OOT, which maximizes information from kinematic distributions by exploiting interference patterns between SM and SMEFT amplitudes.

Our analysis demonstrates that beam polarization plays a crucial role in enhancing sensitivity to various operators. When comparing our results with current bounds from the CMS experiment at $\sqrt{s} = 13$ TeV and $\mathcal{L}_{\text{int}} = 138 \text{ fb}^{-1}$, we find that the ILC setup, with polarized beams, yields significantly tighter constraints—improving the bounds on CP -even operators by factors ranging from 1.5 to 10. This improvement stems from the linear interference term between the SM and the CP -even SMEFT contributions, which scales as $1/\Lambda^2$ and benefits strongly from the clean environment and polarization of the e^+e^- collider.

On the other hand, the sensitivity to CP -odd operators is comparatively weaker than that of CMS. This is attributed to the fact that CP -odd operators contribute dominantly at quadratic order in $1/\Lambda^2$, resulting in suppressed effects at the amplitude level. Consequently, our observable—being inclusive in nature—lacks sensitivity to CP -odd interference effects. This highlights the need for a more targeted analysis involving CP -sensitive observables such as angular asymmetries, triple-product correlations, or differential distributions optimized for CP -violation.

Finally, we present projections at higher integrated luminosity of $\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$ for two different ILC running scenarios; an unpolarized beam and a combination of two complementary polarized beam configurations ($+30\%$, -80%) and (-30% , $+80\%$), with integrated luminosity of $\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$ each. We observe that different polarization choices enhance sensitivity to different subsets of operators due to their distinct helicity structures. The combination of the two polarization configurations, with $\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$, yields the strongest bounds across the operator basis effectively exploiting the polarization-enhanced interference terms. These results reinforce the power of polarized e^+e^- collisions at the ILC in probing new physics in the Higgs sector with high precision.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: ML-BASED INSIGHTS

We train three BDT models using the LightGBM framework; one for the unpolarized case and two for the polarized beam configurations $(P_{e^+}, P_{e^-}) = (+30\%, -80\%)$ and

$(-30\%, +80\%)$. For each model, the dataset is split into equal halves for training and testing. The kinematic variables (features) used for training are as follows. Invariant mass of the dilepton system (M_{ll}) and the $b\bar{b}$ system (M_{bb}), recoil mass of the dilepton system (m_{Recoil}), angular separation between the leptons ($\Delta\eta_{ll}$, ΔR_{ll}), angular separation between the b -jets ($\Delta\eta_{bb}$, ΔR_{bb}), kinematic variables of the dilepton system, transverse momentum (p_T^l), pseudorapidity (η_{ll}), and azimuthal angle (ϕ_{ll}), kinematic variables of the $b\bar{b}$ system (p_T^{bb} , η_{bb} , and ϕ_{bb}), angular separation between the dilepton and $b\bar{b}$ systems ($\Delta\eta_{ll,bb}$, $\Delta R_{ll,bb}$). The recoil mass of the dilepton system is defined as

$$m_{\text{Recoil}} = \sqrt{s - 2\sqrt{s}E_{ll} + M_{ll}^2}, \quad (\text{A1})$$

where E_{ll} is the total energy of the two leptons. The definitions of the other variables follow standard usage in collider physics literature. The polar angle of the final state, reconstructed from the dilepton system, is defined as

$$\cos\theta_{\text{Recoil}} = \frac{p_z^l}{|\vec{p}_{ll}|}, \quad (\text{A2})$$

where p_z^l is the longitudinal momentum of the dilepton system and $|\vec{p}_{ll}|$ is its total three-momentum magnitude. All input features are normalized using the StandardScaler module. The BDT models are trained with equal class weights for signal and background events, while testing is performed using weights proportional to the true event yields. The SHAP values for the trained models are shown in Fig. 5. Among all features, the recoil mass m_{Recoil} emerges as the most powerful discriminator. Therefore, we choose to replace the BDT model with a simple optimized selection cut

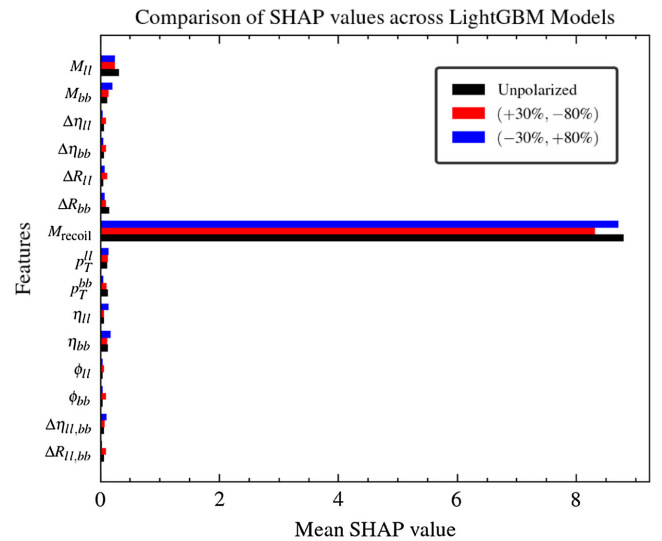


FIG. 5. SHAP values for BDT models trained for signal-background segregation.

on m_{Recoil} , achieving similar classification performance with greater interpretability.

APPENDIX B: COUPLINGS IN Zh DIFFERENTIAL CROSS SECTION AS OPTIMAL OBSERVABLE

As stated before, the differential cross section for $e^+e^- \rightarrow Zh$ serves as the observable to estimate the NP couplings (WCs) optimally in ILC. In the limit, where background contamination are assumed negligible, the

observable can be expressed as

$$\begin{aligned} \mathcal{O}(\phi) &= \left. \frac{d\sigma}{d\phi} \right|_{\text{observed}} = \epsilon_s \left. \frac{d\sigma}{d\phi} \right|_{\text{s,theory}} \\ &= \sum_i g_i f_i(\phi). \end{aligned} \quad (\text{B1})$$

Upon a seminumeric evaluation, with $\sqrt{s} = 250$ GeV, $m_Z = 91$ GeV, and $m_h = 125$ GeV, the couplings attached to the phase space variables as in Eq. (12), are furnished in Eq. (B2).

$$\begin{aligned} g_0^L &= 2.0 \times 10^{-3} + 1.8 \times 10^{-3} C_{HW} + 7.7 \times 10^{-5} C_{HWB} + 4.4 \times 10^{-4} (C_{HW})^2 - 4.7 \times 10^{-4} (C_{HB})^2 \\ &\quad + 7.7 \times 10^{-7} (C_{HWB})^2 - 2.3 \times 10^{-4} C_{HW} C_{HB} - 9.4 \times 10^{-6} C_{HB} C_{HWB} + 8.4 \times 10^{-5} (C_{H\bar{W}})^2 \\ &\quad + 5.4 \times 10^{-6} (C_{H\bar{B}})^2 + 1.5 \times 10^{-7} (C_{H\bar{W}\bar{B}})^2 - 4.2 \times 10^{-5} C_{H\bar{W}} C_{H\bar{B}} + 7.0 \times 10^{-7} C_{H\bar{W}} C_{H\bar{W}\bar{B}} \\ &\quad - 1.8 \times 10^{-6} C_{H\bar{B}} C_{H\bar{W}\bar{B}}, \\ g_0^L &= 2.0 \times 10^{-3} + 1.8 \times 10^{-3} C_{HW} + 7.7 \times 10^{-5} C_{HWB} + 4.4 \times 10^{-4} (C_{HW})^2 - 4.7 \times 10^{-4} (C_{HB})^2 \\ &\quad + 7.7 \times 10^{-7} (C_{HWB})^2 - 2.3 \times 10^{-4} C_{HW} C_{HB} - 9.4 \times 10^{-6} C_{HB} C_{HWB} + 8.4 \times 10^{-5} (C_{H\bar{W}})^2 \\ &\quad + 5.4 \times 10^{-6} (C_{H\bar{B}})^2 + 1.5 \times 10^{-7} (C_{H\bar{W}\bar{B}})^2 - 4.2 \times 10^{-5} C_{H\bar{W}} C_{H\bar{B}} + 7.0 \times 10^{-7} C_{H\bar{W}} C_{H\bar{W}\bar{B}} \\ &\quad - 1.8 \times 10^{-6} C_{H\bar{B}} C_{H\bar{W}\bar{B}}, \\ g_1^L &= 3.2 \times 10^{-19} C_{HW} - 3.2 \times 10^{-19} C_{HB} - 5.4 \times 10^{-20} C_{HWB} - 5.4 \times 10^{-20} (C_{HW})^2 - 2.7 \times 10^{-20} (C_{HB})^2 \\ &\quad - 1.3 \times 10^{-19} C_{HW} C_{HB} - 4.7 \times 10^{-20} C_{HW} C_{HWB} + 2.4 \times 10^{-20} (C_{H\bar{W}})^2 + 2.4 \times 10^{-20} (C_{H\bar{B}})^2 \\ &\quad + 2.5 \times 10^{-21} (C_{H\bar{W}\bar{B}})^2 - 4.7 \times 10^{-20} C_{H\bar{W}} C_{H\bar{B}} - 1.4 \times 10^{-20} C_{H\bar{W}} C_{\Phi\bar{W}\bar{B}} + 1.4 \times 10^{-20} C_{\Phi\bar{B}} C_{\Phi\bar{W}\bar{B}}, \\ g_2^L &= -3.7 \times 10^{-4} - 5.4 \times 10^{-20} C_{\Phi W} + 5.4 \times 10^{-20} C_{\Phi B} + 1.4 \times 10^{-20} C_{\Phi WB} + 8.4 \times 10^{-5} (C_{\Phi W})^2 \\ &\quad - 5.4 \times 10^{-6} (C_{\Phi B})^2 - 1.5 \times 10^{-7} (C_{\Phi WB})^2 - 4.3 \times 10^{-5} C_{\Phi W} C_{\Phi B} - 7.0 \times 10^{-6} C_{\Phi W} C_{\Phi WB} \\ &\quad - 1.8 \times 10^{-6} C_{\Phi B} C_{\Phi WB} + 8.4 \times 10^{-5} (C_{\Phi\bar{W}})^2 + 5.4 \times 10^{-6} (C_{\Phi\bar{B}})^2 + 1.5 \times 10^{-7} (C_{\Phi\bar{W}\bar{B}})^2 \\ &\quad - 4.3 \times 10^{-5} C_{\Phi\bar{W}} C_{\Phi\bar{B}} - 7.0 \times 10^{-6} C_{\Phi\bar{W}} C_{\Phi\bar{W}\bar{B}} + 1.8 \times 10^{-6} C_{\Phi\bar{B}} C_{\Phi\bar{W}\bar{B}}, \\ g_0^R &= 1.4 \times 10^{-3} + 1.0 \times 10^{-4} C_{HW} + 9.1 \times 10^{-4} C_{HB} + 8.6 \times 10^{-4} C_{HWB} + 1.9 \times 10^{-6} (C_{HW})^2 \\ &\quad + 1.5 \times 10^{-4} (C_{HB})^2 + 1.3 \times 10^{-4} (C_{HWB})^2 + 3.6 \times 10^{-7} (C_{H\bar{W}})^2 + 2.8 \times 10^{-5} (C_{H\bar{B}})^2 \\ &\quad + 2.5 \times 10^{-5} (C_{H\bar{W}\bar{B}})^2 + 3.3 \times 10^{-5} C_{HW} C_{HB} + 3.2 \times 10^{-5} C_{HW} C_{HWB} \\ &\quad + 2.8 \times 10^{-4} C_{HB} C_{HWB} + 6.3 \times 10^{-6} C_{H\bar{W}} C_{H\bar{B}} + 6.0 \times 10^{-6} C_{H\bar{W}} C_{H\bar{W}\bar{B}} + 5.2 \times 10^{-5} C_{H\bar{B}} C_{H\bar{W}\bar{B}}, \\ g_1^R &= -1.1 \times 10^{-19} C_{HW} + 1.1 \times 10^{-19} C_{HB} + 1.1 \times 10^{-19} C_{HWB} - 5.4 \times 10^{-20} (C_{HW})^2 + 6.1 \times 10^{-20} (C_{HB})^2 \\ &\quad + 2.0 \times 10^{-20} (C_{HWB})^2 + 2.4 \times 10^{-20} (C_{H\bar{W}})^2 + 1.0 \times 10^{-20} (C_{H\bar{B}})^2 + 2.5 \times 10^{-21} (C_{H\bar{W}\bar{B}})^2 \\ &\quad + 1.4 \times 10^{-19} C_{HW} C_{HB} + 3.4 \times 10^{-20} C_{HW} C_{HWB} - 6.8 \times 10^{-21} C_{HB} C_{HWB} - 4.7 \times 10^{-20} C_{H\bar{W}} C_{H\bar{B}} \\ &\quad - 1.4 \times 10^{-20} C_{H\bar{W}} C_{H\bar{W}\bar{B}} + 2.7 \times 10^{-20} C_{H\bar{B}} C_{H\bar{W}\bar{B}}, \\ g_2^R &= -2.8 \times 10^{-4} - 2.7 \times 10^{-20} C_{HW} + 2.7 \times 10^{-20} C_{HB} - 1.4 \times 10^{-20} C_{HWB} + 3.6 \times 10^{-7} (C_{HW})^2 \\ &\quad + 2.8 \times 10^{-5} (C_{HB})^2 + 2.5 \times 10^{-5} (C_{HWB})^2 + 3.6 \times 10^{-7} (C_{H\bar{W}})^2 + 2.8 \times 10^{-5} (C_{H\bar{B}})^2 \\ &\quad + 2.5 \times 10^{-5} (C_{H\bar{W}\bar{B}})^2 + 6.3 \times 10^{-6} C_{HW} C_{HB} + 6.0 \times 10^{-6} C_{HW} C_{HWB} + 5.2 \times 10^{-5} C_{HB} C_{HWB} \\ &\quad + 6.3 \times 10^{-6} C_{H\bar{W}} C_{H\bar{B}} + 6.0 \times 10^{-6} C_{H\bar{W}} C_{H\bar{W}\bar{B}} + 5.2 \times 10^{-5} C_{H\bar{B}} C_{H\bar{W}\bar{B}}. \end{aligned} \quad (\text{B2})$$

Note that they are quadratic in C_i with cross-terms of different Wilson coefficients coming from different operators. The optimal χ^2 analysis is done based on the above parametrization.

TABLE IV. 95% CL bounds on CP -even (*top half*) and CP -odd (*bottom half*) operator coefficients from Zh production at the FCC-ee 240 GeV vs ILC 250 GeV. For polarized case, we choose $\mathfrak{L}_{\text{int}} = 1000 \text{ fb}^{-1}$ for polarization setups (+30%, -80%) and (-30%, +80%) each.

WCs	FCC-ee (240 GeV 10500 fb^{-1})	ILC (250 GeV 2000 fb^{-1})	
		Unpolarized	Polarized ($\pm 30\%, \mp 80\%$)
C_{HW}	$[-0.02, +0.02]$	$[-0.06, +0.06]$	$[-0.04, +0.04]$
C_{HWB}	$[-0.03, +0.03]$	$[-0.12, +0.12]$	$[-0.08, +0.08]$
C_{HB}	$[-0.06, +0.06]$	$[-0.28, +0.23]$	$[-0.07, +0.07]$
$C_{H\bar{W}}$	$[-0.47, +0.47]$	$[-0.98, +0.98]$	$[-0.83, +0.83]$
$C_{H\bar{W}B}$	$[-0.86, +0.86]$	$[-1.80, +1.80]$	$[-1.42, +1.42]$
$C_{H\bar{B}}$	$[-0.74, +0.74]$	$[-1.55, +1.55]$	$[-1.32, +1.32]$

APPENDIX C: COMPARISON WITH FCC-EE

The electron-positron collision stage of the Future Circular Collider (FCC-ee) [28] is expected to run at a

CM energy of $\sqrt{s} = 240 \text{ GeV}$ with a projected integrated luminosity of $\mathcal{L}_{\text{int}} = 10500 \text{ fb}^{-1}$. However, the setup is expected to collide unpolarized beams only. In this section, we draw a comparison between the 95% CL limits on the SMEFT operators for ILC 250 GeV with $\mathcal{L}_{\text{int}} = 2000 \text{ fb}^{-1}$ for the unpolarized and polarized setup discussed in Table III, with the limits from the FCC-ee run, tabulated in Table IV. The higher luminosity of the FCC-ee results in more sensitive bounds compared to the ILC runs.

APPENDIX D: CORRELATED SENSITIVITIES

The two parameter χ^2 sensitivity contours are shown in Fig. 6 for both the unpolarized and the polarized beam configurations discussed in Table III. For the two-parameter case, the 95% CL limit corresponds to $\chi^2 = 5.99$. The comparison between the unpolarized and combined setups clearly demonstrates the enhanced sensitivity achieved through beam polarization, emphasizing its crucial role in constraining SMEFT operators at future e^+e^- colliders.

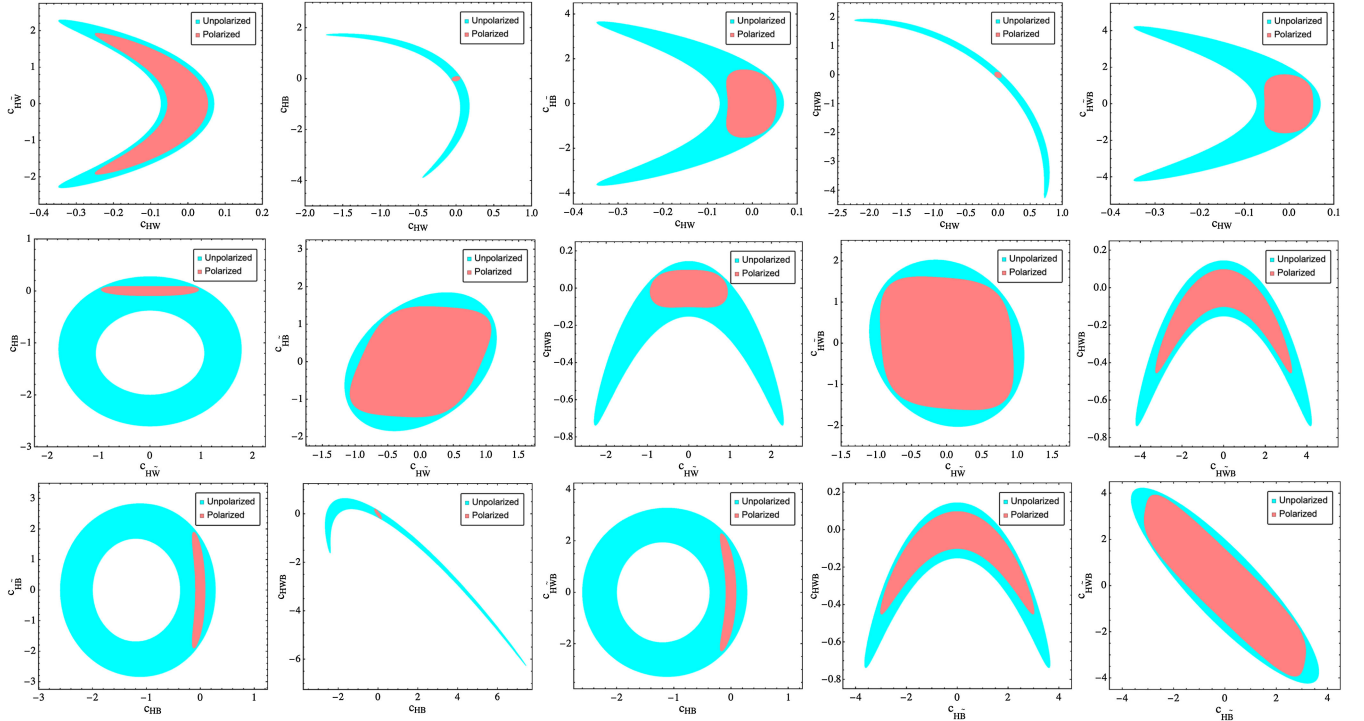


FIG. 6. Two parameter optimal sensitivity plots from Zh production at the ILC 250 GeV for unpolarized and combined setup. For the unpolarized setup, $\mathfrak{L}_{\text{int}} = 2000 \text{ fb}^{-1}$, and for the polarized setup, each polarization configuration viz. (+30%, -80%) and (-30%, +80%) with $\mathfrak{L}_{\text{int}} = 1000 \text{ fb}^{-1}$, combines to a integrated luminosity of $\mathfrak{L}_{\text{int}} = 2000 \text{ fb}^{-1}$.

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