

Belle II constraints on the nonminimal universal extra dimensional model

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Recent measurements by the Belle II Collaboration report a branching ratio for the decay $B^+ \rightarrow K^+ \nu \bar{\nu}$ with a 3.5σ significance and deviating by 2.7σ from Standard Model expectation. In this article, we interpret this experimental result through the lens of a nonminimal universal extra dimensional model, where the mass profiles and interactions among various Kaluza-Klein states differ remarkably from those in the minimal scenario. Our analysis indicates that, according to the model's parameters, the lower limit on the inverse of the compactification radius has been elevated to approximately 900 GeV. In contrast, conducting the same analysis within the model variant in which all boundary terms are nullified does not permit the determination of a lower bound on the inverse compactification radius.

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I. INTRODUCTION

The identification of the Higgs boson by the Large Hadron Collider (LHC) in 2012 represented a significant advancement for the Standard Model (SM) of particle physics [1,2]. Furthermore, experimental data across diverse categories have consistently aligned with the SM predictions for several decades. Nevertheless, there are still unresolved challenges, including the mystery of dark matter (DM), the baryon asymmetry of the universe, and the origin of neutrino mass, which the SM does not adequately address. Additionally, in flavor physics, experimental findings for various observables reveal significant discrepancies when compared with SM predictions. As a result, a fundamental objective of particle physics is to identify new particles and interactions, collectively referred to as “new physics” (NP), with the aim of developing definitive solutions to the aforementioned challenges. There are two main strategies for investigating NP. The first strategy, positioned at the high-energy frontier, involves the direct production of these new degrees of freedom. Conversely, the second strategy, positioned at the high-precision frontier, emphasises the examination of the indirect virtual effects connected to these novel particles. In this latter approach, NP would endeavor to bridge the disparity between the predictions of the SM and the results obtained from experimental measurements.

Focusing on the second approach to identifying NP, observables related to flavour changing neutral current (FCNC) processes—defined by quark transitions that proceed without any alteration in electric charge—constitute highly sensitive indicators for physics beyond the SM (BSM). This sensitivity arises because FCNC processes are strongly suppressed within the SM framework due to the Glashow-Iliopoulos-Maiani (GIM) mechanism. Consequently, numerous FCNC-decay related observables have been measured by various experimental collaborations, including Belle, *BABAR*, and LHCb. In several cases, these experimental findings have revealed significant deviations from SM predictions. Such discrepancies have been explored within a variety of BSM theoretical frameworks, including supersymmetry, extradimensional models, extensions involving additional gauge symmetries, and scenarios incorporating discrete symmetries.

Recently, the Belle II Collaboration, utilizing a dataset corresponding to an integrated luminosity of 362 fb^{-1} , reported the first evidence for the decay $B^+ \rightarrow K^+ \nu \bar{\nu}$. The measured branching ratio, obtained by combining results from both inclusive and hadronic tagging methods, is given by [3],

$$\text{Br}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{Belle II}} = [23 \pm 5(\text{stat})_{-4}^{+5}(\text{syst})] \times 10^{-6}. \quad (1)$$

Notably, the measured value exceeds the SM prediction by approximately 2.7σ . This intriguing result distinctly unveils a novel avenue for investigating NP, particularly noteworthy as it may suggest the presence of NP phenomena in the $b \rightarrow s \nu \bar{\nu}$ transitions. Various theoretical frameworks have been proposed to account for the excess observed by the Belle II experiment in terms of NP effects [4–31]. In particular, the decay rate of $B^+ \rightarrow K^+ \nu \bar{\nu}$ can be

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substantially modified in models incorporating particles beyond the SM. Accordingly, the present study interprets the branching ratio $\text{Br}(B^+ \rightarrow K^+ \nu \bar{\nu})$ reported by the Belle II Collaboration [3] within the context of a BSM framework known as the nonminimal universal extra dimension (NMUED) model [32–40].

The universal extra dimensions (UED) [41] model extends the SM by introducing a flat extra spatial dimension (denoted as y) accessible to all SM fields. The dimension y is compactified on a circle (S^1) with a radius (R). In this manifold, the fields are expanded in terms of 4D Kaluza-Klein (KK) modes, where the zero mode corresponds to the relevant SM field. The emergence of chiral fermions within the SM is facilitated by the imposition of a discrete symmetry Z_2 (where y is transformed to $-y$) on the extra spatial dimension. Consequently, this extra dimension is referred to as an S^1/Z_2 orbifold, leading to a physical domain that extends from $y = 0$ to $y = \pi R$. The $y \leftrightarrow -y$ symmetry manifests as a conserved parity, termed KK parity, which is defined as $(-1)^n$, where n represents the KK number and quantifies the discretized momentum along the y direction. KK-parity conservation guarantees that the lightest KK particle (LKP) remains stable and does not decay into SM particles. As a result, the LKP is considered a viable candidate for DM within this theoretical framework [42–49]. Furthermore, various extensions of this model have the potential to address several shortcomings of the SM, including issues related to gauge coupling unification [50–52], neutrino mass [53,54], and the hierarchy of fermion masses [55].

At the n th KK level, the mass of the KK partner corresponding to any SM particle can be represented by the expression $\sqrt{(m^2 + (nR^{-1})^2)}$, where m denotes the zero-mode mass (the mass of the SM particle) and is significantly smaller than R^{-1} . Consequently, the UED framework exhibits a nearly degenerate mass spectrum at each KK level. This characteristic leads to a deficiency in the phenomenological relevance of the UED scenario, particularly in collider experiments. However, radiative corrections can address the issue of mass spectrum degeneracy. These corrections can be categorized into two types: bulk corrections, which are finite and nonzero solely for KK-excitations of gauge bosons, and boundary localized corrections, which are proportional to terms dependent on a logarithmically defined cutoff scale (Λ) [56]. Boundary correction terms can be incorporated as 4D mass, kinetic, and other interaction terms for the KK-excited states at the two fixed boundary points (i.e., $y = 0$ and $y = \pi R$) of the orbifold. The inclusion of such terms is particularly natural within an extradimensional framework like UED, as they serve as counterterms for loop-induced contributions that depend on the cutoff scale Λ . In the minimal UED (MUED) model, a specific assumption is made whereby the boundary terms are selected to ensure that the 5D radiative corrections disappear at the cutoff

scale¹ Λ . Although this assumption is unique, it can be relaxed, allowing for the parametrization of radiative corrections as kinetic, mass, and other interaction terms localized at the two fixed boundary points. This alternative formulation is referred to as the NMUED model. Within this context, in addition to the compactification radius (R), the coefficients of various boundary localized terms (BLTs) are treated as free parameters, which can be constrained by a range of experimental data pertaining to various physical observables. The literature contains numerous phenomenological studies within this framework. For instance, constraints on the coefficients of the BLTs have been derived from analyses of electroweak observables [38,40], the S, T, and U parameters [36,57], DM relic density [58,59], the production and decay of the SM Higgs boson [60], and collider studies from LHC experiments [61–66]. Additional studies have focused on processes such as $Z \rightarrow b\bar{b}$ [67], branching ratios of rare B -meson decay processes (e.g., $B_s \rightarrow \mu^+ \mu^-$ [68], $B \rightarrow X_s \gamma$ [69], and $B \rightarrow X_s \ell^+ \ell^-$) [70], $\mathcal{R}_{D^{(*)}}$ anomalies [71–73], flavor-changing rare top decays [74,75], $\Delta B = 2$ transitions [76], and the unitarity of scattering amplitudes involving KK excitations [77].

This study presents the first comprehensive analysis of the branching ratio $\text{Br}(B^+ \rightarrow K^+ \nu \bar{\nu})$ within the framework of the NMUED model. We analyze the contributions from KK modes to the relevant Feynman diagrams, including on one-loop box diagrams. The functions derived from these diagrams are affected not only by the compactification radius but also by the parameters related to the BLT coefficients. Utilising these functions, we will compute the relevant Wilson Coefficient (WC), which will facilitate the estimation of the corresponding branching ratios within the NMUED framework. By comparing our theoretical predictions with the currently allowed ranges of the branching ratios, we aim to effectively constrain the parameter space of the NMUED model. Additionally, our analysis enables us to establish a lower limit on R^{-1} and to compare this result with findings from our previous research [68–70].

The organization of this paper is outlined as follows. Section II will offer a brief overview of the NMUED model. Following this, Sec. III will discuss the computational considerations related to the $B^+ \rightarrow K^+ \nu \bar{\nu}$ transition within the NMUED framework. Section IV will focus on the presentation of our numerical results. Lastly, Sec. V will provide a summary of the findings.

II. KK-PARITY CONSERVING NMUED FRAMEWORK

This section provides the key characteristics of the NMUED model, specifically those relevant for our current

¹UED is treated as an effective field theory, defined by the presence of a cutoff scale Λ .

investigation. Comprehensive treatments of this framework can be found in the literature [32–39,61–71,76]. KK-parity conservation is maintained by imposing symmetry on the boundary terms at $y = 0$ and $y = \pi R$. This symmetry leads to the presence of a stable LKP, making it a viable DM candidate—most often the first KK-excitation of the

photon. In-depth studies on DM phenomenology within this scenario are available in [59].

We illustrate a 5D fermion action that includes boundary localized kinetic terms (BLKTs), characterized by the coefficient r_f [37,59,68–71,76],

$$S_{\text{fermion}} = \int d^5x [\bar{\Psi}_L i \Gamma^M D_M \Psi_L + r_f \{\delta(y) + \delta(y - \pi R)\} \bar{\Psi}_L i \gamma^\mu D_\mu P_L \Psi_L + \bar{\Psi}_R i \Gamma^M D_M \Psi_R + r_f \{\delta(y) + \delta(y - \pi R)\} \bar{\Psi}_R i \gamma^\mu D_\mu P_R \Psi_R]. \quad (2)$$

In this context, $\Psi_L(x, y)$ and $\Psi_R(x, y)$ represent four-component 5D Dirac fermions, without distinction between quark or lepton fields. It is important to emphasize that the subscripts L and R do not indicate chirality; rather, they are employed solely as a notational convention. Each of these spinors is further decomposed into two-component spinors according to the following scheme [37,59,68–71,76]:

$$\Psi_L(x, y) = \begin{pmatrix} \phi_L(x, y) \\ \chi_L(x, y) \end{pmatrix} = \sum_n \begin{pmatrix} \phi_L^{(n)}(x) f_L^n(y) \\ \chi_L^{(n)}(x) g_L^n(y) \end{pmatrix}, \quad (3)$$

$$\Psi_R(x, y) = \begin{pmatrix} \phi_R(x, y) \\ \chi_R(x, y) \end{pmatrix} = \sum_n \begin{pmatrix} \phi_R^{(n)}(x) f_R^n(y) \\ \chi_R^{(n)}(x) g_R^n(y) \end{pmatrix}. \quad (4)$$

The corresponding wave functions for KK modes take the following form [33,38,59,68–71,76]:

$$f_L^n = g_R^n = N_n^f \begin{cases} \frac{\cos \left[m_{f(n)} \left(y - \frac{\pi R}{2} \right) \right]}{\cos \left[\frac{m_{f(n)} \pi R}{2} \right]} & \text{for } n \text{ even,} \\ -\frac{\sin \left[m_{f(n)} \left(y - \frac{\pi R}{2} \right) \right]}{\sin \left[\frac{m_{f(n)} \pi R}{2} \right]} & \text{for } n \text{ odd,} \end{cases} \quad (5)$$

and

$$g_L^n = -f_R^n = N_n^f \begin{cases} \frac{\sin \left[m_{f(n)} \left(y - \frac{\pi R}{2} \right) \right]}{\cos \left[\frac{m_{f(n)} \pi R}{2} \right]} & \text{for } n \text{ even,} \\ \frac{\cos \left[m_{f(n)} \left(y - \frac{\pi R}{2} \right) \right]}{\sin \left[\frac{m_{f(n)} \pi R}{2} \right]} & \text{for } n \text{ odd.} \end{cases} \quad (6)$$

The KK-mode wave functions are normalized using the modified inner product, accounting for boundary terms as follows [33,38,59,68–71,76]:

$$\left. \begin{aligned} \int_0^{\pi R} dy [1 + r_f \{\delta(y) + \delta(y - \pi R)\}] f_L^m f_L^n \\ \int_0^{\pi R} dy [1 + r_f \{\delta(y) + \delta(y - \pi R)\}] g_R^m g_R^n \end{aligned} \right\} = \delta^{nm}; \quad \left. \begin{aligned} \int_0^{\pi R} dy f_R^m f_R^n \\ \int_0^{\pi R} dy g_L^m g_L^n \end{aligned} \right\} = \delta^{nm}, \quad (7)$$

yielding the normalization constant [33,38,59,68–71,76],

$$N_n^f = \sqrt{\frac{2}{\pi R}} \left[\frac{1}{\sqrt{1 + \frac{r_f^2 m_{f(n)}^2}{4} + \frac{r_f}{\pi R}}} \right]. \quad (8)$$

The KK-mass $m_{f(n)}$ is determined from [33,59,68–71,76],

$$\frac{r_f m_{f(n)}}{2} = \begin{cases} -\tan \left(\frac{m_{f(n)} \pi R}{2} \right) & \text{for } n \text{ even,} \\ \cot \left(\frac{m_{f(n)} \pi R}{2} \right) & \text{for } n \text{ odd.} \end{cases} \quad (9)$$

In this context, we aim to examine the Yukawa interactions, as the heavy mass of the top quark is instrumental in amplifying the quantum effects addressed in the current study. The Yukawa interaction in 5D, including boundary localized contributions characterized by r_y , takes the form [68–71,76],

$$S_{\text{Yukawa}} = - \int d^5x [\lambda_5' \bar{\Psi}_L \tilde{\Phi} \Psi_R + r_y \{\delta(y) + \delta(y - \pi R)\} \lambda_5' \bar{\phi}_L \tilde{\Phi} \chi_R + \text{H.c.}], \quad (10)$$

where $\tilde{\Phi} = i\sigma_2 \Phi^*$ and $\Phi = (\phi^+)$ is the 5D Higgs doublet. λ_5' denotes the 5D coupling strength associated with the Yukawa interaction for the third generation.

We can expand the actions [given in Eqs. (2) and (10)] using Eqs. (3) and (4), which consequently yield a mass matrix at the KK level n for the third-generation quarks [68–71,76],

$$- \begin{pmatrix} \overline{\phi}_L^{(n)} & \overline{\phi}_R^{(n)} \end{pmatrix} \begin{pmatrix} m_{f(n)} \delta^{nm} & m_t \mathcal{J}_1^{nm} \\ m_t \mathcal{J}_2^{mn} & -m_{f(n)} \delta^{mn} \end{pmatrix} \begin{pmatrix} \chi_L^{(m)} \\ \chi_R^{(m)} \end{pmatrix} + \text{H.c.} \quad (11)$$

Here, m_t denotes the mass of the SM top quark, while $m_{f(n)}$ is obtained from the solution of the transcendental equations presented in Eq. (9). The overlap integrals are defined by [68–71,76],

$$\mathcal{J}_1^{nm} = \left(\frac{1 + \frac{r_f}{\pi R}}{1 + \frac{r_y}{\pi R}} \right) \times \int_0^{\pi R} dy [1 + r_y \{\delta(y) + \delta(y - \pi R)\}] g_R^m f_L^n,$$

and

$$\mathcal{J}_2^{nm} = \left(\frac{1 + \frac{r_f}{\pi R}}{1 + \frac{r_y}{\pi R}} \right) \times \int_0^{\pi R} dy g_L^m f_R^n.$$

The integral $\mathcal{J}_1^{nm}$ is nonzero for both cases where $n = m$ and $n \neq m$. Nonetheless, in the limit as r_y approaches r_f , this integral yields a value of 1 when $n = m$ and zero when $n \neq m$. Conversely, the integral $\mathcal{J}_2^{nm}$ is nonzero exclusively for the case where $n = m$ and also equals 1 when $r_y = r_f$. It is important to note that, in our analysis, we have opted for the condition of equality ($r_y = r_f$) in order to avoid the complexities associated with mode mixing and to facilitate the construction of a more straightforward fermion mixing matrix [67–71]. This criterion² has been uniformly implemented across the entirety of our analysis.

Under the aforementioned equality condition ($r_y = r_f$), the mass matrix, as presented in Eq. (11), can be readily diagonalized through the application of the following biunitary transformations for both the left-handed and right-handed fields [68–71,76]:

$$\begin{aligned} U_L^{(n)} &= \begin{pmatrix} \cos \alpha_{tn} & \sin \alpha_{tn} \\ -\sin \alpha_{tn} & \cos \alpha_{tn} \end{pmatrix}, \\ U_R^{(n)} &= \begin{pmatrix} \cos \alpha_{tn} & \sin \alpha_{tn} \\ \sin \alpha_{tn} & -\cos \alpha_{tn} \end{pmatrix}. \end{aligned} \quad (12)$$

Here, $\alpha_{tn} [= \frac{1}{2} \tan^{-1}(\frac{m_t}{m_{f(n)}})]$ is defined as the mixing angle. The gauge eigenstates $\Psi_L(x, y)$ and $\Psi_R(x, y)$ can be expressed clearly in terms of mass eigenstates T_L^1 and T_L^2 through the subsequent relationships [68–71,76],

$$\begin{aligned} \phi_L^{(n)} &= \cos \alpha_{tn} T_{tL}^{1(n)} - \sin \alpha_{tn} T_{tL}^{2(n)}, \\ \phi_R^{(n)} &= \sin \alpha_{tn} T_{tL}^{1(n)} + \cos \alpha_{tn} T_{tL}^{2(n)}, \\ \chi_L^{(n)} &= \cos \alpha_{tn} T_{tR}^{1(n)} + \sin \alpha_{tn} T_{tR}^{2(n)}, \\ \chi_R^{(n)} &= \sin \alpha_{tn} T_{tR}^{1(n)} - \cos \alpha_{tn} T_{tR}^{2(n)}. \end{aligned} \quad (13)$$

The mass eigenstates $T_t^{1(n)}$ and $T_t^{2(n)}$ exhibit the same mass eigenvalue at each KK level. Note that, for the n th KK level, the mass eigenvalue is expressed as $M_{t(n)} \equiv \sqrt{m_t^2 + m_{f(n)}^2}$.

The gauge and scalar sectors are now considered. The kinetic terms of the gauge and scalar fields receive boundary corrections with respective coefficients r_W , r_B , and r_ϕ . These can be incorporated into the 5D actions as [38,67–71,74,76,78],

$$\begin{aligned} S_{\text{gauge}} &= -\frac{1}{4} \int d^5x [W_{MN}^a W^{aMN} \\ &\quad + r_W \{\delta(y) + \delta(y - \pi R)\} W_{\mu\nu}^a W^{a\mu\nu} \\ &\quad + B_{MN} B^{MN} + r_B \{\delta(y) + \delta(y - \pi R)\} B_{\mu\nu} B^{\mu\nu}], \end{aligned} \quad (14)$$

and

$$\begin{aligned} S_{\text{scalar}} &= \int d^5x [(D_M \Phi)^\dagger (D^M \Phi) \\ &\quad + r_\phi \{\delta(y) + \delta(y - \pi R)\} (D_\mu \Phi)^\dagger (D^\mu \Phi)]. \end{aligned} \quad (15)$$

The 5D field strength tensors are defined as:

$$W_{MN}^a = \partial_M W_N^a - \partial_N W_M^a - \tilde{g}_2 \epsilon^{abc} W_M^b W_N^c, \quad (16)$$

$$B_{MN} = \partial_M B_N - \partial_N B_M. \quad (17)$$

The gauge fields $W_M^a (\equiv W_\mu^a, W_4^a)$ and $B_M (\equiv B_\mu, B_4)$ ($M = 0, 1 \dots 4$) correspond to the $SU(2)_L$ and $U(1)_Y$ gauge symmetries, respectively. The 5D covariant derivative takes the form,

$$D_M = \partial_M + i\tilde{g}_2 \frac{\sigma^a}{2} W_M^a + i\tilde{g}_1 \frac{Y}{2} B_M, \quad (18)$$

with 5D gauge coupling constants $\tilde{g}_2$ and $\tilde{g}_1$. The generators of the gauge groups $SU(2)_L$ and $U(1)_Y$ are expressed as $\frac{\sigma^a}{2}$ (where $a \equiv 1, 2, 3$) and $\frac{Y}{2}$, respectively. Each of the gauge and scalar fields associated with the aforementioned actions [Eqs. (14) and (15)] can be represented through suitable KK wave functions as follows [67–71,74,76,78]:

$$\begin{aligned} V_\mu(x, y) &= \sum_n V_\mu^{(n)}(x) a^n(y), \\ V_4(x, y) &= \sum_n V_4^{(n)}(x) b^n(y), \end{aligned} \quad (19)$$

²Nevertheless, it is generally permissible to utilize unequal coefficients for the boundary terms associated with kinetic and Yukawa interactions in the context of fermions.

and

$$\Phi(x, y) = \sum_n \Phi^{(n)}(x) h^n(y), \quad (20)$$

where, both the 5D $SU(2)_L$ and $U(1)_Y$ gauge bosons are typically represented by the notation (V_μ, V_4) .

For the neutral gauge bosons, additional complexity arises due to mixing between B and W^3 components in both the bulk and boundary sectors. Diagonalization of the

corresponding kinetic terms is only tractable if one assumes³ $r_W = r_B$ [67–71,74,78], which simplifies the neutral sector similarly to the MUED case. Under this condition, the first KK-level mixing of $B^{(1)}$ and $W^{3(1)}$ yields $\gamma^{(1)}$ and $Z^{(1)}$. The photon's first KK excitation, $\gamma^{(1)}$, is stable due to KK parity and acts as the LKP, making it an excellent DM candidate [59].

To proceed consistently with quantization, the gauge fixing terms must also include the boundary terms. This leads to a gauge-fixing action of the form [67–71,74,76,78],

$$\begin{aligned} S_{\text{gf}} = & -\frac{1}{\xi_y} \int d^5x [\partial^\mu W_\mu^+ + \xi_y (\partial_y W_4^+ + iM_W \phi^+ [1 + r_V(\delta(y) + \delta(y - \pi R))])]^2 \\ & -\frac{1}{2\xi_y} \int d^5x [\partial^\mu Z_\mu + \xi_y (\partial_y Z_4 - M_Z \chi [1 + r_V(\delta(y) + \delta(y - \pi R))])]^2 \\ & -\frac{1}{2\xi_y} \int d^5x [\partial^\mu A_\mu + \xi_y \partial_y A_4]^2. \end{aligned} \quad (21)$$

Here, M_W and M_Z are the usual SM masses of $W^\pm$ and Z bosons. For a comprehensive examination of the gauge fixing action and mechanism within the framework of NMUED, we direct the reader to the work [78]. The action presented in Eq. (21) is both complex and crucial for the NMUED context, particularly as we aim to compute one-loop diagrams, which are essential for the current analysis, utilizing the Feynman gauge. Due to the BLKTs, the extra dimensional profiles of the fields acquire a nonuniform weight, necessitating adjustments in gauge fixing. This nonhomogeneity requires the introduction of a y -dependent gauge fixing parameter, denoted as ξ_y , as depicted in previous studies [67–71,74,76,78],

$$\xi = \xi_y (1 + r_V \{\delta(y) + \delta(y - \pi R)\}), \quad (22)$$

where ξ remains independent of y . This relationship can be interpreted as a form of renormalization of the gauge fixing parameter, as the BLKTs effectively serve as counterterms, accommodating the unknown ultraviolet corrections that arise in loop calculations. This redefinition [given in Eq. (22)] allows one to interpret ξ_y as the bare gauge fixing parameter and ξ as the renormalized parameter, taking values like 0 (Landau gauge), 1 (Feynman gauge), or ∞ (unitary gauge).

The condition $r_V = r_\phi$ is essential to ensure the gauge fixing procedure works consistently in the presence of BLKTs. Under this requirement, the KK masses of the gauge and scalar fields are identical [i.e., $m_{V^{(n)}} (= m_{\phi^{(n)}})$], and the mass of the charged gauge bosons at the n th KK level is [67–71,74,76,78],

$$M_{W^{(n)}} = \sqrt{M_W^2 + m_{V^{(n)}}^2}. \quad (23)$$

Likewise, in the 't-Hooft Feynman gauge, the Goldstone bosons $G^{\pm(n)}$ that correspond to $W_\mu^{\pm(n)}$ possess the same mass $M_{W^{(n)}}$ [67–71,74,76,78].

The KK decomposition of interaction terms involves overlap integrals that arise from integrating the y -dependent wave functions. These integrals modify the effective couplings between zero-mode SM fields and KK states, thereby distinguishing NMUED predictions from those of MUED. Explicit forms for these overlap integrals are provided in Appendix A. Further discussions are available in [67–71,74,76,78].

III. $B \rightarrow K^{(*)} \nu \bar{\nu}$ TRANSITION IN THE NMUED SCENARIO

The decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$ can be described at the quark level by the transition $b \rightarrow s \nu \bar{\nu}$. The effective Hamiltonian⁴ that regulates this transition, as discussed in the literature [79–81], can be appropriately adjusted within the framework of the NMUED model. Hence, utilizing the descriptions proposed in [82], the effective Hamiltonian can be written as follows:

³Nevertheless, it is generally permissible to consider the scenario where $r_W \neq r_B$. In this context, the mixing between the B and W^3 fields, both in the bulk and at the boundary points, results in the emergence of off-diagonal elements within the neutral gauge boson mass matrix.

⁴Note that in the present study, the NP scale is significantly higher than the B -meson mass scale.

$$\mathcal{H}_{\text{eff}}^{b \rightarrow s\nu\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(-\frac{X(x_t, r_f, r_V, R^{-1})}{s_W^2} \right) \times \frac{e^2}{16\pi^2} [\bar{s}\gamma_\mu P_L b] [\bar{\nu}\gamma^\mu (1 - \gamma_5)\nu] + \text{H.c.} \quad (24)$$

In the above equation G_F is designated as the Fermi constant. The product $V_{tb} V_{ts}^*$ is the Cabibbo Kobayashi Maskawa (CKM) entry appropriate for the transition $b \rightarrow s\nu\bar{\nu}$. The factor $(-\frac{X}{s_W^2})$ plays the role of WC that incorporates next-to-leading-order quantum chromodynamics (QCD) corrections [83–85] and two-loop electroweak corrections [86]. The value of $s_W^2 (= \sin^2 \theta_W)$ is 0.231 [87], where θ_W is the Weinberg angle.

For the relevant transition ($B^+ \rightarrow K^+ \nu\bar{\nu}$), the overall contribution—comprising both SM and KK-mode effects—in the NMUED framework is described by the function $X(x_t, r_f, r_V, R^{-1})$, which can be decomposed into two functions as shown below,

$$X(x_t, r_f, r_V, R^{-1}) = C(x_t, r_f, r_V, R^{-1}) + B^{\nu\bar{\nu}}(x_t, r_f, r_V, R^{-1}). \quad (25)$$

In the above expression, the function $C(x_t, r_f, r_V, R^{-1})$ arises from penguin (Fig. 1) and self-energy (Fig. 2) diagrams and can be expressed as,

$$C(x_t, r_f, r_V, R^{-1}) = C_0(x_t) + \sum_{n=1}^{\infty} C_n(x_{t(n)}, x_{u(n)}), \quad (26)$$

where, $C_0(x_t)$ represents the corresponding SM contribution [82,88]:

$$C_0(x_t) = \frac{x_t}{8} \left[\frac{x_t - 6}{x_t - 1} + \frac{3x_t + 2}{(x_t - 1)^2} \ln x_t \right]. \quad (27)$$

The pertinent contributions to the KK mode arising from the NMUED framework are represented by the second term, which is derived from the Z-penguin diagrams and self-energy diagrams, as illustrated in Figs. 1 and 2. The function $C_n(x_{t(n)}, x_{u(n)})$ can be expressed as follows [82]:

$$C_n(x_{t(n)}, x_{u(n)}) = F(x_{t(n)}) - F(x_{u(n)}). \quad (28)$$

The functions $F(x_{t(n)})$ and $F(x_{u(n)})$ are derived from the contributions of the modes $T_t^{1(n)}$, $T_t^{2(n)}$, and $T_u^{1(n)}$, $T_u^{2(n)}$, respectively. The function $F(x_{t(n)})$ is expressed in the following manner [82]:

$$F(x_{t(n)}) = \sum_{i=1}^8 F_i(x_{t(n)}) + \left(\frac{1}{2} - \frac{1}{3} \sin^2 \theta_W \right) \sum_{i=1}^2 \Delta S_i(x_{t(n)}). \quad (29)$$

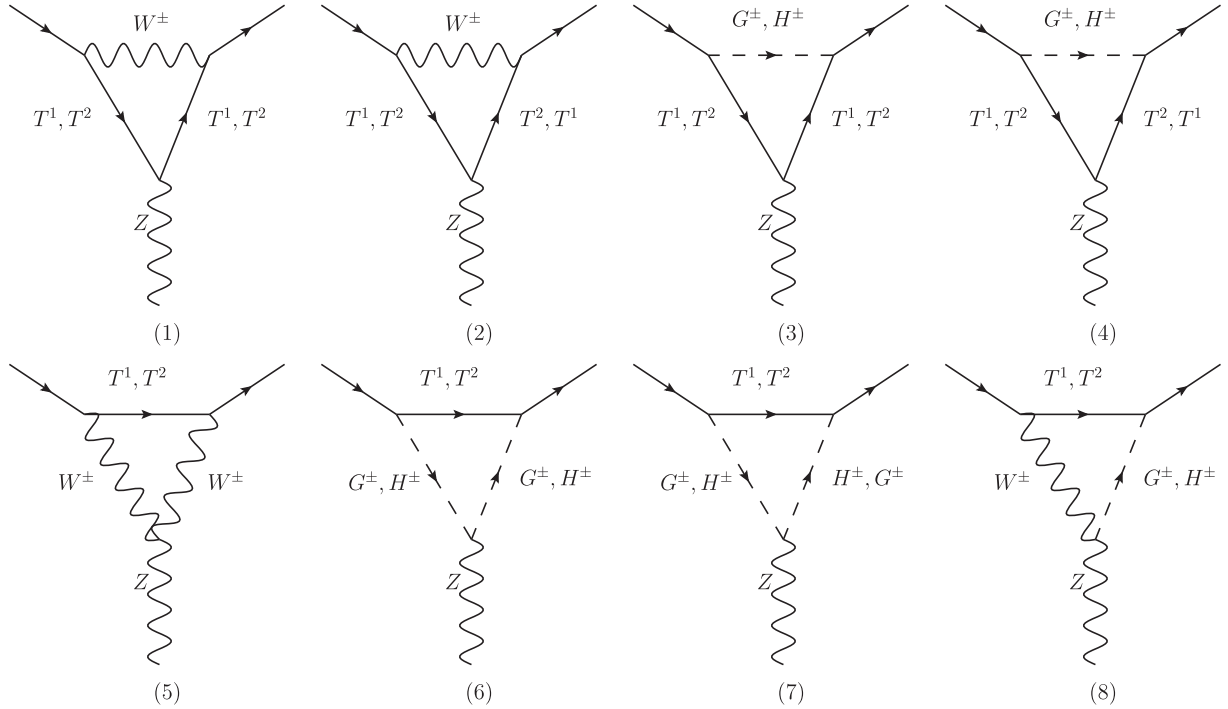
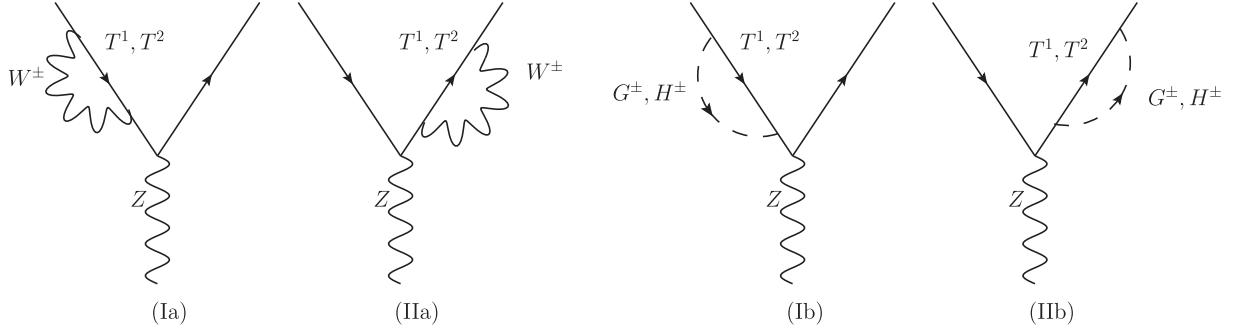


FIG. 1. Decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$ receives the contributions from the Z-penguin diagrams.


 FIG. 2. Self-energy diagrams contribute to the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$.

Furthermore, F_i represents the contributions arising from the Z-penguin diagrams (1–8 in Fig. 1), while ΔS_i denotes the contributions from self-energy diagrams [(Ia) and (IIb) in Fig. 2], which are essential for the computation of the electroweak counterterms. The function $F(x_{u(n)})$ is defined as follows:

$$F(x_{u(n)}) = F(x_{t(n)})|_{x_t \rightarrow 0}. \quad (30)$$

In the aforementioned equation [Eq. (28)], the terms $F(x_{u(n)})$ are subtracted to account for the contributions

arising from the KK excitations of the first two generations of quarks in the Z-penguin and self-energy diagrams, utilizing the GIM mechanism. In the context of the NMUED framework, the expression for $F(x_{t(n)})$ [given in Eq. (31)] is significantly distinct from that in the UED model, primarily due to the inclusion of the overlap integrals I_1^n and I_2^n [68]. The corresponding expressions can be found in Appendix A, specifically in Eqs. (A13) and (A14).

Within the NMUED framework, the function $F(x_{t(n)})$ is represented by the following expression:

$$\begin{aligned} F(x_{t(n)}) = & \left[\frac{1}{8} \left\{ -\ln M_{t(n)}^2 - \frac{3}{2} + h_q(x_{t(n)}) \right\} - \frac{1}{4} x_{t(n)} h_q(x_{t(n)}) \frac{m_{f(n)}^2}{M_{t(n)}^2} \right] (I_1^n)^2 - \frac{3}{4} \left[-\ln M_{W(n)}^2 - \frac{1}{6} - x_{t(n)} h_w(x_{t(n)}) \right] (I_1^n)^2 \\ & - \frac{1}{2} h_w(x_{t(n)}) \frac{m_{f(n)}^2}{M_{W(n)}^2} (I_1^n)^2 + \left[\frac{1}{16} \left\{ -\ln M_{t(n)}^2 - \frac{1}{2} + h_q(x_{t(n)}) \right\} (I_2^n)^2 - \frac{1}{8} x_{t(n)} h_q(x_{t(n)}) \left\{ \frac{m_{f(n)}^4}{M_{t(n)}^2 m_{V(n)}^2} + \frac{m_t^4}{M_{t(n)}^2 M_W^2} \right\} (I_1^n)^2 \right] \\ & - \frac{1}{16} [2(I_2^n)^2 + x_t (I_1^n)^2] \left[-\ln M_{W(n)}^2 + \frac{1}{2} - x_{t(n)} h_w(x_{t(n)}) \right] + \frac{1}{8} \left[-\frac{1}{2} \left\{ \frac{1+x_{t(n)}}{1-x_{t(n)}} + \frac{2x_{t(n)}^2 \ln x_{t(n)}}{(1-x_{t(n)})^2} \right\} - \ln M_{W(n)}^2 \right] (I_1^n)^2 \\ & + \frac{1}{16} [(I_2^n)^2 + x_t (I_1^n)^2] \left[\frac{1}{2} \left\{ \frac{1-3x_{t(n)}}{1-x_{t(n)}} - \frac{2x_{t(n)}^2 \ln x_{t(n)}}{(1-x_{t(n)})^2} \right\} - \ln M_{W(n)}^2 \right]. \end{aligned} \quad (31)$$

The formulations for h_q and h_w can be found in Appendix A [Eqs. (A9) and (A10)].

The function $B^{\nu\bar{\nu}}(x_t, r_f, r_V, R^{-1})$, presented in Eq. (25), is defined by⁵:

$$\begin{aligned} B^{\nu\bar{\nu}}(x_t, r_f, r_V, R^{-1}) = & -4B_0(x_t) \\ & + \sum_{n=1}^{\infty} B_n^{\nu\bar{\nu}}(x_{t(n)}, x_{u(n)}, x_{e(n)}). \end{aligned} \quad (32)$$

⁵We would like to point out that a similar expression for the UED model is available in [82].

The function $B_0(x_t)$ represents the box diagram contribution within the SM [82,88],

$$B_0(x_t) = \frac{1}{4} \left[\frac{x_t}{1-x_t} + \frac{x_t}{(x_t-1)^2} \ln x_t \right]. \quad (33)$$

The second term represents the overall contribution from the KK modes, derived from the box diagrams shown in Fig. 3. The function $B_n^{\nu\bar{\nu}}(x_{t(n)}, x_{u(n)}, x_{e(n)})$ is defined as follows:

$$\begin{aligned} B_n^{\nu\bar{\nu}}(x_{t(n)}, x_{u(n)}, x_{e(n)}) = & G(x_{t(n)}, x_{e(n)}) \\ & - G(x_{u(n)}, x_{e(n)}). \end{aligned} \quad (34)$$

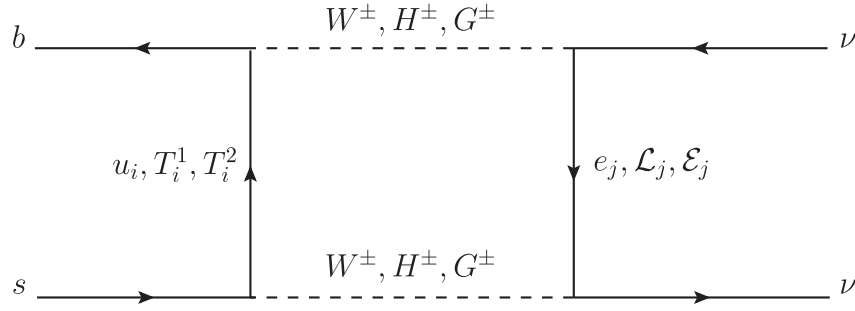


FIG. 3. Box diagrams that play a role in the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$. The indices “ i ” and “ j ” represent fermion flavor indices.

Within the NMUED framework, the expression for $G(x_{t(n)}, x_{e(n)})$ is written as⁶:

$$G(x_{t(n)}, x_{e(n)}) = (I_1^n)^4 \left[-\frac{M_W^2}{M_{W(n)}^2} U(x_{t(n)}, x_{e(n)}) + \frac{1}{2} \frac{M_W^2 M_{f(n)}^2 m_{f(n)}^2}{M_{W(n)}^6} \tilde{U}(x_{t(n)}, x_{e(n)}) \right. \\ \left. + \frac{1}{2} \frac{M_W^2 m_{f(n)}^2}{M_{W(n)}^6} \left(\frac{M_W^2 m_{f(n)}^2}{m_{V(n)}^2} - m_t^2 \right) \tilde{U}(x_{t(n)}, x_{e(n)}) - \frac{1}{8} \frac{M_W^2 m_{f(n)}^2}{M_{W(n)}^6} \left(\frac{M_W^2 m_{f(n)}^2}{m_{V(n)}^2} - m_t^2 \right) U(x_{t(n)}, x_{e(n)}) \right. \\ \left. - \frac{1}{16} \frac{M_W^2 M_{f(n)}^2 m_{f(n)}^2}{M_{W(n)}^6} U(x_{t(n)}, x_{e(n)}) - \frac{1}{16} \frac{M_W^2}{M_{W(n)}^6} \left(\frac{M_{f(n)}^4}{m_{V(n)}^4} + m_t^2 m_{f(n)}^2 \right) U(x_{t(n)}, x_{e(n)}) \right], \quad (35)$$

and the function $G(x_{u(n)}, x_{e(n)})$ is defined as follows:

$$G(x_{u(n)}, x_{e(n)}) = G(x_{t(n)}, x_{e(n)})|_{x_t \rightarrow 0}. \quad (36)$$

The formulations for the functions U and $\tilde{U}$ are presented in Appendix A [Eqs. (A21) and (A22)].

As shown in Eq. (34), the expression for $B_n^{\nu\bar{\nu}}$ incorporates contributions from KK excitations of the first two quark generations into the analytical framework.

In all the previous expressions, $x_t = \frac{m_t^2}{M_W^2}$, $x_{V(n)} = \frac{m_{V(n)}^2}{M_W^2}$, and $x_{f(n)} = \frac{m_{f(n)}^2}{M_W^2}$, where $m_{V(n)}$ and $m_{f(n)}$ can be derived from the solution of the transcendental equation given in Eq. (9).

To this end, we are now equipped to evaluate the branching fraction of the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$ in the NMUED framework. The differential branching fraction for the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$ can be expressed as follows [79–81]:

$$\frac{d\text{Br}}{dq^2}(B^+ \rightarrow K^+ \nu \bar{\nu}) = \tau_{B^+} \frac{G_F^2 \alpha_{\text{em}}^2 \lambda_K^{3/2} X^2}{256\pi^5 m_B^3 S_W^4} |V_{tb} V_{ts}^*|^2 [f_+(q^2)]^2, \quad (37)$$

⁶To the best of our knowledge, the current article presents, for the first time, the expression derived from the box diagrams (shown in Fig. 3) relevant to the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$ in the context of the NMUED model.

where $\alpha_{\text{em}} (= \frac{e^2}{4\pi})$ is the fine structure constant and $\lambda_K = m_B^4 + m_K^4 + q^4 - 2(m_B^2 m_K^2 + m_K^2 q^2 + m_B^2 q^2)$. $0 < q^2 \leq (m_B - m_K)^2$ represents the invariant mass of the dineutrinos. Moreover, $f_+(q^2)$ denotes the vector form factor for the $B^+ \rightarrow K^+$ transition, which has been discussed in Appendix C for convenience. The total branching fraction can be obtained by integrating the differential branching fraction over the entire kinematically allowed region.

It is important to highlight a significant aspect of this article. While the primary objective is to analyze the recent Belle II data [3] within the framework of the NMUED, the effective Hamiltonian presented in Eq. (24) for the $b \rightarrow s \nu \bar{\nu}$ transition also governs several other important observables, for example the branching fraction of the decay process $B \rightarrow K^* \nu \bar{\nu}$ [denoted as $\text{Br}(B \rightarrow K^* \nu \bar{\nu})$]. Furthermore, it imposes a constraint derived from the upper limit of the branching fraction reported by the Belle Collaboration [89]. Consequently, the following discussion will focus on the evaluation of the observable.

Analogous to the previous case, the branching fraction for the decay process ($B \rightarrow K^* \nu \bar{\nu}$) can be expressed as [79–81],

$$\frac{d\text{Br}}{dq^2}(B \rightarrow K^* \nu \bar{\nu}) = \tau_{B^0} \frac{G_F^2 \alpha_{\text{em}}^2 \lambda_{K^*}^{1/2} X^2}{256\pi^5 m_B^3 S_W^4} |V_{tb} V_{ts}^*|^2 \mathbb{F}(q^2), \quad (38)$$

with $\lambda_{K^*} = m_B^4 + m_{K^*}^4 + q^4 - 2(m_B^2 m_{K^*}^2 + m_{K^*}^2 q^2 + m_B^2 q^2)$, and the function $\mathbb{F}(q^2)$ is defined as,

$$\mathbb{F}(q^2) = 2q^2(m_B + m_{K^*})^2[A_1(q^2)]^2 + 64m_{K^*}^2m_B^2[A_{12}(q^2)]^2 + \frac{2q^2\lambda_{K^*}}{(m_B + m_{K^*})^2}[V(q^2)]^2. \quad (39)$$

For the sake of convenience, the form factors $A_1(q^2)$, $A_{12}(q^2)$, and $V(q^2)$, associated with the $B \rightarrow K^*$ transition, are presented in Appendix C. The overall branching fraction can be determined by integrating the differential branching fraction across the entire kinematically permissible region.

Before we proceed to the numerical analysis, we would like to emphasize some important issues. It has been previously noted that the presence of various BLTs in the NMUED action results in nontrivial modifications to the KK masses and couplings (altered by I_1^n and I_2^n) associated with KK excitations, in contrast to their UED counterparts.⁷ Therefore, it is not feasible to derive the function X_n (defined as $X_n = C_n + B_n^{\nu\bar{\nu}}$) in the NMUED context merely by rescaling the corresponding values from the UED model [82]. Consequently, we have independently calculated the KK-mode contributions within the NMUED framework utilizing Feynman diagrams (illustrated in Figs. 1–3). Furthermore, it is evident from Eqs. (31) and (35) that the KK-mode contributions in the NMUED model differ significantly from those in the UED formulation. However, if we set the boundary terms to zero (i.e., $r_V = 0$ and $r_f = 0$), we can readily recover the results of the UED model from our expressions. Moreover, as the KK number increases (i.e., for larger values of n), the masses of the fields $T_t^{1(n)}$, $T_t^{2(n)}$, $T_u^{1(n)}$, and $T_u^{2(n)}$ become nearly degenerate. This mass degeneracy leads to a suppression of the function X_n , for higher values of n . Therefore, under the NMUED framework, only a limited number of terms in the KK-mode summation have a substantial impact. Additionally, it is important to note that in the one-loop Feynman diagram calculations, we only consider interactions where the zero-mode field couples to pairs of KK excitations with identical KK numbers. In the context of KK-parity conserving NMUED, it is also possible to have nonzero interactions involving KK excitations with KK numbers n , m , and p , provided that $n + m + p$ is an even integer. Nevertheless, we have explicitly confirmed that the final results remain largely unchanged even when accounting for all potential off-diagonal interactions [67–70,76].

IV. NUMERICAL ANALYSIS

In the present study, we have, for the first time, calculated the KK contributions to the WC of the $B^+ \rightarrow K^+\nu\bar{\nu}$ (and also $B \rightarrow K^*\nu\bar{\nu}$) transition within the framework of the NMUED scenario. The function $X(x_t, r_f, r_V, R^{-1})$,

as defined in Eq. (25), encapsulates the dependence on the KK masses of both gauge bosons and fermions in the NMUED context. Furthermore, in light of the analysis regarding the impact of the SM Higgs mass on vacuum stability within the UED model [90], we have considered the sum of KK contributions up to ten KK levels.⁸ This approach contrasts with previous studies, which typically included contributions from 20 to 30 KK levels. Subsequently, we combined the total KK contributions with those from the SM.⁹ Additionally, due to the convergent nature of the KK summation in UED type models with a single extra spatial dimension, particularly in the context of one-loop calculations [91], the numerical results are unlikely to vary significantly based on the inclusion of a greater number of KK levels when evaluating the KK contributions for the loop diagrams [68–70,76]. Moreover, it is important to note that the function $X(x_t, r_f, r_V, R^{-1})$ essentially serves as the WC for the specific $b \rightarrow s\nu\bar{\nu}$ transition. Consequently, following the calculation of this function within the framework of the NMUED model, we have evaluated the numerical values of the observables associated with the $b \rightarrow s\nu\bar{\nu}$ transitions by employing the package FLAVIO [92]. This package serves exclusively as a numerical tool for the evaluation of flavor observables, depending upon the provision of the pertinent low-energy WC. All short-distance effects specific to the NMUED model are analytically incorporated into modified loop functions that contribute to the $b \rightarrow s$ effective Hamiltonian. These functions are independently calculated within the NMUED model and subsequently supplied as external inputs to FLAVIO [92].

A. Constraints and permissible range for BLT parameter values

In this analysis, we emphasize the constraints that have been taken into account.

- (i) In the context of the NMUED model, extensive analyses have been conducted on various rare B -decay processes characterized by FCNC, particularly focusing on the $b \rightarrow s$ transition. Notable decay processes include $B_s \rightarrow \mu^+\mu^-$ [68], $B \rightarrow X_s\gamma$ [69], and $B \rightarrow X_s\ell^+\ell^-$ [70]. These decay modes have been instrumental in investigating potential NP scenarios. It is significant to note that the expressions for these observables rely on a common set of parameters, namely r_V , r_f , and R^{-1} , which are also

⁷It is important to highlight that, concerning the NMUED model, the quantity of Feynman diagrams corresponds exactly to that calculated in the UED model, as elaborated in Ref. [82].

⁸Electroweak vacuum stability [90] constrains the cutoff scale in UED-like models to $\Lambda R = \mathcal{O}(5-10)$, as KK contributions drive the Higgs quartic coupling negative at higher scales. This implies that only a few low-lying KK modes contribute, making truncated KK sums theoretically consistent for phenomenological analyses.

⁹For the mass of the SM $W^\pm$ boson, we utilized $M_W = 80.369$ GeV [87], and for the mass of the SM top quark, we adopted $\bar{m}_t(m_t) = 172.56$ GeV [87].

relevant to the observables analyzed in the current study. Consequently, we incorporate the branching ratios of these rare decay processes as constraints in our analysis. Furthermore, in this article we have derived the theoretical predictions for the branching ratios $\text{Br}(B_s \rightarrow \mu^+\mu^-)$, $\text{Br}(B \rightarrow X_s\gamma)$, and $\text{Br}(B \rightarrow X_s\ell^+\ell^-)$ as outlined in [68,69,70], utilizing the package FLAVIO [92]. The corresponding relevant updated experimental data have been obtained from the references [87,93,94], respectively.

- (ii) Electroweak precision tests (EWPTs) play a crucial role in constraining various forms of BSM physics. By employing the method of correcting the Fermi constant G_F at the tree level, one can derive corrections to the Peskin-Takeuchi parameters S , T , and U within the framework of the NMUED model. This characteristic distinguishes the NMUED scenario from the minimal version of the UED model, where such corrections arise at one-loop processes. A comprehensive analysis of EWPT within the NMUED model has been presented in the Refs. [68,71]. In the NMUED model, the S , T , and U parameters are expressed as functions of r_V , r_f , and R^{-1} . Following the methodology outlined in the aforementioned Refs. [68,71], we have incorporated EWPT as a constraint in our analysis.

In this context, we aim to examine the range of values for the BLT parameters utilized in our current analysis. Generally, the BLT parameters can assume both negative and positive values. Nonetheless, as demonstrated by Eq. (8), when the ratio r_f/R is assigned the value $-\pi$, the zero-mode solution becomes divergent. Furthermore, for values of r_f/R lower than $-\pi$, the zero-mode fields display properties analogous to those of ghost fields. Consequently, values of the BLT parameters that fall below $-\pi R$ should be excluded from consideration. Nevertheless, for the sake of thoroughness, we have incorporated some negative values of the BLT parameters in our numerical analysis. However, based on the study of electroweak precision data [68,71], a large portion of negative values of the BLT parameters has been disfavored.

In the numerical analysis, all theoretical and experimental constraints outlined above are applied concurrently. During the parameter space exploration, any parameter point that fails to meet at least one of these constraints is discarded from further analysis. Only those parameter points that comply with the entire set of constraints are retained and subsequently employed to compute the relevant flavor observables and to produce the numerical results presented in the figures, such as Fig. 4. This methodological approach is consistent with the procedure utilized in our previous investigations of B -physics observables within the NMUED framework, as detailed in Refs. [68–70,76].

B. Results

At this stage, considering the previously outlined constraints, our objective is to depict the parameter space that complies with the 99% confidence level (CL) bounds for the allowed range of the branching ratio of the decay process $B^+ \rightarrow K^+\nu\bar{\nu}$, while simultaneously respecting the upper limit imposed on the branching ratio of the decay $B \rightarrow K^*\nu\bar{\nu}$. In the current version of the NMUED framework, we have three independent free parameters: the inverse radius of compactification R^{-1} , and the dimensionless scaled BLT parameters for bosons $R_V (= r_V/R)$ and fermions $R_f (= r_f/R)$. This can be elucidated as follows: as the values of the BLT parameters increase, the KK masses decrease, which in turn enhances the loop function X derived from the one-loop Feynman diagrams depicted in Figs. 1–3. Consequently, to counterbalance this enhancement, it is necessary to increase the values of R^{-1} , since the KK masses increase with larger values of R^{-1} .

Another significant aspect of the NMUED scenario is that, as a result of EWPT, the region where $R_V \approx R_f$ and higher values of R^{-1} are generally disfavored for most negative values of the BLT parameters [68,71]. In the subsequent sections, for illustrative purposes, we have selected several sets of nonzero scaled BLT parameters (R_V, R_f) that are permissible under the different constraints considered in our analysis.

Figure 4 comprises six panels that illustrate the relationship between the branching ratio $\text{Br}(B^+ \rightarrow K^+\nu\bar{\nu})$ and the parameter R^{-1} across various combinations of BLT parameters, including cases where these parameters are set to zero. When a specific curve intersects the 99% CL of the permissible range for the experimental value, we can derive the lower limits for R^{-1} . These lower limit values are presented in Table I in units of GeV. The data in this table indicate that for multiple selections of BLT parameters, we have determined lower limits for R^{-1} . For each specific BLT configuration, a lower limit on R^{-1} is established only when the corresponding theoretical curve crosses the upper experimental constraint. For instance, with $R_V = 8$ and $R_f = 16$, our analysis yields a lower limit of $R^{-1} \approx 711$ GeV. In contrast, employing a different set of BLT parameters, specifically $R_V = 24$ and $R_f = 24$, results in a lower limit of $R^{-1} \approx 902$ GeV. At the limit of KK-mode decoupling, all observable values reach saturation and exhibit no dependence on R^{-1} .

Furthermore, it is important to clarify the impact of the $\text{Br}(B \rightarrow K^*\nu\bar{\nu})$ constraint on the derived lower limits of R^{-1} , particularly in the context of the bounds presented in Table I. In general, removing a constraint enlarges the allowed parameter space, although the resulting bounds on specific parameters may either strengthen or weaken depending on the interplay among different observables. In analyses involving multiple observables, each observable can probe distinct regions of the parameter space with

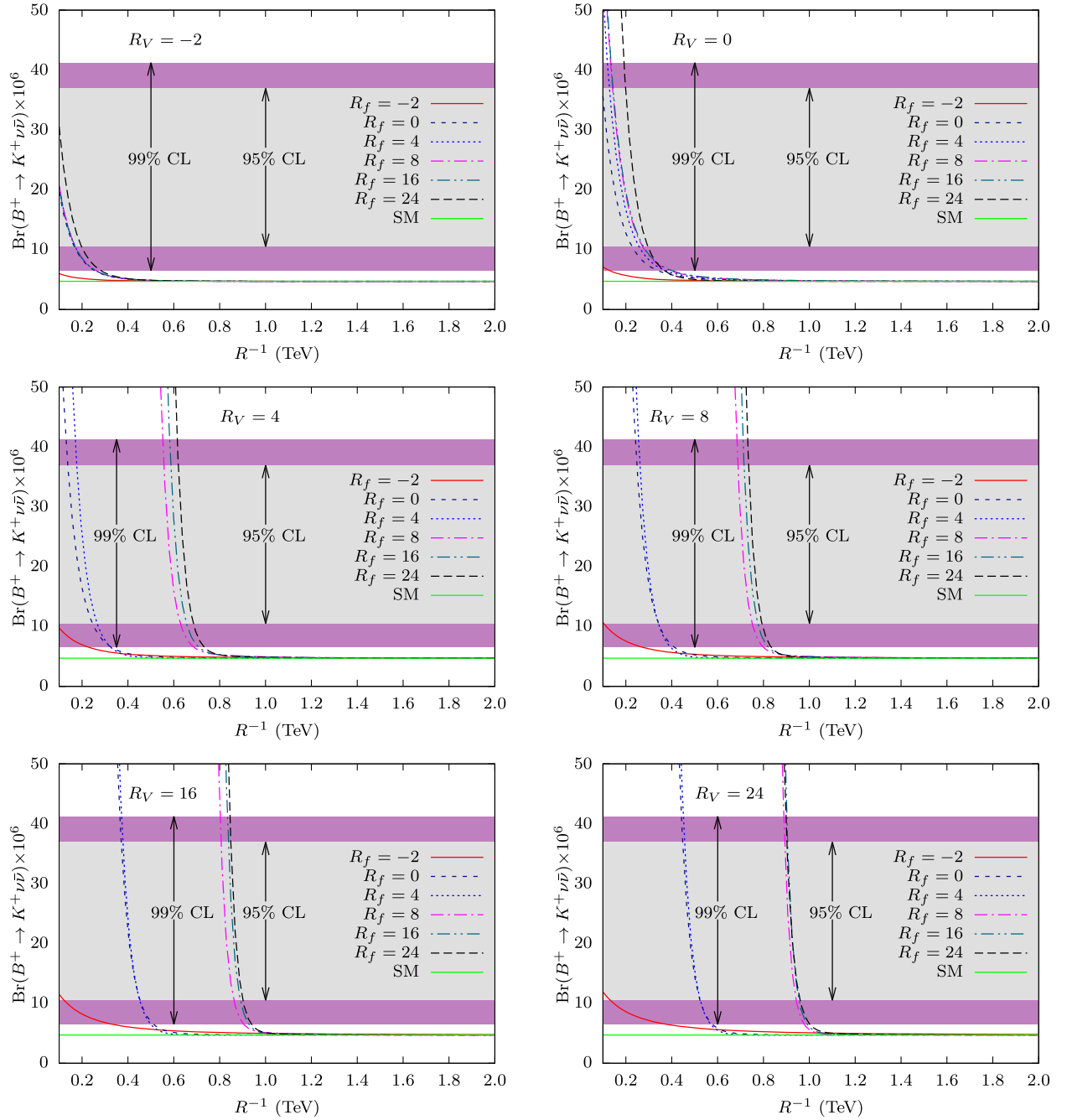


FIG. 4. Branching ratio of the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$ is analyzed as a function of R^{-1} (in TeV) for various values of $R_f (= r_f/R)$. The six panels correspond to distinct values of $R_V (= r_V/R)$. The figure exclusively presents parameter points that fulfill all the constraints outlined in Sec. IV A. In the computation of the WCs, contributions from up to ten KK-levels are incorporated through different loop functions. The horizontal grey (purple) band illustrates the 95% (99%) confidence interval of the experimentally determined branching ratio.

varying sensitivities. In the present study, the experimental upper limit on $\text{Br}(B \rightarrow K^* \nu \bar{\nu})$, as reported by the Belle Collaboration [89], favors regions corresponding to relatively smaller values of R^{-1} . Consequently, this constraint is comparatively less restrictive in the region preferred by the remaining observables, and its inclusion extends the combined allowed parameter space toward lower values of

R^{-1} . When this constraint is removed, the allowed region is instead determined by the remaining observables, which tend to prefer comparatively larger values of R^{-1} . As a result, although the total allowed parameter space is formally enlarged, the region favored by the combined analysis shifts, leading to a marginal increase in the effective lower bound on R^{-1} . This behavior reflects the

TABLE I. Lower bounds on the inverse compactification radius, R^{-1} (in GeV), have been derived from the branching ratio of the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$. These constraints are presented for various values of the BLT parameters, corresponding to the 95% and 99% CL intervals of the experimental measurements reported by the Belle II collaboration [3]. For the 99% CL interval, the analysis additionally incorporates the constraint arising from the upper limit on the branching ratio of the decay $B \rightarrow K^* \nu \bar{\nu}$, as reported by the Belle collaboration [89]. The bounds on R^{-1} are extracted based on the experimental branching ratio specified in Eq. (1), and consequently, they carry an overall uncertainty on the order of 30%, predominantly stemming from the experimental measurement uncertainties.

R_f	$R_V = 4$		$R_V = 8$		$R_V = 16$		$R_V = 24$	
	95% CL	99% CL	95% CL	99% CL	95% CL	99% CL	95% CL	99% CL
0	137	131	248	242	371	365	447	441
4	176	171	262	255	378	374	457	451
8	557	552	688	684	807	803	892	888
16	588	583	715	711	840	834	903	898
24	619	615	736	731	848	845	905	902

nontrivial interplay between different observables, rather than a simple monotonic dependence on the number of constraints. For the specified sets of BLT parameters, the resulting lower bounds on R^{-1} are approximately 715 GeV and 905 GeV, respectively, both of which remain consistent with the 95% confidence level allowed range for the branching ratio of $B^+ \rightarrow K^+ \nu \bar{\nu}$.

For the purpose of comparison, it is pertinent to note that in the UED scenario, electroweak precision measurements and direct collider searches [95] generally establish a lower limit on the compactification scale of approximately $R^{-1} \sim \mathcal{O}(1 \text{ TeV})$. However, these limits are known to be influenced by the specific mass spectrum and decay patterns involved [95]. In contrast, within the NMUED framework, the inclusion of BLTs can significantly alter the KK-mass spectrum and interaction strengths. This modification permits a relaxation of the aforementioned bounds, potentially lowering the R^{-1} to a few hundred GeV, depending upon suitable selections of BLT parameters. Within this context, our analysis reveals that the theoretical predictions for the branching ratio of the decay $B^+ \rightarrow K^+ \nu \bar{\nu}$ remain consistent with current experimental constraints across the entire parameter space that satisfies all imposed conditions. Therefore, although this decay channel offers valuable complementary insights, it does not, in the present analysis, independently yield the most stringent lower bound on R^{-1} when compared to the results presented in [76]. Nonetheless, the obtained bounds remain broadly consistent with those reported in [68–70,76].

Additionally, for the sake of completeness, the six panels in Fig. 4 depict the variation of $\text{Br}(B^+ \rightarrow K^+ \nu \bar{\nu})$ for both zero and negative values of the BLT parameters. However, since the branching ratio curves corresponding to these combinations of BLT parameters do not intersect the upper limits (for both 95% CL and 99% CL) of the allowed experimental results, we are unable to establish lower limits for R^{-1} in these cases. Therefore, Table I provides the lower

limit values of R^{-1} (for both 95% CL and 99% CL) corresponding to the combinations of BLT parameters where $\text{Br}(B^+ \rightarrow K^+ \nu \bar{\nu})$ intersects the 95% and 99% CL of the permissible range of the experimental value.

In this context, we wish to address the lower bounds on R^{-1} within the framework of UED, particularly in light of the current analysis concerning the branching ratio of the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$. Our analysis yields UED results when the BLT parameters are set to zero, specifically for $R_V = R_f = 0$. Under these conditions, the KK mass for the n th KK level is expressed as nR^{-1} , while the overlap integrals I_1^n and I_2^n equal 1. Consequently, the function X_n simplifies to its UED form. We have verified that when the BLT parameters are nullified, the expression for the function X_n aligns precisely with that presented in Ref. [82].¹⁰ However, our current analysis reveals a significant contrast from our previous findings [68–71,76]. Specifically, for the case where R^{-1} is evaluated with $R_V = R_f = 0$, the branching ratio curve fails to intersect the upper limits (for both 95% and 99% CL) of the permissible experimental values. Consequently, we are unable to establish a lower limit for R^{-1} in the context of the branching ratio results for the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$.

Focusing on a specific panel that represents a positive value of R_V , it is evident that both vanishing and negative BLT parameters do not impose any lower limit on R^{-1} ; thus, our analysis is confined to relevant positive BLT parameters. If we consider a vertical line corresponding to a fixed value of R^{-1} , it becomes apparent that the branching ratio decreases as R_f increases. This trend is expected, as the masses of the KK states diminish with increasing BLT parameters. Consequently, a lower KK mass results in a

¹⁰In the work by Buras *et al.* [82], the authors did not account for any radiative corrections to the KK masses in their analysis, resulting in the KK mass for the n th KK mode being nR^{-1} .

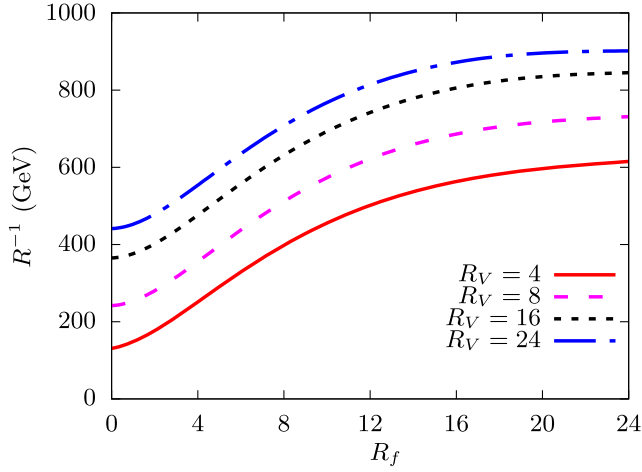


FIG. 5. This figure illustrates the exclusion contours derived from the branching ratio of the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$ within the $R_f - R^{-1}$ plane for four distinct values of R_V . The exclusion curves are constructed based on the lower limit values of R^{-1} , while incorporating contributions from up to ten KK levels in various loop functions necessary for the computation of WCs. The regions below each specific curve (corresponding to a fixed R_V) have been excluded based on the experimental measurements of the branching ratio with 99% CL.

enhanced branching ratio. Therefore, to achieve a constant branching ratio at elevated values of R^{-1} , it is necessary to utilize higher values of R_f . This effect is further amplified by increasing values of R_V .

In Fig. 5, we present the parameter space that has been excluded based on the experimentally determined branching ratio for the decay process $B^+ \rightarrow K^+ \nu \bar{\nu}$. The figure illustrates contours corresponding to four distinct values of R_V within the $R_f - R^{-1}$ plane. The region below every contour line is ruled out with 99% CL. The characteristics of these contours can be elucidated by referring to Fig. 4.

V. SUMMARY

In this study, we analyze the Kaluza-Klein contributions to the transitions $B \rightarrow K^{(*)} \nu \bar{\nu}$ within a specific framework of $(4 + 1)$ -dimensional universal extra dimensions, wherein all Standard Model particles are permitted to propagate along a additional spatial dimension. This analysis is conducted in the context of BLTs, whose coefficients are parametrized to account for the unknown radiative corrections affecting the masses and couplings of the Kaluza-Klein modes. The inclusion of these boundary terms leads to nontrivial modifications of the masses and coupling strengths in the four-dimensional effective theory, deviating from the minimal formulation of the universal extra dimensional model. By employing two distinct types of BLT parameters—namely, r_V , which denotes the coefficients of boundary terms associated with the gauge and Higgs sectors, and r_f , which characterizes the coefficients of

boundary terms related to fermions and Yukawa interactions—we investigate the $B \rightarrow K^{(*)} \nu \bar{\nu}$ transitions within the nonminimal universal extra dimensional framework.

The effective Hamiltonian governing the $B \rightarrow K^{(*)} \nu \bar{\nu}$ transitions can be formulated in terms of four-fermion interactions, with the coefficients of these interactions being characterized by appropriate Wilson coefficients. Through the analysis of one-loop Feynman diagrams, as illustrated in Figs. 1–3, we have computed the coefficient associated with the operator responsible for the $B \rightarrow K^{(*)} \nu \bar{\nu}$ transitions. Furthermore, by employing the Glashow-Iliopoulos-Maiani mechanism, we have incorporated contributions from all three generations of quarks into our evaluation. Additionally, in light of a recent study that connects the Higgs boson mass with the cutoff in a universal extra dimensional framework [90], we have included contributions from up to ten Kaluza-Klein modes in our calculations. The Kaluza-Klein contributions derived from the relevant one-loop Feynman diagrams have been combined with the corresponding contributions from the Standard Model (zero-mode) to provide a comprehensive analysis.

Following an assessment of the function X , derived from one-loop Feynman diagrams, we have calculated the branching fraction for the decay $B^+ \rightarrow K^+ \nu \bar{\nu}$ within the framework of the nonminimal universal extra dimensional scenario. By comparing our theoretical predictions with the 99% confidence level intervals of the experimentally allowed ranges [3], we have successfully imposed constraints on the parameter space of the present version of the nonminimal universal extra dimensional model. Additionally, we take into account the upper limit on the branching fraction for the decay $B \rightarrow K^* \nu \bar{\nu}$ as reported by the Belle collaboration [89], which serves as a significant constraint. This decay, occurring at the quark level, is governed by the same transition and characterized by the same Wilson coefficient as that involved in the decay $B^+ \rightarrow K^+ \nu \bar{\nu}$.

It has been previously indicated that in the case of vanishing BLT parameters (i.e., $R_f = R_V = 0$), the outcomes align with those of the minimal version of the universal extra dimensional scenario. Consequently, we have endeavored to establish a lower bound on R^{-1} within the context of the minimal universal extra dimensional scenario under the condition of vanishing BLT parameters. However, our numerical analysis clearly demonstrates that the curve representing the branching ratio for the decay $B^+ \rightarrow K^+ \nu \bar{\nu}$ (when $R_f = R_V = 0$) does not intersect with the upper limit of the experimental data. As a result, we are unable to ascertain a lower limit for R^{-1} in the universal extra dimensional scenario. This finding stands in stark contrast to our previous investigations in the realm of B physics.

In the context of the nonminimal universal extra dimensional scenario, the incorporation of various nonvanishing BLT parameters allows for an enhancement of the lower

limit on R^{-1} . For instance, by selecting a specific set of BLT parameters, namely $R_V = 8$ and $R_f = 16$, our analysis yields a lower limit of $R^{-1} = 710.98$ GeV. Conversely, utilizing another set of BLT parameters, $R_V = 24$ and $R_f = 24$, results in a lower limit of $R^{-1} = 901.78$ GeV. These lower limit values are consistent with those derived from our previous investigations into rare decays of B mesons [68–70,76]. Furthermore, it is noteworthy that the lower limits of R^{-1} for the same BLT parameter sets may experience a slight increase if the upper limit on the branching fraction for the decay $B \rightarrow K^* \nu \bar{\nu}$, as reported by the Belle collaboration, is removed.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: THE VARIOUS FEYNMAN DIAGRAMS THAT ARE NOT CONTRIBUTING TO THE $B \rightarrow K^{(*)} \nu \bar{\nu}$

1. The Z-penguin and self-energy contributions to the functions C_n , shown in Figs. 1 and 2, are given below

The subsequent expressions, obtained from Fig. 1, correspond to the contributions of the Z-penguin diagrams to the functions C_n ,

$$F_1(x_{f(n)}) = \frac{1}{8} \left(c_{fn}^4 + s_{fn}^4 - \frac{4}{3} s_W^2 \right) \left[\Delta + \ln \frac{\mu^2}{M_{f(n)}^2} - \frac{3}{2} + h_q(x_{f(n)}) - 2x_{f(n)} h_q(x_{f(n)}) \right] (I_1^n)^2, \quad (\text{A1})$$

$$F_2(x_{f(n)}) = \frac{1}{4} c_{fn}^2 s_{fn}^2 \left[\Delta + \ln \frac{\mu^2}{M_{f(n)}^2} - \frac{3}{2} + h_q(x_{f(n)}) + 2x_{f(n)} h_q(x_{f(n)}) \right] (I_1^n)^2, \quad (\text{A2})$$

$$F_3(x_{f(n)}) = \frac{1}{16M_{W(n)}^2} \left[((m_1^{(f)})^2 + (m_3^{(f)})^2) \left(c_{fn}^2 - \frac{4}{3} s_W^2 \right) + ((m_2^{(f)})^2 + (m_4^{(f)})^2) \left(s_{fn}^2 - \frac{4}{3} s_W^2 \right) \right] \times \left[\Delta + \ln \frac{\mu^2}{M_{f(n)}^2} - \frac{1}{2} + h_q(x_{f(n)}) - 2x_{f(n)} h_q(x_{f(n)}) \right], \quad (\text{A3})$$

$$F_4(x_{f(n)}) = -\frac{1}{8M_{W(n)}^2} (m_1^{(f)} m_2^{(f)} + m_3^{(f)} m_4^{(f)}) c_{fn} s_{fn} \left[\Delta + \ln \frac{\mu^2}{M_{f(n)}^2} - \frac{1}{2} + h_q(x_{f(n)}) + 2x_{f(n)} h_q(x_{f(n)}) \right], \quad (\text{A4})$$

$$F_5(x_{f(n)}) = -\frac{3}{4} c_W^2 \left[\Delta + \ln \frac{\mu^2}{M_{W(n)}^2} - \frac{1}{6} - x_{f(n)} h_w(x_{f(n)}) \right] (I_1^n)^2, \quad (\text{A5})$$

$$F_6(x_{f(n)}) = -\frac{1}{16M_{W(n)}^4} [(1 - 2s_W^2) M_W^2 + 2c_W^2 m_{V(n)}^2] ((m_1^{(f)})^2 + (m_2^{(f)})^2) + ((1 - 2s_W^2) m_{V(n)}^2 + 2c_W^2 M_W^2) ((m_3^{(f)})^2 + (m_4^{(f)})^2) \left[\Delta + \ln \frac{\mu^2}{M_{W(n)}^2} + \frac{1}{2} - x_{f(n)} h_w(x_{f(n)}) \right], \quad (\text{A6})$$

$$F_7(x_{f(n)}) = \frac{M_W m_{V(n)}}{8M_{W(n)}^4} (m_1^{(f)} m_3^{(f)} + m_2^{(f)} m_4^{(f)}) \left[\Delta + \ln \frac{\mu^2}{M_{W(n)}^2} + \frac{1}{2} - x_{f(n)} h_w(x_{f(n)}) \right], \quad (\text{A7})$$

$$F_8(x_{f(n)}) = \frac{M_{f(n)}}{2M_{W(n)}^4} [(s_W^2 M_W^2 - c_W^2 m_{V(n)}^2) (m_1^{(f)} c_{fn} + m_2^{(f)} s_{fn}) + M_W m_{V(n)} (m_3^{(f)} c_{fn} + m_4^{(f)} s_{fn})] h_w(x_{f(n)}) I_1^n, \quad (\text{A8})$$

with the functions h_q and h_w defined as,

$$h_q(x) = \frac{1}{1-x} + \frac{\ln x}{(1-x)^2}, \quad (\text{A9})$$

$$h_w(x) = \frac{1}{1-x} + \frac{x \ln x}{(1-x)^2}. \quad (\text{A10})$$

The following expressions, derived from Fig. 2, represent the self-energy diagram contributions to the functions C_n :

$$\Delta S_1(x_{f(n)}) = \frac{1}{4} \left[\Delta - \frac{1}{2} \left\{ \frac{1+x_{f(n)}}{1-x_{f(n)}} + \frac{2x_{f(n)}^2 \ln x_{f(n)}}{(1-x_{f(n)})^2} \right\} - \ln \frac{M_{W(n)}^2}{\mu^2} \right] (I_1^n)^2, \quad (\text{A11})$$

$$\Delta S_2(x_{f(n)}) = \frac{1}{8} [(I_2^n)^2 + (I_1^n)^2 x_f] \left[\Delta + \frac{1}{2} \left\{ \frac{1-3x_{f(n)}}{1-x_{f(n)}} - \frac{2x_{f(n)}^2 \ln x_{f(n)}}{(1-x_{f(n)})^2} \right\} - \ln \frac{M_{W(n)}^2}{\mu^2} \right]. \quad (\text{A12})$$

Here, $\Delta = \frac{2}{\epsilon} + \ln 4\pi - \gamma_E$, $D = 4 - \epsilon$.

Note that the quantities I_1^n and I_2^n , which appear in all the Feynman vertices, are the overlap integrals given in Eqs. (A13) and (A14), respectively [68],

$$I_1^n = 2 \sqrt{\frac{1 + \frac{r_V}{\pi R}}{1 + \frac{r_f}{\pi R}}} \left[\frac{1}{\sqrt{1 + \frac{r_f^2 m_{f(n)}^2}{4} + \frac{r_f}{\pi R}}} \right] \left[\frac{1}{\sqrt{1 + \frac{r_V^2 m_{V(n)}^2}{4} + \frac{r_V}{\pi R}}} \right] \frac{m_{V(n)}^2}{(m_{V(n)}^2 - m_{f(n)}^2)} \frac{(r_f - r_V)}{\pi R}, \quad (\text{A13})$$

$$I_2^n = 2 \sqrt{\frac{1 + \frac{r_V}{\pi R}}{1 + \frac{r_f}{\pi R}}} \left[\frac{1}{\sqrt{1 + \frac{r_f^2 m_{f(n)}^2}{4} + \frac{r_f}{\pi R}}} \right] \left[\frac{1}{\sqrt{1 + \frac{r_V^2 m_{V(n)}^2}{4} + \frac{r_V}{\pi R}}} \right] \frac{m_{V(n)} m_{f(n)}}{(m_{V(n)}^2 - m_{f(n)}^2)} \frac{(r_f - r_V)}{\pi R}. \quad (\text{A14})$$

2. The box diagram contributions to the functions $B_n^{\nu\bar{\nu}}$, as depicted in Fig. 3, are given by

$$G_{WW(n)} = -\frac{M_W^2}{M_{W(n)}^2} U(x_{f(n)}, x_{e(n)}) (I_1^n)^4, \quad (\text{A15})$$

$$G_{WG(n)} = \frac{1}{2} \frac{M_W^2 M_{f(n)} m_{e(n)}}{M_{W(n)}^6} [m_1^{(f)} c_{f(n)} + m_2^{(f)} s_{f(n)}] m_1^{(e)} \tilde{U}(x_{f(n)}, x_{e(n)}) (I_1^n)^2, \quad (\text{A16})$$

$$G_{WH(n)} = \frac{1}{2} \frac{M_W^2 M_{f(n)} m_{e(n)}}{M_{W(n)}^6} [m_3^{(f)} c_{f(n)} + m_4^{(f)} s_{f(n)}] m_3^{(e)} \tilde{U}(x_{f(n)}, x_{e(n)}) (I_1^n)^2, \quad (\text{A17})$$

$$G_{GH(n)} = -\frac{1}{8} \frac{M_W^2}{M_{W(n)}^6} [m_1^{(f)} m_3^{(f)} + m_2^{(f)} m_4^{(f)}] m_1^{(e)} m_3^{(e)} U(x_{f(n)}, x_{e(n)}), \quad (\text{A18})$$

$$G_{GG(n)} = -\frac{1}{16} \frac{M_W^2}{M_{W(n)}^6} [(m_1^{(f)})^2 + (m_2^{(f)})^2] (m_1^{(e)})^2 U(x_{f(n)}, x_{e(n)}), \quad (\text{A19})$$

$$G_{HH(n)} = -\frac{1}{16} \frac{M_W^2}{M_{W(n)}^6} [(m_3^{(f)})^2 + (m_4^{(f)})^2] (m_3^{(e)})^2 U(x_{f(n)}, x_{e(n)}). \quad (\text{A20})$$

The definitions of the functions U and $\tilde{U}$ are given by,

$$U(x_t, x_u) = \frac{x_t^2 \log x_t}{(x_t - x_u)(1 - x_t)^2} + \frac{x_u^2 \log x_u}{(x_u - x_t)(1 - x_u)^2} + \frac{1}{(1 - x_u)(1 - x_t)}, \quad (\text{A21})$$

$$\tilde{U}(x_t, x_u) = \frac{x_t \log x_t}{(x_t - x_u)(1 - x_t)^2} + \frac{x_u \log x_u}{(x_u - x_t)(1 - x_u)^2} + \frac{1}{(1 - x_u)(1 - x_t)}. \quad (\text{A22})$$

APPENDIX B: FEYNMAN RULES FOR $B \rightarrow K^{(*)}\nu\bar{\nu}$ IN THE NMUED FRAMEWORK

The relevant Feynman rules for various interaction vertices are listed below, assuming all momenta and fields to be incoming. While similar Feynman rules are discussed in [82], they have been appropriately modified in the context of the NMUED model, as presented in [68].

- (1) $Z^\mu W^{\nu\pm} S^\mp: \frac{g_2}{c_W M_{W(n)}} g_{\mu\nu} C$, where C is expressed as follows:

$$\begin{aligned} Z^\mu W^{(n)+} G^{(n)-}: C &= -s_W^2 M_W^2 + c_W^2 m_{V(n)}^2, \\ Z^\mu W^{(n)-} G^{(n)+}: C &= s_W^2 M_W^2 - c_W^2 m_{V(n)}^2, \\ Z^\mu W^{(n)+} H^{(n)-}: C &= -i M_W m_{V(n)}, \\ Z^\mu W^{(n)-} H^{(n)+}: C &= i M_W m_{V(n)}. \end{aligned} \quad (B1)$$

- (2) $Z^\mu S_1^\pm S_2^\mp: \frac{ig_2}{2c_W M_{W(n)}^2} (k_2 - k_1)_\mu C$, where C is expressed as follows:

$$\begin{aligned} Z^\mu G^{(n)+} G^{(n)-}: C &= -(c_W^2 - s_W^2) M_W^2 - 2c_W^2 m_{V(n)}^2, \\ Z^\mu H^{(n)+} H^{(n)-}: C &= -2c_W^2 M_W^2 - (c_W^2 - s_W^2) m_{V(n)}^2, \\ Z^\mu G^{(n)+} H^{(n)-}: C &= i M_W m_{V(n)}, \\ Z^\mu G^{(n)-} H^{(n)+}: C &= -i M_W m_{V(n)}. \end{aligned} \quad (B2)$$

In this context, the scalar fields are presented as $S \equiv H, G$.

- (3) $Z^\mu (k_1) W^{\nu+} (k_2) W^{\lambda-} (k_3):$
- $$ig_2 c_W [g_{\mu\nu} (k_2 - k_1)_\lambda + g_{\mu\lambda} (k_1 - k_3)_\nu + g_{\lambda\nu} (k_3 - k_2)_\mu]. \quad (B3)$$

- (4) $Z^\mu \bar{f}_1 f_2: \frac{ig_2}{6c_W} \gamma_\mu (P_L C_L + P_R C_R)$, where C_L and C_R are expressed as follows:

$$\begin{aligned} Z^\mu \bar{u}_i u_i: \begin{cases} C_L = 3 - 4s_W^2, \\ C_R = -4s_W^2, \end{cases} & Z^\mu \bar{d}_j d_j: \begin{cases} C_L = -3 + 2s_W^2, \\ C_R = 2s_W^2, \end{cases} \\ Z^\mu \bar{\nu}_i \nu_i: \begin{cases} C_L = 3, \\ C_R = 0, \end{cases} & Z^\mu \bar{e}_j e_j: \begin{cases} C_L = -3 + 6s_W^2, \\ C_R = 6s_W^2, \end{cases} \\ Z^\mu \bar{T}_i^{1(n)} T_i^{1(n)}: \begin{cases} C_L = -4s_W^2 + 3c_{\text{in}}^2, \\ C_R = -4s_W^2 + 3c_{\text{in}}^2, \end{cases} & Z^\mu \bar{T}_i^{2(n)} T_i^{2(n)}: \begin{cases} C_L = -4s_W^2 + 3s_{\text{in}}^2, \\ C_R = -4s_W^2 + 3s_{\text{in}}^2, \end{cases} \\ Z^\mu \bar{T}_i^{1(n)} T_i^{2(n)}: \begin{cases} C_L = -3s_{\text{in}} c_{\text{in}}, \\ C_R = 3s_{\text{in}} c_{\text{in}}, \end{cases} & Z^\mu \bar{T}_i^{2(n)} T_i^{1(n)}: \begin{cases} C_L = -3s_{\text{in}} c_{\text{in}}, \\ C_R = 3s_{\text{in}} c_{\text{in}}, \end{cases} \end{aligned} \quad (B4)$$

- (5) $S^\pm \bar{f}_1 f_2 = \frac{g_2}{\sqrt{2} M_{W(n)}} (P_L C_L + P_R C_R)$, where C_L and C_R are expressed as follows:

$$\begin{aligned} G^+ \bar{u}_i d_j: \begin{cases} C_L = -m_i V_{ij}, \\ C_R = m_j V_{ij}, \end{cases} & G^- \bar{d}_j u_i: \begin{cases} C_L = -m_j V_{ij}^*, \\ C_R = m_i V_{ij}^*, \end{cases} \\ G^{(n)+} \bar{T}_i^{1(n)} d_j: \begin{cases} C_L = -m_1^{(i)} V_{ij}, \\ C_R = M_1^{(i,j)} V_{ij}, \end{cases} & G^{(n)-} \bar{d}_j T_i^{1(n)}: \begin{cases} C_L = -M_1^{(i,j)} V_{ij}^*, \\ C_R = m_1^{(i)} V_{ij}^*, \end{cases} \\ G^{(n)+} \bar{T}_i^{2(n)} d_j: \begin{cases} C_L = m_2^{(i)} V_{ij}, \\ C_R = -M_2^{(i,j)} V_{ij}, \end{cases} & G^{(n)-} \bar{d}_j T_i^{2(n)}: \begin{cases} C_L = M_2^{(i,j)} V_{ij}^*, \\ C_R = -m_2^{(i)} V_{ij}^*, \end{cases} \\ G^+ \bar{\nu}_i e_j: \begin{cases} C_L = 0, \\ C_R = m_j \delta_{ij}, \end{cases} & G^- \bar{e}_j \nu_i: \begin{cases} C_L = -m_j \delta_{ij}, \\ C_R = 0, \end{cases} \\ G^{(n)+} \bar{\nu}_i \mathcal{L}_j^{(n)}: \begin{cases} C_L = 0, \\ C_R = m_1^{(j)} \delta_{ij}, \end{cases} & G^{(n)-} \bar{\mathcal{L}}_j^{(n)} \nu_i: \begin{cases} C_L = -m_1^{(j)} \delta_{ij}, \\ C_R = 0, \end{cases} \\ G^{(n)+} \bar{\nu}_i \mathcal{E}_j^{(n)}: \begin{cases} C_L = 0, \\ C_R = -m_2^{(j)} \delta_{ij}, \end{cases} & G^{(n)-} \bar{\mathcal{E}}_j^{(n)} \nu_i: \begin{cases} C_L = m_2^{(j)} \delta_{ij}, \\ C_R = 0, \end{cases} \\ H^{(n)+} \bar{T}_i^{1(n)} d_j: \begin{cases} C_L = -m_3^{(i)} V_{ij}, \\ C_R = M_3^{(i,j)} V_{ij}, \end{cases} & H^{(n)-} \bar{d}_j T_i^{1(n)}: \begin{cases} C_L = -M_3^{(i,j)} V_{ij}^*, \\ C_R = m_3^{(i)} V_{ij}^*, \end{cases} \end{aligned}$$

$$\begin{aligned}
H^{(n)+}\bar{T}_i^{2(n)}d_j: & \begin{cases} C_L = m_4^{(i)}V_{ij}, \\ C_R = -M_4^{(i,j)}V_{ij}, \end{cases} & H^{(n)-}\bar{d}_jT_i^{2(n)}: & \begin{cases} C_L = M_4^{(i,j)}V_{ij}^*, \\ C_R = -m_4^{(i)}V_{ij}^*, \end{cases} \\
H^{(n)+}\bar{\nu}_i\mathcal{L}_j^{(n)}: & \begin{cases} C_L = 0, \\ C_R = m_3^{(j)}\delta_{ij}, \end{cases} & H^{(n)-}\bar{\mathcal{L}}_j^{(n)}\nu_i: & \begin{cases} C_L = -m_3^{(j)}\delta_{ij}, \\ C_R = 0, \end{cases} \\
H^{(n)+}\bar{\nu}_i\mathcal{E}_j^{(n)}: & \begin{cases} C_L = 0, \\ C_R = -m_4^{(j)}\delta_{ij}, \end{cases} & H^{(n)-}\bar{\mathcal{E}}_j^{(n)}\nu_i: & \begin{cases} C_L = m_4^{(j)}\delta_{ij}, \\ C_R = 0. \end{cases}
\end{aligned} \tag{B5}$$

In this context, the fermion fields are presented as $f \equiv u, d, T_i^1, T_i^2, \nu, e, \mathcal{L}_\nu, \mathcal{L}_e, \mathcal{E}$.

(6) $W^{\mu\pm}\bar{f}_1f_2: \frac{ig_2}{\sqrt{2}}\gamma_\mu P_L C_L$, where C_L is expressed as follows:

$$\begin{aligned}
W^{\mu+}\bar{u}_id_j: & C_L = V_{ij}, & W^{\mu-}\bar{d}_ju_i: & C_L = V_{ij}^*, \\
W^{\mu(n)+}\bar{T}_i^{1(n)}d_j: & C_L = I_1^n c_{\text{in}} V_{ij}, & W^{\mu(n)-}\bar{d}_jT_i^{1(n)}: & C_L = I_1^n c_{\text{in}} V_{ij}^*, \\
W^{\mu(n)+}\bar{T}_i^{2(n)}d_j: & C_L = -I_1^n s_{\text{in}} V_{ij}, & W^{\mu(n)-}\bar{d}_jT_i^{2(n)}: & C_L = -I_1^n s_{\text{in}} V_{ij}^*, \\
W^{\mu+}\bar{\nu}_ie_j: & C_L = \delta_{ij}, & W^{\mu-}\bar{e}_j\nu_i: & C_L = \delta_{ij}, \\
W^{\mu(n)+}\bar{\nu}_i\mathcal{L}_j^{(n)}: & C_L = I_1^n \delta_{ij}, & W^{\mu(n)-}\bar{\mathcal{L}}_j^{(n)}\nu_i: & C_L = I_1^n \delta_{ij}, \\
W^{\mu(n)+}\bar{\nu}_i\mathcal{E}_j^{(n)}: & C_L = 0, & W^{\mu(n)-}\bar{\mathcal{E}}_j^{(n)}\nu_i: & C_L = 0.
\end{aligned} \tag{B6}$$

The mass parameters $m_x^{(i)}$ are defined by the following expressions:

$$\begin{aligned}
m_1^{(i)} &= I_2^n m_{V^{(n)}} c_{\text{in}} + I_1^n m_i s_{\text{in}}, \\
m_2^{(i)} &= -I_2^n m_{V^{(n)}} s_{\text{in}} + I_1^n m_i c_{\text{in}}, \\
m_3^{(i)} &= -I_2^n i M_W c_{\text{in}} + I_1^n i \frac{m_{V^{(n)}} m_i}{M_W} s_{\text{in}}, \\
m_4^{(i)} &= I_2^n i M_W s_{\text{in}} + I_1^n i \frac{m_{V^{(n)}} m_i}{M_W} c_{\text{in}}.
\end{aligned} \tag{B7}$$

Here, m_i denotes the mass of the zero-mode *up-type* fermion, while $c_{\text{in}} = \cos(\alpha_{\text{in}})$ and $s_{\text{in}} = \sin(\alpha_{\text{in}})$, with α_{in} defined earlier.

The mass parameters $M_x^{(i,j)}$ are given by the following expressions:

$$\begin{aligned}
M_1^{(i,j)} &= I_1^n m_j c_{\text{in}}, \\
M_2^{(i,j)} &= I_1^n m_j s_{\text{in}}, \\
M_3^{(i,j)} &= I_1^n i \frac{m_{V^{(n)}} m_j}{M_W} c_{\text{in}}, \\
M_4^{(i,j)} &= I_1^n i \frac{m_{V^{(n)}} m_j}{M_W} s_{\text{in}},
\end{aligned} \tag{B8}$$

where m_j denotes the mass of the zero-mode *down-type* fermion.

APPENDIX C: FORM FACTORS RELEVANT TO THE $B \rightarrow K^{(*)}\nu\bar{\nu}$ TRANSITIONS

This appendix provides a brief overview of the $B \rightarrow K^{(*)}$ form factors relevant to the rare B -meson decays studied herein.

1. Details of the form factors for $B^+ \rightarrow K^+\nu\bar{\nu}$ transition

The long-range effects associated with the hadronic dynamics of the decay process $B^+ \rightarrow K^+\nu\bar{\nu}$ are characterized by the subsequent matrix elements [96,97],

$$\begin{aligned}
\langle K^+(p') | \bar{s}\gamma^\mu b | B^+(p) \rangle \\
= f_+(s)(p+p')^\mu + [f_0(s) - f_+(s)] \frac{m_B^2 - m_K^2}{q^2} q^\mu.
\end{aligned} \tag{C1}$$

In this context, the relevant form factors are denoted as f_+ and f_0 . Additionally, we define $q = p - p'$ and $s = \frac{q^2}{m_B^2}$. It is noteworthy that the contributions from the f_0 term are negligible in the expression for the differential branching ratio [refer to Eq. (37)], primarily due to the extremely small masses of the neutrinos. The updated parametrization of the form factors for the $B^+ \rightarrow K^+$ transition, as presented by the FLAG collaboration [98], is available for reference.

2. Details of form factors for $B \rightarrow K^* \nu \bar{\nu}$ transitions

The matrix elements for the relevant operators for $B(p) \rightarrow K^*(k)$ transitions in terms of momentum transfer ($q = p - k$) dependent form factors can be written as [99,100],

$$\begin{aligned} \langle K^*(k) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle \\ = \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho k^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} - i \epsilon_\mu^* (m_B + m_{K^*}) A_1(q^2) \\ + i(2p - q)_\mu (\epsilon^* \cdot q) \frac{A_2(q^2)}{m_B + m_{K^*}} \\ + i q_\mu (\epsilon^* \cdot q) \frac{2m_{K^*}}{q^2} [A_3(q^2) - A_0(q^2)]. \end{aligned} \quad (C2)$$

In this context, ϵ_μ denotes the polarization vector of the K^* meson, while $V(q^2)$ and $A_{0,1,2,3}(q^2)$ represent the associated form factors. The definition provided utilizes $A_3(q^2)$, whereas Eq. (39) employs $A_{12}(q^2)$. Both of these form factors are interconnected with $A_1(q^2)$ and $A_2(q^2)$ through

the following relationships:

$$A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2), \quad (C3)$$

$$\begin{aligned} A_{12}(q^2) = \frac{(m_B + m_{K^*})(m_B^2 - m_{K^*}^2 - q^2)}{16m_B m_{K^*}^2} A_1(q^2) \\ - \frac{\lambda_{K^*}}{16m_B m_{K^*}^2 (m_B + m_{K^*})} A_2(q^2). \end{aligned} \quad (C4)$$

At the specific point of $q^2 = 0$, these form factors satisfy the relations: $8m_B m_{K^*} A_{12}(0) = (m_B^2 - m_{K^*}^2) A_0(0)$ and $A_3(0) = A_0(0)$. It is noteworthy that the pseudoscalar form factor $A_0(q^2)$ does not influence the decay rate in the limit of massless neutrinos. Consequently, three independent form factors—namely V , A_1 , and A_2 —are essential for the computation of the decay process $B \rightarrow K^* \nu \bar{\nu}$. An updated parameterization of the form factors pertinent to the $B \rightarrow K^*$ transition can be found in Ref. [100].

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